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Annals of Operations Research
https://doi.org/10.1007/s10479-026-07084-2

ORIGINAL RESEARCH

Portfolio optimization with minimum assets and dividend yield constraints

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Received: 9 December 2024 / Accepted: 26 January 2026

Abstract
This paper develops and evaluates three portfolio optimization strategies for Real Estate Investment Trusts (FIIs), Brazilian Depositary Receipts (BDRs), and Brazilian stocks traded on the Brazilian stock exchange (B3). The aim is to guide Brazilian investors by exploring investment opportunities across these distinct financial products. FIIs are analyzed for their tax-free dividends and mandatory profit distributions, BDRs for their international exposure and ease of diversification, and stocks for their potential in capital raising and profit sharing. The research employs three optimization techniques based on Markowitz's theory but incorporates innovative constraints such as entropy, mutual information, high-order moments, dividend yield, and ticker balancing. The study also integrates the TOPSIS methodology with entropy-weighted financial variables to enhance portfolio selection. The findings indicate that the newly developed Model 3, which includes dividend yield and ticker balancing, outperforms traditional Markowitz and modified strategies (Model 1 and Model 2) in terms of risk and return. Despite the superior performance of the Naïve strategy in certain contexts, Model 3 shows significant advantages, particularly in stock portfolios with higher returns and lower risks. In the context of FIIs and BDRs, the results suggest that while Model 3 is competitive, BDRs present additional risks due to currency fluctuations. The study concludes that innovative constraints in portfolio optimization can provide substantial benefits to investors, offering a novel contribution to modern portfolio theory. Future research should explore different asset allocation constraints and the combined impact of multiple financial variables on portfolio performance.

Keywords Portfolio optimization · Dividend yield · Minimum assets · Markowitz · Quantitative finance · Entropy

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Published online: 10 February 2026

JEL Classification G11 · G5 · C61 · C63

1 Introduction

This paper develops three types of portfolio optimization strategies for three different products traded on the Brazilian stock exchange (B3): Real Estate Investment Trusts (FIIs in Portuguese), Brazilian Depositary Receipts (BDRs), and Brazilian stocks. The portfolio optimization involves three separate strategies that will be applied to each product type individually, without mixing them together, providing guidance to Brazilian investors seeking investment opportunities through the many options available to them. Traditional portfolio optimization models, such as Markowitz's Modern Portfolio Theory (MPT), often fail to account for the complexities of real-world investment scenarios, where investors must balance multiple conflicting criteria.

FIIs are financial products designed to invest resources in real estate ventures such as warehouses, commercial buildings, shopping centers, and other real estate investments. These products distribute dividends to their shareholders and are traded as investment funds, offering the benefit of tax-free dividends for investors (Consentino et al., 2011). Additionally, as noted by Consentino et al., one of the requirements for FIIs is the semiannual distribution of 95% of the net profit. In this analysis, all types of FIIs offered by B3 will be considered in the creation of the optimized portfolios.

The second type of financial product that will be utilized in this study to construct an optimized portfolio is BDRs (Brazilian Depositary Receipts), which are contracts that represent shares traded on foreign exchanges. According to the Brazilian stock exchange (B3), there are various types of BDRs available and traded in the stock market. However, for the purpose of this work, the focus will be on two specific types: sponsored and non-sponsored BDRs. Sponsored BDRs involve the active participation of a foreign company in the issuance process, where the company cooperates with a Brazilian depositary institution to create the product. On the other hand, non-sponsored BDRs are issued and maintained by Brazilian depositary institutions without direct involvement from the foreign company. The main difference between these types is the level of participation from the foreign entity. Both sponsored and non-sponsored BDRs provide investors with the opportunity to gain exposure to international companies and diversify their portfolios without the complexities of investing directly in foreign securities. By including these two types of BDRs in the analysis of portfolio optimization strategies, this study aims to clarify investment outcomes and explore the potential benefits of these instruments for Brazilian investors.

The final financial product to be analyzed is stocks, which, according to B3, provide companies with a means to raise capital for their future investments. By selling a portion of their ownership to shareholders, companies enable investors to participate in the distribution of profits. For this study, both common and preferred stocks were considered in constructing the portfolios. Common stocks grant shareholders voting rights, allowing them to influence decision-making in shareholder assemblies. Preferred stocks, on the other hand, offer advantages such as higher dividend payouts and priority in capital reimbursement. However, the variations in characteristics among these three financial assets (FIIs, BDRs, and stocks) do not significantly impact the portfolio construction process. Despite their distinct features, such as real estate investments for FIIs, international exposure for BDRs, and voting rights or priority in capital reimbursement for stocks, they all share the common characteristics of price variation and the ability to be bought and sold in the stock market. These commonalities make them tradable assets, providing investors with the opportunity to capitalize on price fluctuations and engage in buying and selling activities. Therefore, these differences do not significantly affect the objectives and outcomes of this study.

To achieve the objective of optimizing investment returns while effectively managing risks, this study employs three distinct portfolio optimization techniques applied to the three financial products mentioned previously. These techniques are all based on the foundation of Markowitz (1952)'s theory, with the integration of new constraints and variables for optimization. In the first approach (Model 1), instead of relying on covariance and variance as measures of risk, entropy and mutual information are utilized. The advantage of using entropy and mutual information is their independence from the assumption of normality for the underlying random variable returns, unlike variance and covariance (Novais et al., 2022). The second portfolio approach (Model 2) incorporates high-order moments such as kurtosis and skewness, along with entropy of returns, to account for additional risk variables not directly considered in the mean–variance portfolio (Gonçalves et al., 2022).

The focal point of this study is the final portfolio approach (Model 3), which serves as the main objective of this research. This approach incorporates two critical constraints: dividend yield and ticker balancing and aims to compare its performance with the outcomes of the other two portfolio optimization strategies. Dividend yield represents the level of dividend income generated by investments and provides investors with a reliable source of earnings. It is a measure of the annual dividends received from an investment relative to its price, and it plays a significant role in determining the intrinsic value of a stock (Foerster & Sapp, 2005). Additionally, ticker balancing addresses concentration risk by requiring a minimum number of different financial products (tickers) in the portfolio, promoting diversification and reducing reliance on any single stock or asset.

From an Operations Research (OR) and decision-analytics perspective, this study contributes a unified, practitioner-oriented framework that links multi-criteria asset screening with implementable portfolio optimization. Specifically, we first rank and preselect candidate assets using entropy-weighted TOPSIS, and then construct portfolios through a set of benchmark optimization models, culminating in Model 3, which augments the classical mean–variance formulation with two practical constraints: a minimum dividend yield requirement and a minimum number of assets to control concentration risk. This integrated workflow (i.e. data, ranked universe, feasible portfolios and out of sample evaluation) is designed to support investor decision-making with transparent trade-offs between capital gains, income generation, and diversification.This study addresses critical questions in the field of portfolio optimization: Can the inclusion of dividend yield in the diversification problem provide more reliable returns compared to relying solely on the mean return derived from price variations? Additionally, can setting a minimum number of tickers mitigate the issue of portfolio concentration as identified by Markowitz? By employing these strategies across various financial products, the outcomes will enhance diversification and guide Brazilian investors in their capital market investments, balancing risk and returns. The significance of this research lies in its potential to offer a more robust framework for portfolio management by incorporating dividend yield and enforcing ticker balancing.

2 Literature review

Portfolio optimization is a well-researched area in finance, and existing literature can be broadly categorized into several types. One prominent category involves quantitative approaches that utilize statistical and mathematical models to balance risk and return in stock portfolios. Examples include multi-task learning models that predict risk and return simultaneously. Another category focuses on stock rank prediction methods, combining industry attributes and price data to rank stocks for portfolio selection. A third category is machine learning-based optimization, which uses algorithms to predict stock prices and optimize portfolios based on these predictions. This approach is often combined with traditional mean–variance optimization to enhance prediction accuracy and portfolio performance. Scoring and screening models represent another stream of research where algorithms select stocks based on predefined criteria, mimicking fund managers' selection processes. Finally, online portfolio strategies leverage ensemble learning algorithms to adapt to market trends and optimize portfolios in real-time.

Portfolio optimization has evolved to address practical investment constraints, such as cardinality restrictions and dividend yield considerations. Cardinality constraints, which limit the number of assets in a portfolio, are widely used to manage selection complexity and transaction costs. Cesarone et al. (2013) introduced the Limited Asset Markowitz (LAM) model, which integrates cardinality constraints to maintain an optimal balance between risk and return while addressing computational challenges associated with enforcing asset limits. More broadly, multiobjective evolutionary algorithms (MOEAs) have been used to approximate efficient frontiers in cardinality-constrained portfolio optimization problems (CCPOP), as exact methods often struggle with the NP-hard nature of these problems (Liagkouras & Metaxiotis, 2018). Hybrid encoding techniques and constraint-handling mechanisms in MOEAs have proven effective in solving large-scale CCPOP instances (Liagkouras & Metaxiotis, 2018). Similarly, dividend yield constraints have played a significant role in portfolio selection, particularly in multiobjective optimization models that align with investment strategies focused on income stability (Xidonas et al., 2018).

Kolm et al. (2014) discuss the importance of dividend yield constraints in portfolio construction, emphasizing their role in balancing risk and return while ensuring a stable income stream. Prior research has demonstrated that incorporating dividend yield constraints can enhance portfolio stability, particularly in long-term investment strategies (Xidonas et al., 2018). While cardinality and dividend yield constraints have been examined individually in the literature, their joint application alongside ticker balancing has not been extensively explored. Model 3 builds upon this foundation by integrating both cardinality and dividend yield constraints within a unified framework, ensuring a structured approach to diversification, risk management, and income generation. This novel combination enhances existing portfolio optimization techniques by providing a more comprehensive method for balancing return objectives with real-world investment constraints.

This paper primarily falls into the category of quantitative portfolio optimization, extending traditional mean–variance optimization by incorporating innovative constraints and variables to better manage risk and enhance returns. Specifically, it develops three portfolio optimization strategies for Real Estate Investment Trusts (FIIs), Brazilian Depositary Receipts (BDRs), and Brazilian stocks traded on the Brazilian stock exchange (B3). The research employs Markowitz's foundational theory, integrating new constraints such as entropy, mutual information, high-order moments, dividend yield, and ticker balancing. By leveraging these advanced techniques, the study aims to provide Brazilian investors with optimized investment opportunities across diverse financial products, contributing to the broader field of quantitative portfolio optimization.

