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Estimating market power under a nonparametric analysis: evidence from the Chinese real estate sector

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Abstract

The traditional Lerner index is limited in its capacity to estimate the level of competition in the economic sector from the perspective that it mainly focuses on the overall level of market power for each individual decision-making unit. Recently, Fukuyama and Tan (J Oper Res Soc, 73:445–453, 2022) estimated the Lerner index by applying the nonparametric data envelopment analysis (DEA) to calculate the marginal cost, which is an important component in the estimation of the Lerner index. Our study further extends the study of Fukuyama and Tan (J Oper Res Soc, 73:445–453, 2022) by estimating the marginal cost under the DEA in a multi-product setting. Our proposed methodology benefits from the ability to find positive marginal costs for all the products and specifies all decision-making units are profit maximizers. In order to achieve this, the marginal cost is estimated by referring to the nearest point on the best practice cost-efficient frontier for the profit-maximizing firms. We then apply our innovative method to the Chinese real estate industry. The result shows that the Chinese real estate industry has higher market power in the residential commodity housing market than that in the commodity housing market. This is also the case for different geographical areas in China. Overall, for both of these two different markets, the level of market power experiences a level of volatility.

Keywords Data envelopment analysis · Aggregate Lerner index · Nearest targets · Real estate

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1 Introduction

The nonparametric data envelopment analysis (DEA) was introduced by Charnes et al. (CCR) in 1978 as a tool for efficiency and productivity measurement of the decision-making units. The namely DEA CCR model assumes constant returns to scale, while Banker et al. (BCC) in 1984 further extended the CCR model, the so-called DEA BCC model can decompose the overall efficiency into pure technical efficiency and scale efficiency by assuming variable returns to scale. The DEA models have been widely applied across different economic sectors including the banking industry (Fukuyama and Tan, 2022b); education industry (Mozaffari et al. 2022); hospitality industry (Tan and Despotis, 2021); Transportation industry (Omrani et al. 2022a); healthcare industry (Omrani et al. 2022b); energy industry (Chu et al. 2021); among others. Both the traditional DEA and various model advancements have been used by previous studies in the estimation of efficiency in the real estate industry, including the traditional DEA CCR and BBC models (Li et al. 2019; Nicholson and Stevens, 2021); DEA Malmquist approach (Chen et al. 2020); Hybrid DEA and Malmquist total factor productivity index (Jin et al. 2015); DEA super-efficiency model (Zheng et al. 2011); Slack-based DEA model (Yang and Fang, 2020); Dynamic network DEA (Yang et al. 2019); Slack-based super-efficiency model (Wang et al. 2020); hierarchy slacks-based DEA (Mills et al. 2021), and combination of grey methodology and DEA (Wang et al. 2021).

According to Tan (2016a), market power mainly measures the extent to which a specific firm can set the price level of its products above the marginal cost. Besides the traditional concentration ratio and Herfindahl–Hirschman index, the Lerner index has been widely applied in the empirical literature to measure the level of competition/market power (Aghion et al. 2015; Ariss, 2010; Leroy and Lucotte, 2017; Tan et al. 2021). The traditional Lerner index benefits from the advantage over the Panzar-Roose H statistic (Tan, 2016b) by being able to generate the level of market power for each decision-making unit rather than the level of competition/market power on an annual basis for the whole industry. However, it still suffers from the disadvantage of its inability to derive the level of market power for different markets/products. Very limited attempts have been made to investigate market power in the real estate industry.[1] Building on the traditional Lerner index, attempts have been made to develop this index in different ways, including (1) the use of a semiparametric smooth coefficient model (Diallo, 2015); the main advantage of this method lies in the fact that it can increase the flexibility of the functional form in estimating the Lerner index, however, it suffers from the same disadvantages as the traditional Lerner index that it does not attempt to address the market power in a multi-product context; (2) Scale-elasticity based marginal cost (Tsionas et al. 2018). Instead of estimating the cost functions, the study proposed a nonlinear system of simultaneous equations, through the estimation of which, the level of cost efficiency and scale elasticity can be derived at the same time,

[1] As far as we are concerned, there has not been any study in estimating market power in the real estate industry published in any English language journal.

the latter of which was then used to derive the Lerner index. In addition, the study considered a multi-product context in estimating the Lerner index. The main limitation of the study lies in the fact that its method allows for non-profit maximizing DMUs across different years of the period. It is an important piece of information that can be used by the firm's management. (3) Fukuyama and Tan (2022a) used the nonparametric DEA for the estimation of the Lerner index. The main limitation of Fukuyama and Tan (2022a) lies in the fact that it only considered a single-output empirical scenario although this study contributes to the literature by applying the nonparametric DEA in estimating the multi-output marginal costs. Finally, Shaffer and Spierdijk (2020) proposed to use the non-homothetic generalized Leontief (NHT-GL) cost function to estimate Lerner indices and applied it to the banking sector. Different from the previous estimations of Lerner indices, this measurement discussed the estimation of Lerner indices in a multi-product context and the proposed translog cost function can reduce the number of cases for negative Lerner indices, however, it still cannot completely remove the negative values of Lerner indices, which is not in accordance with the definition and properties of the Lerner index. This indicates that the estimation still suffers from limitations. The proposal and solution are needed to provide an estimation that would be able to completely remove the negativity issue of Lerner indices, and at the same time, the method disallows non-profit maximizing.

We propose a DEA approach for estimating the Lerner index, which uses a modified nonparametric cost function. It is well-known that the DEA-based cost function not only suffers from the existing multiple optima (with zero marginal costs for some products), but also some observed product prices can be less than the estimated marginal costs in practice. In the latter case, the Lerner index becomes negative and may not be acceptable from the monopoly and/or oligopoly theoretic perspective. However, it may be difficult to obtain practically useful prices which are directly comparable to the estimated shadow (multipliers) prices, although the observed prices are supposedly unitless. We usually construct the firm-specific prices according to the same price construction procedure for all firms (for example by dividing expenditure by the relevant input quantity). Hence, we develop an optimization method and handle the issue of the possible negative Lerner index by controlling for the multiplier values, so that the estimates do not exceed the observed input prices. Moreover, our DEA framework has at least two more desirable characteristics: (a) the Lerner index estimation is carried out without assuming any specific functional structure, which is generally unknown, and (b) the estimation procedure disallows non-profit maximizing behaviour.

The real estate enterprises took a large amount of bank loans and played a key role in determining the level of financial stability in China. A lower level of market power for those small and medium-sized real estate enterprises will induce them to undertake a higher level of risk-taking behavior, which would increase the probability of bank loan default and further result in financial instability. Therefore, improving the level of market power in the Chinese real estate enterprises would be the priority for the government. Before finding ways to improve the level of market power, we need to clearly understand the level of market power in the Chinese real estate industry. This would purely rely on an accurate and comprehensive analysis of the degree of market power. The current study not only contributes to the estimation of market power in the real estate industry in general and the Chinese real estate industry specifically from the empirical perspective, but our model provides significant contributions from the operational research perspective.

2 Notation and methodology.

2.1 Lerner index in a multi-product setting: preliminaries

To start with, let $\mathbf{x} = \left(x_1, \dots, x_N\right)^{\mathrm{T}} \in \mathbb{R}^N_+$ and $\mathbf{y} = \left(y_1, \dots, y_M\right)^{\mathrm{T}} \in \mathbb{R}^M_+$ represent non-negative column vectors of N different inputs and M different products, respectively, where superscript T indicates the transposing operator. The production possibility set (PPS) is denoted as

$$PPS = \left\{ \begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix} \middle| \begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix} \text{ is a technologically feasible input/product vector} \right\}. \tag{1}$$

With respect to (1), the standard or unrestricted cost function is denoted as

$$\overline{C}(\mathbf{y}, \mathbf{w}) = \min_{\mathbf{x}} \left\{ \mathbf{w}^{\mathrm{T}}\mathbf{x} \middle| \begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix} \in PPS \right\} \tag{2}$$

where $\mathbf{w} = \left(w_1, \dots, w_N\right)^{\mathrm{T}} \in \mathbb{R}^N_{++}$ is a positive column vector of N input prices and the inner product $\overline{c} = \mathbf{w}^{\mathrm{T}}\mathbf{x} = w_1x_1 + \cdots + w_Nx_N$ is total input cost. Let

$$\overline{MC}_m(\mathbf{y}, \mathbf{w}) = \frac{\partial \overline{C}(\mathbf{y}, \mathbf{w})}{\partial y_m} \tag{3}$$

be the marginal cost of product m, and let p_m be the price of product m. Then the Lerner sub-index for product m is written as

$$\overline{L}_m = \frac{P_m - \overline{MC}_m(\mathbf{y}, \mathbf{w})}{P_m}, m = 1, \dots, M \tag{4}$$

