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Free surface air entrainment and single-bubble movement in supercritical open-channel flow

Item Type	Article
Authors	Wei, W.;Xu, W.;Deng, J.;Guo, Yakun
Citation	Wei W, Xu W, Deng J et al (2020) Free surface air entrainment and single-bubble movement in supercritical open-channel flow. Journal of Hydraulic Engineering. 146(7): 04020050.
DOI	https://doi.org/10.1061/(ASCE)HY.1943-7900.0001769
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Download date	2026-05-10 02:14:38
Link to Item	https://bradscholars.brad.ac.uk/handle/10454/17610.2

18 and advance the knowledge of our understanding of the macroscopic and microscopic air–water
19 properties in hydraulic engineering.

20 **Keywords:** free surface; air entrainment; air bubble; open channel flow

21

22 **Introduction**

23 Air entrainment takes place naturally through free surface of high-speed flows in hydraulic engineering,
24 as shown in Fig. 1. Vertical continuous air concentration profile is found with different shapes as mean
25 void fraction increases in flow direction (Straub and Anderson 1958). This conveyed air transported with
26 water greatly increases the bulk volume of the flow, greatly affecting the hydraulic structure design
27 (Falvey 1980; Ervine 1998). Such air entrained in water flow reduces the air-water mixture density and
28 can change the energy dissipation downstream (Xu et al. 2004; Wei et al. 2013; Guo 2014; Chanson
29 2015). The presence of air bubbles near structure walls can reduce or prevent cavitation damage caused
30 by high-speed flows (Wilhelms et al. 2005; Frizell et al. 2013; Xu et al. 2015). In addition, water quality
31 is affected by the excess of air introduced into water (Gameson 1957; Bung and Valero 2018). Moreover,
32 air bubbles can alter flow turbulence properties, boundary layer thickness (Castro-Orgaz and Hager 2010;
33 Castro-Orgaz 2012), shear stress, and free surface momentum transfer (Wang et al. 1990; Guo et al. 1999;
34 Yang and Dou 2010; Balachandar and Eaton 2010).

35 The mechanism of the air entrainment is identified with primary causative influence from free
36 surface turbulence, where an accepted threshold of the intensity of eddy turbulence is surpassed (Pagliara
37 et al. 2011; Pfister and Hager 2011; Bung 2013). The deformation of a local free surface is considered as

38 the direct “carrier”, resulting in individual bubbles entrained into the water flow. Volkart (1980) observed
39 the swelling of the free water surface, indicating that air bubble entrainment might be caused by water
40 droplets falling back to the free surface. Rein (1998) applied turbulent vortices theory to analyze the air
41 bubble entrainment generation. However, his theory was not supported by the laboratory investigation
42 (Straub and Anderson 1958). The air bubble size distribution using these methods differ from that
43 measured and observed (Pumphrey and Bjørnø1 1989; Medwin et al. 1990; Oguz and Prosperetti 1990;
44 Cole and Liow 2004). Brocchini and Peregrine (2001) described a wide range of free surface
45 deformations and investigated the effects of gravity, surface tension, and turbulence kinetics on free
46 surface breakup and bubbling. Valero and Bung (2016, 2018a) proposed a linkage between free surface
47 distortion and the perturbation breakup in high speed flows, and established a kinematic and dynamic
48 consideration. This highlighted the importance of microscopic surface shape and size scale in the
49 generation of the air–water mixture.

50 So far, the process of air entrainment in high-speed free surface flows has not been fully understood
51 and it is difficult to accurately predict macroscopic air–water properties, such as mixture flow depth, air
52 content, and air concentration distribution in hydraulic engineering. From the point of view of
53 microscopic level, air–water mixture across the flow depth is mainly characterized by the bubble motion
54 in water. The vertical motion, expressed by bubble rising velocity (Haberman and Morton 1954), is
55 important for the air–water diffusion process in open channel flows. Comolet (1979) developed a theory
56 for describing the rising velocity for different bubble size spectrum in still water. However, the situation
57 in flowing water could be very different. Once entrained into the flowing water, individual air bubbles
58 move longitudinally and vertically in two dimensional diffusions, which is more complicated than that

59 in still water (Culligan et al. 2006). This complex flow phenomenon is caused by the interactions of
60 inertia, drag, buoyancy and turbulent eddy diffusion, and their effects on coalescence and break-up in the
61 turbulent flow. The description of bubble diffusion behavior could affect the prediction of the aeration
62 development downstream and the cross-sectional distribution of the air concentration (Falvey and Ervine
63 1988). Toombes and Chanson (2007) and Felder and Chanson (2014) applied the intrusive conductivity
64 probes to detect the air-water interface transfer, while the bubble rising velocity was inferred indirectly
65 to describe the time-averaged air–water diffusion behaviors in two–phase flows (Chanson 1993; Kramer
66 2004). The hypothetical inferences make the basic diffusion theory of two–phase flows be limited to
67 specific applications (Wood 1984; Chanson and Toombes 2002; Valero and Bung 2018b). Until now,
68 there is little information on the temporal and spatial movements of air bubbles around the inception
69 aeration area in high-speed flows. Air bubble behaviors in the air–water mixture process from the visual
70 observation on the microscopic movements within free-surface open channel flows are not yet fully
71 understood.

72 Given limited information and previous shortcomings of analysis, the aim of the present study is to
73 provide new insights into potential consequences of free surface air entrainment generation and bubble
74 diffusion in open channel flows. To this end, a high-speed camera system is used to record the visual
75 processes of entrained bubble generation and movement within supercritical free surface flows. Recoded
76 local free surface entrapment and bubble entrainment images are used to analyze the shape and size of
77 air bubbles. The movement properties of air bubbles, such as rising and penetration in the water flow are
78 obtained for a wide range of bubble size. Furthermore, the present investigation analyzes the effect of
79 bubble diffusion relationship on the aeration development prediction in high speed open channel flows.

80 **Experimental Design and Conditions**

81 Laboratory experiments are conducted to investigate the air entrainment and single bubble movement in
82 open channel flows. Experimental flume is a rectangular glass channel having the dimension of 8.6 m
83 (long) \times 0.4 m (wide) \times 0.6 m (high), as shown in Fig. 2. Water enters into a horizontal channel from the
84 inlet tank and then flows through a crest transition section connected to a straight channel downstream
85 (see Fig. 2(b)). The straight channel slope α is kept constant of 36° for all tests to enhance the flow
86 acceleration and air entrainment occurrence in the experiments. Five water flow discharges, namely $Q =$
87 $0.109 \text{ m}^3/\text{s}$, $0.117 \text{ m}^3/\text{s}$, $0.119 \text{ m}^3/\text{s}$, $0.131 \text{ m}^3/\text{s}$ and $0.137 \text{ m}^3/\text{s}$, are conducted. The flow discharge is
88 measured by a rectangular-thin-weir downstream. Given the extremely rapid variation in shape of the
89 free surface and air bubbles (on the order of 1 – 10 ms), and the relevant sub-millimeter length scale
90 (Strotos et al. 2016), a high-speed camera system is used to capture and record the entire process of free
91 surface deformation and single bubble movement. The high-speed camera-based data acquisition system
92 consists of a high-speed video camera (MotionPro Y3-class, Integrated Design Tools Inc., USA) with a
93 Nikkor lens, transferring captured image signals to a computer. The shooting scope is $30 \times 12 \text{ cm}^2$, with
94 an image area of 1280×324 pixels. All the free surface bubble entrainment and bubble movements for
95 each case are captured and analyzed in this shooting scope. The camera is set with an adjustment fixing
96 mechanism to measure free surface flow along the 30 cm streamwise length. The lens is adjusted to the
97 same elevation as the free water surface, and is perpendicular to the sidewall of channel. High-speed
98 images are taken at 3000 frames per second with an exposure time of 0.0003 s and a sample duration of
99 3 s. The focus plane is set approximately 3 mm inside from the sidewall. A uniform lighting is provided

100 by an 18 W constant current LED without stroboscopic pulse from the opposite side of the channel, which
101 ensures that a clear luminous beam to the camera through the free water surface is achieved. The stable
102 illumination facilitates the capture and determination of the air–water interface position.