Traditional portfolio optimization relies on variance and covariance to assess risk, as established by Markowitz (1952). However, alternative risk measures, such as entropy, have been proposed to better capture uncertainty in financial returns. Philippatos and Wilson (1974) introduced entropy as a measure of market risk, demonstrating its effectiveness in portfolio selection by accounting for both return dispersion and systemic uncertainty. Unlike variance, entropy provides a more holistic view of risk by incorporating the unpredictability of return distributions. Further advancing this perspective, Maasoumi and Racine (2002) employed a metric entropy measure to assess nonlinear dependencies in stock market returns. Their findings suggest that entropy-based models outperform traditional correlation-based methods in detecting complex return structures, making them particularly useful for multi-objective optimization problems. The study also highlights entropy's potential as a model evaluation criterion, enhancing portfolio selection strategies by improving risk assessment and return predictability. These insights align with the present study's approach to portfolio optimization, which integrates entropy-weighted financial variables. By leveraging entropy in combination with other constraints—such as dividend yield and ticker balancing—this research extends the existing literature on constrained portfolio optimization and multi-objective decision-making.

The optimization of investment portfolios is a crucial aspect of financial decision-making, and the literature presents a diverse array of methodologies and strategies aimed at enhancing portfolio performance while managing risk. Several studies focus on integrating technical analysis indicators with portfolio optimization models to enhance performance. Barroso et al. (2021) propose a fusion between technical analysis and multi-objective portfolio optimization, demonstrating improved performance in the Brazilian stock market. Similarly, Silva et al. (2024) utilize the Support Vector Machine model for preselecting assets before optimization, achieving superior returns and faster recovery post-drawdown periods. Freitas and Junior (2023) present the Stock Network Portfolio Allocation (SNPA) algorithm, leveraging complex network modeling and random walk techniques for automatic recommendation of stock portfolios, presenting evidence of dominant performance across multiple datasets and market indices.

Machine learning plays a significant role in portfolio optimization, as demonstrated by various papers. Barua and Sharma (2023) incorporate fear and greed indicators into the Black-Litterman model, outperforming traditional strategies across different investment periods. Vasconcelos et al. (2022) introduce a self-tuning portfolio-based Bayesian optimization method, enhancing exploration and performance without manual tuning of hyperparameters. Moreover, the consideration of higher-order moments and information entropy adds complexity to portfolio optimization models. Gonçalves et al. (2022) expand portfolio optimization models to include skewness, kurtosis, and information entropy, aiming to capture market uncertainty and improve decision-making. Ashfaq et al. (2021) explore multi-objective optimization with higher-order risk measures, providing insights into the dynamics of international portfolio allocation among BRICS economies.

In response to the COVID-19 pandemic's impact on financial markets, Navarro et al. (2023) develop a method combining technical analysis, machine learning, and mean—variance optimization for stock market optimization, showcasing efficient strategies amidst market volatility. Robust portfolio optimization techniques also emerge as a significant area of research, addressing uncertainty and conservatism in portfolio construction. Min et al. (2021) propose robust portfolio models under ellipsoidal uncertainty sets, leveraging machine learning for parameter estimation and demonstrating superior performance over traditional models. Sehgal and Mehra (2020) introduce a robust portfolio optimization model involving second-order stochastic dominance constraints, yielding optimal portfolios ideal for rational and risk-averse investors.

Silva and de Almeida Filho (2023) introduce a novel approach to index tracking optimization by combining generative adversarial networks (GANs) with genetic algorithms.

Their study explores the impact of GAN-generated market simulations on portfolio performance, highlighting the potential of synthetic data in constructing portfolios with more realistic constraints. On the other hand, de Melo et al. (2022) propose a multi-objective model predictive control (MPC) strategy for portfolio optimization, considering cardinality constraints and multiple risk measures. Their approach dynamically readjusts portfolios based on market expectations, outperforming traditional methods and investment funds, even during crisis periods like the COVID-19 pandemic. Ramos et al. (2023) conduct a comprehensive comparison of risk measures for portfolio optimization under cardinality constraints using linear programming. They evaluate seven risk measures and find that portfolios optimized for expected loss deviation demonstrate superior performance in terms of diversification and risk management, indicating the efficacy of simple linearized optimization models. Moreover, Kamrud et al. (2023) analyze logistics competition between the US and Brazil for soybean shipments to China, employing an optimized Monte Carlo simulation approach to inform trading strategies amid volatile and risky logistical functions and costs.

Lastly, Mendonça et al. (2020) propose a multi-attribute decision-making approach to portfolio optimization, considering different investor profiles and preferences. Their study compares the performance of various decision-making methods and optimization models using computational simulations, demonstrating the robustness of their approach across different investor profiles and market conditions. It is evident that the diversity of approaches to portfolio optimization ranges from traditional techniques to methodologies incorporating machine learning and advanced optimization algorithms.

While these studies offer valuable insights and methodologies, several limitations and gaps remain unaddressed. Many existing studies primarily focus on integrating traditional financial indicators and machine learning models but often overlook practical constraints such as dividend yield and ticker balancing, which are crucial for real-world applications. Moreover, despite the focus on emerging markets in some studies, there is a lack of comprehensive approaches that combine multiple advanced techniques to address the unique characteristics of different financial products specifically tailored for the Brazilian market.

This study addresses these gaps by developing three innovative portfolio optimization strategies for Real Estate Investment Trusts (FIIs), Brazilian Depositary Receipts (BDRs), and Brazilian stocks traded on the Brazilian stock exchange (B3). By integrating entropy, mutual information, high-order moments, dividend yield, and ticker balancing into the optimization models, this research provides a novel contribution to the field. These advanced techniques not only enhance the robustness and performance of the portfolios but also offer practical solutions tailored to the Brazilian market. The novelty of this study lies in its comprehensive approach, combining traditional and advanced optimization methods with practical constraints, providing valuable insights and guidance for Brazilian investors in their investment decisions.

3 Methodology

To identify the best stocks, BDRs, and FIIs, this study employs the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), a robust framework for multi-criteria decision-making developed by Hwang and Yoon (1981). TOPSIS evaluates alternatives by comparing their attributes with ideal and worst-case scenarios using Euclidean distances, providing a clear ranking based on proximity to the ideal and distance from the worst solution. In detailed Tables 1 and 2, the variables used in TOPSIS are meticulously listed, emphasiz-

Table 1 Financial indicators for stocks and BDRs used in TOPSIS analysis. Symbol column shows abbreviations, Indicators describe each metric, and Criteria (±) indicate whether higher or lower values are preferred for portfolio selection

Symbol	Indicators	Criteria
P / E	Price / Earnings	–
PEG RATIO	(Price/Earnings) / Annual earnings per share growth	–
P / BV	Price / Book Value ratio	–
EV / EBITDA	Enteprise value / EBITDA	–
EV / EBIT	Enteprise value / EBIT	–
P / EBITDA	Price / EBITDA	–
P / EBIT	Price / EBIT	–
EPS	Earnings per share	+
P / ASSET	Price / Asset	–
BVPS	Book value per share	+
P / SR	Price / Sales Ratio	–
P / Working Capital	Price / Working Capital	–
P / Net Current Assets	Price/ Net Current Assets	–
Net Debt / Equity	Net Debt / Equity	–
Net Debt / EBITDA	Net Debt / EBITDA	–
Net Debt / EBIT	Net Debt / EBIT	–
Equity / Assets	Equity / Assets	+
Liabilities / Assets	Liabilities / Assets	–
Current Ratio	Current assets / Current liabilities	+
Gross Margin	Gross Profit / Net Revenue	+
EBITDA Margin	EBITDA	+
EBIT Margin	EBIT	+
Net Margin	Net Profit / Net Revenue	+
ROE	Net Profit / Shareholders' Equity	+
ROA	Net Profit / Asset	+
ROIC	(EBIT – taxes) / Invested capital	+
Asset Turnover	Net Sales / Assets	+
CAGR Revenue 5 years	(Ending Revenue / Beginning Revenue) $^{1/5}$–1	+
CAGR Profit 5 years	(Ending Profit / Beginning Profit) $^{1/5}$–1	+

ing transparency and reproducibility in the evaluation process (The results are in Appendix A: Tables 12 and 13). A distinctive feature of this study is the novel application of variable entropy as weights. Entropy calculation involves discretizing continuous variables, determining category probabilities, and normalizing entropy values. Higher entropy signifies indicator variability and importance, while lower entropy suggests consistency. Thus, entropy calculation enhances decision-making by highlighting the relative importance of each variable.

This section also explores three modified Markowitz models: Model 1, Model 2, and the primary focus, Model 3. Model 1 and Model 2 act as vital benchmarks for assessing Model 3's performance. Understanding the equations defining Model 1 and Model 2 is essential to appreciate Model 3's significance in the broader context of analysis and decision-making.

