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Article

A New No Equilibrium Fractional Order Chaotic System, Dynamical Investigation, Synchronization and Its Digital Implementation

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Abstract: In this paper, a new fractional order chaotic system without equilibrium is proposed, analytically and numerically investigated, and numerically and experimentally tested. The analytical and numerical investigation were used to describe the system dynamical behaviors including, the system equilibria, the chaotic attractors, the bifurcation diagrams and the Lyapunov exponents. Based on the obtained dynamical behaviors, the system can excite hidden chaotic attractors since it has no equilibrium. Then, a synchronization mechanism based on the adaptive control theory has been developed between two identical new systems (master and slave). The adaptive control laws are derived based on synchronization error dynamics of the state variables for the master and slave. Consequently, the update laws of the slave parameters are obtained, where the slave parameters are assumed to be uncertain and estimate corresponding to the master parameters by the synchronization process. Furthermore, Arduino Due boards were used to implement the proposed system in order to demonstrate its practicality in real-world applications. The simulation experimental results are obtained by MATLAB and the Arduino Due boards respectively, where a good consistent between the simulation results and the experimental results. indicating that the new fractional order chaotic system is capable of being employed in real-world applications.

Keywords: Fractional order, Dynamics, chaotic, system, synchronization, Arduino Due.

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1. Introduction

Research on chaotic systems has had a significant practical effect since Lorenz established chaos theory in 1963 [1]. Over the last few decades, nonlinear phenomena in chaos have been widely employed in engineering, science, and applied mathematics [2][3][4]. Chaos systems with hidden attractors have been the focus of recent research. Self-excited attractors and hidden attractors have been classed as chaotic attractors in dynamical systems since the seminal article by Leonov et al is investigated [5]. The unstable equilibrium points (system equilibria) are responsible for exciting the basin of attraction in the self-excited chaotic [6]. An attractor, on the other hand, is said to be hidden if its basin of attraction does not intersect with any of the small neighborhoods of the unstable equilibrium [7]. Hidden chaotic attractors are chaotic attractors in dynamical systems with stable equilibria, no equilibrium, and surfaces or lines of equilibria [8].

Chaotic complex systems with hidden attractors are vital in a wide range of scientific and engineering fields, such as bridge wings design [9], induction motors for drilling [10], chemical reactors systems [11], aircraft control systems [12], memristive circuits [13], encryption [14], PLL systems [15], weather prediction systems [16], and secure communication schemes [17].

Hidden attractors research in the past has primarily concentrated on integer-order dynamical systems. There have been several studies on complicated chaotic systems with hidden chaotic attractors, such as in [18][19][20][21][22][23]. The fractional order derivative and fraction order integration calculus have recently received a lot of attention, owing to the fractional calculus providing more accurate models than the integer order [24].

Fractional calculus can be thought of as an expansion of traditional calculus as a branch of mathematical analysis. Due to its possible applications in a variety of domains, the study of fractional calculus has recently gotten a lot of attention [25]. Fractional calculus can describe many systems in transdisciplinary domains. Furthermore, the fractional-order model can provide an explicit description of the physical process and provide additional insight into it [26]. Fractional calculus can be used in control, bio-engineering, oscillators, analog filters, circuit theory, image processing encryption systems, and chemistry [27]. Fractional order chaotic models have a more complex dynamical behavior than integer models (since they include the fractional order parameter as well as the original system characteristics); as a result, they are important in secure communications systems [28][29].

The synchronization technique of chaos is based on the principle that two chaotic systems may develop on different attractors, but when synchronized, they begin on different attractors and later follow a single trajectory. When the trajectories of two systems are matched, this synchronization is achieved between these two systems [30]. The control and synchronization techniques of the fractional order chaotic systems can be considered the fundamental challenge of using these systems in many applications as robotics, cryptography, mechanical, and secure communication applications [31][32]. To control and synchronize the fractional-order chaotic systems, a variety of control and synchronization techniques have been developed such as, as active control [33], sliding mode control [34], adaptive control [35], passive control [36], and impulsive control [37].

We suggested a new 3D fractional order chaotic system with no equilibrium in this paper, so it can excite hidden chaotic attractors. The system dynamical behaviors including, the system equilibria, the chaotic attractors, the bifurcation diagrams and the Lyapunov exponents has been analytically and numerically investigated. Then, two identical new systems, one working as the master (drive) and the other as the slave (response), have been used to develop a synchronization mechanism based on adaptive control theory.

Based on Lyapunov stability theory, the adaptive control laws have been derived, that is responsible for the slave state variables (slave trajectories) track and aligned the equivalent trajectories in the slave side. Consequently, the update laws for updating the uncertain slave parameter have been obtained. By this scenario, the slave well synchronous the master. The MATLAB was used to verify our work, testing, and results. Additionally, Arduino Due boards have been used to implement a workable hardware electronic circuit for that new system.

The paper rest is organized as following; The basic mathematical background of the fractional order systems is introduced in section 2. Section 3 introduces a new fractional order chaotic system and determines its chaotic attractors classes, as well as the system equilibria. In section 4, the suggested system's dynamical behavior properties are investigated using Lyapunov exponents and bifurcation diagrams. In Section 5, we establish a synchronization approach between two identical new systems, using Lyapunov stability to drive the adaptive control laws and the update laws for achieving the synchronization mechanism and estimating the uncertain slave parameters respectively. In Section 6, we use the Arduino Due boards to implement the real electronic circuit for the suggested system. In section 7, we conclude this paper.