103 The two-dimensional air concentration distribution at the central line of tested channel is measured
104 using a phase-detection needle probe (CQY-Z8a Measurement Instrument, China, Wei et al. 2016). The
105 tip consists of an internal platinum needle with a diameter 0.05 mm, and the two tips are aligned in the
106 flow streamwise direction with a distance 9.28 mm. The sampling rate is set as 200 kHz with a sampling
107 duration of 10 s. It should be noted that the aeration property difference between the central and near
108 sidewall regions cannot be avoided due to the sidewall effect on flow velocity characteristics. Thus, the
109 present experimental measurement aims to explore the general aeration intensity and corresponds to
110 classical air-water mixture experiments. Typical air concentration profiles for various flow conditions
111 are plotted in Fig. 3, which are in good agreement with characteristic self-aerated flows in open channel
112 (Hager 1991; Chanson 1997). The average air concentration at flow cross-section C_{mean} is deduced by
113 integrating over the flow depth between channel bottom and the mixture flow surface h_{90} where local
114 concentration is $C = 90\%$

$$C_{\text{mean}} = \frac{1}{h_{90}} \int_0^{h_{90}} C \, dh \quad (1)$$

115 The characterized clear water depth h_w is obtained as:

$$h_w = h_{90} \cdot (1 - C_{\text{mean}}) \quad (2)$$

116 The average flow velocity is then calculated as:

$$V = \frac{Q}{W \cdot h_w} \quad (3)$$

117 where W = the channel width. Detailed experimental conditions are listed in Table 1. The aeration
 118 expression of the observation region refers to all tested conditions with $C_{\text{mean}} = 5.6\% - 7.9\%$. This
 119 indicates that the free surface air entrainment is relatively weak and the aeration layer at the free surface
 120 region is thin with a large clear water layer at the flow cross-section. Under these conditions, the free
 121 surface deformation and air–water mixture can be moderated for visual observation. The average free
 122 surface velocity V_{fs} is provided to show the surrounding velocity in the air-water mixture area.
 123 Considering the thin aeration layer in each case, the V_{fs} is obtained from the average value of air-water
 124 velocities over the air-water mixture layer.

125 For the characterized dimensionless flow parameters, Reynolds number is defined as $Re = Vh_w / \nu$,
 126 and Weber number We is defined as $We = \rho V^2 h_w / \sigma$, where ν = the kinematic viscosity, ρ = water density
 127 and σ = the surface tension. Previous studies showed that the scale effect of air concentrations and bubble
 128 transport in aerated open channel flows was limited provided that Re and $We^{0.5}$ were larger than 1.0
 129 $\times 10^5$ and 110, respectively (Boes and Hager 2003; Pfister and Chanson 2014). For the experimental
 130 conditions carried out in this study, the minimal values of Re and $We^{0.5}$ are 2.73×10^5 and 124.48,
 131 respectively, demonstrating that the scale effect can be neglected.

132 A series of instantaneous images are recorded continuously. The two-dimensional air–water interface
 133 profile is determined on the basis of the difference in luminance between air and water (Bung 2013;
 134 Besagni et al. 2016). A ruler is placed above the free surface inside the flume with the identical slope of
 135 channel bottom and is used as the reference calibration of conversion factor between pixels and model

136 millimeter scales. The camera is appropriately adjusted to focus on the ruler plane. The pixels have a
137 horizontal and vertical resolution of 10^{-1} mm. The raw movies are used to identify the bubble entrainment
138 and generation caused by local free surface deformation based on a time series of air–water interface
139 evolution. This ensures that the specific entrained bubble is linked to the spatial and temporal behaviors
140 of corresponding deformed free surface. Owing to the hollow-shape of local two-dimensional free
141 surface, as shown in Fig. 4, the length and depth of entrapped deformation are extracted to describe the
142 free surface deformation quantitatively. The local air-water interface is detected to determine the surface
143 edge. Because the entrapped shape is not symmetrical, the depth y of an entrapped part is defined as the
144 distance between apex and the mid-point of the length L .

145 For the individual bubble movement of each case, the specific position is tracked per time interval.
146 It is difficult to avoid the air-water interface transverse movement around the focus plane due to the
147 random occurrence of forward and backward movement in the transverse direction, but it is assumed that
148 such movement is small in the present 30 cm length shooting scope. The weak aeration level near the
149 free surface and high sampling frequency can reduce the effect of different planes in the determination
150 of a specific bubble entrainment. Because the shape of air bubbles is irregular, individual bubble size is
151 considered as a spherical particle with the equivalent diameter being d_{ab} for the convenience of
152 description and analysis. Although there are many individual air bubbles in such low air concentration
153 region, the amount of air bubble entrainment case with full and clear free surface deformation process is
154 limited. This is mainly due to the fact that the disturbing surroundings are complicated, such as adjacent
155 free surface movements and intruding following air bubbles. To better understand the bubble entrainment

156 at the free water surface and bubble movement, the bubble size recorded and analyzed has a range from
157 1 mm to 10 mm for the flow conditions investigated.

158

159 **Results and Discussions**

160 *Free Surface Air Entrainment and Bubble Generation*

161 Fig. 5 shows a typical time series of images from a visual process of free surface entrainment and bubble
162 entrainment for $V = 5.5$ m/s in which grey-dark shade represents air while white color represents water.
163 Initially, the free surface is relatively smooth and flat with a wave-like shape ($t = 0.0 - 2.1$ ms). When a
164 disturbance towards inside the water flow acts on the surface, an “entrapped air” appears ($t = 2.4 - 4.5$
165 ms). The free surface is clearly higher than the entrapped air center and the surface becomes unstable and
166 shrank at a middle position ($t = 4.8 - 6.0$ ms). This entrainment continues to develop and eventually an
167 air bubble is formed and entrained into water ($t = 6.3 - 7.2$ ms). The determination of an individual
168 bubble is dependent on the initial appearance with a visual and continuous air–water interface below the
169 free surface. For this process at $t = 7.2$ ms, the air–water interface around approximately spherical
170 boundary of this bubble can be seen clearly in the image, separating from the water surface. In this
171 process, the air bubble size of the equivalent diameter is about $d_{ab} = 2.5$ mm.

172 The visual process shows that air bubble entrainment in the open channel flow is the evolution of
173 free surface entrainment deformation. Fig. 6 is the observed (a) and the schematic diagram (b), to show
174 the process of free surface air entrainment. Such air entrainment at free surface has two stages:
175 development and entrainment. This process accompanies with the size variations of deformed surface

176 including the distance length L between the two sides of entrapment surface and depth y of entrapment
177 shape. The ratio of L/y describes the entrapment deformation shape. Small L/y represents the deeper
178 penetration of entrapment surface. Initially, the length and depth of a local free surface remain
179 approximately the same. Once the local free surface is disturbed by a vertical fluctuation, such as the
180 turbulence, water droplet, streamwise velocity slip etc., the entrapment shape of free surface is generated
181 as the inception of air entrainment process. During the development stage, the entrapment cavity
182 develops with both the length and depth increasing. The value of L/y decreases significantly, indicating
183 that the entrapment cavity penetrates deeper into water. During the entrainment stage, the remarkable
184 feature is the shrink of entrapped free surface, which is considered to be unstable. The entrapped surface
185 at both sides shrinks. Air entrapped in the cavity is entrained as an individual bubble when the entrapped
186 free surface gets enclosed in the shrink deformation process. The increase of y with decrease of L
187 indicates the as the bubble separates from free surface and enters into water, the free surface rebounds
188 and returns back to the inception state.