If $\overline{L}_m = 0$ with $P_m = \overline{MC}_m(\mathbf{y}, \mathbf{w})$, then the decision-making unit (DMU) has no market power. Under profit maximization, $\overline{L}_m$ takes the value between zero and one. To show this, consider the following profit maximization problem:

$$\max_{\mathbf{x},\mathbf{y}} \left\{ \mathbf{p}(\mathbf{y})^{\mathrm{T}}\mathbf{y} - \mathbf{w}^{\mathrm{T}}\mathbf{x} | (\mathbf{x}, \mathbf{y}) \in PPS \right\} = \max_{\mathbf{y}} \left\{ \mathbf{p}(\mathbf{y})^{\mathrm{T}}\mathbf{y} - \overline{C}(\mathbf{y}, \mathbf{w}) \right\} \tag{5}$$

where the product price vector $\mathbf{p}(\mathbf{y})$ depends upon $\mathbf{y}$. The first-order conditions of the right-hand side of (5) lead to

$$\frac{p_m - \overline{MC}_m(\mathbf{y},\mathbf{w})}{p_m} = -\frac{1}{\varepsilon_m(\mathbf{y})}, \quad m = 1, \ldots, M. \tag{6}$$

where $\varepsilon_m(\mathbf{y})$ is the inverse of partial elasticity of demand with respect to product m, i.e., $\varepsilon_m(\mathbf{y}) = \left(\frac{\partial p_m(\mathbf{y})}{\partial y_m}\frac{y_m}{p_m(\mathbf{y})}\right)^{-1} \leq 0$. For a single product case, the right-hand side of (5) becomes the single product inverse demand function. Equation (6) and non-positivity of $\varepsilon_m(\mathbf{y})$ indicates the standard Lerner sub-index $\overline{L}_m \in [0, 1]$ and hence, the profit maximizing DMU will not operate at the region where the marginal cost is less than zero or $\varepsilon_m(\mathbf{y})$ is greater than zero, indicating that $\overline{L}_m$ cannot be greater than one. If $0 < \overline{L}_m < 1$, then the DMU has some market power (the larger $\overline{L}_m$ is, the higher the market power).

Let $R_m = p_m y_m$ be the revenue from product m and let $R = \sum_{m'=1}^{M} p_{m'} y_{m'} = \sum_{m'=1}^{M} R_{m'}$ be the total revenue. Utilizing these notations, we define the revenue weight for product m as $W_m = \frac{R_m}{\sum_{m'=1}^{M} R_{m'}}$. We aggregate the standard Lerner sub-indexes (4) by using revenue shares as the weight. We denote a standard aggregate Lerner index, $\overline{AL}$ as:

$$\overline{AL} = \sum_{m=1}^{M} W_m \overline{L}_m \tag{7}$$

where $\sum_{m=1}^{M} W_m = 1$. The standard aggregate Lerner index (7) is not only less than or equal to one, but also greater than or equal to zero under profit maximization, like Lerner sub-indexes (4).

Now we consider a nonparametric DEA implementation of the standard cost function. Suppose that there are J DMUs, each DMU_j of which consumes a non-negative input vector $\mathbf{x}_j = \left(x_{1j}, \ldots, x_{Nj}\right)^{\mathrm{T}} \in \mathbb{R}_+^N$ to produce a non-negative product vector $\mathbf{y}_j = \left(y_{1j}, \ldots, y_{Mj}\right)^{\mathrm{T}} \in \mathbb{R}_+^M$. Let $\boldsymbol{\lambda} = \left(\lambda_1, \ldots, \lambda_J\right)^{\mathrm{T}} \in \mathbb{R}_+^J$ be a non-negative vector of intensities in linear programming and let 0 be an appropriate dimensional column vector with all elements being zero. A DEA production possibility set is constructed as

$$\left\{\begin{pmatrix}\mathbf{x}\\ \mathbf{y}\end{pmatrix} \middle| \sum_{j=1}^{J} \mathbf{x}_j \lambda_j \leq \mathbf{x}, \sum_{j=1}^{J} \mathbf{y}_j \lambda_j \geq \mathbf{y}, \sum_{j=1}^{J} \lambda_j = 1, \mathbf{x} \geq \mathbf{0}, \mathbf{y} \geq \mathbf{0}, \boldsymbol{\lambda} \geq \mathbf{0}\right\}, \tag{8}$$

where $\sum_{j=1}^{J} \lambda_j = 1$ allows for variable returns to scale. Using (8), the envelopment form of the DEA cost function (Färe et al. 1994) is constructed by

$$\overline{C}(\mathbf{y}, \mathbf{w}) = \min_{\mathbf{x},\boldsymbol{\lambda}} \left\{\mathbf{w}^{\mathrm{T}}\mathbf{x} \middle| \sum_{j=1}^{J} \mathbf{x}_j \lambda_j \leq \mathbf{x}, \sum_{j=1}^{J} \mathbf{y}_j \lambda_j \geq \mathbf{y}, \sum_{j=1}^{J} \lambda_j = 1, \quad \mathbf{x} \geq \mathbf{0}, \quad \boldsymbol{\lambda} \geq \mathbf{0}\right\}, \tag{9}$$

The multiplier representation of (9) is written as

$$\max_{\mathbf{v},\mathbf{u},\omega} \left\{ \mathbf{u}^{\mathrm{T}}\mathbf{y} + \omega \middle| -\mathbf{v}^{\mathrm{T}}\mathbf{x}_j + \mathbf{u}^{\mathrm{T}}\mathbf{y}_j + \omega \le 0 \ (\forall j), \ \ \mathbf{v} \le \mathbf{w}, \ \ \mathbf{v} \ge \mathbf{0}, \ \ \mathbf{u} \ge \mathbf{0}, \ \ \omega : \ \text{free} \right\} \tag{10}$$

Using the standard cost function, we consider the standard indirect output possibility correspondence, which is a mapping $\overline{IP} : \mathbb{R}^N_+ \to \overline{IP}(\mathbf{w}/\bar{c}) \subseteq \mathbb{R}^M_+$, where

$$\overline{IP}(\mathbf{w}/\bar{c}) = \left\{ \mathbf{y} \in \mathbb{R}^M_+ \middle| \overline{C}\left(\mathbf{y}, \mathbf{w}/\bar{c}\right) \le 1 \right\} \tag{11}$$

where $\overline{C}\left(\mathbf{y}, \mathbf{w}/\bar{c}\right) \le 1 \Leftrightarrow \overline{C}(\mathbf{y}, \mathbf{w}) \le \bar{c}$ from the definition of the standard cost function. Therefore, $\overline{C}\left(\mathbf{y}, \mathbf{w}/\bar{c}\right) = \overline{C}(\mathbf{y}, \mathbf{w})/\bar{c}$ is the standard cost efficiency. For a detailed account of (11), see Shephard (1970), which was further developed by Färe et al. (1994), and Färe and Primont (2006). For some applications of (11), see Grosskopf et al. (1999), Fukuyama and Weber (2005) and Fukuyama and Tan (2021).

As stated earlier, when the multiplier form of the standard cost function is used to estimate marginal costs, two issues should be considered: the unsatisfied profit maximization conditions and alternate solutions with zero product multiplier values. We develop an optimization method for dealing with these issues in the following two sub-sections.

2.2 Empirical procedure for coping with profit maximization

To avoid any possible unsatisfaction of profit maximization conditions, we implement the following multiplier restrictions:

$$\mathbf{u} \le \mathbf{p}. \tag{12}$$

It is usually necessary to adapt the multipliers (shadow prices) because the prices of various products are preferably unitless for comparison purposes, but it is sometimes difficult to find such prices in empirical analysis. Appending (12) in the multiplier form of (10), we obtain the following adapted cost function, which implements the price data to satisfy the profit maximization conditions, as

$$C(\mathbf{y}, \mathbf{w}, \mathbf{p}) = \max_{\mathbf{v},\mathbf{u},\ \omega} \left\{ \mathbf{u}^{\mathrm{T}}\mathbf{y} + \omega \middle| \begin{array}{c} -\mathbf{v}^{\mathrm{T}}\mathbf{x}_j + \mathbf{u}^{\mathrm{T}}\mathbf{y}_j + \omega \le 0 \ (\forall j), \ \ \mathbf{v} \le \mathbf{w}, \\ \mathbf{u} \le \mathbf{p}, \ \ \mathbf{v} \ge \mathbf{0}, \ \ \mathbf{u} \ge \mathbf{0}, \ \ \omega : \mathit{free} \end{array} \right\}. \tag{13}$$

The corresponding envelopment form is written as

$$C(\mathbf{y}, \mathbf{w}, \mathbf{p}) = \min_{\mathbf{x},\mathbf{d},\boldsymbol{\lambda}} \left\{ \mathbf{w}^{\mathrm{T}}\mathbf{x} + \mathbf{p}^{\mathrm{T}}\mathbf{d} \middle| \begin{array}{c} \sum_{j=1}^{J} \mathbf{x}_j\lambda_j \le \mathbf{x}, \ \sum_{j=1}^{J} \mathbf{y}_j\lambda_j \ge \mathbf{y} - \mathbf{d}, \\ \mathbf{d} \ge \mathbf{0}, \ \ \sum_{j=1}^{J} \lambda_j = 1, \ \ \mathbf{x} \ge \mathbf{0}, \ \ \boldsymbol{\lambda} \ge \mathbf{0} \end{array} \right\}. \tag{14}$$

The difference between $C(\mathbf{y}, \mathbf{w}, \mathbf{p})$ and $\overline{C}(\mathbf{y}, \mathbf{w})$ is that the inner product $\mathbf{p}^{\mathrm{T}}\mathbf{d}$ and the variable $\mathbf{d}$ appear in the objective function and the product constraints, respectively, for the adapted cost function case, but do not for the standard cost function.