2. Fundamentals of Fractional Order Systems

Integer-order calculus is mathematically extended to fractional calculus. While it offers the advantages of integer order calculus, it also has its own logic and laws. In the definitions of fractional calculus, the Caputo, Riemann–Liouville (RL), and Grunwald–Letnikov (GL) concepts are commonly used [38].

The Gamma function noted by $\Gamma(.)$, which is defined as in Equation (1), is the basic function used in fractional order calculus [39].

$$\Gamma(n) = \int_{0}^{+\infty} e^{-t} t^{n-1}\, dt \quad ; \quad n > 0 \tag{1}$$

Where, $\Gamma(1) = 1$, $\Gamma(0) = +\infty$

The fractional order calculus introduced by Caputo is stated as follows in Equation (2) [40].

$$ {}_{t_0}D_t^q f(t) = \begin{cases} \dfrac{1}{\Gamma(k-q)} \displaystyle\int_{t_0}^{t} \dfrac{f^{(k)}(\tau)}{(t-\tau)^{q-k+1}} d\tau ; & k-1 < q < k \\ \dfrac{d^k f(t)}{dt^k} \quad ; & q = k \end{cases} \tag{2}$$

Where k is the first integer number that is greater than or equal to fractional q-order.

The following Equation (3), gives the fractional integral operator (J^q) established by Riemann-Liouville for order ($q \geq 0$) of function ($f(t)$) [41].

$$J^q f(t) = \begin{cases} \dfrac{1}{\Gamma(q)} \displaystyle\int_{0}^{t} (t - \tau)^{q-1} f(t) d\tau \;; & q < 0 \\ f(t) \quad ; & q = 0 \end{cases} \tag{3}$$

As shown in Equation (4), Grunwald–Letnikov approaches the fractional derivative [42].

$$D^q x(t) = f(x,t) = \lim_{h \to 0} h^{-q}\, (-1) \sum_{j=0}^{t/h} (-1) \binom{q}{j} x(t - jh) \tag{4}$$

Where, h is the step size.

3. A New Fractional Order Chaotic Model

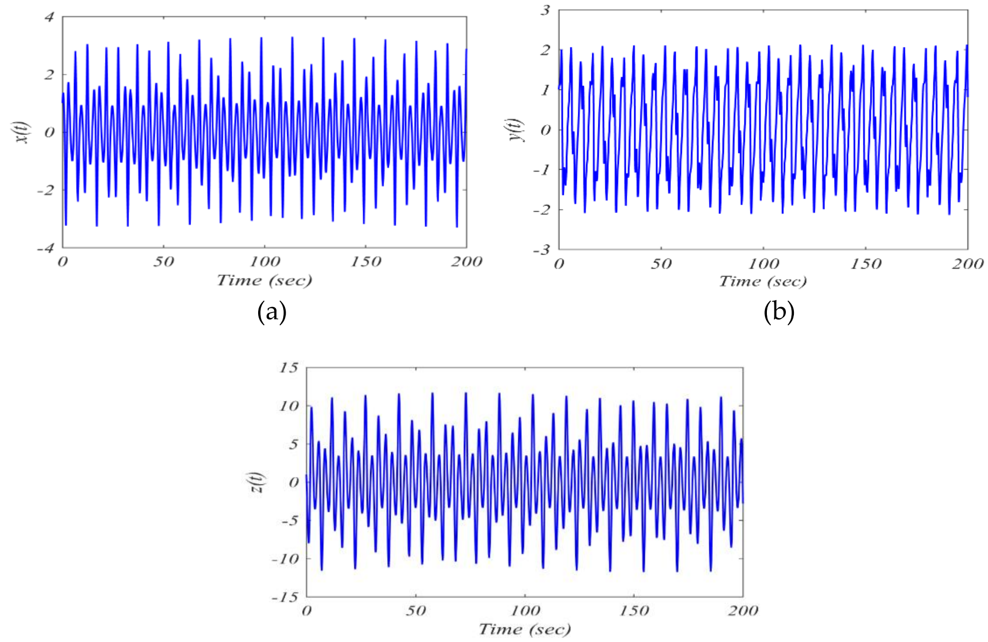
Fractional order chaotic systems are a category of nonlinear systems that, in addition to the fundamental characteristics of integer order chaotic systems, have extra features like as extreme complexity and severe sadness behavior [43]. The following state Equation (5) can be used to describe the mathematical model of the new three-dimensional fractional order chaotic system proposed in this paper.

$$\frac{d^q x}{dt^q} = az + xy; \; \frac{d^q y}{dt^q} = b - x^2; \; \frac{d^q z}{dt^q} = -cx \tag{5}$$

Where, the state variables are x, y, and z, the positive constant system parameters are a, b, and c, and q present the fractional order ($0<q<1$).