189 The above analysis confirms that the air entrainment in open channel flows is resulted from the
190 unstable surface deformation. Because several forces, such as velocity fluctuations, gravity, surface
191 tension and surrounding pressure (Valero 2019), act on the entrapped deformed surface, it is difficult to
192 remain the stable distortion once the entrapment cavity penetrates deeply. Fig. 7 plots 10 air entrainment
193 processes for present 5 flow conditions and two bubble sizes generated for each case. In Fig. 7(a) and
194 (b), the instantaneous development of the entrapment surface in the length direction can be obtained by
195 $v_L' = dL/dt$, and the entrapment surface penetrating into the water with an instantaneous velocity v_y' ,
196 defined as $v_y' = dy/dt$. Both of the two velocities are introduced to describe the detailed geometry variation

197 in the unstable shape deformation process. The time mean velocities of $(v_L')_{\text{mean}}$ and $(v_y')_{\text{mean}}$ are obtained
 198 from the visual process of free surface entrapment and bubble entrainment. The fluctuations of v_L' and
 199 v_y' show a strong unstable free surface instability. The values of $(v_L')_{\text{mean}}$ are mainly greater than that of
 200 $(v_y')_{\text{mean}}$, indicating that the penetration of entrapment surface into the water is the key feature in the
 201 deformation process. The positive and negative mean values of v_L' indicates the length variation of
 202 entrapment development depend on specific bubble entrainment cases. Compared with the local free
 203 surface mean velocity, both of $(v_L')_{\text{mean}}$ and $(v_y')_{\text{mean}}$ are generally smaller than V_{fs} on the order of 10% –
 204 20%. In terms of the variation rate of L/y , its development may be expressed as:

$$\frac{d(L/y)}{dt} = 6 \cdot \left(\tanh \frac{t}{t^*} - 1 \right) \quad (4)$$

205 The regression coefficient of $R^2 = 0.565$. The normalization of Eq. (4) includes t/t^* , where t^* is a referred
 206 time scale, suggested as $t^* = 1$ ms in the present conditions. From Eq. (4) follow $d(L/y)/dt = -0.216$ at t
 207 $= 2$ ms and $d(L/y)/dt = -0.004$ at $t = 4$ ms, respectively. The data trend indicates that the entrapment shape
 208 variation underneath the surface in the bubble entrainment process depends on the time duration. For
 209 time duration smaller than about 4 ms, the L/y is a variable, while it can be considered as a constant for
 210 time duration exceeding 4 ms. This is important for the basic theory in which the length and depth scale
 211 are the key geometry parameters affecting the kinematics and dynamic analysis of free surface turbulent
 212 deformation. For the upstream free surface perturbation analysis (Valero and Bung 2018a), the linear
 213 relationship between the length and amplitude of the submerged body can be established for the time
 214 duration greater than 4 ms. For shorter time duration of a free surface break-up and air entrainment
 215 process, the effect of geometry variation should be further considered.

216 In Fig. 7(d), T_0 is the total time of the development and entrainment duration. The end of entrainment
217 stage is determined on the basis of the photo frame order in which a single bubble with clear air–water
218 interface all around separates from the surface entrapped it. The rapid decrease in L/y in the initial
219 duration (i.e. $t/T_0 < 0.2$) illustrates that the free-surface deformation develops rapidly. It is seen from
220 Figure 7 that the data in the $t/T_0 < 0.2$ are a bit scatter, which may be ascribed to the fact that the difference
221 of the random disturbance and strong free surface interaction exist during the initial development stage.
222 The variation trend then becomes gradual for the rest of time, i.e. $0.2 < t/T_0 < 1.0$. Fig. 7 also demonstrates
223 that the variation of L/y with dimensionless time for different bubble size is similar. Analysis of the ten
224 bubble generation data shows that the relationship between L/y and t/T_0 follows a power law:

$$L/y = m \cdot (t/T_0)^{-n} \quad (5)$$

225 Using the experimental data, the coefficient m and exponent n in equation (5) can be determined as 1.6
226 and 0.5, respectively, with the regression coefficient of $R^2 = 0.912$. Fig. 7 confirms the uniformity of the
227 free surface entrainment deformation and bubble entrainment process.

228 Be analogy to the previous analysis about the free surface perturbation breakups (Valero and Bung
229 2016; Valero and Bung 2018a), the ratio of L/y can be used to express the entrainment steepness. Valero
230 and Bung (2018a) deduced that the ratio of length to perturbation amplitude (the radial height above the
231 mean free surface) should be below a limiting steepness for the inception of distortion breakup and the
232 onset of air entrainment. For the deformation shape underneath the mean free surface, they made a
233 hypothesis of $y = 0.5L$ (expressed in the present parameters) for the perturbation geometry as it developed
234 from a vertical velocity fluctuation on the free surface. This is consistent with the entrainment stage of
235 this study where the entrainment shape becomes shrunk with bubble generation. The development stage

236 is a necessary condition for bubble entrainment generation. Whether the bubble is entrained into water is
237 dependent on the entrainment stage in which the entrapment distortion becomes significant within the
238 L/y getting decreased to 2. Analysis shows that the surface deformation corresponds to the turbulence
239 intensity and aeration level in self-aerated open channel flow. The highly aerated area sustains much
240 small. The result demonstrates that the entrapment deformation evolution of free surface in open channel
241 flow is an essential process for the air bubble generation.

242 Based on the entrapment characteristics of free surface deformation development and air
243 entrainment, the length L_m and depth y_m of the entrapped shape at the end of developing period before
244 the shrink are used as the scale parameters to describe the threshold shape of local surface (i.e. the shape
245 at $t = 4.8$ ms in Fig. 5). Fig. 8 shows the relation of surface entrapment size scales and the entrained
246 bubble size. Comparing with the entrapped deformation quantitatively, the skew distribution in the $d_{ab} <$
247 L_m and $d_{ab} < y_m$ regions indicates that most air bubble equivalent sizes are smaller than the deformation
248 size at both the longitudinal and lateral directions. Moreover, it is seen from Figure 8 that the finally
249 formed air bubble size ranges between 0.2 and 2 times of the final deformation size for the parameter
250 ranges tested, that is, $0.2L_m < d_{ab} < 2L_m$, and $0.2y_m < d_{ab} < 2y_m$. This is owing to the shrink feature of free
251 surface entrapment deformation before the bubble entrainment. Both the disturbance effect and the
252 enclosed position seem to be random and unpredictable. On the other hand, with the increase of entrained
253 bubble size, the corresponding ranges of L_m and y_m decrease. For example, for small air bubbles ($d_{ab} < 6$
254 mm), the orders of magnitude of L_m and y_m range from 1 mm to 14 mm and 1 mm to 10 mm, respectively.
255 While for large air bubble ($d_{ab} > 6$ mm), the order of the magnitude of both L_m and y_m changes to 4 mm
256 to 10 mm. These two features demonstrate that the bubble size is affected by the absolute size scale of

257 deformation.

258 Fig. 9 shows the relationship between L_m/y_m and d_{ab} . It is seen from Fig. 9 that air bubble size
259 depends on the entrainment deformation shape of free surface. In the present experimental observation,
260 most entrapped shapes of free surface have “long-shallow” type with $L_m/y_m > 1.0$. The “short-deep” type
261 of entrapped free surface for $L_m/y_m < 1.0$ occurs mainly for small bubble ($d_{ab} < 6$ mm). For the large
262 bubble generation ($d_{ab} > 6$ mm), the associated value of L_m/y_m is mainly within the range of 1.0 – 1.5.
263 For $L_m/y_m > 1.5$, the entrained bubble size starts to decrease with the increase of L_m/y_m . This means that
264 in addition to the absolute size scale effect, a moderate value of L_m/y_m with a slightly “long-shallow” type
265 favors the generation of the large air bubble in the free surface evolution process.

266 To capture the air bubble entrainment process, the local air concentration in the flow needs to be
267 relatively low for the purpose of the visual clear-water. The amount probability of different bubble size
268 is plotted in Fig. 10(a), where the probability of d_{ab} from 1 to 2 mm is represented by the column labeled
269 1 mm, and column labeled 6 mm represents the total probability of bubble diameter being larger than 6
270 mm. Note that air bubbles which are smaller than 1 mm are indeed observed, however, the formation
271 process for this size scale cannot be clearly observed due to the experimental shooting limitations. The
272 probability of small air bubbles is greater than that of large bubble. Comparing the present study with the
273 previous intrusive tests on air bubble size distribution in lower aerated region (Chanson 1997), the
274 observed bubble size distribution in this study is more skewed, with a preponderance of small bubble
275 with $d_{ab} < 6$ mm. The values of mean air bubble size d_{mean} for intrusive and visual tests are different, as
276 shown in Fig. 10(b). The ratio γ of intrusive air chord length over visual bubble size can exceed two,
277 which indicates the intrusive results overestimate the individual bubble scale in self-aerated flows. This

278 can be explained as following. Firstly, it should be noted that the minimum bubble size detected by the
279 intrusive measurement depends on the instrument tip diameter and a reasonable sample rate to avoid
280 aliasing (Chanson 2013). The diameter of needle tip should be much smaller than the smallest bubble
281 size in the air-water flow and the sampling rate should be set according to the flow velocity. However, it
282 is difficult to avoid completely the interface aliasing for small bubble detection. Secondly, the intrusive
283 measurement mainly detects the continuous signal of air–water interface in streamwise direction,
284 therefore, it is difficult to distinguish the deformed surface and the individual air bubble in the air–water
285 mixture region, especially for the process containing both entrapment deformation and bubble
286 entrainment. The size L_m of entrapped surface is generally larger than the final size d_{ab} of the entrained
287 air bubble, thus, the probability of enclosed two-surface sides at lower positions near the entrapped apex
288 is much greater than that for higher enclosed positions. Consequently, the entrained bubble properties
289 (e.g. the size and amount probability) depend on both the size scale and the shape of entrapped free
290 surface. The bubble size distribution obtained from the intrusive measurement cannot represent the
291 bubble size characteristics generated from the free surface air entrainment.