We refer to the sum, $\mathbf{w}^{\mathrm{T}}\mathbf{x}+\mathbf{p}^{\mathrm{T}}\mathbf{d}$, as the cost which is an "adapted" cost due to the constraint (12). When a firm is adapted cost efficient, we have $c=\mathbf{w}^{\mathrm{T}}\mathbf{x}+\mathbf{p}^{\mathrm{T}}\mathbf{d}$. Otherwise, $c>\mathbf{w}^{\mathrm{T}}\mathbf{x}+\mathbf{p}^{\mathrm{T}}\mathbf{d}$.

At the optimum, the optimized objection functions of (13) and (14) are the same, i.e.,

$$C(\mathbf{y},\mathbf{w},\mathbf{p})=\mathbf{u}^{*\mathrm{T}}\mathbf{y}+\omega=\mathbf{w}^{\mathrm{T}}\mathbf{x}^{*}+\mathbf{p}^{\mathrm{T}}\mathbf{d}^{*}$$

where asterisk * shows the optimality of (13) and (15). Since the observed input cost $c>0$ is greater than or equal to the minimum adapted cost, we have $c\geq\mathbf{u}^{*\mathrm{T}}\mathbf{y}+\omega=\mathbf{w}^{\mathrm{T}}\mathbf{x}^{*}+\mathbf{p}^{\mathrm{T}}\mathbf{d}^{*}\Rightarrow 0\geq\mathbf{w}^{\mathrm{T}}\mathbf{x}^{*}+\mathbf{p}^{\mathrm{T}}\mathbf{d}^{*}-c$.

Using this result, we formulate the Lagrange function for (13) as follows:

$$\begin{aligned}\mathcal{L}(\mathbf{v},\mathbf{u},\omega,\mathbf{t},\mathbf{e},\mathbf{d},a)=&\,\mathbf{u}^{\mathrm{T}}\mathbf{y}+\omega-\sum_{j=1}^{J}t_j\left(-\mathbf{v}^{\mathrm{T}}\mathbf{x}_j+\mathbf{u}^{\mathrm{T}}\mathbf{y}_j+\omega\right)\\&-\sum_{n=1}^{N}e_n\left(v_n-w_n\right)-\sum_{m=1}^{M}d_m\left(u_m-p_m\right)-a\left(c-\mathbf{w}^{\mathrm{T}}\mathbf{x}-\mathbf{p}^{\mathrm{T}}\mathbf{d}\right)\end{aligned}\tag{15}$$

Hence, we can write

$$\begin{aligned}\mathcal{L}(\mathbf{v}^*,\mathbf{u}^*,\omega^*,\mathbf{t}^*,\mathbf{e}^*,\mathbf{d}^*,a^*)=&\,\mathbf{u}^{*\mathrm{T}}\mathbf{y}+\omega-\sum_{j=1}^{J}t_j^*\left(-\mathbf{v}^{*\mathrm{T}}\mathbf{x}_j+\mathbf{u}^{*\mathrm{T}}\mathbf{y}_j+\omega^*\right)\\&-\sum_{n=1}^{N}e_n^*\left(v_n^*-w_n\right)-\sum_{m=1}^{M}d_m^*\left(u_m^*-p_m\right)-a^*\left(c-\mathbf{w}^{\mathrm{T}}\mathbf{x}^*-\mathbf{p}^{\mathrm{T}}\mathbf{d}^*\right)\end{aligned}\tag{16}$$

Since the optimal objective function value of (13) is equal to $C(\mathbf{y},\mathbf{w},\mathbf{p})$ in (14), we have $C(\mathbf{y},\mathbf{w},\mathbf{p})=\mathcal{L}^*(\mathbf{y},\mathbf{w},\mathbf{p})=\mathcal{L}(\mathbf{v}^*,\mathbf{u}^*,\omega^*,\mathbf{t}^*,\mathbf{e}^*,\mathbf{d}^*,a^*)$. Applying the envelop theorem, we obtain

$$MC_m(\mathbf{y},\mathbf{w},\mathbf{p})=\frac{\partial C(\mathbf{y},\mathbf{w},\mathbf{p})}{\partial y_m}=u_m^*\tag{17}$$

which is the marginal cost for product m for (13) and indicates that the evaluated DMU's marginal cost with respect to product m can be obtained by $u_m^*\geq 0$ (part of the solution to (13)). Similar to the standard cost function case, the Lerner sub-indexes, L_m, as well as the aggregate Lerner index, AL, for the product-price restricted cost function case are obtained as follows:

$$AL=\sum_{m=1}^{M}W_mL_m,\ L_m=\frac{P_m-MC_m(\mathbf{y},\mathbf{w};\mathbf{p})}{P_m},\quad m=1,\ldots,M\tag{18}$$

Note that Fukuyama and Tan (2022a) reported a linkage between the standard cost function and marginal cost and provided an illustration within a single product setting. The weak complementary slackness conditions state

$$\left\{\begin{array}{ll} \lambda_j^*\left(\mathbf{v}^{*\mathrm{T}}\mathbf{x}_j - \mathbf{u}^{*\mathrm{T}}\mathbf{y}_j - \omega^*\right) = 0, & j = 1, \dots, J \\ x_n^*\left(w_n - v_n^*\right) = 0, & n = 1, \dots N \\ d_m^*\left(p_m - u_m^*\right) = 0, & m = 1, \dots M \\ v_n^*\left(x_n^* - \sum_{j=1}^{J} x_{nj}\lambda_j^*\right) = 0, & n = 1, \dots N \\ u_m^*\left(\sum_{j=1}^{J} y_{mj}\lambda_j^* - y_m\right) = 0, & m = 1, \dots M \\ a^*\left(c - \mathbf{w}^{\mathrm{T}}\mathbf{z}^* - \mathbf{p}^{\mathrm{T}}\mathbf{d}^*\right) = 0 & \end{array}\right\} \tag{19}$$

If strong complementary slackness is assumed, then the last two sets of constraints in (19) yield the following results:

$$\left\{\begin{array}{l} u_m^* > 0 \Leftrightarrow \sum_{j=1}^{J} y_{mj}\lambda_j^* - y_m = 0 \\ u_m^* = 0 \Leftrightarrow \sum_{j=1}^{J} y_{mj}\lambda_j^* - y_m > 0 \end{array}\right\} \tag{20}$$

which indicates that we can identify whether the DMU has market power in the m-th product market. The vector inequality $\mathbf{u} \leq \mathbf{p}$ in (12), derived from the assumption of profit maximization, indicates that each product multiplier does not exceed the respective observed product price. Note that this vector inequality leads to nonnegativity of the aggregate Lerner index as well as the Lerner sub-indexes in (17).

2.3 Empirical approach for implementing positive Lerner index

In order to handle the issue of multiple optima with zero values of product multipliers as well as the product price adaptations, we extend the indirect output possibility correspondence, which is a mapping $\overline{IP} : \mathbb{R}_{++}^N \to \overline{IP}(\mathbf{w}/\bar{c}) \subseteq \mathbb{R}_+^M$, where

$$\overline{IP}(\mathbf{w}/\bar{c}) = \left\{\mathbf{y} \in \mathbb{R}_+^M | \bar{C}(\mathbf{y}, \mathbf{w}/\bar{c}) \leq 1\right\} = \left\{\mathbf{y} \in \mathbb{R}_+^M \left| \begin{array}{c} \sum_{j=1}^{J} \mathbf{x}_j\lambda_j \leq \mathbf{x}, \sum_{j=1}^{J} \mathbf{y}_j\lambda_j \geq \mathbf{x}, \\ (\mathbf{w}/\bar{c})^{\mathrm{T}}\mathbf{x} \leq 1, \\ \sum_{j=1}^{J} \lambda_j = 1, \quad \mathbf{x} \geq \mathbf{0}, \quad \lambda \geq \mathbf{0} \end{array}\right.\right\} \tag{21}$$

into the adapted indirect output possibility correspondence, which is a mapping $IP : \mathbb{R}_{++}^N \times \mathbb{R}_{++}^M \to IP(w/c, p/c) \subseteq \mathbb{R}_+^M$, where