The system (5) exhibits chaotic behavior throughout a wide range of a, b, and c parameter values, as well as fractional order (q). The system parameters for the numerical simulation are a=0.5, b=1.8, c=8, and q = 0.99, with initial conditions (x(0), y(0), z(0)) = (1, 1

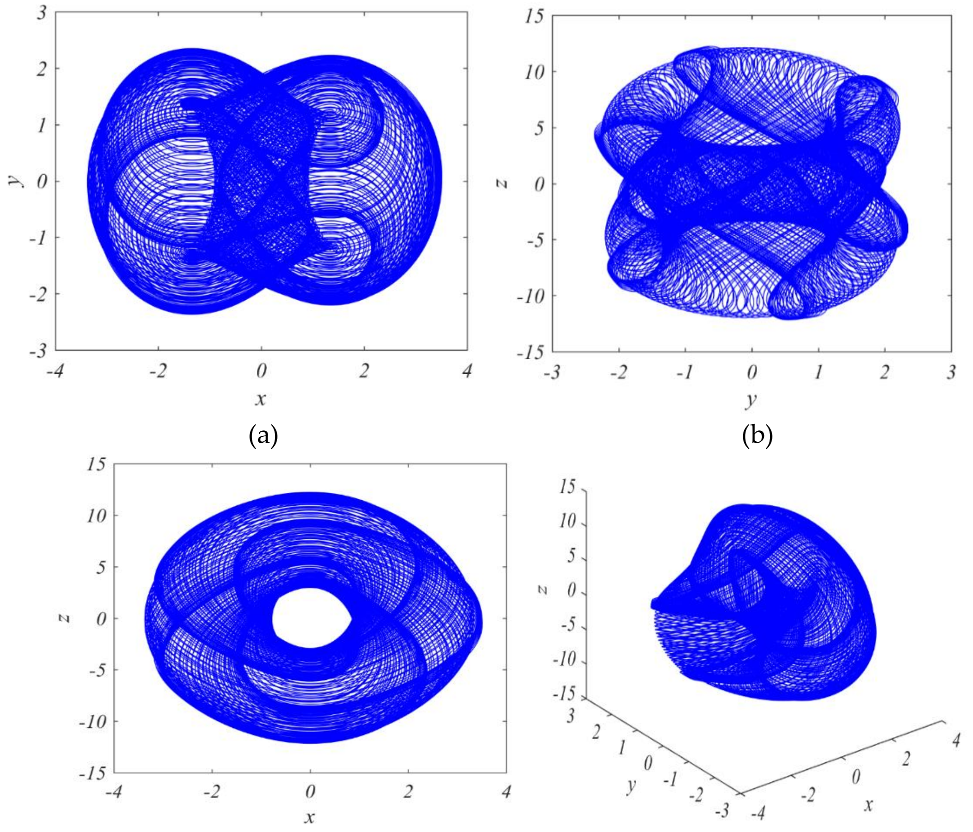
,1). In 2017 Roberto Garrappa [44], introduced a method for solving the nonlinear fractional order systems, where the simulation results of systems (5) are obtained based on Roberto Garrappa method with step size (h=0.005). As illustrated in Figure 1 and Figure 2, the relevant time series of the system states and phase portraits as projections on various planes are obtained respectively.



(a) (b)

(c)

Figure 1. States time series of proposed system; (a) x(t); (b) y(t); (c) z(t)



(a) (b)

(c) (d)

Figure 2. System (5) phase portraits; (a) x-y chaotic attractor; (b) y-z chaotic attractor; (c) x-z chaotic attractor; (d) three-dimensional chaotic attractor.

For determining the equilibrium points (equilibria) of the proposed fractional order chaotic system the state equations (5) are equaled by zero as following in Equation (6):

$$\frac{d^q x}{dt^q} = az + xy = 0; \quad \frac{d^q y}{dt^q} = b - x^2 = 0; \quad \frac{d^q z}{dt^q} = -cx = 0 \tag{6}$$

As can be noted in Equation (6), there are an inconsistent in the solving of the Equation (6), where the state variable x=0 can be obtained from the third term in the Equation (6), in the same time that is impossible in solving the second term in the same Equation (6), because it’s impossible that the constant b equal zero in the solving process of Equation (6). Therefore, the Equation (6) has no solution, that lead the proposed fractional order chaotic system (5) without equilibrium points, i.e., the new fractional order chaotic system can excite hidden chaotic attractors.

4. The System Dynamical Analyses

Generally, the bifurcation diagrams and Lyapunov exponents are the main two dynamical tools that can be investigate the dynamical behaviors of the nonlinear chaotic systems [45]. In this section, the bifurcation diagrams and the Lyapunov exponents are numerically investigated by using MATLAB.

4.1. Bifurcation Diagrams

The bifurcation diagrams are important means in the nonlinear dynamics and chaos theory. In In this work, for exploring the system dynamical behavior by the bifurcation diagrams, the state variable y(t) of the suggested system is plotted in contradiction of the system parameter b, and with respect to the system fractional order (q).

The influence of the system parameter a on the system dynamical behavior are obtained by the bifurcation diagrams as illustrated in Figure 3. Where Roberto Garrappa method with step size (h=0.005) and another original program programmed by us have been used for plotting the bifurcation diagrams. The other system parameters are chosen as a=0.5 and, c=8, with initial conditions with initial conditions (x(0), y(0), z(0)) = (1, 1, 1) and fractional order (q=0.98). As can be seen in Figure 3, the new system exhibits chaotic behavior when the parameter b>0.