Table 2 Financial indicators for FIIs used in TOPSIS analysis. Criteria (±) indicate optimal direction for each indicator in the multi-criteria evaluation

Symbol	Indicators	Criteria
N° investors	Number of investors	+
N° shares	Number of issued shares	+
Assets	Assets	+
Net worth	Net worth	+
Net asset / shares	Net asset value per share	+
Total liquidity	Total liquidity requirement	–
Cash	Available cash	+
G bonds	Government bonds	+
P bonds	Private bonds	+
Total invested	Total invested	+
Assets and rights	Assets and rights	+
Lands	Lands	+
Finished IP	Finished income properties	+
IP under construction	Income properties under construction	+
FP for sale	Finished properties for sale	+
Other assets and rights	Other assets and rights	+
Shares	Shares	+
FII	Real Estate Investment Fund	+
Shares of Companies for FIIs	Shares of Companies within the activities permitted for FIIs	+
Stocks of Companies for FIIs	Stocks of Companies within the activities permitted for FIIs	+
CRI	Real Estate Receivables Certificate	+
LCI	Real Estate Credit Letters	+
Receivables	Receivables	+
Rental receivables	Rental receivables	+
Receivables PS	Receivables from property sales	+
Other receivables	Other receivables	+
Payables	Payables	–
Distributable income	Distributable income	+
Administrative FP	Administrative fee payable	–
Performance FP	Performance fee payable	–
Obligations PA	Obligations for property acquisition	–
Obligations RS	Obligations for receivables securitization	–
Provisions	Provisions for contingencies	–
Other payables	Other payables	–

The following equations succinctly represent Model 1, emphasizing its key features:

$$MinW^T\Sigma W \tag{1}$$

$$s.t.$$

$$\sum_{i=1}^{n} W_i = 1 \tag{2}$$

$$\overline{R}_p = W_i R_i \tag{3}$$

$$W_i \geq 0 \tag{4}$$

In Model 1, we retain the classic Markowitz model's structure but modify the covariance matrix. By introducing entropy and mutual information in Eq. 5, we replace the traditional covariance matrix, enhancing portfolio construction by capturing dependencies and non-linear relationships among asset returns.

$$\Sigma = \begin{bmatrix} H_1 & \ldots & MI_{n1} \\ MI_{12} & \ldots & MI_{n2} \\ MI_{1n} & \ldots & H_n \end{bmatrix} \tag{5}$$

In Eq. 5, H_1 represents the entropy of asset 1, and MI_{12} denotes the mutual information between asset 1 and asset 2, extending to n assets. Novais et al. (2022) note that this approach addresses non-normality challenges in returns, demonstrating superior performance in certain scenarios. The second model (Model 2), a MSVKE model used as a benchmark for Model 3, incorporates higher moments and portfolio entropy to reduce return variability and improve predictability (Philippatos & Wilson, 1974). The equations below provide a concise representation of the multi-objective function that will be solved using a non-linear optimization approach:

$$Min\sigma_p^2 = W^T\Sigma W \tag{6}$$

$$MaxS_p = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} E\left(R_i R_j R_k\right) W_i W_j W_k}{\left(\sum_{i=1}^{n}\sum_{j=1}^{n} W_i W_j cov_{i,j}\right)^{3/2}} \tag{7}$$

$$MinK_p = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} E\left(R_i R_j R_k R_l\right) W_i W_j W_k W_l}{\left(\sum_{i=1}^{n}\sum_{j=1}^{n} W_i W_j cov_{i,j}\right)^{4/2}} \tag{8}$$

$$MinH_p = -\sum_{x=1}^{n} W_x ln W_x \tag{9}$$

$$S.t.$$

$$\sum_{i=1}^{n} W_i = 1 \tag{10}$$

$$\overline{R}_p = W_i R_i \tag{11}$$

$$W_i \geq 0 \tag{12}$$

Model 3, the primary focus of this study, enhances the MV model with two key modifications: a constraint on the minimum number of tickers and a minimum dividend yield requirement. The equations below illustrate Model 3, building on Markowitz's framework while introducing constraints that contribute to modern portfolio theory (MPT).

$$Min W^T \Sigma W \tag{13}$$

$$S.t.$$

$$\sum_{i=1}^{n} W_i = 1 \tag{14}$$

$$\overline{R}_p = W_i R_i \tag{15}$$

$$W_i \geq 0 \tag{16}$$

$$\sum_{i=1}^{n} (W_i > 0) = n_p \tag{17}$$

$$W^T y \geq \overline{R}_{yield} \tag{18}$$

The cardinality constraint 3 is implemented through a continuous formulation rather than explicit binary decision variables. Specifically, a lower bound is imposed on portfolio weights, such that each selected asset must receive at least a minimum allocation (e.g., 1%), and the sum of active weights ensures that the portfolio contains at least a predefined number of assets (20 or 30). This formulation avoids the introduction of binary variables and allows the optimization problem to be solved efficiently using nonlinear programming techniques.

Model 1 relaxes normality assumptions through entropy measures, while Models 2 and 3 retain mean–variance framework assumptions. Key parameter choices directly influence results: higher DY constraints increase income but may reduce capital gains by excluding growth stocks; larger minimum asset counts (30 vs 20) enhance diversification but may dilute returns. DY constraints should be chosen wisely to balance optimization feasibility with meaningful income targets. Equations 17 and 18 in Model 3 introduce constraints for diversification and minimum dividend yield. Unlike traditional approaches focused on weight and risk, this innovative method incorporates investor-defined inputs, ensuring diversification and accommodating individual preferences. By considering these inputs, the model enhances portfolio optimization, providing a personalized tool for investors to align portfolios with their preferences and risk tolerance.

4 Experimental setup

This study applies the Model 3 to stocks, BDRs, and FIIs, comparing outcomes with Model 1 and Model 2, alongside mean–variance portfolio (MV) and Naïve models. Asset selection involves web scraping with Python libraries (BeautifulSoup, Selenium) to collect tickers from B3. For stocks, tickers ending in 3 (common shares), 4, 5, 6 (preferred shares), and 11 (combination of common and preferred shares) were gathered; for BDRs, codes ending in 34, 35 (non-sponsored BDRs), and 32 (sponsored BDRs) were collected; and for FIIs, tickers with the ending code 11 were obtained. Web scraping on the StatusInvest website provided financial data, emphasizing indicators like P/E ratio and PEG. To ensure data completeness and quality, the analysis focuses exclusively on tickers with complete financial variables; assets with missing or incomplete data were excluded from the sample to avoid potential biases in portfolio construction.

Once all the financial indicators are collected from the StatusInvest website, three separate matrices will be created, each containing the data for a specific product type. To address the presence of outliers, a winsorization technique will be applied to the matrices, limiting extreme values to the 2.5th and 97.5th percentiles. With the processed financial data at hand, the next step involves the careful selection of the optimal tickers for each product type based on their financial metrics.

The study employs the TOPSIS algorithm for ticker selection in portfolio optimization, emphasizing entropy-based weights. A density graph visually compares TOPSIS scores (Cj +), evaluating how entropy-based weights impact ticker score variability (Appendix A, Figs. 4, 5, 6, 7). This investigation focuses on entropy as a weight determination method, using equal weights as a reference for potential advantages.

After computing the weights based on the entropy of the financial variables, the TOPSIS algorithm requires additional information to determine the best tickers to select. Specifically, it needs to know which variables are to be maximized and which columns are to be minimized. Table 1 presents the financial indicators collected from StatusInvest for BDRs and stocks, along with a comprehensive explanation of each column. The first column provides the abbreviation of the indicators, making it easier to identify each variable. The second column offers a brief explanation of the meaning and significance of each financial indicator, aiding readers in understanding their relevance. The third column specifies the criteria for each indicator, indicating whether an increase (+) or decrease (-) in the variable is more favorable in the portfolio selection process, which are crucial inputs for the TOPSIS algorithm.

Table 2 showcases the variables for FIIs, following the same structural format as Table 1. However, the financial indicators are specific to real estate stocks, providing valuable insights into the characteristics of these assets.

After determining TOPSIS scores and their order, the selected tickers' historical daily data is crucially verified on Yahoo Finance. Ensuring reliability, various criteria are assessed, including positive price values, units below R$1,000.00, dynamic price variations, minimal null values, a minimum of two years of data, and price changes over fifteen consecutive days. This thorough data check guarantees the inclusion of reliable and relevant stocks in the final portfolio of the top 100 tickers for each financial product.

With all the tickers' prices collected, the next step involves transforming the daily prices into daily returns for each stock. These return vectors are then merged together, creating a matrix of size $(i \times n)$, where (i) represents the number of trading days in the collected period, and (n) represents the number of tickers selected in the portfolio. It is important to note that only the top 100 tickers with the highest TOPSIS scores are considered for portfolio construction. Equation 1 serves as an example of the matrix that all the models will utilize to implement their respective strategies. This matrix represents the returns of the selected tickers over time, providing the necessary data for further analysis and optimization.

$$\begin{pmatrix} Data & ticket_1 & ticket_2 & \dots & ticket_n \\ 1 & R1_1 & R2_1 & \dots & Rn_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ i & R1_i & R2_i & \dots & Rn_i \end{pmatrix} \tag{19}$$

After completing the portfolio selection process and generating the returns matrix (Eq. 19), this study will proceed to apply and compare the Model 3 models with all other ones like Model 1, Model 2, and the others. The optimization computations for Model 1, Model 2, and MV will be performed using the scipy library, which can solve the quadratic variance optimization and handle additional constraints using the SQLSP method. For Model 3, the pyomo library, using the solver IPOPT designed for solving multi-objective problems, will be utilized. The dataset will be divided, with 80% allocated to the in-sample set and 20% to the out-of-sample set. The portfolio evaluation will encompass key metrics like the Sharpe (1964) ratio, Sortino (1991) ratio, standard deviation, downside risk, daily returns, and cumulative returns for each model. It's important to note that the models will be compared independently for each financial product. Additionally, for Model 3, variations in dividend yield constraints will be explored using a percentile-based approach specific to each product. After selecting the top 100 tickers through TOPSIS scores, the dividend yield distribution of these assets is analyzed to establish appropriate constraint values. The DY constraints are strategically chosen to align closely with the 75th percentile and approach the upper limits observed within each dataset, balancing the solver's capacity to meet all constraints without overly restricting the optimization process. This methodology ensures realistic outcomes and avoids infeasible results that could arise from setting constraints at maximum observed values, which might clash with other model restrictions such as the minimum asset requirements. The minimum number of assets parameter will be tested with two values: twenty and thirty, in line with Newbould and Poon's (1993) recommendation for effective diversification. The minimum percent adopted in the Model 3 strategy is 1% to give flexibility to the optimization and not restrict so much the assets in the strategy. So, there will be a total of 10 portfolios for each financial product, providing valuable insights into the applied strategies.

5 Results

The results reported in this section are obtained by applying the portfolio optimization Model 3 to the top 100 assets selected in the previous step using the entropy-weighted TOPSIS procedure.