$$\begin{aligned} IP(\mathbf{w}/c, \mathbf{p}/c) &= \left\{\mathbf{y} \in \mathbb{R}_+^M | C(\mathbf{y}, \mathbf{w}/c, \mathbf{p}/c) \leq 1\right\} \\ &= \left\{\mathbf{y} \in \mathbb{R}_+^M \left| \begin{array}{c} \sum_{j=1}^{J} \mathbf{x}_j\lambda_j \leq \mathbf{x}, \sum_{j=1}^{J} \mathbf{y}_j\lambda_j \geq \mathbf{y} - \mathbf{d}, \\ (\mathbf{w}/c)^{\mathrm{T}}\mathbf{x} + (\mathbf{p}/c)^{\mathrm{T}}\mathbf{d} \leq 1, \sum_{j=1}^{J} \lambda_j = 1, \\ \mathbf{d} \geq \mathbf{0}, \mathbf{x} \geq \mathbf{0}, \lambda \geq \mathbf{0} \end{array}\right.\right\} \end{aligned} \tag{22}$$

where $c(\geq \mathbf{wx}+\mathbf{py})$ is a prespecified positive value as stated above. The main difference between (21) and (22) is that the vector $\mathbf{d}$ appears in (22) but does not in (21). From the constraints in (21) and (22), we have $\sum_{j=1}^{J}\mathbf{y}_j\lambda_j \geq \mathbf{y} \geq \mathbf{y}-\mathbf{d}$ and $\left[\mathbf{wx} \leq \bar{c} \Rightarrow \mathbf{wx}+\mathbf{pd} \leq \bar{c}+\mathbf{pd} \leq c\right]$, yielding $\overline{IP}(\mathbf{w}/c) \subseteq IP(\mathbf{w}/c, \mathbf{p}/c)$. Hence, $C(\mathbf{y}, \mathbf{w}/c, \mathbf{p}/c) \leq \overline{C}(\mathbf{y}, \mathbf{w}/c)$. Note that the adapted unit cost function or the adapted cost efficiency is used as the indirect technology or adapted indirect output possibility set.

Now we are ready to establish the following two propositions.

Proposition 1 *The adapted DEA-based indirect technology, $IP(\mathbf{w}/c, \mathbf{p}/c)$ is a convex set, and $C(\mathbf{y}, \mathbf{w}, \mathbf{p})$ is quasi-convex in $\mathbf{y}$.*

Proposition 1 establishes the convexity of the adapted indirect output possibility set, which is equivalent to the quasi-convexity of the adapted cost function.[2] The proof of Proposition 1 is given in "Appendix A1".

Proposition 2 *Let $\partial_o(IP(\mathbf{w}/c, \mathbf{p}/c))$ be the set of all adapted cost-efficient outputs in $IP(\mathbf{w}/c, \mathbf{p}/c)$, which dominate the output vector $\mathbf{y}_o$ of DMU_o. Then, we have: $\mathbf{y} \in \partial_o(IP(\mathbf{w}/c, \mathbf{p}/c))$ if and only if*

$$\exists\lambda_k \geq 0,\ t_k \geq 0,\ b_k \in \{0,1\},\ (k \in CE),\ \mathbf{x} \geq \mathbf{0},\ a \geq 0,\ \mathbf{v} \geq \mathbf{0},\ \mathbf{u} \geq \mathbf{0},\ \mathbf{s}^x \geq \mathbf{0},\ \mathbf{s}^y \geq \mathbf{0},\ \omega \text{ free in sign}$$

such that

$$\begin{aligned}
&\sum_{k\in CE} x_{nk}\lambda_k = x_n - s_n^x, \quad n = 1,\ldots,N, \mathbf{u} \leq \mathbf{p},\\
&y_m = \sum_{k\in CE} y_{mk}\lambda_j = y_{mo} - d_m + s_m^y, m = 1,\ldots,M,\\
&(\mathbf{w}/c)^{\mathrm{T}}\mathbf{x} + (\mathbf{p}/c)^{\mathrm{T}}\mathbf{d} \leq 1,\\
&\mathbf{w}^{\mathrm{T}}\mathbf{x} + \mathbf{p}^{\mathrm{T}}\mathbf{d} = \mathbf{u}^{\mathrm{T}}\mathbf{y}_o + \omega\\
&1 = \sum_{k\in CE} \lambda_k\,,\\
&\quad -\sum_{n\in N} v_n x_{nk} + \sum_{m\in M} u_m y_{mk} + \omega + t_k = 0, k \in CE,\\
&\quad -v_n + \left(\frac{w_n}{c}\right)a \geq 0, n = 1,\ldots,N,\\
&\quad -u_m + \left(\frac{p_m}{c}\right)a \geq 0, m = 1,\ldots,M,\\
&v_n \geq \frac{1}{2Ng_n^x}, n = 1,\ldots,N,\\
&u_m \geq \frac{1}{2Mg_m^y}, m = 1,\ldots,M,\\
&\lambda_k \leq \mathbb{M}b_k,\ \ t_k \leq \mathbb{M}\left(1-b_k\right), k \in CE
\end{aligned}$$

[2] Since the standard indirect output possibility set is non-convex under the regularity conditions of Shephard (1970), it is common to assume that the standard indirect output possibility set is convex.

where $\mathbb{M}$ *is a large positive quantity.*

Proposition 2 can be established by adapting Aparicio et al. (2007) and Zhu et al. (2018). Proposition 2 does not guarantee the satisfaction of profit maximization conditions. In the empirical section, we reported the number of the DMUs which fail to satisfy these conditions.

Now we obtain product multipliers according to the following two-step procedure.

Step 1 Identify the adapted cost-efficient DMUs based upon (14) or (13). The cost efficiency of the assessed DMU_o, denoted $CEff_o$, is estimated as

$$CEff_o = \frac{C(\mathbf{y},\mathbf{w})}{\bar{c}_o}. \tag{23}$$

which represents the ratio of minimum adapted cost to the observed adapted cost. The scalar $\bar{c}_o = \mathbf{w}^{\mathrm{T}}\mathbf{x}_o$ is DMU_o's observed total cost. The cost efficiency of DMU_o refers to the ability to make use of fewer resources to achieve the same product levels. If $CEff_o = 1$, i.e., $C(\mathbf{y}_o, \mathbf{w}) = \bar{c}_o$, then the DMU_o is cost-efficient. If $0 < CEff_o < 1$, i.e.,$C(\mathbf{y}_o, \mathbf{w}) < \bar{c}_o$, then the DMU_o is cost inefficient.

Let CE be an index set of adapted cost-efficient DMUs (observations) with the adapted cost-efficiency score being a value of one and let $\mathbb{M}$ be some positive large number. Using these notations, we introduce Step 2 as follows:

Step 2 Solve the following mixed integer linear programming problem, which minimizes the cost-based slacks (mdCBS) relative to the restricted output possibility set.

$$mdCBS(\mathbf{y}_o, \mathbf{w}/c_o, \mathbf{p}/c_o) = \min\left\{ \frac{1}{2N}\sum_{n=1}^{N}\frac{s_n^x}{g_n^x} + \frac{1}{2M}\sum_{m=1}^{M}\frac{s_m^y}{g_m^y} \;\middle|\; \begin{array}{c} \sum_{k\in CE}\mathbf{x}_k\lambda_k = \mathbf{x} - \mathbf{s}^x, \mathbf{x} \ge \mathbf{0},\ \ \mathbf{s}^x \ge \mathbf{0}, \\ \sum_{k\in CE}\mathbf{y}_k\lambda_k = \mathbf{y}_o - \mathbf{d} + \mathbf{s}^y,\ \ \mathbf{d} \ge \mathbf{0},\ \ \mathbf{s}^y \ge \mathbf{0}, \\ (\mathbf{w}/c_o)^{\mathrm{T}}\mathbf{x} + (\mathbf{p}/c_o)^{\mathrm{T}}\mathbf{d} \le 1, \\ \mathbf{w}^{\mathrm{T}}\mathbf{x} + \mathbf{p}^{\mathrm{T}}\mathbf{d} = \mathbf{u}^{\mathrm{T}}\mathbf{y}_o + \omega \\ \sum_{k\in CE}\lambda_k = 1,\ \boldsymbol{\lambda} \ge \mathbf{0}, \\ -\mathbf{v}^{\mathrm{T}}\mathbf{x}_k + \mathbf{u}^{\mathrm{T}}\mathbf{y}_k + \omega + t_k = 0,\ k \in CE, \\ (\mathbf{w}/c_o)a \ge \mathbf{v}, \mathbf{v} \ge \mathbf{0}, \\ (\mathbf{p}/c_o)a \ge \mathbf{u}, \mathbf{u} \ge \mathbf{0}, \\ v_n \ge \frac{1}{2Ng_n^x},\ \ n = 1,\dots,N \\ u_m \ge \frac{1}{2Mg_m^y},\ \ m = 1,\dots,M \\ \lambda_k \le \mathbb{M}b_k,\ \ t_k \le \mathbb{M}(1 - b_k),\ \ k \in CE \\ b_k : \text{binary variable for } k \in CE \\ \mathbf{u} \le \mathbf{p} \end{array} \right\} \tag{24}$$

where g_n^x and g_m^y represent, respectively, the n-th input and the m-th product directions that are at the discretion of the researcher or DMUs, and $\mathbb{M}$ is a large number. In Eq. (24), the input target and the product target vectors at the adapted cost frontier are obtained as $\sum_{j \in E} \mathbf{x}_j \lambda_j^*$ and $\sum_{j \in E} \mathbf{y}_j \lambda_j^*$, respectively. In their efficiency measurement, Fukuyama and Weber (2009) employed the objective function such as that of (21), that is,

$$\frac{1}{2N} \sum_{n=1}^{N} \frac{s_n^x}{g_n^x} + \frac{1}{2M} \sum_{m=1}^{M} \frac{s_m^y}{g_m^y}$$