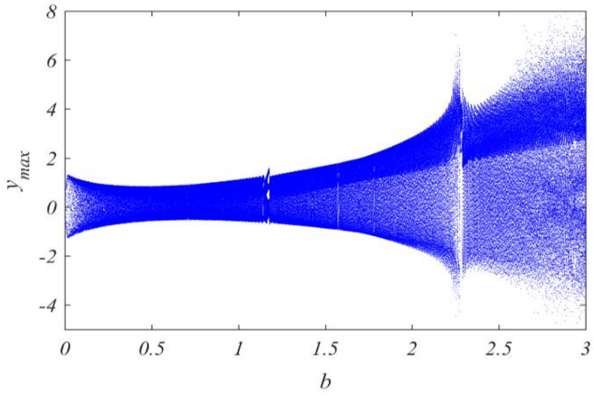


Figure 3. The Bifurcation diagram verse system parameter (b)

Then choose the fractional order (q) to be the bifurcation parameter and fixed the parameters a = 0.5, b = 1.8 and c = 8 with the initial conditions (1, 1, 1), the dynamical behavior of system (5) is obtained by the bifurcation diagrams as shown in Figure 4. As can be noted from the bifurcation diagram in Figure 4, the chaotic behavior is excited when the system fractional order q > 0.965. It can be seen from Figure 3 and Figure 4, that the system (5) displays various bifurcation topological patterns.

This result shows that the selected system fractional order determines the generation of the hidden chaotic attractor.

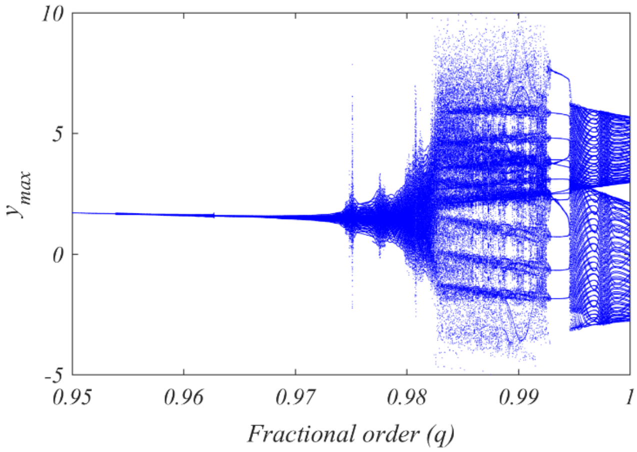


Figure 4. The bifurcation diagram verses the system fractional order (q)

4.2. Lyapunov Exponents

The Lyapunov exponents are calculated to strongly indicate the new system exhibits the chaoticity phenomenon, at least one positive Lyapunov exponents in the nonlinear dynamics systems that ensure as these system exhibit chaos [46].

Figure 5, demonstrates the Lyapunov exponents with respect to the time (1000 seconds), where the system parameters are chosen as a=0.5 b=1.8 and, c=8, with initial conditions with initial conditions (x(0), y(0), z(0)) = (1, 1, 1) and fractional order (q=0.98). The consistent Lyapunov exponents are obtained as Le1= 0.2384, Le2= -0.2859, and Le3= -0.3681. The existence of positive Lyapunov exponent (Le3) is enough to prove the system (5) can exhibit the chaos. Do not forget, because Le1 + Le2 + Le3 = -0.3256, system (5) is dissipative i.e., the new system state trajectories converge into a weird attractor in the end.

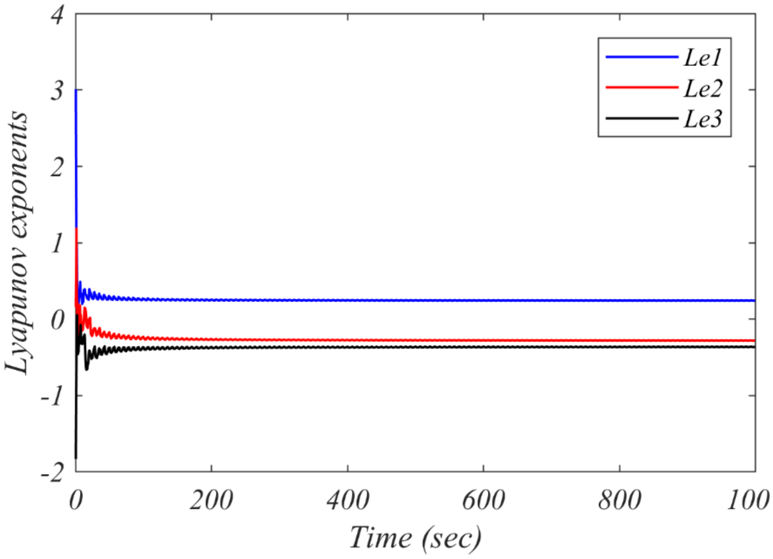


Figure 5. Lyapunov exponents corresponding to time

In addition, Lyapunov exponents are calculated whit respect to changing the fractional order as q ∈ [0.95,1] as shown in Figure 6. The used system parameters are as mentioned in the first Lyapunov exponents calculations. As can be seen in Figure 6, the Lyapunov exponents are found as Le1= 0.0287, Le2= 0.0019, and Le3= -0.0017. That clearly indicates chaotic attractors in the proposed system.