5.1 Stocks

Following the computation of TOPSIS scores and their arrangement from highest to lowest, we identify the top 100 tickers that adhere to specific predefined rules in the process of collecting prices from them. This process ensures data consistency before the application of the models. The primary portfolio model in this study (Model 3) incorporates a more extensive set of input parameters compared to the others. Table 3 presents the percentile distribution of DY within the sample:

Following the methodology described in Sect. 4.1, the dividend yield constraints for Model 3 were set at ten, twelve, and fourteen percent for stocks. Although imposing a minimum dividend yield ensures that portfolios generate a baseline level of income, it does

Table 3 Dividend yield percentiles for the top 100 stocks selected via TOPSIS. These values guide the DY constraint settings (10%, 12%, 14%) for Model 3, targeting the 75th percentile range

Percentiles	DY (%)
0th	0
25th	3
50th	5.1
75th	10.9
100th	15.4

Table 4 Portfolio models applied to stock optimization. Model 3 variants differ in minimum asset count (20 vs 30) and dividend yield constraints (10%, 12%, 14%)

Label	Model
Model 1	Mean-entropy
Model 2	Mean Variance Skewness Kurtosis Entropy
Model 3 20n 10DY	Mean–Variance with constraints of 20 assets and 10% dividend yield
Model 3 20n 12DY	Mean–Variance with constraints of 20 assets and 12% dividend yield
Model 3 20n 14DY	Mean–Variance with constraints of 20 assets and 14% dividend yield
Model 3 30n 10DY	Mean–Variance with constraints of 30 assets and 10% dividend yield
Model 3 30n 12DY	Mean–Variance with constraints of 30 assets and 12% dividend yield
Model 3 30n 14DY	Mean–Variance with constraints of 30 assets and 14% dividend yield
Naive	Equal weights in the 100 tickers (1% in each one)
MV	Mean–variance

not necessarily maximize total returns. The results show that Model 3 portfolios with higher dividend constraints often underperform in cumulative returns compared to lower-dividend portfolios. This suggests that high-dividend-yield stocks may not always align with high-growth opportunities. Investors seeking income stability may prefer Model 3, while those targeting capital appreciation might favor a more flexible allocation approach.

By adopting this range for DY, our aim was to provide the solver with greater flexibility in navigating the challenging task of balancing this metric with other stringent constraints, particularly the minimum asset requirements within the portfolio. This deliberate selection was intended to strike a delicate balance, ensuring the solver's capacity to meet all constraints without overly restricting the optimization process. With the dividend yield defined, Table 4 showcases the models that will apply the optimization process using the returns of the selected tickers.

Table 5 showcases statistical calculations encompassing all models, their corresponding risk metrics, and daily mean returns. The table is organized by the highest to lowest Sharpe index, separated by a dashed black line indicating the in-sample and out-of-sample sections.

Analyzing risk, the standard deviation and the downside risk of the in-sample data are higher than those of the out-of-sample data. This difference could be attributed to the length of the collected periods in both sets. The in-sample data spans a significantly larger number of days compared to the out-of-sample data, resulting in increased volatility across the period.

Moving to portfolio analysis, we observe distinct portfolio compositions across different models: Model 1 holds only six tickers, and Model 2 contains merely two.[1] Remarkably, both exhibit the highest standard deviation within the in-sample and out-of-sample datasets. Similarly, the MV portfolio consists of only eight weights, contributing to higher risk compared to all Model 3 models and the Naive model. It's evident that these riskier portfolios

[1] We do not report the table because of space and word limits, it is available upon request from the correponding author.

Table 5 Daily results of all stocks' portfolios, sorted by Sharpe index, separating IN and Out of sample

Portfolio	Sample	DY	Return	Standard Deviation	Downside Risk	Sharpe	Sortino
MV	IN	9,60%	0,17%	1,42%	0,86%	0,093	0,154
Model 3 20n 10DY	IN	10,00%	0,16%	1,28%	0,79%	0,092	0,15
Model 3 30n 10DY	IN	10,00%	0,15%	1,23%	0,75%	0,088	0,143
Model 3 20n 12DY	IN	12,00%	0,14%	1,21%	0,74%	0,086	0,14
Model 2	IN	10,10%	0,20%	1,99%	1,14%	0,082	0,144
Model 3 30n 12DY	IN	12,00%	0,13%	1,14%	0,71%	0,079	0,127
Model 1	IN	5,70%	0,19%	1,94%	1,19%	0,076	0,125
Model 3 20n 14DY	IN	14,00%	0,12%	1,28%	0,81%	0,062	0,098
Model 3 30n 14DY	IN	14,00%	0,09%	1,11%	0,73%	0,045	0,068
Naive	IN	6,70%	- 0,04%	1,17%	0,89%	- 0,072	- 0,095
Model 3 30n 14DY	OUT	14,00%	0,21%	0,79%	0,48%	0,203	0,338
Naive	OUT	6,70%	0,23%	0,98%	0,58%	0,186	0,315
Model 3 30n 12DY	OUT	12,00%	0,19%	0,80%	0,50%	0,17	0,272
Model 3 20n 14DY	OUT	14,00%	0,21%	0,96%	0,60%	0,164	0,263
Model 3 20n 12DY	OUT	12,00%	0,18%	0,83%	0,52%	0,158	0,254
Model 3 30n 10DY	OUT	10,00%	0,17%	0,86%	0,53%	0,135	0,22
Model 3 20n 10DY	OUT	10,00%	0,16%	0,90%	0,56%	0,122	0,196
MV	OUT	9,60%	0,16%	0,99%	0,63%	0,111	0,175
Model 2	OUT	10,10%	0,16%	1,21%	0,80%	0,09	0,137
Model 1	OUT	5,70%	0,04%	1,31%	0,96%	- 0,01	- 0,014

possess a lower number of assets, aligning with the principles of diversification in the literature review. The relationship between portfolio risk and asset count suggests a behavioral pattern, emphasizing how fewer assets correlate with higher inherent risk in the portfolio. However, an interesting deviation emerges with the Naive portfolio, which, despite having a significant number of assets, demonstrates a similar standard deviation and downside risk compared to Model 3 models. This observation aligns with the essence of modern portfolio theory, suggesting that systematic risk tends to converge as events impacting the market affect it as a whole. This potentially highlights the limitation of solely relying on increased asset count for risk mitigation, as other systemic factors can influence overall portfolio risk.

Looking at the results of the out-of-sample data, the Model 3 models, which are the innovative strategy and the main focus of this work, have lower risks compared to the traditional MV portfolio and the Model 1 and Model 2 approaches, showing a good alternative for Brazilian investors who are risk-averse. Additionally, when comparing all the Model 3 models against the Naive portfolio, it's noteworthy that every Model 3 model demonstrates a lower standard deviation. Interestingly, among these, only the Model 3 20n 14DY model exhibits a slightly higher downside risk than the Naive. This shows the effectiveness of Model 3 in controlling risk.

According to the returns, the Model 3 models consistently outperform Model 1, Model 2, and MV models, especially in the out-of-sample analysis. They exhibit an opposite trend to these traditional portfolios in underestimating values in the in-sample data. Across the board, Model 3 models display higher returns and lower risk, reflected in superior Sharpe and Sortino ratios compared to standard models. Surprisingly, when compared to the Naive model, all Model 3 models fall short in mean daily returns, as evident in Table 5. The results indicate that while Model 3 provides superior risk-adjusted returns compared to Model 1 and Model 2, it does not consistently outperform the Naive portfolio. This suggests that a simple diversification strategy—where equal weights are assigned across a broad set of assets—can be just as effective as more complex optimization models in certain market conditions. This aligns with findings from DeMiguel et al. (2009), who demonstrated that naive diversification often competes with more sophisticated portfolio selection techniques. A natural interpretation is that the Naive strategy is applied after a strong preselection step: because TOPSIS already identifies the top 100 candidates using multiple financial attributes, a simple 1/N allocation can inherit much of the diversification and quality filtering embedded in the curated universe. In contrast, Model 3 deliberately imposes minimum dividend-yield and minimum-asset requirements, which can restrict exposure to low-dividend/high-growth stocks and reduce upside in periods when capital gains dominate. For this reason, Model 3 should be viewed as a policy and decision-support portfolio: it exchanges some growth potential for implementable guarantees (income floor and concentration control) and typically improves portfolio stability and risk-adjusted performance, even when mean daily returns are not the highest. These results imply that while Model 3 offers structured risk management through diversification constraints, investors must weigh the trade-off between a rules-based approach and simpler diversification methods.

Figure 1 illustrates the volatility inherent in the Naive approach when observing returns over time. This volatility underscores why the Model 3 30n 14DY model boasts a higher Sharpe and Sortino index compared to the Naive, making it an optimal choice for investors seeking a more balanced portfolio in terms of returns and risk. Interestingly, the Naive portfolio only outperforms the risk-free (CDI) near May 2023. In contrast, the Model 3 models surpass the CDI earlier, around the end of March, and demonstrate more stable returns compared to the Naive.

Additionally, when investors assess returns in the capital market, they not only consider portfolio appreciation but also the dividend yield (DY). Analyzing both aspects of returns, the Model 3 models present higher DY than the Naive, aligning with the objective of maintaining a minimum DY, which is an aspiration for many investors. Despite the Naïve's higher daily returns, it falls short in DY compared to the Model 3 models. This emphasizes how the Model 3 30n 14DY stands out as the optimal choice for investors, showcasing the innovative and efficient approach of combining DY restrictions with a minimum number of assets.