A pair of (20) and (24) form an adapted cost function extension of the two-step procedures of Aparicio et al. (2007) and Zhu et al. (2018). Since $C(\mathbf{y}, \mathbf{w}/c, \mathbf{p}/c) \leq 1 \Leftrightarrow C(\mathbf{y}, \mathbf{w}, \mathbf{p}) \leq c$ from the definition of the adapted cost function, we have

$$\frac{\partial C(\mathbf{y}, \mathbf{w}/c_o, \mathbf{p}/c_o)}{\partial y_m} = u_m^*\left(\mathbf{y}, \mathbf{w}/c_o, \mathbf{p}/c_o\right) \ \ and \ \ \frac{\partial C(\mathbf{y}_o, \mathbf{w}, \mathbf{p})}{\partial y_m} \times \frac{1}{c_o} = u_m^*\left(\mathbf{y}_o, \mathbf{w}, \mathbf{p}\right) \times \frac{1}{c_o}, \tag{25}$$

hence

$$u_m^* = u_m^*\left(\mathbf{y}_o, \mathbf{w}, \mathbf{p}\right) = u_m^{**}\left(\mathbf{y}, \mathbf{w}/c_o, \mathbf{p}/c_o\right) \times c_o \tag{26}$$

where $u_m^{**}\left(\mathbf{y}, \mathbf{w}/c_o, \mathbf{p}/c_o\right)$ are the estimates from (24).

Our two-step procedure for estimating the adapted cost function constructs the adapted cost-efficient frontier implicitly (or indirectly) and then provides product multipliers $\mathbf{u}^*$, hence the Lerner sub-indexes and the multi-product index in (17).

Remark 1 Since (24) possesses all constraints associated to the multiplier and envelopment forms of the adjusted cost function $C(\mathbf{y}, \mathbf{w}, \mathbf{p})$, the set of all constraints in (24) ensures that the projection point of the inputs and products is on the adapted cost-efficient frontier $\partial_o(IP(\mathbf{w}/c, \mathbf{p}/c))$. Consequently, the multipliers associated with products are all positive due to the optimization problem (24).

Remark 2 For the fixed levels of $\mathbf{w}$, as well as $\mathbf{p}$, the adapted cost-efficient frontier is constructed by spanning the adapted cost-efficient DMUs' input and product vectors, while keeping $C(\mathbf{y}, \mathbf{w}, \mathbf{p}) = c$.

3 Data

The real estate sector is connected to many other industries such as steel, glass, wood, and home appliances. It is a leading industry that drives economic growth. For every one-unit increase in the value-added of the real estate sector in China, the total driving impact on each industry is 1.416 units (Wang and Liu 2004). According to the data provided by the National Bureau of Statistics in China, the real estate industry's added value continues to increase as a proportion of GDP, from 4.61 in 2008 to 6.6% in 2018. However, the real estate companies are facing increasing inventory pressure and financial pressure. The real estate industry's development model that relies on real estate development to obtain high profits is unsustainable, and it is in urgent need of transformation and upgrading to achieve sustainable development. The housing issue is not only an economic issue, but also an important livelihood issue, a common concern of the government, enterprises, and the people. As a leading industry, how the real estate industry can reasonably use natural resources and the ecological environment while achieving its own healthy and sustainable development and still provide the basis for the development of other industries is the focus of academic research.

The continuous improvement of the market system and the shift of land policy from free allocation to bidding, auction and listing have made the institutional barriers to entry continue to weaken, and more and more companies have begun to enter the real estate market. A growing number of listed companies have diversified into the real estate field. In 1998, the real estate marketization reforms and a period of rapid development had come for the real estate enterprises. In 1986, the total number of real estate companies nationwide was only 1991. In 1998, it increased to 24,378, and by 2018 it had increased to 97,938. It is worth mentioning that the real estate market was dominated by state-owned enterprises and collective enterprises during the early stage of the market development. The market share occupied by state-owned enterprises was 32.6% in 1998. Because of the increase in the participation of private enterprises derived from the deepening of the real estate reform, the market share of state-owned enterprises experienced a continuous decline. In 2018, the proportion of state-owned enterprises was only 0.8%.

We collected the data of 31 provinces/cities in China from the National Bureau of Statistics in China over the period 2002–2018. The 31 provinces/cities are further divided into three different geographical areas (Eastern area, Central area and Western area). According to the classification provided by the National Bureau of Statistics of China, the Eastern area includes Beijing, Tianjin, Hebei, Shanghai, Jiangsu, Zhejiang, Fujian, Shandong, Guangdong, Hainan, Liaoning, and Heilongjiang; the Central area includes Shanxi, Anhui, Jiangxi, Henan, Hubei, and Hunan, and the Western area includes Inner Mongolia, Guangxi, Chongqing, Sichuan, Guizhou, Yunnan, Tibet, Shan Xi, Gansu, Ningxia, and Xinjiang.

In order to estimate different types of Lerner indices, we need inputs, input prices, products, and product prices according to the models described in the methodology section. This study considers two inputs, which are land areas purchased by the real estate development enterprises, and real estate enterprises' completed housing areas. We consider the land areas purchased by the real estate development enterprises as one input due to the fact that they can be regarded as the raw materials, based on which the real estate enterprises will build the houses, however, the completed housing area is different from the land areas purchased due to the fact that the former will be much smaller than the latter and the difference is attributed to the extent to which the enterprises can fully use the raw materials. Therefore, these two different inputs reflect different aspects of company operation and management. The two related input prices are expenses for purchasing one square meter of land area, and real estate development enterprises' completed housing costs. There are two products included in the current

Table 1 Descriptive statistics of the variables used in the analysis

Variables	Observations	Mean	Standard deviation	Minimum	Maximum
Inputs					
Input 1	527	1109.722	872.9	1.18	4207.74
Input 2	527	2485.783	2045.68	9.19	10,512.57
Input prices					
Input price 1	527	0.46	1.04	0.012	12.77
Input price 2	527	0.21	0.094	0.036	0.62
Products					
Product 1	527	3145.579	2979.16	9.56	15,958.81
Product 2	527	2795.44	2659.467	9.56	13,522.51
Product prices					
Product price 1	527	0.5	0.41	0.11	3.41
Product price 2	527	0.48	0.42	0.09	3.74

Input 1 is land areas purchased by the real estate development enterprises; input 2 is real estate enterprises' completed housing area, input price 1 is expenses for purchasing one square meter of land area by the real estate development enterprises; input price 2 is real estate development enterprises' completed housing costs; product 1 is sales area for commodity housing; product 2 is residential commodity houses' sales area; product price 1 is per square meter revenue of commodity housing; product price 2 is per square meter sales revenue of residential commodity houses. Both input 1 and input 2 are in the unit of 10 thousand square meters. Both input prices and output prices are in ratios, while both product 1 and product 2 are in the unit of 10 thousand square meters.

study: sales area for commodity housing and residential commodity houses' sales area; the related two product prices are per square meter revenue of commodity housing as well as per square meter sales revenue of residential commodity houses. Different types of outputs (houses) will be generated by the real estate enterprises including commodity housing, residential commodity housing, villa and high-end apartment, commodity house for offices, houses for business use, and other commodity houses. Based on the statistics provided by the National Bureau of Statistics in China, the commodity houses and residential houses are the most two important types of outputs (houses) generated by the real estate enterprises, as reflected by the volumes of sales areas and the volumes of sales. Since the adapted cost c is implied by adapting the standard cost function with (12), we calculate it as the sum of the observed cost and observed revenue, i.e., $c_o = \mathbf{w}^{\mathrm{T}}\mathbf{x}_o + \mathbf{p}^{\mathrm{T}}\mathbf{y}_o$.

Table 1 shows the descriptive statistics of the variables used in the analysis. The table shows that there is a small difference in the level of product prices. In comparison, there are big differences in terms of inputs and products, while per square meter revenue of houses sold has quite small variations.