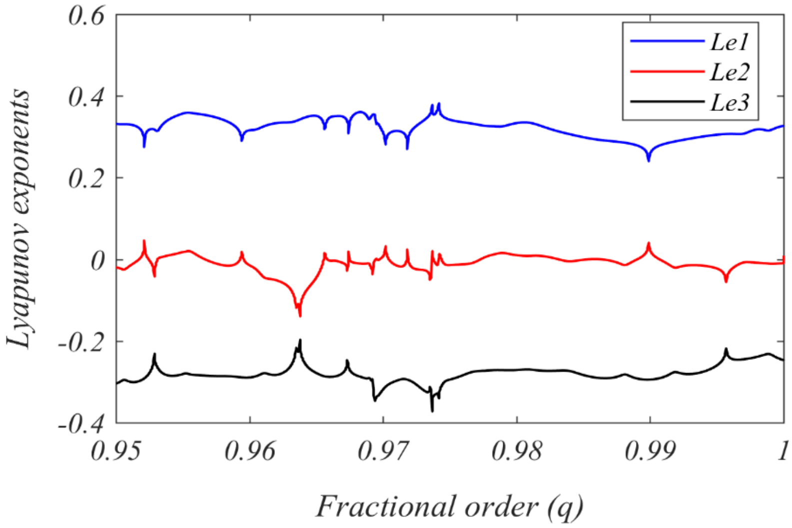


Figure 6. Lyapunov exponents corresponding to system (5) fractional order (q)

5. Adaptive Synchronization of Two New Fractional Order Chaotic Systems

Particularly, because the fractional order chaotic complex systems have an extra complexity in its dynamical behavior and cannot be described by the classical mathematical methods (its deterministic systems), as these systems are wide capable to be used in secure communication, image processing, cryptography systems [47]. Therefore, the fractional order chaotic system synchronization mechanisms have much attentions that is because excessive importance in applications extending in many fields as physics, engineering, computer science, biology, economics and brain science. The drive-response (master-slave) form is considered the basic configuration of the chaos synchronization mechanism, where the trajectories of the slave chaotic system must track the trajectories of the master chaotic system. Several methods have been developed to attain the chaos synchronization in fractional-order chaotic systems as these mentioned in the introduction term.

In this work, we developed an adaptive synchronization technique in order achieve the synchronization between two identical new systems, where the first acts the master (drive) system and the second acts the slave (response) system. Based on Lyapunov theory the synchronization controllers, and the slave parameter estimation laws has been derived based. The adaptive control laws will drive the slave so its trajectories track the analogues master trajectories and the slave parameter estimation laws are used for updating the uncertain slave parameters corresponding to the analogous known master parameters.

The adaptive synchronization technique has many gorgeous benefits, as good transient performance, rapid dynamics responses, and robust for system parameter variations and initial conditions.

5.1. Adaptive Controller Design Process

In this subsection, we well design the adaptive controller for achieving the synchronization between two identical new fractional order chaotic systems, the master and slave state equations are presented by Equations (7) and (8), respectively:

$$\frac{d^q x_m}{dt^q} = a_m z_m + x_m y_m;\ \ \frac{d^q y_m}{dt^q} = b_m - {x_m}^2;\ \ \frac{d^q z}{dt^q} = -c_m x_m \tag{7}$$

$$\frac{d^q x_s}{dt^q} = a_s(t) z_s + x_s y_s + u_1;\ \ \frac{d^q y_s}{dt^q} = b_s(t) - {x_s}^2 + u_2;\ \ \frac{d^q z}{dt^q} = -c_s(t) x_s + u_3 \tag{8}$$

Where, are u_1, u_2, and u_3 are the adaptive synchronization controllers that want to be de-signed, a_m,b_m, and c_m are the known master parameters, and $a_s(t)$, $b_s(t)$, and $c_s(t)$ present the uncertain slave parameters that must be estimated. The master- slave synchronization errors can be determined by Equation (9).

$$e_x = x_s - x_m;\ \ e_y = y_s - y_m;\ \ e_z = z_s - z_m \tag{9}$$

That results, the dynamic errors are determined as in Equation (10).

$$\frac{d^q e_x}{dt^q} = a_2 e_z + x_s y_s - x_m y_m + z_m e_a + u_1;\ \ \frac{d^q e_y}{dt^q} = e_b - {x_s}^2 + {x_m}^2 + u_2;\ \ \frac{d^q e_z}{dt^q} = c_2 e_x + x_m e_c + u_3 \tag{10}$$

Where e_a, e_b and e_c are the errors of master-slave parameters and can be determined as in Equations (11).

$$e_a = a_s(t) - a_m;\quad e_b = b_s(t) - b_m;\quad e_c = c_s(t) - c_m \tag{11}$$

That results the dynamics of the parameter errors can be calculated by Equation (12).

$$\dot{e}_a = \dot{a}_s(t)\ ;\quad \dot{e}_b = \dot{b}_s(t);\quad \dot{e}_c = \dot{c}_s(t) \tag{12}$$

We used the Lyapunov strategy for designing the adaptive controllers to verify the adaptive master-slave synchronization mechanism, consequently the update laws for estimating the uncertain parameters are obtained. Therefore, the quadratic positive Lyapunov function are used as in Equation (13).

$$V(e_x, e_y, e_z, e_a, e_b, e_c) = \frac{1}{2}(e_x^2 + e_y^2 + e_z^2 + e_a^2 + e_b^2 + e_c^2) \tag{13}$$

That results the Lyapunov function dynamics are obtained as in Equation (14).