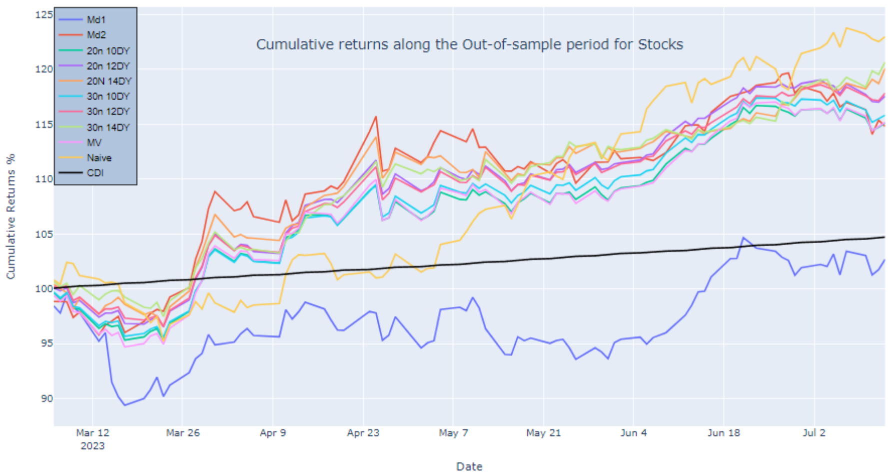

Fig. 1 Out-of-sample cumulative daily returns for stock portfolios. Model 3 portfolios (varying DY constraints) demonstrate more stable growth patterns than Model 1. The horizontal dashed line represents the CDI risk-free rate

Table 6 Dividend yield percentiles for the top 100 BDRs. Lower DY values (3%, 3.5%, 4%) reflect the characteristic lower dividend payouts of international stocks

Percentiles	DY (%)
0th	0
25th	0.4
50th	1.8
75th	3.5
100th	4.5

5.2 BDRs

We collected the top 100 tickers based on their highest TOPSIS scores to serve as assets for our modeling and subsequent analysis. Our evaluation of the dividend yield (DY) percentiles for the BDRs led us to choose values of 3, 3.5, and 4 for the Model 3 models. These values strategically mirror the 75th percentile and the maximum value within our dataset. Table 6 delineates the percentiles of the 100 BDRs' tickers:

It's evident in Table 6 that foreign stocks tend to yield lower dividends compared to the Brazilian stock market. This observation aligns with findings presented in articles such as Zagonel et al. (2018), which expound on how Brazilian taxation rates are comparatively lower than those in the USA. Additionally, research like that of Lozano and Caltabiano (2015) supports this notion, indicating that the Brazilian stock market tends to offer higher dividend yields in contrast to the UK market. With the selection of DY inputs completed, we now have all the models ready for comparison. Table 7 outlines the models:

After our model selection, we conducted optimization on the 100 chosen tickers, defining the weights for each asset.[2] Table 8 presents the daily results of these models, sorted from the

[2] Please see appendix B for detail.

Table 7 Portfolio models applied to BDR optimization with dividend yield constraints adjusted for lower international dividend payouts (3%, 3.5%, 4%)

Label	Model
Model 1	Mean-entropy
Model 2	Mean Variance Skewness Kurtosis Entropy
Model 3 20n 3DY	Mean–Variance with constraints of 20 assets and 3% dividend yield
Model 3 20n 3.5DY	Mean–Variance with constraints of 20 assets and 3.5% dividend yield
Model 3 20n 4DY	Mean–Variance with constraints of 20 assets and 4% dividend yield
Model 3 30n 3DY	Mean–Variance with constraints of 30 assets and 3% dividend yield
Model 3 30n 3.5DY	Mean–Variance with constraints of 30 assets and 3.5% dividend yield
Model 3 30n 4DY	Mean–Variance with constraints of 30 assets and 4% dividend yield
Naive	Equal weights in the 100 tickers (1% in each one)
MV	Mean–variance

highest Sharpe ratio to the lowest, and categorizes them into in-sample and out-of-sample periods:

Appendix B shows that Model 3 can be solved efficiently using nonlinear programming, with runtimes close to those of the standard mean–variance model and without the computational burden associated with binary-variable cardinality constraints. This confirms that the proposed formulation achieves diversification control while preserving numerical scalability.

Examining daily results in Table 8 reveals a stark contrast between returns in a controlled setting (in-sample) and real-life scenarios (out-of-sample), with negative returns indicating challenges for Brazilian investors in BDRs. Despite the Sharpe and Sortino indices showing negative numbers, comparing strategies becomes complicated, as higher risk could yield higher indices with negative returns. Relying on standard deviation and downside risk to gauge risks, Model 3 models show reduced risks compared to Model 1 and Model 2 models in standard deviation, but an interesting shift occurs in downside risk. Both Model 3 20n 4DY and 30n 4DY models perform less favorably than the Model 1 model, possibly due to Model 1's asset allocation within BDR portfolios comprising 11 assets compared to six in the stock portfolio. This divergence extends to the MV variance portfolio, with superior risk metrics in BDRs compared to Model 3 models, highlighting the impact of diversified asset allocation on risk mitigation in these strategies.

The best portfolio in terms of risk assessment turns out to be the Naive strategy, displaying the highest return alongside the lowest risk metrics. This outcome is a positive reflection of the portfolio selection method, which identifies the best assets based on the TOPSIS score. Unexpectedly, despite employing 30 assets, the Model 3 models couldn't surpass the Naive model concerning risk metrics. This contrast hints at the persistent high level of idiosyncratic risk within this scenario, even when utilizing a minimum constraint of 30 assets. It suggests that the Naive approach might offer a more substantial diversification effect than the limitations imposed by this model. Interestingly, in the stock analysis, one of the Model 3

Table 8 Daily results of all BDRs' portfolios, sorted by Sharpe index, separating IN and Out of sample

Portfolio	Sample	DY	Return	Standard Deviation	Downside Risk	Sharpe	Sortino
MV	IN	1,8%	0,17%	1,17%	0,72%	0,114	0,185
Model 3 20n 3DY	IN	3,0%	0,17%	1,24%	0,79%	0,106	0,164
Model 1	IN	2,4%	0,19%	1,52%	0,98%	0,103	0,159
Model 3 30n 3DY	IN	3,0%	0,16%	1,21%	0,78%	0,099	0,155
Model 3 20n 3.5DY	IN	3,5%	0,16%	1,32%	0,86%	0,097	0,148
Model 3 30n 3.5DY	IN	3,5%	0,15%	1,28%	0,83%	0,089	0,137
Model 3 20n 4DY	IN	4,0%	0,16%	1,44%	0,95%	0,084	0,127
Model 3 30n 4DY	IN	4,0%	0,14%	1,41%	0,94%	0,077	0,115
Model 2	IN	2,7%	0,21%	2,34%	1,52%	0,076	0,116
Naive	IN	2,0%	0,02%	1,22%	0,78%	- 0,010	- 0,015
Model 1	OUT	2,4%	- 0,04%	1,36%	0,87%	- 0,070	- 0,109
MV	OUT	1,8%	- 0,02%	0,97%	0,64%	- 0,075	- 0,113
Naive	OUT	2,0%	- 0,01%	0,80%	0,58%	- 0,080	- 0,111
Model 3 20n 4DY	OUT	4,0%	- 0,06%	1,22%	0,90%	- 0,093	- 0,127
Model 3 30n 4DY	OUT	4,0%	- 0,06%	1,18%	0,87%	- 0,096	- 0,130
Model 3 20n 3.5DY	OUT	3,5%	- 0,06%	1,10%	0,81%	- 0,104	- 0,140
Model 3 20n 3DY	OUT	3,0%	- 0,05%	1,00%	0,74%	- 0,105	- 0,143
Model 3 30n 3.5DY	OUT	3,5%	- 0,06%	1,07%	0,79%	- 0,107	- 0,144
Model 3 30n 3DY	OUT	3,0%	- 0,06%	0,99%	0,73%	- 0,112	- 0,151
Model 2	OUT	2,7%	- 0,18%	1,70%	1,30%	- 0,138	- 0,181

models, using 30 assets as the minimum, effectively mitigated the idiosyncratic risk, showing lower risks than the Naive model.

In terms of returns, it's noteworthy that all portfolios have displayed negative returns, showcasing a significant contrast between the in-sample and out-of-sample results. This unusual trend could be attributed to the devaluation of the Dollar relative to the Real, indicating to Brazilian investors how the appreciation of the Real can impact stock prices. Figure 2 illustrates the cumulative returns of all strategies, showing the Dollar's variation against the Real (black dashed line).

The depreciation of the Dollar against the Real is evident in the cumulative returns pattern across all portfolios. For Brazilian investors, buying foreign stocks can be affected by changes

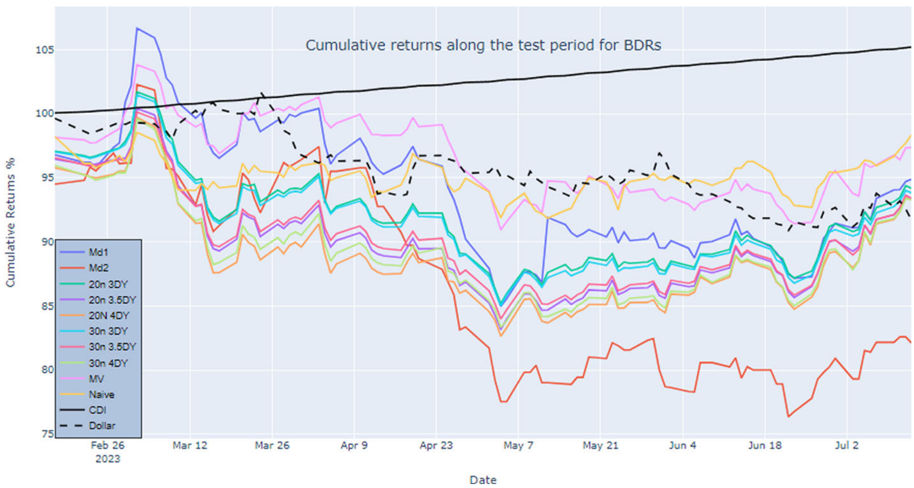

Fig. 2 Out-of-sample cumulative daily returns for BDR portfolios with US Dollar variation (black dashed line). Currency appreciation during the testing period negatively impacts all strategies, with portfolios underperforming the CDI risk-free rate (horizontal dashed line)

in currency value, impacting how much they earn from those investments. During the data collection period, the risk-free rate surpassed all strategies. Among the portfolios, the Naive and MV portfolios performed relatively better compared to the rest.

Regarding the dividend yield (DY) of the models, while Model 3's restrictions slightly outperform others, they aren't significantly better. In cases of negative returns, this restriction isn't distinctive enough when choosing an investment strategy. Examining the best BDR portfolios, both Naive and MV portfolios hover near a 2% DY, while the minimum requirement in Model 3 models is 3%, representing a 1% difference. However, examining the graph, Naive and MV portfolios show cumulative returns nearing 98% at the end of the out-of-sample period, whereas Model 3 models fall below 95%. Additionally, the risk associated with Model 3 models surpasses that of Naive and MV, indicating that in this scenario, Model 3 models perform worse than MV and Naive.