Table 2 Cost efficient provinces/cities and number of non-profit-maximizing DMUs

year	IDs of adapted cost-efficient regions	IDs of standard cost-efficient regions	No. of Non-PM regions in product market 1 (IDs)	No. of Non-PM regions in product market 2 (IDs)
2002	#10, #19, #23	#10, #12, #19, #22, #23	0	0
2003	#8, #19	#8, #12, #19	0	0
2004	#1, #10, #19, #23	#1, #10, #12, #19, #23	0	0
2005	#6, #8, #12, #19, #23	#6, #8, #12, #19, #22, #23	0	0
2006	#4,#5,#8,#12,#15,#19,#22	#4, #5, #8, #12, #15, #19, #22	0	0
2007	#8, #19, #22, #23	#8, #19, #22, #23	0	0
2008	#5, #8, #16, #19,	#5, #8, #16, #19, #22, #23	0	0
2009	#5, #8, #19, #22, #23	#5, #8, #19, #22, #23	0	0
2010	#5, #8, #16, #19	#5, #8, #16, #19, #22	0	0
2011	#5, #8, #16, #19	#5, #8, #16, #19	0	0
2012	#5, #8, #12, #19, #26	#5,#8,#12,#16,#19,#22,#26	0	0
2013	#8, #11, #12, #16, #19, #22	#8, #11, #12, #16, #19, #22	0	0
2014	#5, #8, #12, #14, #19	#5, #8, #12, #14, #19, #22	0	0
2015	#12, #14, #19	#12, #14, #19	0	0
2016	#12, #14, #17, #19	#12, #14, #17, #19	0	0
2017	#7, #8, #12, #14, #15, #19	#7, #8, #12, #14, #15, #19	0	0
2018	#7, #14, #15, #17, #19	#7, #12, #14, #15, #17, #19	0	0

Non-PM Non-profit maximizing

Table 3 Multiplier estimates of three DMUs in 2009

Province/city	Year	p1	u1	p2	u2	AL
Xinjiang	2009	0.26040	0.07544	0.24661	0.00019	0.8383
Hunan	2009	0.26428	0.10749	0.25168	0.00012	0.7788
Beijing	2009	1.37990	0.29978	1.32243	0.00013	0.8695

p1 and p2 represent the price levels for product 1 (commodity housing) and product 2 (residential commodity housing), u1 and u2 are marginal costs (multipliers) for product 1 and product 2, and AL represents the aggregate Lerner index

4 Results

Table 2 gives the cost-efficient DMUs according to the standard and adapted restricted cost functions. Columns 2 and 3 represent two types of cost-efficient cities/provinces: one is based on the adapted cost function and the other is on the standard cost function. The table indicates that all the adapted cost-efficient cities/provinces are standard cost-efficient. Columns 4 and 5 confirms that all cities are maximizing their profits as our proposed framework suggests. In terms of the commodity housing market, different cities across different years are not maximizing their profits. More specifically, in 2005, there are two cities: Shanghai and Beijing. Jiangsu and Shandong are not maximizing their profits in 2007 and 2008, respectively. In 2009, Shanghai and Sichuan are not maximizing their profits. There are two cities in 2010 that are not maximizing their profits, including Sichuan and Shan Xi. Shanxi, Sichuan, Hunan, and Henan are not maximizing their profits in 2011. Tibet is not maximizing its profits in 2012, 2013, and 2016, and, finally, Shanxi is not maximizing its profit in 2014. In terms of the residential housing market, we only find the cities that are not maximizing their profits in 2002, 2003, 2004, 2015, and 2018. In 2002, they are Qinghai, Hunan, Jiangxi, Shanxi, Tibet, and Inner Mongolia. The first four cities are not profit-maximizing in 2003 as well. Shan Xi, Tibet, and Henan are not profit-maximizing in 2003, 2015, and 2018, respectively.

Before presenting the overall aggregate Lerner index and the Lerner sub-indices for the sample and the ones based on geographical classifications, we provide the results of the price levels and marginal costs of three different provinces/cities to further show and confirm our contributions to the literature by providing positive marginal costs (multipliers) in a multi-product context. Table 3 provides the empirical result for three provinces/cities including Xinjiang, Jiangxi, and Beijing. Xinjiang is in the Western area, Jiangxi is in the central area, and Beijing is in the Eastern area. To illustrate, consider Xinjiang in 2009. The product multipliers satisfy the constraints (12), i.e., $p1=0.2604>u1=0.07544$, and $p2=0.24661>u2=0.00019$. The Lerner sub-indexes of product markets 1 and 2 are 0.7103 and 0.9992, respectively. For the same year, in terms of Jiangxi, $p1=0.26428>u1=0.10749$, and $p2=0.25168>u2=0.00012$. The Lerner sub-indices of product markets 1 and 2 are 0.5933 and 0.9995, respectively. Finally, for Beijing in 2009, $p1=1.37990>u1=0.29978$, and $p2=1.32243>u2=0.00013$. The Lerner sub-indices of product markets 1 and 2 are 0.7827 and 0.9999, respectively.

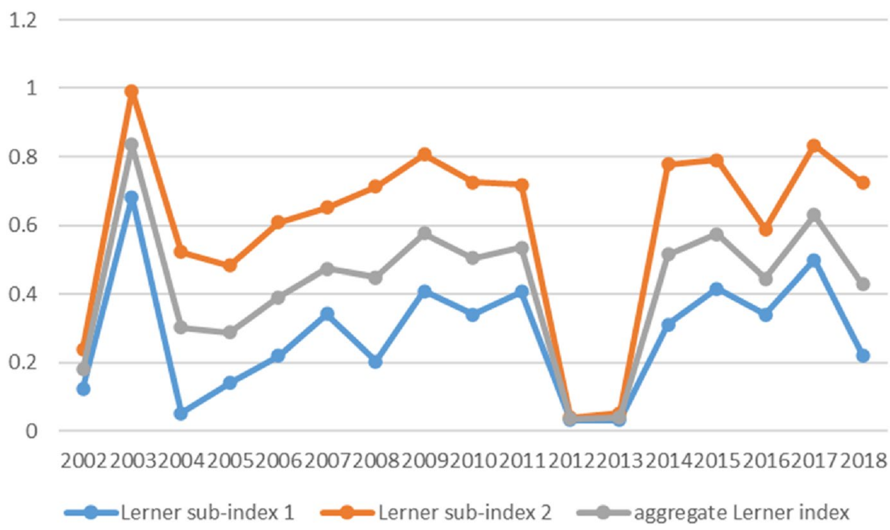

Fig. 1 Lerner sub-index 1, Lerner sub-index 2 and revenue-share weighted aggregate Lerner index

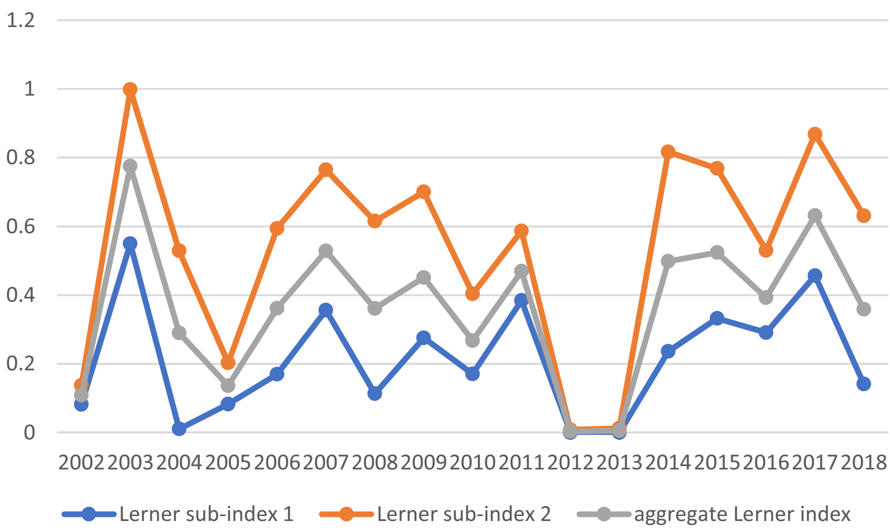

Fig. 2 Lerner sub-index 1, Lerner sub-index 2 and revenue-share weighted aggregate Lerner index in the Eastern area

To estimate the multipliers of products in (21), we set the direction vector as the observations. We first look at the Lerner indices over the examined period of 2002–2018, the results of which are reported in Fig. 1. The figure shows that, in a general trend, the Lerner sub-index 2 is the highest, followed by the aggregate Lerner index and the Lerner sub-index 1. This result means that the Chinese real estate enterprises compete much more strongly in commercial commodity housing