$$\dot{V} = (e_x \frac{d^q e_x}{dt^q} + e_y \frac{d^q e_y}{dt^q} + e_z \frac{d^q e_z}{dt^q} + e_a \dot{e}_a + e_b \dot{e}_b + e_c \dot{e}_c) \tag{14}$$

Equations (10-12) are substituted in Equation (14) to get the following Equation (15).

$$\begin{aligned} \dot{V} = {} & (e_x(a_2 e_z + x_s y_s - x_m y_m + z_m e_a + u_1) + e_y(e_b - {x_s}^2 + {x_m}^2 + u_2) \\ & + e_z(c_2 e_x + x_m e_c + u_3) + (a_s(t) - a_m)\dot{a}_s(t) + (b_s(t) \\ & - b_m)\dot{b}_s(t) + (c_s(t) - c_m)\dot{c}_s(t) \end{aligned} \tag{15}$$

Thus, Equation (15) has been used to design the synchronization controllers as in the following Equation (16).

$$u_1 = -k_x e_x - a_s e_z - y_s z_s + y_m z_m\ ;\ \ u_2 = -k_y e_y + {x_s}^2 - {x_m}^2\ ;\ \ u_3 = -k_z e_z - c_s e_x \tag{16}$$

In Equation (16), k_x, k_y, and k_z present positive constants, and the uncertain slave system parameters included (a_s, b_s, and c_s) are estimated by updated laws as in the following Equation (17).

$$\dot{a}_s(t) = -z_m e_x;\ \ \dot{b}_s(t) = -e_y;\ \ \dot{c}_s(t) = -x_m e_z \tag{17}$$

Finally, the following Equation (18) is obtained by substituting Equation (16) and Equation (17) in Equation (15).

$$\dot{V} \ = -k_x {e_x}^2 \ \ - k_y {e_y}^2 - k_z y^2 \tag{17}$$

As can be seen from Equation (17), its negative definite function [48]. As a result, given any initial conditions, the synchronization state errors and the master-slave parameter estimation error converge to zero exponentially with respect to time.

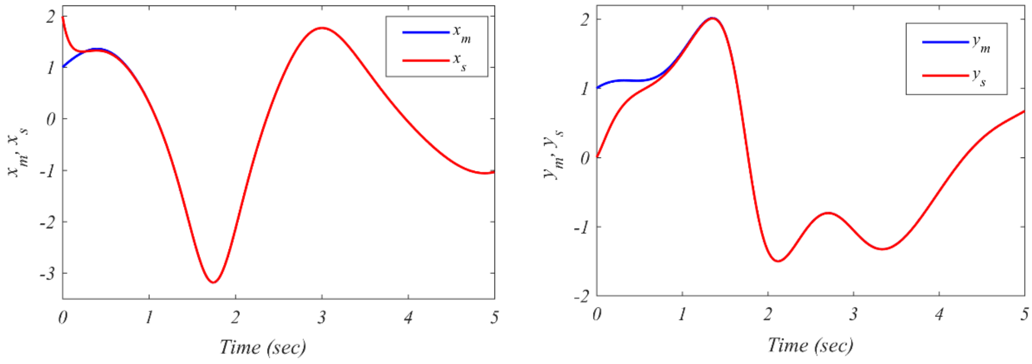
5.2. Simulation results

Numerical studies using the MATLAB platform are used to confirm the efficiency of the suggested synchronization strategy. Table 1, shows the values of the master and slave parameters, fractional orders and initial conditions utilized to simulate the aforementioned synchronization technique.

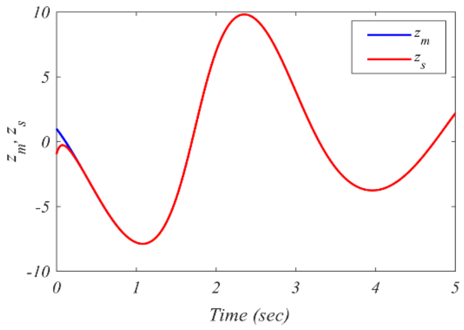
Table 1. Master and slave parameters values

Master System		**Slave System**	
Parameter	**Value**	**Parameter**	**Value**
a_m	0.5	$a_s(t)$	Estimated
b_m	1.8	$b_s(t)$	Estimated
c_m	8	$c_m(t)$	Estimated
fractional order (q)	0.98	fractional order (q)	0.98
$X_m(0)$	1	$X_s(0)$	2
$y_m(0)$	1	$y_s(0)$	0
$z_m(0)$	1	$z_s(0)$	-1

The slave trajectories (state variables) effectively follow the master trajectories based on the derived adaptive control laws in Equation (16) as shown in Figure 7. Although initial conditions have different signs and values, the simulation results demonstrate that the master and slave state variables have been synchronized in a short time, indicating that the developed controller is efficient. In Figure 8, the synchronization errors (e_x, e_y, and e_z) are illustrated. It can be seen; the synchronization errors rapidly (in less than 1.5 seconds) decrease to zero values exponentially. As illustrated in Figure 9, these uncertain slave parameters have been appropriately estimated to the corresponding master parameters using the update laws in Equation (17).