292

293 ***Bubble Diffusion in Water Flow***

294 After a single bubble is entrained into water, it moves downstream with water flow. In vertical direction,
295 the air bubble can either rise towards the free surface or penetrate deeply into water, as shown in Fig. 11,
296 in which some images are systematically skipped during this process to reduce the overlap effect of the
297 particles. The bubble movement across this interval period (Δt) is approximated as linear motion, and

298 the component velocities ΔV_{ax} and ΔV_{ay} are deduced using respectively the longitudinal and vertical
299 centroid displacement over the time interval. Based on the bubble movement and the variation of
300 instantaneous velocity in Fig. 12, ΔV_{ax} and ΔV_{ay} fluctuate around 0 for both rising and penetrating
301 processes in the low-aerated region. Thus, in the present study, this means that the single bubble's
302 diffusion path for weak air–water mixture can be simplified as a linear motion.

303 Applying a linear approximation to the bubble movement, the streamwise velocity V_{ax} , rising
304 velocity V_{ay} and penetration velocity V_{ay}' can be estimated from the centroid displacement in both
305 streamwise direction Δx and vertical direction Δy over the same observed period. The order of magnitude
306 of individual bubble size ranges from 1 mm to 10 mm. The characteristic velocities of different bubble
307 sizes are shown in Fig. 13. In the streamwise direction, V_{ax} is approximately the same as the free surface
308 velocity across the range of the bubble size, indicating that the following behavior of bubble moved with
309 water flow downstream is well performed. For the time average process, there is no prominent slipping
310 between single entrained bubbles and the flowing free surface of water bulk. In the vertical direction, the
311 V_{ay} and V_{ay}' generally range from 0.01 m/s to 0.50 m/s. For small bubble size ($d_{ab} < 6$ mm), the vertical
312 velocity has a wider range (0.01 m/s to 0.50 m/s), while for larger bubble size ($d_{ab} > 6$ mm), the range of
313 vertical velocity becomes narrow (0.10 m/s to 0.50 m/s). Comparing with the effective bubble rising
314 velocity in still water as a function of the bubble size (Haberman and Morton 1954; Falvey 1980), the
315 vertical velocities are mainly smaller across bubble size scales, especially for small air bubble diffusion
316 process.

317 The transport process of air bubbles in the water flow is a basic issue for the high-speed aerated
318 flow. In hydraulic engineering, the previous analysis on air–water mixture in the open channel flow was

319 normally established based on the indirect hypothesis of air bubble transport (Wood 1991; Chanson 1993;
 320 Kramer 2004). The air bubble motion at vertical direction is characterized as the rising velocity $V_r \cdot \cos\alpha$,
 321 representing the air phase spilling over the water flow. The value of V_r can be indirectly deduced by
 322 continuity theory (Chanson 1993) and diffusion theory (Liang and Wang 1982; Yuan and Xiang 1988) or
 323 air profile gradient (Kramer 2004). Fig. 14 shows the average values of bubble rise and penetration
 324 velocities $((V_{ay})_{\text{mean}}, (V_{ay'})_{\text{mean}})$ from the present study of direct observation. In Fig. 14, the average flow
 325 velocity V , which can be estimated easily, is used in order to make the results having wide engineering
 326 application. It is seen from Fig. 14 that the $(V_{ay})_{\text{mean}}$ and $(V_{ay'})_{\text{mean}}$ increase with the increase of water flow
 327 velocity. The variation trends of $(V_{ay})_{\text{mean}}$ and $(V_{ay'})_{\text{mean}}$ with V are approximately the same, which can be
 328 best fitted as following:

$$(V_{ay})_{\text{mean}} = (V_{ay'})_{\text{mean}} = (0.023 \cdot V + 0.046) \cdot \cos\alpha \quad (6)$$

329 where the channel slope α is set to reflect the vertical component of bubble motion. Table 2 shows the
 330 comparison of the computed values from Eq. (6) with the previous studies. Good agreement in Table 2
 331 indicates that the present relationship can represent the basic bubble motion in free surface air
 332 entrainment and air–water mixture development in open channel flows. Moreover, the same variation
 333 trends of bubble rising and penetration processes indicate that the two characterized bubble motions
 334 should be equally considered.

335 ***Application in Hydraulic engineering***

336 For a self-aerated flow on a spillway in hydraulic engineering, the prediction of gradually varied
 337 aeration region downstream of the inception location of free surface air entrainment relates to the accurate

338 recognition of detailed air bubble diffusion process. Based on the air phase continuity in an air–water
 339 flow along a channel of constant slope, Wood (1991) and Chanson (1993) proposed an equation for the
 340 gradual variation of air quantity entrainment along the spillway:

$$\frac{1}{(1-C_e)^2} \cdot \ln\left(\frac{1-C_{\text{mean}}}{C_e-C_{\text{mean}}}\right) - \frac{1}{(1-C_e) \cdot (1-C_{\text{mean}})} = \frac{V_r \cdot \cos \alpha}{V_0} \cdot \frac{x}{d_0} + K_0 \quad (7)$$

341 where x = the distance from the self-aeration inception, V_0 and d_0 = the flow velocity and depth at the
 342 self-aeration inception, respectively. The coefficient K_0 can be determined as,

$$K_0 = \frac{1}{1-C_e} \cdot \left(\frac{1}{1-C_e} \cdot \ln \frac{1}{C_e} - 1 \right) \quad (8)$$

343 where C_e = the cross-sectional averaged air concentration for the uniform air–water mixture flow, which
 344 is a function of the channel slope and can be determined by (Hager 1991)

$$C_e = 0.75 \cdot (\sin \alpha)^{0.75} \quad (9)$$

345 Equation (9) is valid for $7.5^\circ \leq \alpha \leq 75^\circ$. As the channel bed slope is easy to determine, in order to obtain
 346 the C_{mean} variation along the spillway, the key factor for solving the Eq. (7) is the characterized bubble
 347 velocity V_r . So far, little knowledge on V_r has been available, making it difficult to obtain an explicit
 348 solution of C_{mean} . However, the value of V_r may be determined empirically from each specific
 349 experimental result (Chanson 1993). Assuming that V_0 equal to V in Eq. (6) and applying this in Eq. (7),
 350 the C_{mean} can then be solved explicitly. Fig. 15 shows the comparison of the calculated C_{mean} with the
 351 prototype Aviemore dam observation (Cain 1978) and laboratory experiment (Xi 1988). Good agreement
 352 between the calculated and measured/observed C_{mean} confirms that the microscopic bubble movement
 353 obtained from the visual observation in self-aerated open channel flows can accurately predict the air–
 354 water two–phase flow and improve the knowledge of the link between the macroscopic and microscopic

355 air–water properties in hydraulic engineering.

356 The application about the self-aeration development in open channel flows shows that the
357 characteristic bubble velocity is a key parameter in the air-water mixture process. Firstly, the bubble
358 motion process in still fluid is affected by mechanism factors, including inertial, drag, buoyancy, and
359 fluid kinematic viscosity and density (or air concentration). The decrease of air-water mixture density (or
360 increase of air concentration) can lead to a decrease of the single bubble rising velocity (Chanson 1995,
361 Bennen 2005). Secondly, the turbulent eddy transport force is another important factor for the bubble
362 transport process in high velocity turbulent flows (Falvey and Ervine 1988). The increase of flow velocity
363 enhances turbulent shear and eddy transfer, resulting in significant mass exchange across the flow depth.
364 For very high flow aeration in prototype self-aerated flow (Cain 1978), V_r is almost two times greater
365 than that in scale model flows. Although the high velocity of a self-aerated open channel flow can
366 improve the aeration and decrease the entire flow density, the increase of the bubble velocity inferred
367 from the literature and present observation indicates the bubble vertical transfer is mainly determined by
368 the turbulent eddy transport intensity, and the basic relationships Eq. (6) – (9) can give a reasonable
369 results for some prediction applications.