5.3 FIIs

Continuing the methodology applied to stocks and BDRs, the top 100 FIIs with the highest TOPSIS scores were chosen, and their percentiles were collected as inputs for the Model 3 models. Table 9 displays these percentiles for the selected FIIs:

The DY percentiles for FIIs reveal that they possess the highest DY among all the financial products analyzed in this study. This notably high dividend yield is primarily due to the inherent obligation within this product, especially in the Brazilian market, to distribute dividends (Consentino et al., 2011). Following the percentile-based methodology described in Sect. 4.1, the DY constraints for Model 3 were set at fourteen, fifteen, and sixteen percent for FIIs. Table 10 presents the models designated for optimizing the FIIs tickers:

Table 11 presents the analysis of risk metrics and daily returns, sorted from the highest Sharpe ratio to the lowest, and categorizes them into in-sample and out-of-sample periods:

Table 9 Dividend yield percentiles for the top 100 FIIs. Higher DY values (14%, 15%, 16%) reflect mandatory profit distribution requirements for real estate funds

Percentiles	DY (%)
0th	0
25th	6.4
50th	8.1
75th	11.7
100th	19.6

Table 10 Portfolio models applied to FII optimization with higher dividend yield constraints (14%, 15%, 16%) appropriate for real estate funds

Label	Model
Model 1	Mean-entropy
Model 2	Mean Variance Skewness Kurtosis Entropy
Model 3 20n 14DY	Mean–Variance with constraints of 20 assets and 14% dividend yield
Model 3 20n 15DY	Mean–Variance with constraints of 20 assets and 15% dividend yield
Model 3 20n 16DY	Mean–Variance with constraints of 20 assets and 16% dividend yield
Model 3 30n 14DY	Mean–Variance with constraints of 30 assets and 14% dividend yield
Model 3 30n 15DY	Mean–Variance with constraints of 30 assets and 15% dividend yield
Model 3 30n 16DY	Mean–Variance with constraints of 30 assets and 16% dividend yield
Naive	Equal weights in the 100 tickers (1% in each one)
MV	Mean–variance

Commencing our analysis with risk metrics, similar to the approach taken with stocks and BDRs, the Model 3 models exhibit significantly better risk metrics compared to Model 1 and Model 2. This highlights the robustness imparted by the minimum constraint, mitigating unsystematic risks inherent in the selected assets. In line with the stock analysis, Model 1 and Model 2 perform poorly in the out-of-sample scenario. This performance discrepancy can be attributed to portfolio concentration in which Model 1 holds only nine assets, and Model 2 merely three, which is considerably similar to stock portfolios results. The imposition of minimum asset constraints forces the optimizer to distribute weights across other assets. In Model 1 and Model 2, this distribution might concentrate on higher-return, lower-risk assets from the in-sample data, resulting in notable differences in the out-of-sample performance. The out-of-sample results highlight an important limitation of complex portfolio optimization models: while they perform well in-sample, their out-of-sample performance does not always surpass traditional models. This may be attributed to overfitting—where the model tailors its selection too closely to past data rather than adapting to future market conditions. Investors should therefore be cautious when relying solely on historical optimization models and consider integrating dynamic adjustment strategies that rebalance based on changing market conditions.

Table 11 Daily results of all FIIs' portfolios, sorted by Sharpe index, separating IN and Out of sample

Portfolio	Sample	DY	Return	Standard Deviation	Downside Risk	Sharpe	Sortino
MV	IN	14,4%	0,08%	0,58%	0,33%	0,077	0,136
Model 3 20n 14DY	IN	14,0%	0,08%	0,59%	0,33%	0,072	0,127
Model 3 20n 15DY	IN	15,0%	0,08%	0,63%	0,35%	0,071	0,129
Model 3 20n 16DY	IN	16,0%	0,09%	0,71%	0,38%	0,069	0,13
Model 3 30n 14DY	IN	14,0%	0,08%	0,71%	0,40%	0,064	0,114
Model 3 30n 15DY	IN	15,0%	0,09%	0,77%	0,43%	0,064	0,116
Model 3 30n 16DY	IN	16,0%	0,09%	0,81%	0,42%	0,062	0,118
Model 2	IN	9,5%	0,16%	2,19%	1,42%	0,056	0,086
Model 1	IN	14,6%	0,14%	2,22%	1,04%	0,049	0,104
Naive	IN	9,0%	− 0,01%	0,37%	0,27%	− 0,156	− 0,213
Naive	OUT	9,0%	0,13%	0,33%	0,17%	0,234	0,451
Model 3 20n 14DY	OUT	14,0%	0,06%	0,40%	0,23%	0,037	0,063
MV	OUT	14,4%	0,06%	0,41%	0,23%	0,024	0,042
Model 3 30n 14DY	OUT	14,0%	0,05%	0,45%	0,27%	0,009	0,014
Model 3 20n 15DY	OUT	15,0%	0,05%	0,39%	0,23%	0,004	0,006
Model 3 30n 15DY	OUT	15,0%	0,04%	0,46%	0,28%	− 0,023	− 0,037
Model 3 20n 16DY	OUT	16,0%	0,04%	0,41%	0,25%	− 0,025	− 0,042
Model 3 30n 16DY	OUT	16,0%	0,03%	0,45%	0,28%	− 0,027	− 0,044
Model 1	OUT	14,6%	0,00%	1,25%	0,87%	− 0,041	− 0,058
Model 2	OUT	9,5%	− 0,05%	1,69%	1,26%	− 0,064	− 0,086

In Sects. 5.1, 5.2, and Table 11, we observe the MV portfolio emerging as the top performer in the in-sample analysis, but this scenario changes in the out-of-sample data, aligning with findings by DeMiguel et al. (2009). Despite this shift, the MV strategy still showcases superior risk metrics compared to Model 1 and Model 2 across all financial products, including FIIs. In the comparison involving Model 3 models, the Model 3 20n 14DY model demonstrates superior risk metrics in terms of standard deviation when juxtaposed with the MV strategy. However, both exhibit similar downside risk, which suggests that the imposition of a minimum asset constraint in the Model 3 20n 14DY and in the other Model 3 portfolios wasn't entirely effective in mitigating idiosyncratic risk for this risk-averse portfolio. Upon analysis, the Naive portfolio emerges with the lowest risk among all other FII portfolios, mirroring the results observed in the BDR analysis. This underscores the effectiveness of the TOPSIS score in selecting commendable assets for portfolio inclusion. Even within the stock scenario, while one of the Model 3 models displays lower risk compared to the Naive portfolio, only one managed to outperform the Naive in terms of risk.

In terms of returns, the Model 3 portfolios surpass both Model 1 and Model 2 significantly. While Model 1 and Model 2 perform worse than the risk-free rate, exhibiting negative Sharpe and Sortino indices, some of the Model 3 portfolios also show negative metrics. However, three out of the six Model 3 portfolios exhibit favorable returns coupled with good risk metrics. When compared to the MV portfolio, only the Model 3 20n 14DY portfolio outperforms this one, and this trend remains consistent with BDRs, where the MV portfolio yields higher returns than Model 3 models. Still, in stocks, the majority of Model 3 models outperform the MV portfolio in daily returns. Looking at the Naive, it stands out with the highest daily return across all financial products. This shows that diversification across a hundred assets can indeed yield substantial returns, outperforming all the models in all financial products, disregarding DY returns. Figure 3 shows the cumulative returns of each strategy and those which lose to the risk-free rate in the out-of-sample period (lines below the CDI).

When examining cumulative returns, it becomes evident that the Naive strategy surpasses other models, even outperforming the Model 3 20n 14DY, which stands as the top portfolio among the Model 3 models in terms of Sharpe and Sortino indices. While the Model 3's best model exhibits a higher DY at fourteen percent compared to nine percent in Naive, the cumulative daily return of Naive exceeds at one hundred ten percent against one hundred five percent of the Model 3 20n 14DY. This suggests a similarity in the returns from DY plus cumulative daily return between the Model 3 20n 14DY and Naive. However, the Naive's risk profile is significantly lower than the Model 3 model, indicating that, in this scenario, the Naive portfolio effectively balances risk and return better than the Model 3 model.

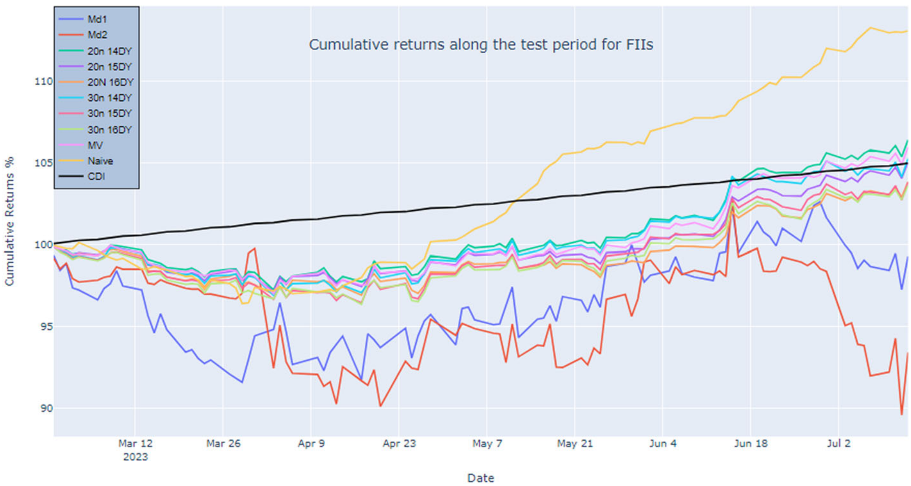

Fig. 3 Out-of-sample cumulative daily returns for FII portfolios. The Naive strategy achieves the highest returns with lowest volatility. Model 3 portfolios maintain higher dividend yields (14–16%) and exhibit higher returns than Model 1 and 2. The horizontal dashed line represents the CDI risk-free rate

6 Conclusion

This paper proposes an integrated OR decision-support framework for portfolio construction and evaluation. The framework combines entropy-weighted TOPSIS for screening and ranking investable assets with a set of portfolio optimization models that progressively incorporate richer risk information and practical implementability constraints. Using three financial products in the Brazilian market (stocks, BDRs, and FIIs) and out-of-sample testing, we show how the proposed workflow translates financial indicators and return data into feasible portfolios with explicit control over diversification, income targets, and risk–return trade-offs.. Importantly, the main contribution is not limited to the Brazilian setting. Rather, it lies in the integration of MCDA and optimization into a single, reproducible pipeline that can be implemented by practitioners: TOPSIS provides a transparent mechanism to curate a consistent investment universe, while the optimization stage produces portfolios that satisfy realistic investor constraints and facilitate direct comparison across strategies. Moreover, an innovative aspect of this research involves the application of TOPSIS, using the entropy of financial variables as algorithm weights, to calculate the TOPSIS score and select the best tickers from each financial product in this work. This innovative approach not only enriches portfolio selection methods but also contributes significantly to the academic sphere, presenting a novel approach to selecting portfolios.