(a) (b)



(c)

Figure 7. Tracking the slave trajectories corresponding to master trajectories

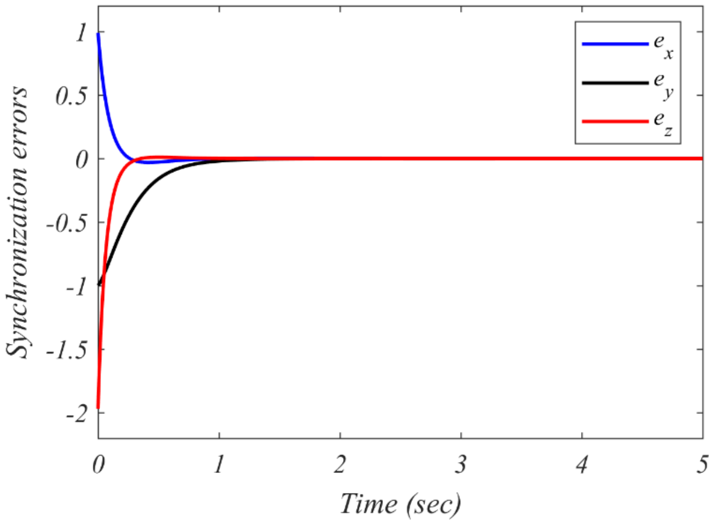


Figure 8. The state variables synchronization errors

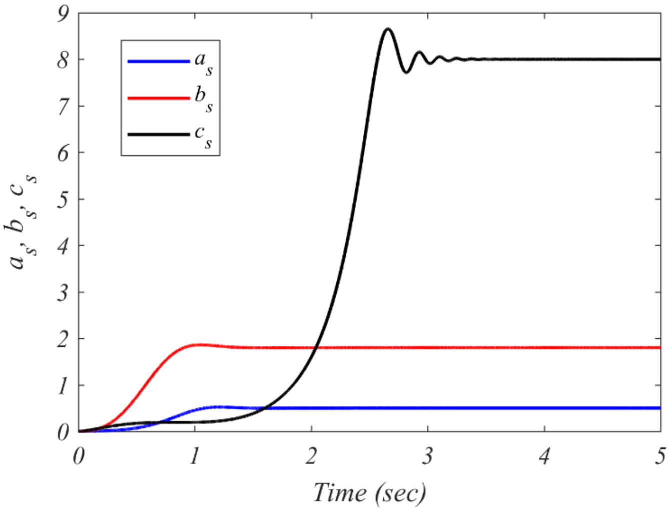


Figure 9. Estimation of uncertain slave parameters corresponding to master trajectories

As can be seen from Figure 9, that the uncertain slave parameters a_s and b_s are rapidly estimated corresponding to the master parameter a_m and b_m respectively (in less than 1.5 sec), that is exactly match to time duration of the synchronization errors to be zero. On the other hand, the time duration was about 3.5 seconds for estimating the third uncertain slave parameter c_s corresponding to the master parameter c_m, where that does not affect the synchronization process. That is because the dynamical behavior of the nonlinear systems is not affected by the same degree of sensitivity for all its parameters as mentioned in [49].

6. Digital Implementation of New Fractional Order Chaotic System

The major goal of the hardware is to test the possibility of implementing fractional order chaotic systems so that they can be employed in real-world applications. Fractional order chaotic systems can be implemented in hardware utilizing a variety of embedded devices, such as microcontrollers, Raspberry Pi, FPGAs, and DSPs as well as its implementation by analog electronic circuit as in [29]. In this work we implemented the new three-dimensional system (5) by using a microcontroller (Arduino DUE) Based on the discrete method as in [50].

Arduino Due is a digital board with Atmel SAM3X8E, ARM Cortex-M3 CPU. It is having wonderful structure for performing the complex arithmetic operation, in briefly it has the following characteristics; 32-bit ARM core microcontroller, 84 MHz clock, 54 digital input/output pins, 12 analog inputs, USB OTG capable connection, 4 UARTs, 2 DAC (digital to analog), power jack 2 TWI, SPI header, JTAG header. The major reasons of using like that microcontroller are; 12 bits resolution for its two peripherals DAC0 and DAC1 and its attractive cost advantage when compared with the FPGAs or DSPs boards. Where the Arduino specified programing language that is similar to C++ language are used for programing ARM microcontroller by the Arduino IDE through the native USB port (serial port) [51]. The hardware implementation of the new fractional order chaotic system is shown in Figure 10.