370 In the present study, the direct observations on the single bubble transport characteristics can be
371 considered as benchmark data for the air-water mixture development in high speed open channel flows.
372 For air-water flows with complex air-water structures, such as bubble clouds and recirculation and local
373 non-hydrostatic pressure condition, the bubble transfer will be influenced by more microscopic
374 interactions (Kobus 1991; Kramer 2005). These complex conditions influence the bubble shape
375 deformation, collision within break-up or coalescence among each other, and slip movement in distorted

376 streamline area, which may result in different performance of bubble motion. Further experiments will
377 be required to investigate the difference of bubble rise velocity between a single bubble and a mixture
378 structure, and to expand the application of air-water diffusion in high speed open channel flows.

379

380 **Conclusions**

381 This study presents laboratory experimental results on the free surface air entrainment and single bubble
382 diffusion in open channel flows. The visual processes of free surface deformation and bubble movement
383 are recorded by using a high-speed camera system. Based on the recorded detailed shape deformation,
384 the free surface air entrainment and bubble generation are described quantitatively. Moreover, the single
385 bubble movement with water flow and its effect on the air–water mixture development in self-aerated
386 open channel flows are analyzed.

387 The free surface entrapment can entrain air into water flow depending on the shape deformation
388 evolution. When the entrapment deformation develops significantly, namely penetrating and distorting
389 steeply below the flow surface, air entrainment occurs by creating individual bubble. For different bubble
390 size generations, the present study shows an approximately identical power law for the evolution trend
391 of free surface entrapment deformation. The entrained bubble size is determined by the deformation size
392 scale, entrapment shape and the unstable enclosed position. A moderate long-shallow type of a local
393 entrapped surface with a large deformation size favors the generation of the large air bubble in the free
394 surface evolution process.

395 For the single bubble motion in the low aerated area of high speed open channel flows, its

396 streamwise velocity is approximately the same as that of water flow, while the vertical velocity for both
397 rise and penetration is smaller than that of the effective bubble rise velocity in still water. The well
398 correlation between the bubble vertical velocity and the average flow velocity confirms that the
399 individual bubble vertical transfer is mainly determined by the turbulent eddy transport intensity. An
400 empirical relationship between the single bubble motion and average flow velocity is established from
401 this study, which provides reasonable agreement with measurements. This demonstrates that the
402 microscopic bubble motion properties can promote the self-aeration prediction.

403 The self-aeration is an unstable interaction process among multiple instability forces on the air–
404 water interface. Air diffusion in the turbulent aerated open channel flow is highly affected by the
405 turbulence transportation. The theoretical framework should be of interest on the study on the linkage
406 between the surface deformation and the turbulence intensity in open channel flows. Further
407 experimental studies on complex bubble motion characteristics are required to better interpret air-water
408 mixture process and to expand application in hydraulic engineering.

409

410 **Data Availability Statement**

411 All data, models, and code generated or used during the study appear in the submitted article.

412

413 **Acknowledgements:** The research reports here is financially supported by National Natural Science
414 Foundation of China (Grant number 51609162), Sichuan Science and Technology Program (Grant
415 number 2019JDTD0007) and the Open funding of the State Key Laboratory of Hydraulics and Mountain
416 River Engineering of Sichuan University (Project No: Skh11809). Comments made by Reviewers have

417 greatly improved the quality of the paper.

418 **Notations**

C	=	air concentration;
C_e	=	average cross-sectional air concentration for uniform air–water mixture flow;
C_{mean}	=	average cross-sectional air concentration along the spillway;
d	=	characterized water flow depth;
d_{ab}	=	equivalent size of single bubble;
d_{mean}	=	average air bubble size
d_0	=	characterized water flow depth at the self-aeration inception;
h	=	flow depth from the channel bottom;
h_{90}	=	mixture flow surface level where local concentration is $C = 0.90$;
K_0	=	coefficient;
L	=	length of surface entrapment;
L_m	=	length of surface entrapment at the end of deformation development period;
m	=	coefficient;
n	=	coefficient;
Q	=	water flow discharge rate;
T_0	=	total time of the development and entrainment periods;
t	=	time;
V	=	average flow velocity;

- V_{ax} = single bubble velocity in streamwise direction;
- V_{ay} = single bubble rising velocity at vertical direction;
- $(V_{ay})_{\text{mean}}$ = average bubble rising velocity at vertical direction;
- V_{ay}' = single bubble penetration velocity at vertical direction;
- $(V_{ay}')_{\text{mean}}$ = average bubble penetration velocity at vertical direction;
- V_{fs} = average free surface velocity;
- V_r = bubble rising velocity in literature;
- V_0 = flow velocity at the self-aeration inception;
- v_L' = instantaneous development of the entrapment surface in the length direction;
- $(v_L')_{\text{mean}}$ = average velocity of entrapment length development;
- v_y' = instantaneous velocity of entrapment surface penetration;
- $(v_y')_{\text{mean}}$ = average velocity of entrapment penetration development;
- W = channel width;
- x = streamwise direction from the self-aeration inception;
- y = depth of surface entrapment;
- y_m = depth of surface entrapment at the end of deformation development period;
- ΔV_{ax} = component velocity over the interval period in streamwise direction;
- ΔV_{ay} = component velocity over the interval period in vertical direction;
- Δx = centroid displacement in streamwise direction;
- Δy = centroid displacement in vertical direction.

α = channel slope;

γ = ratio of average intrusive air chord length over visual bubble size.

419

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Fig. 1. Air–water flows in hydraulic engineering: (a) Jinping-I dam; (b) Wudu spillway (photos taken by Professor Jun Deng).

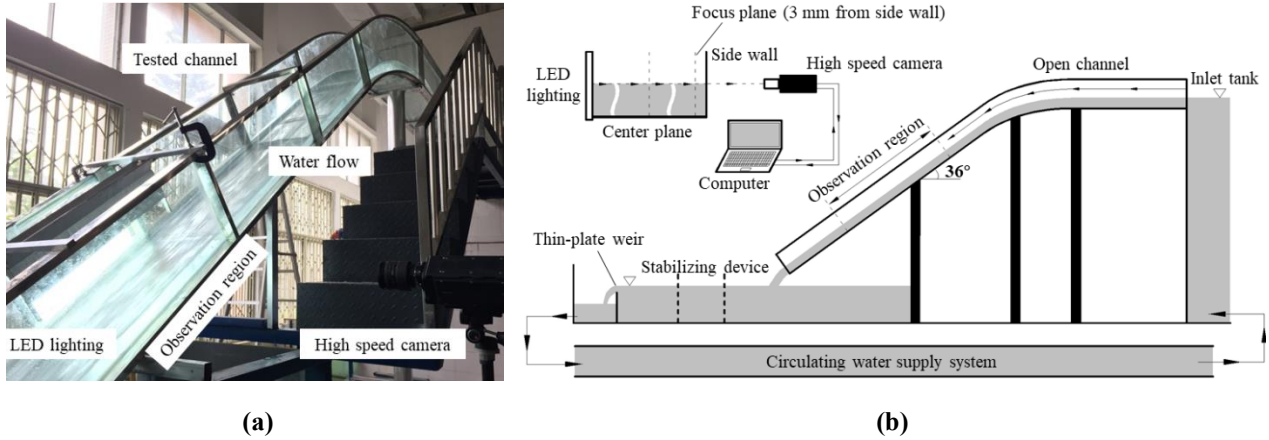


Fig. 2. Experimental facility and arrangement: (a) experimental flume photo; (b) observation arrangement.

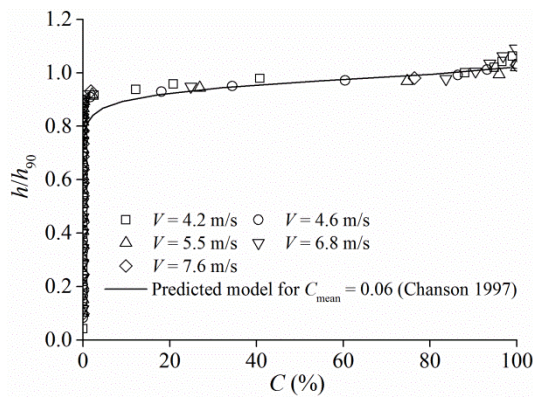


Fig. 3. Air concentration profiles at observation regions.