In assessing the performance across the three financial products, our empirical analysis underscores the superiority of the innovative Model 3 model over Model 1 and Model 2 in terms of risk and return metrics. Notably, the Naive strategy consistently outperforms Model 1, Model 2, and at times even Model 3, showcasing the effectiveness of the TOPSIS portfolio selection in diversifying risk and delivering favorable daily returns. This result is consistent with the two-stage design of our framework: TOPSIS curates a financially attractive set of assets, so 1/N diversification can perform strongly on the already-filtered universe. Model 3, by construction, prioritizes implementability through dividend-yield and diversification constraints, which may reduce exposure to growth-oriented assets in some windows. Nevertheless, Model 3 offers clear decision-support value by producing feasible portfolios with explicit income and concentration controls, features that the Naive strategy does not guarantee. Interestingly, this contradicts Gonçalves et al.'s findings (2022), where the performance of Model 2 was superior to the Naive strategy. Conversely, Novais's study (2022) aligns more closely with our results for Model 1 compared to the Naive strategy, although contrasting with our work as the Model 1 model surpassed the MV portfolio in his study, while here the Model 1 portfolio doesn't exhibit noteworthy performance across the financial variables. The commendable performance of the MV and Naive models can be attributed to our innovative portfolio selection method, emphasizing its effectiveness in identifying optimal assets for each financial product.

The Model 3 models exhibit notably strong performance, particularly evident in the stock section, where the Model 3 30n 14DY portfolio boasts a considerable increase in price and higher dividend yield compared to other models. Furthermore, it demonstrates lower risk metrics, specifically in standard deviation and downside risk. In the FIIs section, the results were commendable and akin to the MV model, lagging behind only the Naive strategy in terms of daily returns and risk metrics. In addition, the minimum DY constraint in this scenario for the Model 3 wasn't sufficient to outpace the combination of the total cumulative return and the DY achieved by the Naive strategy during the period.

An essential consideration for Brazilian investors lies in BDR investments, as they typically yield lower dividend yields compared to stocks and FIIs. Moreover, BDRs are notably sensitive to fluctuations in foreign currency, adding an additional layer of risk that investors need to evaluate. Interestingly, within this context, the Model 1 model may demonstrate slightly less unfavorable daily returns compared to Model 3 models. However, this comes at the cost of lower dividend yields and heightened risk, making it a less preferred choice for certain investors.

This study introduces innovative constraints in portfolio optimization, yet several areas remain open for further exploration. One essential consideration for Brazilian investors is the performance of BDR investments, which typically yield lower dividends compared to stocks and FIIs. Moreover, BDRs are notably sensitive to foreign currency fluctuations, adding an additional layer of risk that investors must evaluate. While Model 1 may demonstrate slightly less unfavorable daily returns in this context, this comes at the cost of lower dividend yields and heightened risk, making it a less preferred choice for certain investors.

Despite the contributions of this study, several limitations should be acknowledged. First, the treatment of missing data through complete case deletion may introduce selection bias, as assets with incomplete financial information were excluded from the analysis. Second, the winsorization technique applied to handle outliers at the 2.5th and 97.5th percentiles, though effective in reducing extreme value influence, may introduce bias by artificially constraining the distribution of financial indicators. Third, while this study compares Model 3 with traditional approaches (MV, Naive) and entropy-based models (Model 1 and Model 2), other portfolio optimization techniques were not considered, which could potentially offer different risk-return profiles.

Future research could experiment with different minimum asset allocations beyond the constraints applied in this study. Exploring variations in the number of assets included in portfolio selection—whether increasing or decreasing the 100-ticker threshold—could offer deeper insights into diversification effects. Additionally, an avenue worth exploring involves integrating multiple asset classes into a unified portfolio, rather than analyzing stocks, BDRs, and FIIs separately. This would allow for a more comprehensive assessment of their combined risk-return characteristics.

Another key challenge lies in accounting for evolving market conditions. Currency fluctuations, regulatory changes, and liquidity constraints may significantly impact portfolio performance, requiring adaptive models that adjust dynamically to macroeconomic conditions. Future studies could incorporate robust optimization techniques or machine learning-based asset selection methods to enhance portfolio resilience.

Furthermore, this study assumes a constant dividend yield over the analysis period, which may not accurately reflect real-world variations. A more refined approach could involve forecasting dividend payments and integrating stochastic models to capture changes in dividend policies. Lastly, extending this research to include other financial products available on the B3, as well as transaction costs and taxation implications, would provide a more realistic framework for investment decision-making.

By addressing these challenges and extending the research in these directions, future studies can enhance the applicability and robustness of portfolio optimization models, offering even greater value to investors (See Appendix Figs. 4, 5, 6, 7 and Tables 12, 13).

Appendix A

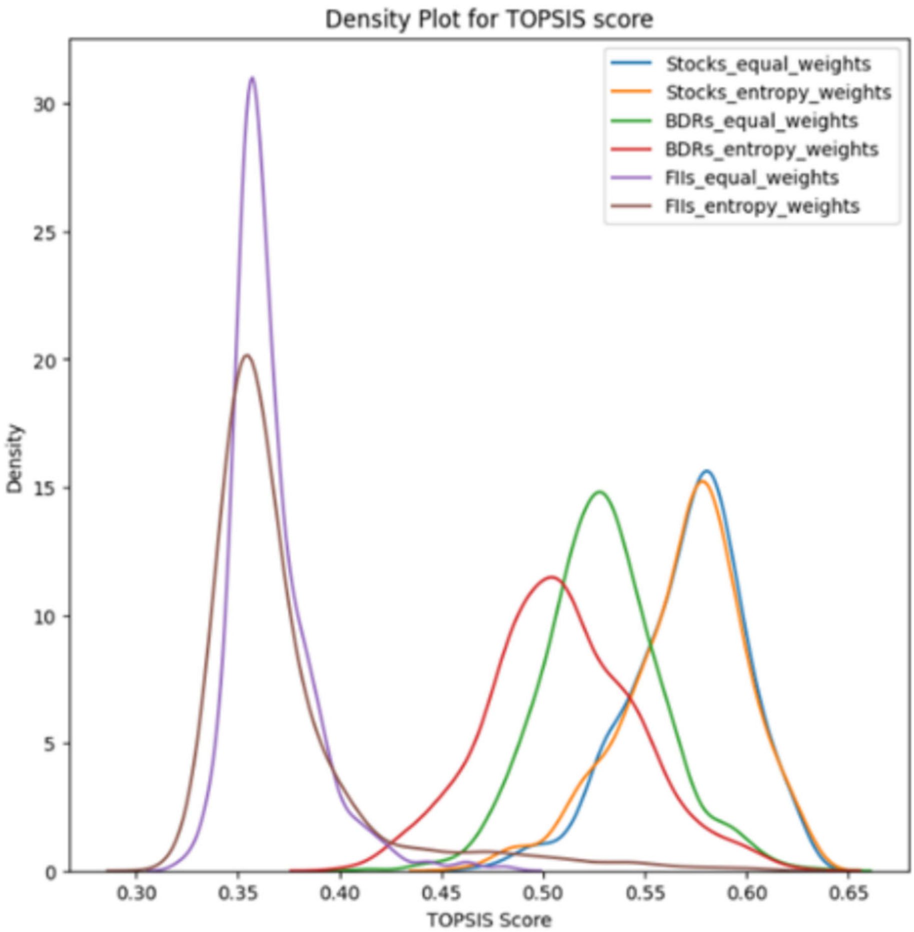

Fig. 4 Density plot for all financial products with equal and entropy weights

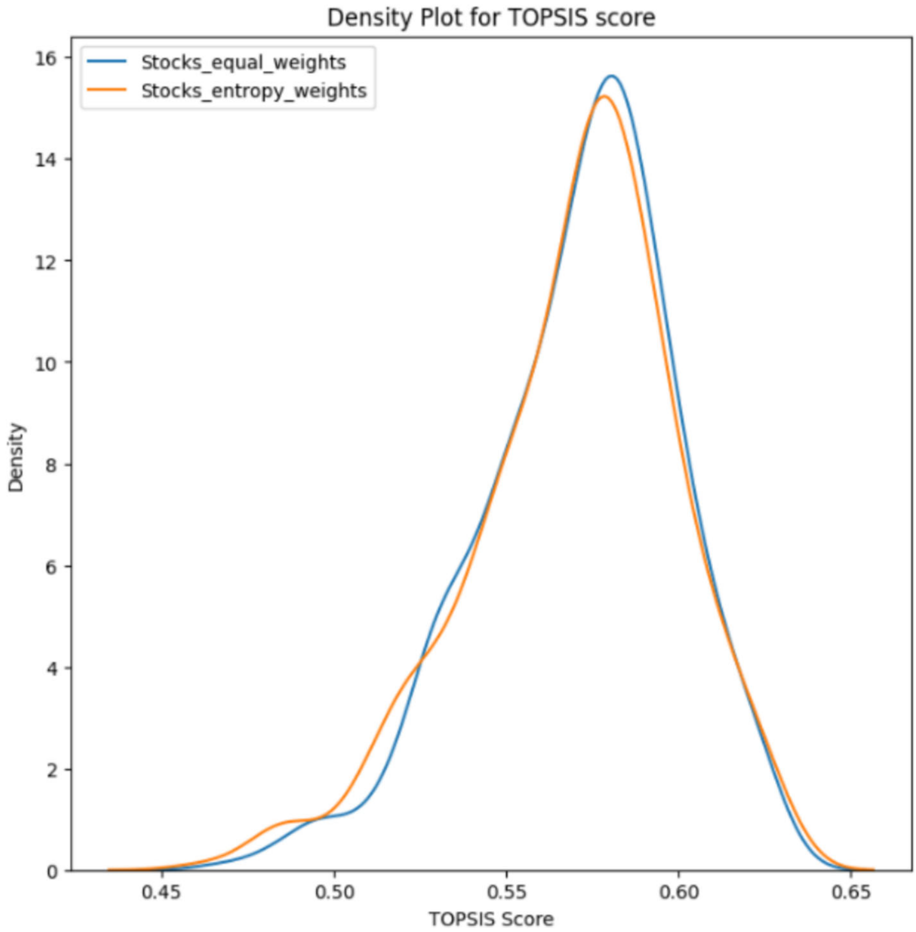

Fig. 5 Density plot for Stocks with equal (blue) and entropy weights (orange)