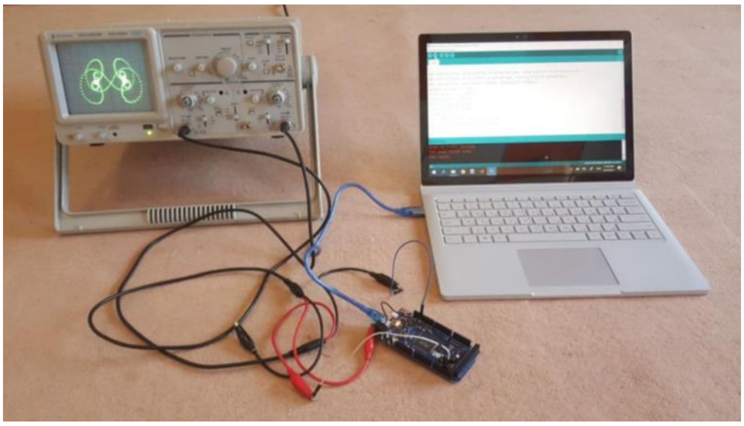
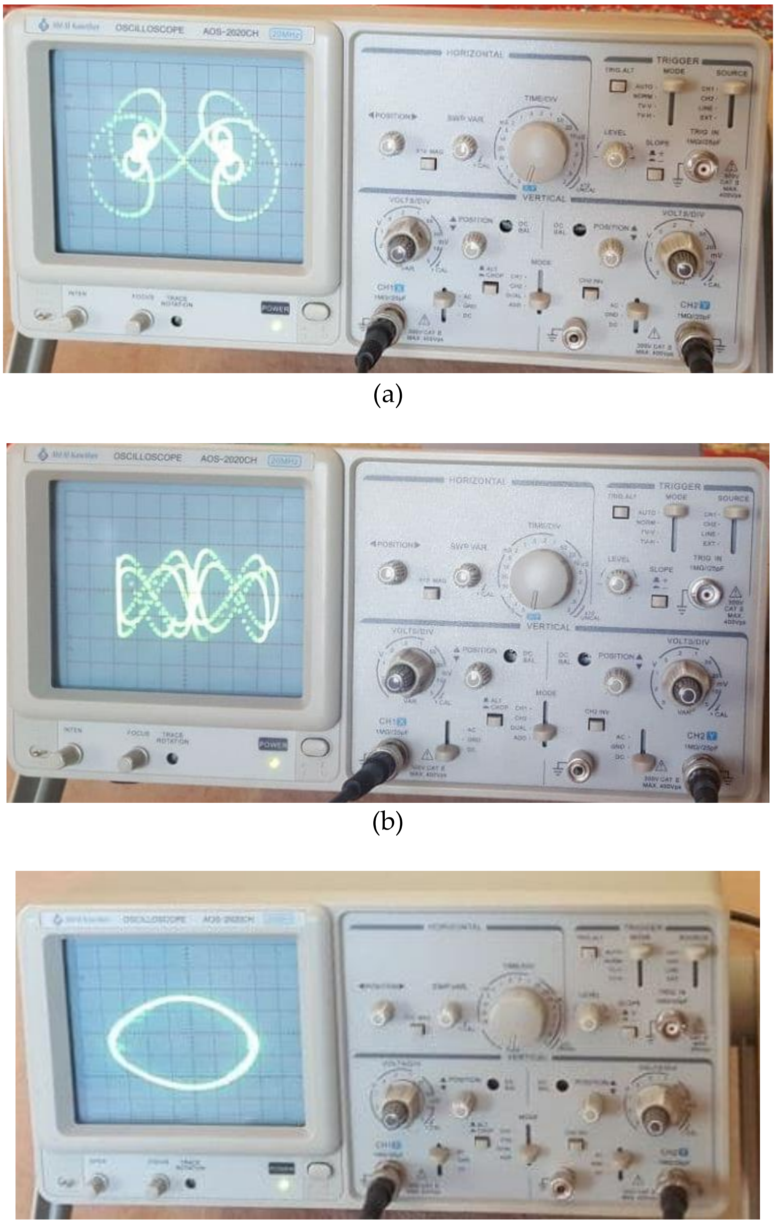


Figure 10. Hardware implementation of the system (5) based om Arduino Due board

The system parameters for the experimental test are a=0.5, b=1.8, c=8, and q = 0.99, with initial conditions (x(0), y(0), z(0)) = (1, 1 ,1). The ADC0 and DAC1 are used to give the system (5) phase portraits of chaotic attractors by analog oscilloscope as displayed in Figure 11. In the fact, because the microcontroller's digital to analog converters (DAC0 and DAC1) operate between 0.5V and 2.7 V, the amplitudes of the simulation results by MATLAB and the experimental results by the Arduino Due will different for the systems (5) state variables (system trajectories). It would be necessary to install an external operational amplification stage in order to achieve the same amplitude values as the computed numerical simulations. Based on the approach that has been used for implementing the system (5) by the Arduino DUE board, there are a level of error about ±1.56% when compared with the simulation results obtained by MATLAB.



(a)

(b)

(c)

Figure 11. The experimental results of the proposed system phase portraits; (a) x-y chaotic attractors, (b) y-z chaotic attractors, (c) x-z chaotic attractors

7. Conclusions

A new three-dimensional nonlinear autonomous system with fractional order and exhibits chaos was suggested. The nonlinear dynamical behaviors of this system were analytically and numerically investigated, where these dynamics are the equilibrium points, chaotic attractors, bifurcation diagrams, and Lyapunov exponents. Because the system has no equilibrium, the observed dynamics show that the system can excite hidden chaotic attractors and displays extremely complex dynamics. After that, an adaptive synchronization strategy was formulated. This synchronization approach is set up between two identical new fractional order chaotic systems, one serving as the master and the other as the slave. The adaptive control principles responsible for synchronization verification have been derived. Also, in order to estimate the unknown slave parameters, the update laws were determined. Finally, in order to show the feasibility of using that proposed system in real world applications, that system has been implemented by Arduino Due boards. The obtained numerical results by MATLAB simulation consistent the experimental results of the hardware implementation, that show the feasibility of the system to be used in the real application fields. The hardware implementation of the system by Arduino Due verify the low-costs implementation compared with implementation by other devices as FPGAs and DSPs which are need high costs. The main advantage of system implementation based on Arduino Due boards are the low costs implementation when compared to alternative devices such as FPGAs and DSPs, which require large costs.

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