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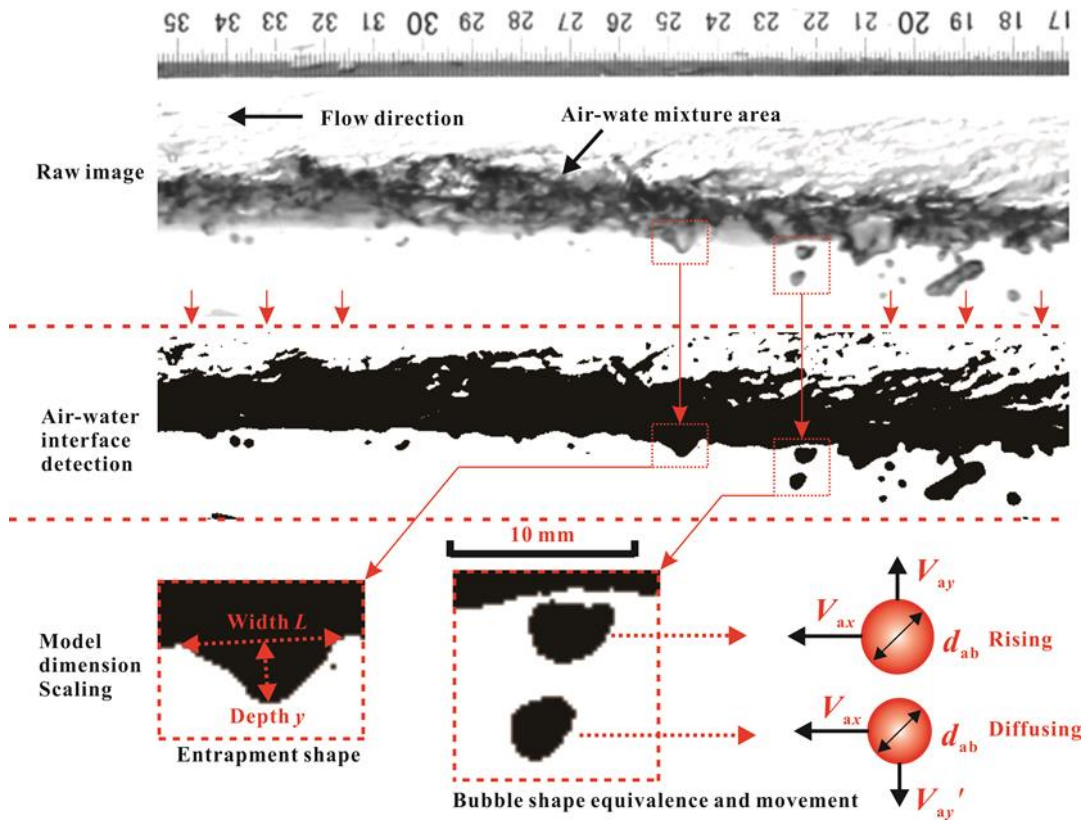


Fig. 4. Image processing for air–water interface and bubble size scales.

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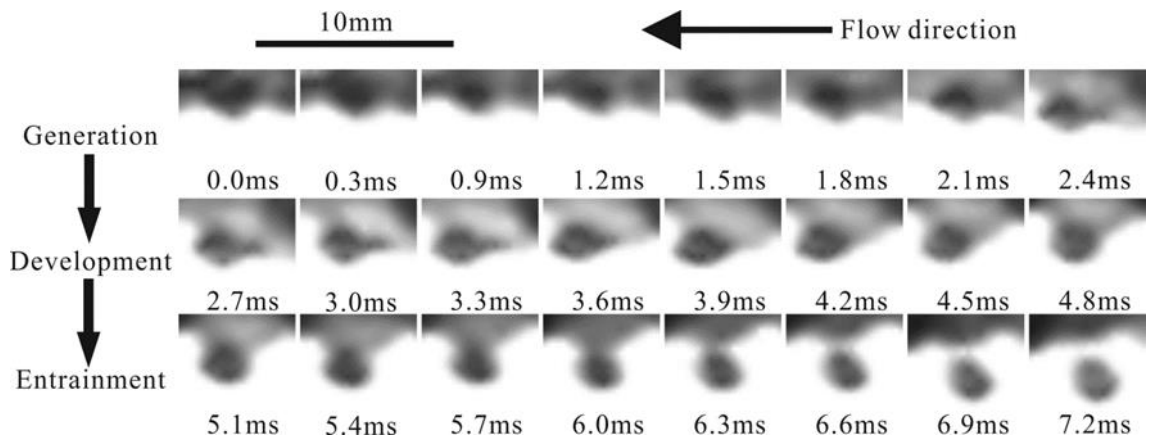
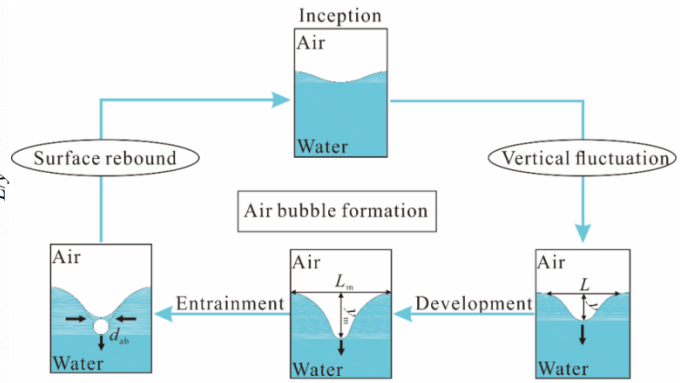
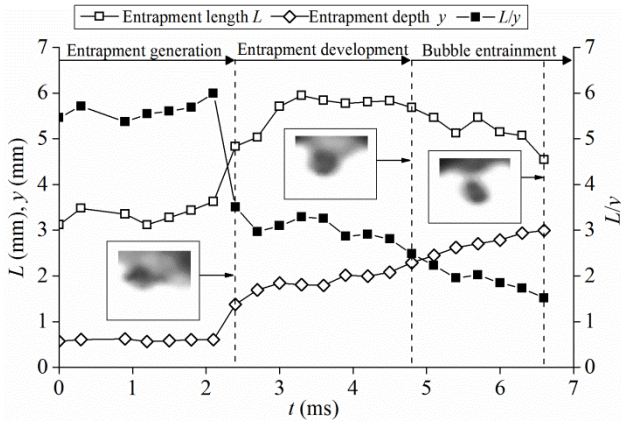


Fig. 5. High speed images of the free surface deformation and bubble entrainment for $V = 5.5$ m/s. Grey-dark shade represents air and white represents water.

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(a)

(b)

Fig. 6. Size scale and entrapped shape variations over time of (a) observation for $V = 5.5$ m/s and $Re = 2.97 \times 10^5$ and (b) schematic diagram showing the processes for bubble formation.