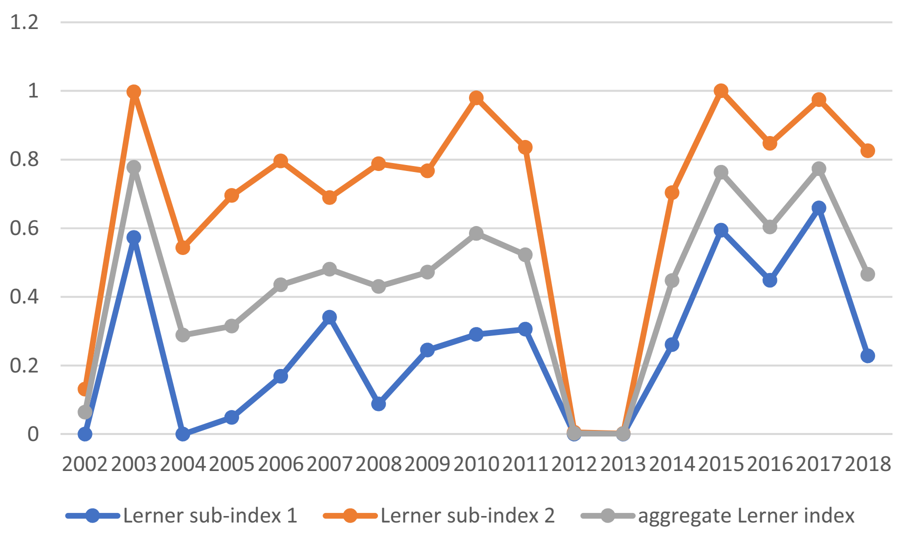

Fig. 3 Lerner sub-index 1, Lerner sub-index 2 and revenue-share weighted aggregate Lerner index in the Central area

than that in residential housing. The higher level of competition in the commercial commodity housing can be explained by the fact that 1) there are many real estate enterprises focusing on commercial commodity housing and most of the enterprises have been listed on stock exchange. The stock market listing of these enterprises not only help them optimize the resource and further improve the corporate governance, the resulted more transparency to the public monitoring and their commitment to key stakeholders will induce more efforts to improve their operations and management. Therefore, they intend to compete more strongly to obtain higher profits not only to provide returns to the existing shareholders, but also attract more potential investors; 2) the real estate enterprises providing commercial commodity housing can be divided into two groups, one belongs to domestic enterprises and the other mainly includes the enterprises with investment outside of mainland China, such as Taiwan, Macau and Hong Kong. The enterprises do not only compete with other ones within their own category, but also compete with the ones from the other category. This further promote the level of competition in this real estate segment. Based on the statistics from Forward-The Economist, the concentration in the commercial commodity housing is quite low. It shows that the 10-enteprise concentration ratio is 10.35% in 2020, the 30-enterprises and 50-enterprise concentration ratios in the same year are 16.56% and 19.31%, respectively. These ratios are even higher than the ones in 2019.

Not only do we consider the market power of the sample, but also, we examine it for three different geographical areas (Eastern area, Central area, and Western area) in China, and Figs. 2, 3, and 4 report the results. We can see that all these three figures show a similar trend in general with the Lerner sub-index 2 being the highest, followed by the aggregate Lerner index and the Lerner sub-index 1. Starting from Fig. 2, we observe that the Lerner sub-index 2 experiences higher volatility

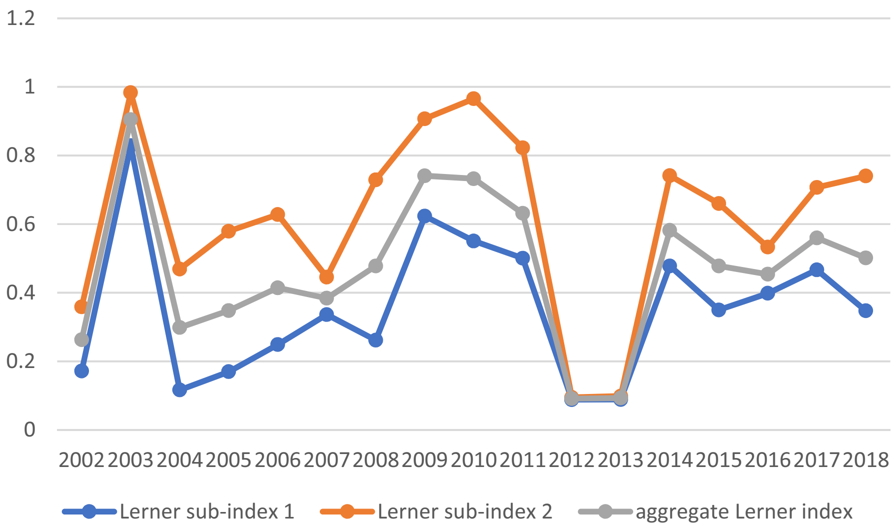

Fig. 4 Lerner sub-index 1, Lerner sub-index 2 and revenue-share weighted aggregate Lerner index in the Western area

compared to the Lerner sub-index 1. This indicates that the level of market power in residential commodity houses varies to a larger extent across the examined period, while there is relatively more stable market power in the commercial commodity housing market. This is also the case for Fig. 3 and Fig. 4. The highest volatility in the residential commodity houses across all three different geographical areas in China can be explained by the fact that as the economic development and growth took place continuously over the examined period, the peoples' increase in disposable income increases their demand on changing the living conditions. This pushed up the house price and further induce the real estate enterprises to compete strongly in this market segment. Because of high profitability of this market segment, it attracted more enterprise to enter into this market segment or the existing real estate enterprises to increase the share of their residential commodity housing in their overall portfolio. Meanwhile, because of higher competition, some of the enterprises were unfortunately dropped out from the market. Both of these two different scenarios contribute to the volatility of market power over this period.

Further to the previous analysis we provided, we look at these three different Lerner indices across different geographical areas in a separate manner. Figures 5, 6, and 7 show the findings. Figure 5 shows that there is no clear comparison in terms of the level of market power among the Eastern, Central, and Western areas when using the sales area for commodity housing as the product; however, we notice that in general, the western area has the highest level of market power up to 2014. Not only do we observe the highest level of market power in the western area in general, but also we find that the market power in the western area experiences the highest volatility. This highest volatility can be attributed to the policy of "developing the western area" in China. As one of the key industries in the economic sector, the real estate industry actively participated and contributed to the development of this area

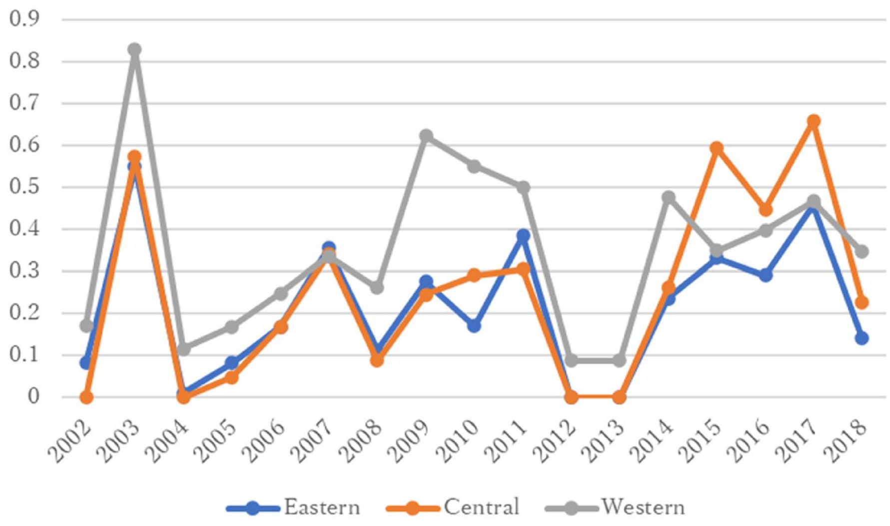

Fig. 5 Lerner sub-index 1 among different areas of China

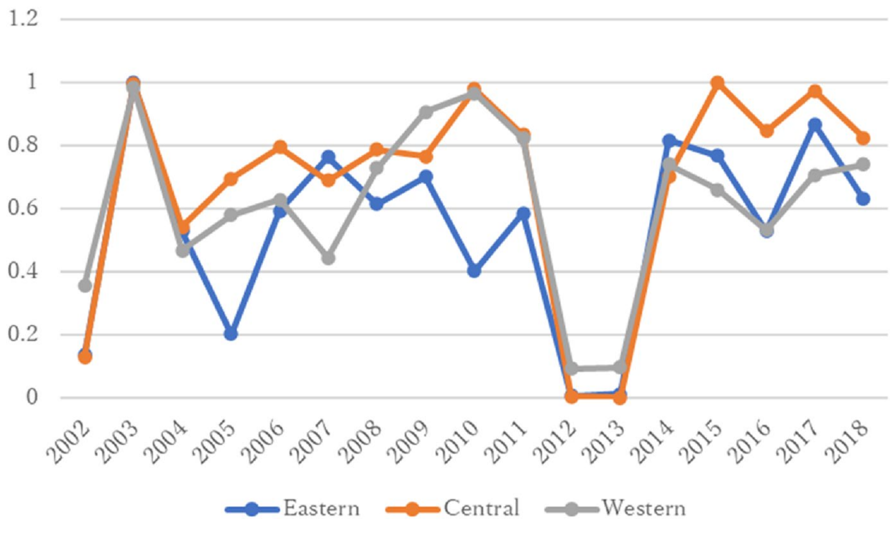

Fig. 6 Lerner sub-index 2 among different areas of China

by gradually increasing their investment in this area. This results in an increase in the level of competition. Because of the co-existence of opportunities and threats, the former is derived from preferential policies granted to the real estate enterprises, which facilitate their increase in profitability; while the latter is mainly attributed to the higher level of competition, which drives out the ones with lower level of competitiveness. Because of entry and exist of the real estate enterprise in this area, the