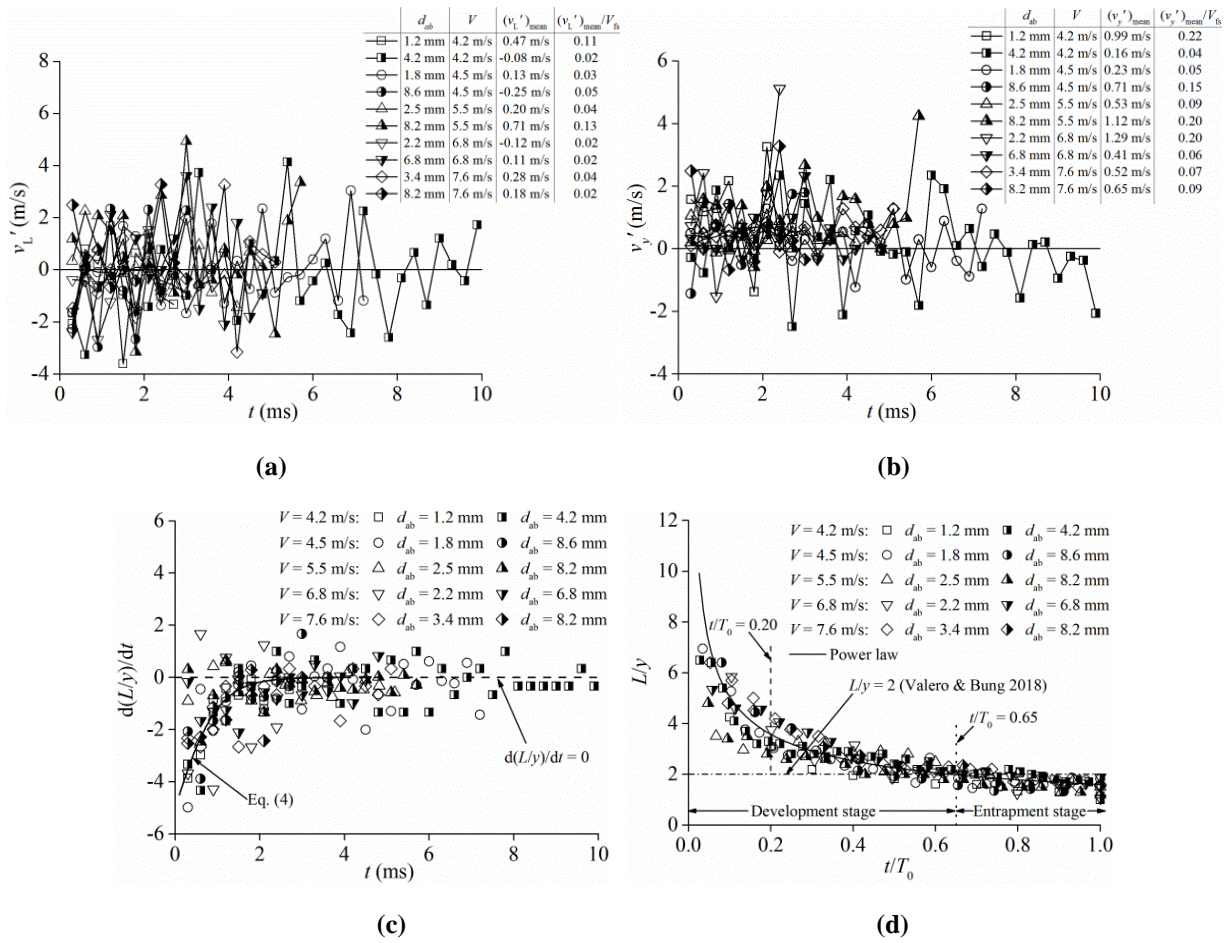


Fig.7. Geometry deformation with time for different bubble size generations: (a) entrapment length; (b) entrapment depth; (c) ratio of length to depth; (d) shape variations with dimensionless time.

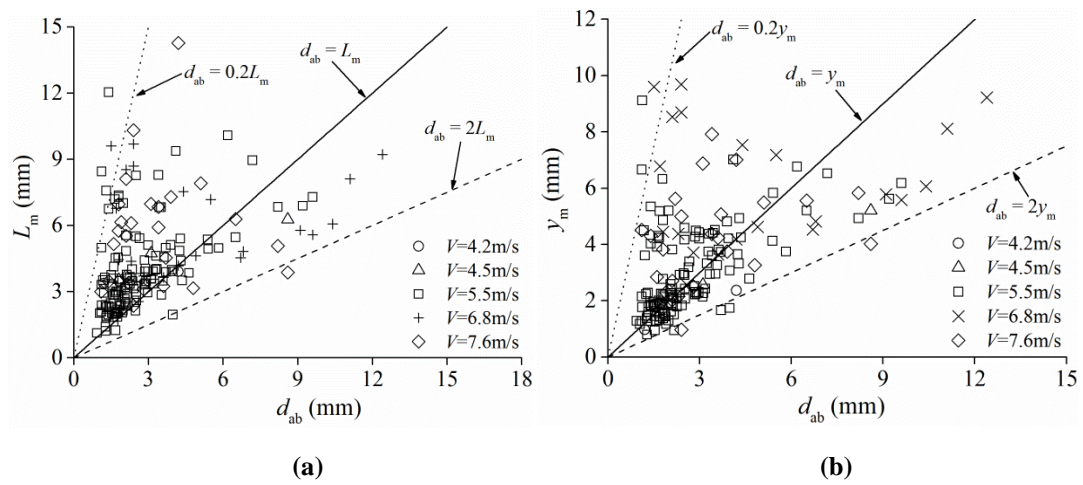


Fig. 8. Relation of (a) entrapped width and (b) depth scale with the entrained bubble size scale.

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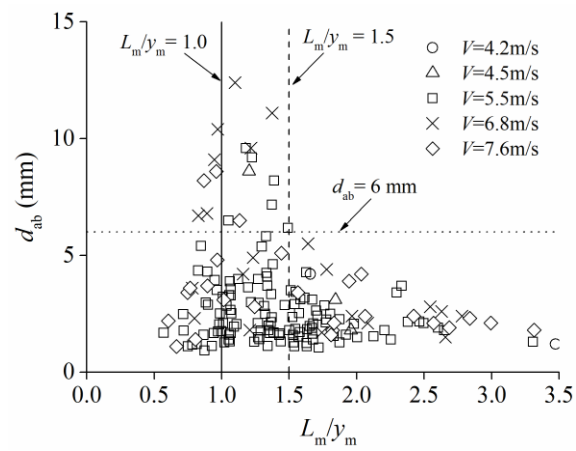


Fig. 9. Effect of entrapped shape on entrained bubble size scale.

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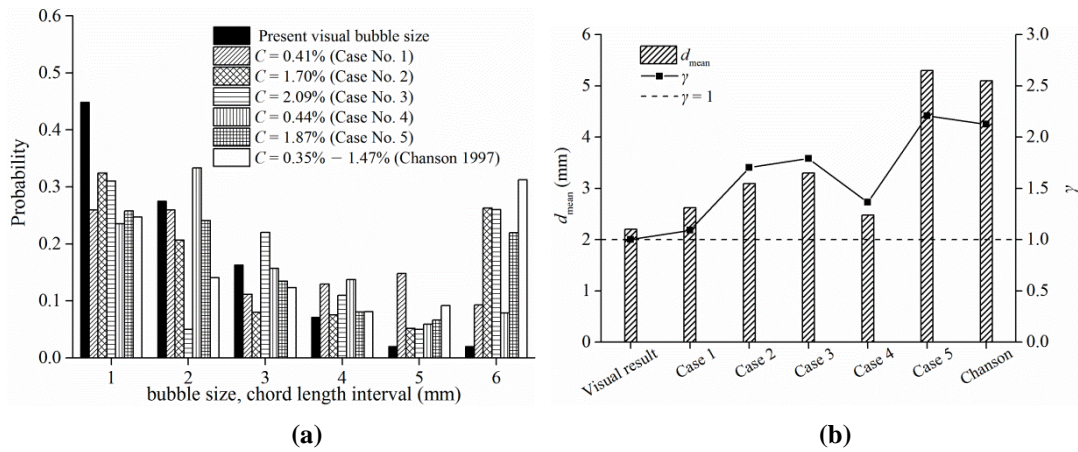


Fig. 10. Comparison of observed bubble size generation with intrusive measurements of bubble chord length: (a) bubble size distribution; (b) average bubble size.

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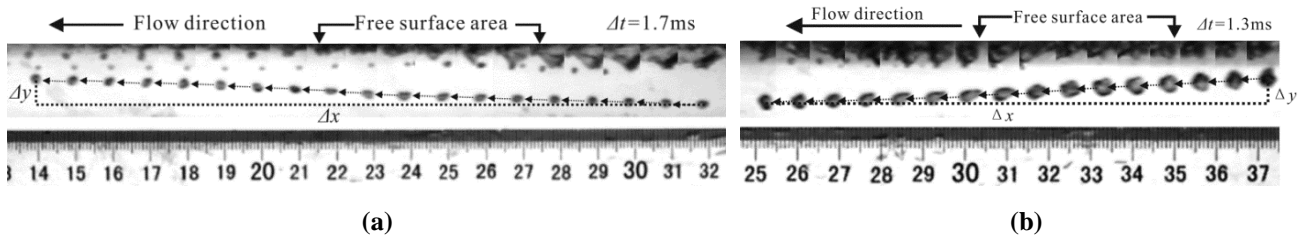


Fig. 11. Single bubble (a) rising ($d_{ab} = 3.0$ mm) and (b) penetration ($d_{ab} = 4.5$ mm) processes in the water flow ($V = 5.5$ m/s).

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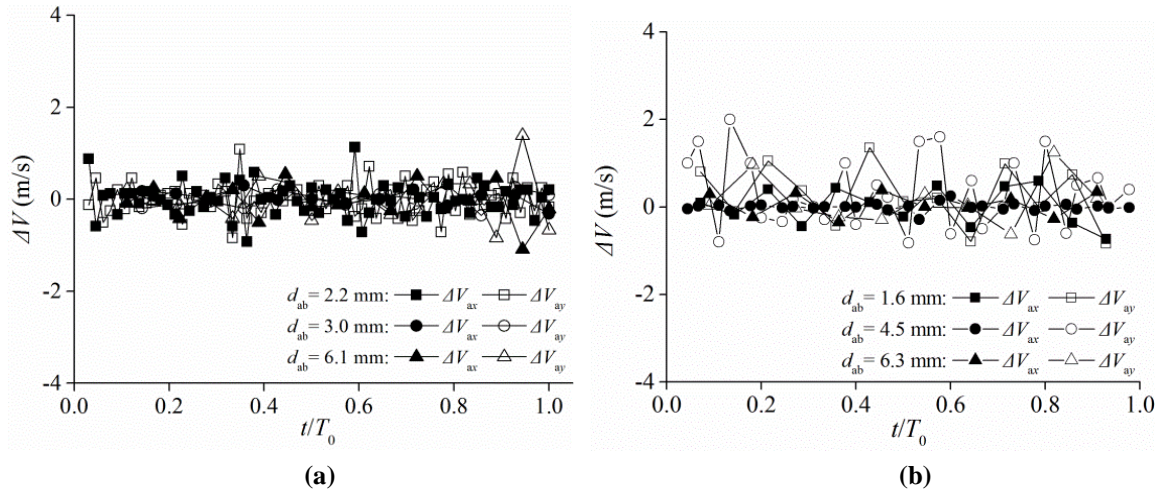


Fig. 12. Time variation of velocity fluctuation in longitudinal (ΔV_{ax}) and vertical (ΔV_{ay}) direction for (a) rising and (b) penetration processes ($V = 5.5$ m/s).

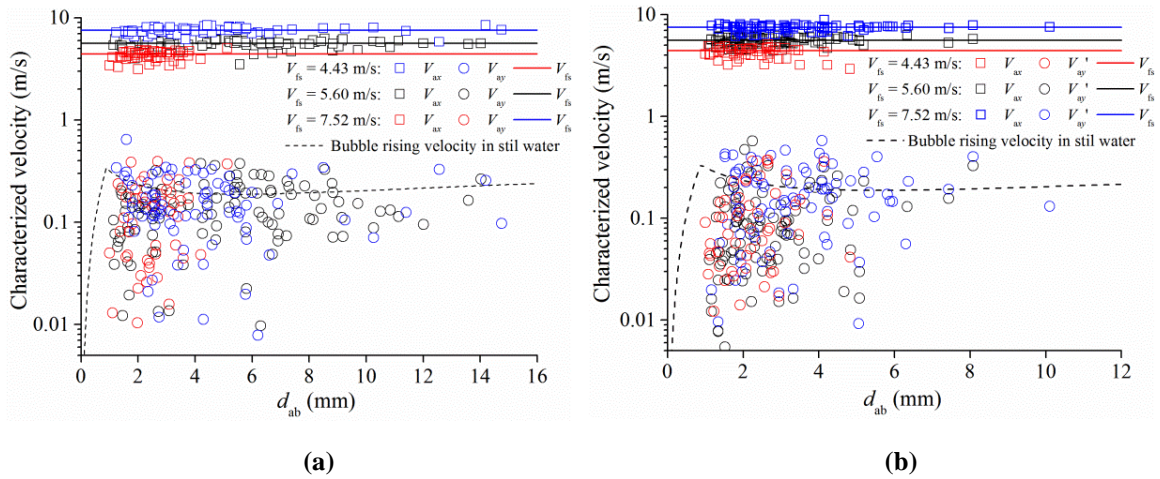


Fig. 13. Characterized velocities of different bubble sizes for (a) rising and (b) penetration processes.

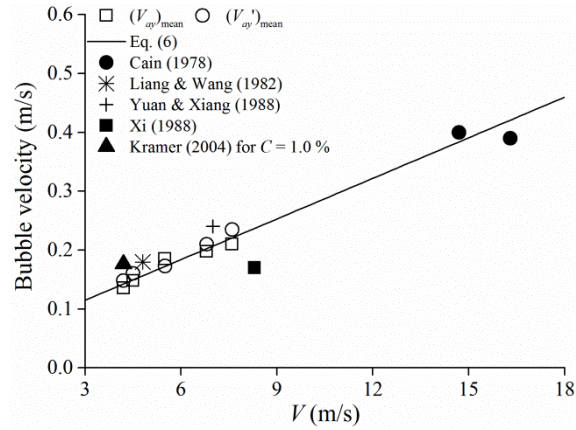


Fig. 14. Effect of flow velocity on characteristic bubble velocities in open channel flows.

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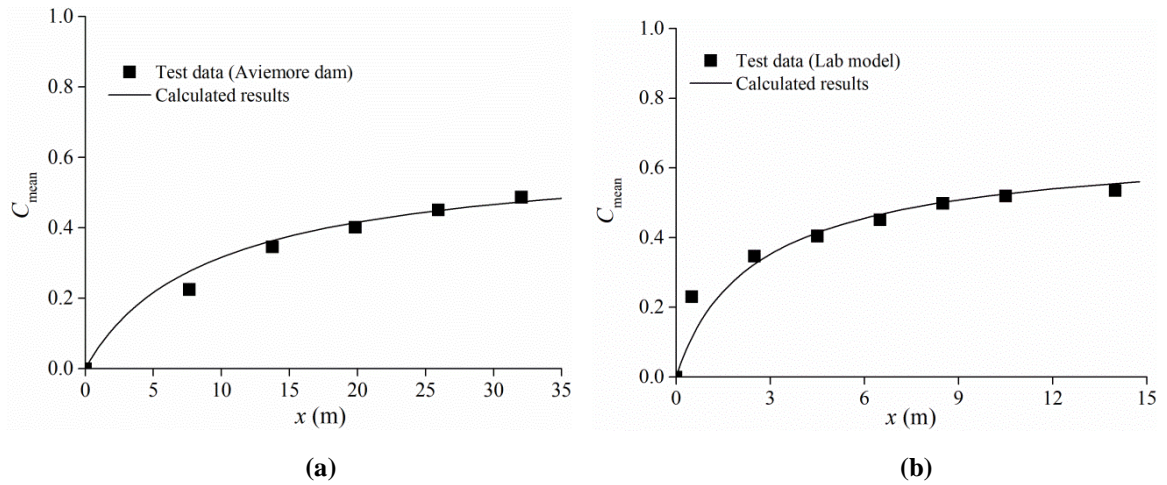


Fig. 15. Comparison of calculated results with Eq. (6) to test data on C_{mean} variations along the air–water open channel flows: (a) Aviemore dam, prototype (Cain 1978); (b) Laboratory experiment (Xi 1988).

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Table 1. Test Program and Flow Parameters.

Case No.	V (m/s)	Q (m ³ /s)	C_{mean} (-)	V_{fs} (m/s)	$Re \times 10^5$ (-)
1	4.2	0.109	0.061	4.43	2.73
2	4.5	0.117	0.074	4.82	2.93
3	5.5	0.119	0.067	5.60	2.97
4	6.8	0.131	0.079	6.58	3.26
5	7.6	0.137	0.056	7.52	3.42

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Table 2. Comparison of Characterized Bubble Velocities with Literature in Open Channel Flows.

No.	Case	V (m/s)	V_r from test inference (m/s)	V_r from Eq. (6) (m/s)
1	Cain (1978)	14.7	0.40	0.38
	Prototype spillway on Aviemore dam, deduced by Chanson (1993)	16.3	0.39	0.42
2	Yuan & Xiang (1988) Laboratory chute test	7.6	0.24	0.22
3	Liang & Wang (1982) Laboratory chute test	4.8	0.18	0.16
4	Xi (1988) Scaled Model spillway on Meishan dam, deduced by Chanson (1993)	8.3	0.17	0.24
5	Kramer (2004) Laboratory chute test for $C = 1.0\%$	4.2	0.18	0.14

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574