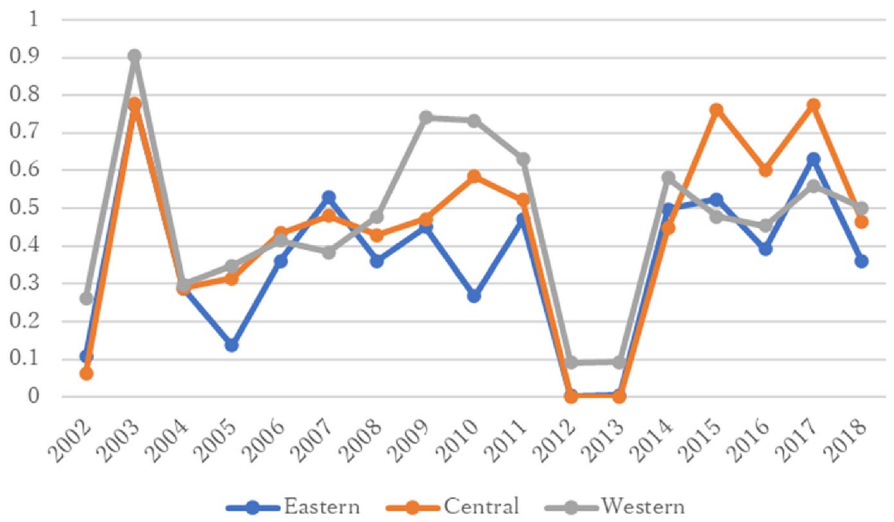

Fig. 7 Revenue-share weighted aggregate Lerner index among different areas of China

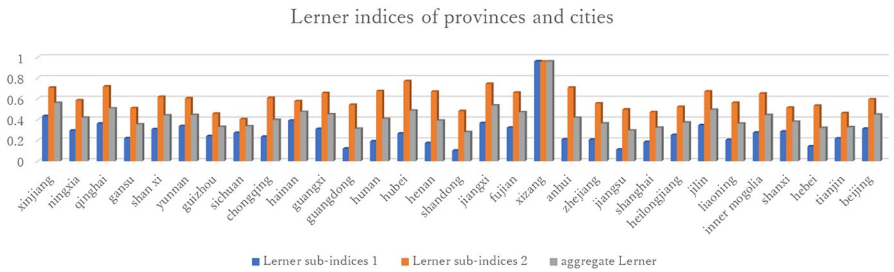

Fig. 8 Lerner indices of all the cities in China

market power experiences the highest level of volatility. In terms of the residential commodity housing, we can not see a clear comparison among the levels of market power in these three areas, while different from Fig. 5, we find that the central area experiences the highest volatility in the residential commodity housing. Considering the results from both Fig. 5 and Fig. 6, we deduce that there is a relatively quite stable market power in the commercial commodity housing in the central area (commodity housing includes both commercial commodity housing and residential commodity housing). Finally, Fig. 7 shows that the western area, in general, has the highest aggregate Lerner indices over the period and has the highest volatility.

Finally, we look at the level of market power for each of the 31 cities in the sample over the period 2002–2018, the findings of which are reported in Fig. 8. In terms of Lerner sub-index 1, we find that Tibet has the highest market power, while the market power in Shandong is the lowest. When looking at the residential housing market, the Lerner sub-index 2 shows that Tibet has the highest market power, while

Sichuan has the lowest market power. Finally, regarding the aggregate Lerner index, the findings show that Tibet has the highest market power and Shandong has the lowest market power.

5 Conclusion

The current paper provides new knowledge and significantly extends the study of Fukuyama and Tan (2022a) by presenting a multi-product Lerner index, which is a revenue-share-weighted index consisting of Lerner sub-indexes within a minimum-distance DEA framework. In order to estimate the DEA-based multi-product index, two issues were considered: one is the implementation of the profit maximization conditions, in which the multi-product Lerner index, as well as the sub-indices, take a value between zero and one, and the other is the issue of having several zero multiplier values. To deal with this circumstance, exploiting Shephard's (1970) indirect output possibility set (see also Färe et al. 1994) and introducing the adapted cost function, we have developed a minimum-distance cost function estimation procedure, in which the adapted cost function is estimated indirectly. In addition, we further build on the work of Fukuyama and Tan (2022a) by addressing the issue of negative values of the Lerner index by proposing a method to provide non-negative marginal costs for all the decision-making units. We applied our innovative approach to the Chinese real estate industry. The result shows that the Chinese real estate industry has higher market power in the residential commodity housing market than that in the commodity housing market. This is also the case for different geographical areas in China. Overall, for both of these two different markets, the level of market power experiences a level of volatility. We finally identify the market power level on a province/city level.

Our results provide important policy implications: (1) in the commercial commodity housing market, the real estate companies should find ways to increase their market power. This would improve the financial stability because higher competition results in higher risk levels. The real estate companies take out a large amount of bank loans, the higher risk increases the probability of loan default and leads to financial instability; (2) across the three geographical areas in China, efforts should be made not only to increase the level of market power in a more consistent and sustainable manner; (3) from a city-level perspective, the small-sized real estate enterprises should further increase their market power to avoid their unfortunate drop-out derived from higher levels of competition; (4) regarding increasing the level of market power, the policy aims to potentially improve the financial stability in the banking industry by reducing the risk-taking behavior of real estate firms. However, a higher level of market power does not necessarily lead to a positive outcome because it does not provide incentives to company managers for performance improvement, the government should consider both these two effects on the real estate sector, and the well-coordinated policies would balance the development of these two industries.

Future research can further develop the existing model by dividing the production process into stages and considering the undesirable product in the production process. Finally, data on different components of non-residential commodity houses can be collected to check the robustness of our results.

Appendix A1: Proof of Proposition 1

Proof of Proposition 1 Let $\left(\mathbf{y}^1, \mathbf{d}^1, \mathbf{x}^1, \boldsymbol{\lambda}^1\right)$ and $\left(\mathbf{y}^2, \mathbf{d}^2, \mathbf{x}^2, \boldsymbol{\lambda}^2\right)$ be two sets of elements in $IP(\mathbf{w}/c, \mathbf{p}/c)$. Define $s_n^{x,1} = -\sum_{j=1}^{J} x_{nj}\lambda_j^1 + x_n^1$ and $s_n^{x,2} = -\sum_{j=1}^{J} x_{nj}\lambda_j^2 + x_n^2$ for all n. For $\mu \in [0,1]$, we have: $0 \le (1-\mu)s_n^{x,1} + \mu s_n^{x,2} = -\sum_{j=1}^{J} x_{nj}\left((1-\mu)\lambda_j^1 + \mu\lambda_j^2\right) + \left((1-\mu)x_n^1 + \mu x_n^2\right)$, yielding $\sum_{j=1}^{J} x_{nj}\left((1-\mu)\lambda_j^1 + \mu\lambda_j^2\right) \le \left((1-\mu)x_n^1 + \mu x_n^2\right)$. From the constraint $(\mathbf{w}/c)\cdot\mathbf{x} + (\mathbf{p}/c)\cdot\mathbf{d} \le 1$ in Eq. (22), we have: $(1-\mu)(\mathbf{w}/c)\cdot\mathbf{x}^1 + \mu(\mathbf{w}/c)\cdot\mathbf{x}^2 + (1-\mu)(\mathbf{p}/c)\cdot\mathbf{d}^1 + \mu(\mathbf{p}/c)\cdot\mathbf{d}^2 \le 1-\mu+\mu = 1$. Since $\mathbf{y}^1, \mathbf{y}^2 \in IP(\mathbf{w}/c, \mathbf{p}/c)$, we can establish $(1-\mu)y_m^1, +\mu y_m^2 \le \sum_{j=1}^{J} y_{mk}\left((1-\mu)\lambda_j^1 + \mu\lambda_j^2\right)$ for all m. Moreover, $\sum_{j=1}^{J}\left((1-\mu)\lambda_j^1 + \mu\lambda_j^2\right) = 1-\mu+\mu = 1$. Consequently, we have: $(1-\mu)\mathbf{y}^1 + \mu\mathbf{y}^2 \in IP(\mathbf{w}/c, \mathbf{p}/c)$. Therefore, $C(\mathbf{y}, \mathbf{w}/c, \mathbf{p}/c)$ is quasi-convex in $(\mathbf{w}/c, \mathbf{p}/c)$. The converse can easily be proved because quasi-convexity of $C(\mathbf{y}, \mathbf{w}/c, \mathbf{p}/c) \Leftrightarrow$ convexity of $IP(\mathbf{w}/c, \mathbf{p}/c)$□.

Data availability statements The data that support the findings of this study are available from the corresponding author upon request.

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