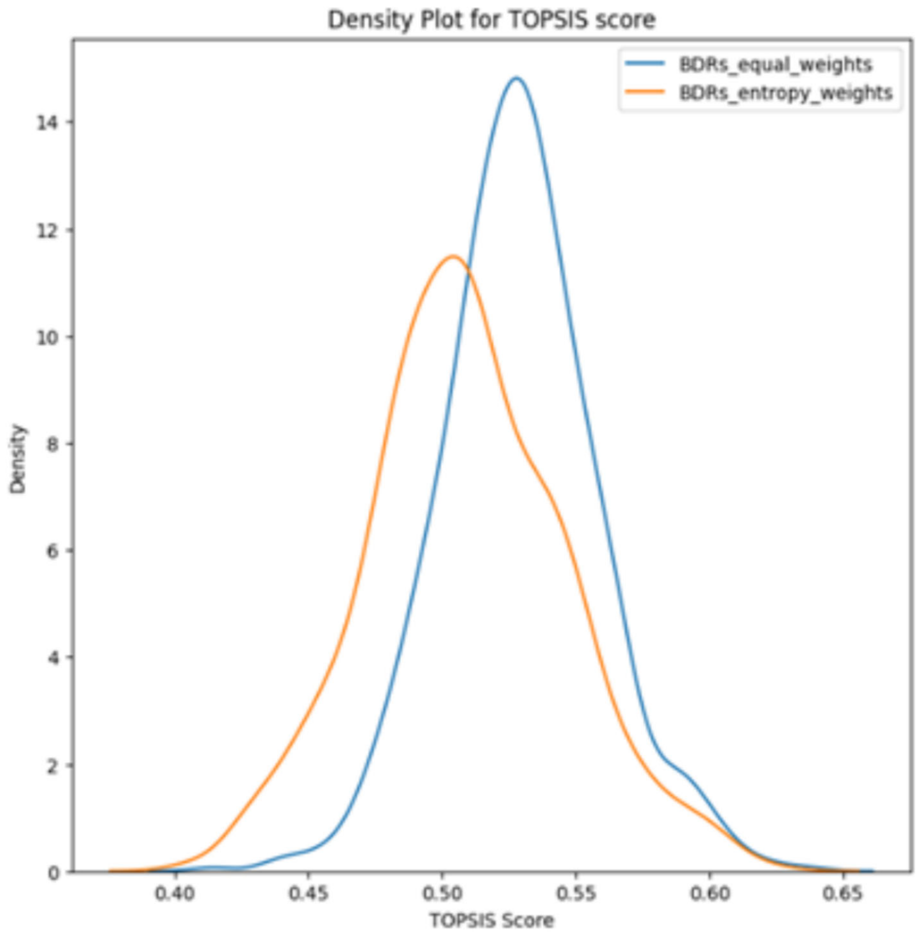

Fig. 6 Density plot for BDRs with equal (blue) and entropy weights (orange)

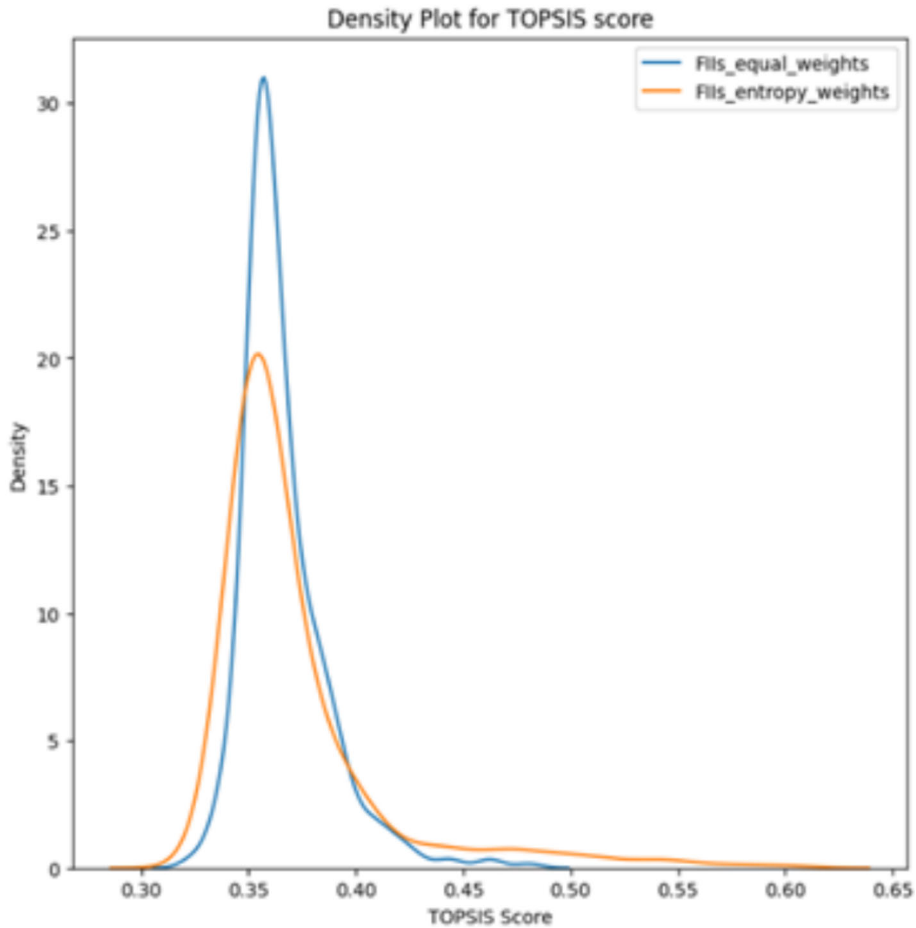

Fig. 7 Density plot for FIIs with equal (blue) and entropy weights (orange)

Table 12 Descriptive statistics for stocks and BDRs, including entropy weights

Symbol	Mean Stocks	Mean BDRs	Std Stocks	Std BDRs	p .25 Stocks	p .50 Stocks	p .75 Stocks	p .25 BDRs	p .50 BDRs	p .75 BDRs	E.w Stocks	E.w BDRs
P / E	7.07	8.79	20.77	21.40	0.00	2.50	11.47	0.00	0.00	14.22	0.0340	0.0286
PEG RATIO	– 0.01	–	0.56	–	0.00	0.00	0.00	–	–	–	0.0240	–
P / BV	1.25	2.63	1.97	5.97	0.00	0.65	1.68	0.00	0.00	2.35	0.0380	0.0215
EV / EBITDA	3.47	12.83	7.40	30.08	0.00	0.00	5.74	5.65	11.89	18.55	0.0310	0.0305
EV / EBIT	6.12	16.39	15.51	39.21	0.00	2.35	9.91	7.33	15.73	25.66	0.0340	0.0333
P / EBITDA	3.18	5.42	6.60	11.18	0.00	0.00	4.41	0.00	0.00	7.34	0.0310	0.0263
P / EBIT	5.83	6.92	14.48	15.60	0.00	2.80	8.40	0.00	0.00	10.53	0.0350	0.0285
EPS	12.78	25.89	31.57	27.64	0.71	7.46	18.27	7.35	17.46	34.88	0.0330	0.0399
P / ASSET	0.53	0.85	0.82	1.67	0.00	0.23	0.69	0.00	0.00	0.94	0.0330	0.0249
BVPS	0.62	3.69	6.94	5.03	0.00	0.51	2.04	0.72	2.63	5.23	0.0300	0.0403
P / SR	1.82	1.82	3.96	3.46	0.00	0.50	1.57	0.00	0.00	2.20	0.0220	0.0258
P / Working Capital	2.51	4.49	10.92	22.27	0.00	0.45	3.89	0.00	0.00	3.08	0.032	0.0220
P / Net Current Assets	– 1.05	– 1.68	1.74	3.85	– 1.17	– 0.39	0.00	– 1.34	0.00	0.00	0.0309	0.0212
Net Debt / Equity (%)	0.55	0.44	1.21	1.30	0.00	0.13	0.74	– 0.15	0.28	0.87	0.0366	0.0394
Net Debt / EBITDA	0.77	0.68	3.21	6.30	0.00	0.00	1.80	– 0.11	1.18	3.07	0.0336	0.0327
Net Debt / EBIT	0.93	1.60	9.75	7.32	– 0.55	0.86	3.55	– 0.32	1.58	4.27	0.0333	0.0370
Equity / Assets	0.21	0.34	0.60	0.22	0.13	0.32	0.51	0.19	0.35	0.51	0.0378	0.0471
Liabilities / Assets	0.78	0.64	0.62	0.22	0.47	0.65	0.86	0.49	0.65	0.80	0.0376	0.0474
Current Ratio	1.88	1.62	1.99	1.40	0.85	1.39	2.20	0.81	1.31	2.03	0.0374	0.0404
Gross Margin (%)	32.76	49.87	25.53	26.89	17.08	29.01	46.21	30.40	46.94	69.94	0.0466	0.0482
EBITDA Margin (%)	8.34	21.37	27.92	21.25	0.00	3.14	18.58	9.33	20.16	33.71	0.0331	0.0439
EBIT Margin (%)	8.85	14.22	45.87	20.05	1.53	10.21	20.21	6.63	14.75	24.48	0.0331	0.0414
Net Margin (%)	1.03	8.81	55.69	18.05	– 0.79	5.45	13.74	3.90	10.14	18.01	0.0292	0.0393
ROE (%)	– 2.14	10.08	47.54	36.26	– 4.00	7.32	16.99	3.94	11.87	22.95	0.0342	0.0351
ROA (%)	0.88	4.65	12.23	8.35	– 0.79	2.40	6.73	1.07	4.46	9.08	0.0407	0.0415
ROIC (%)	4.91	9.33	23.23	13.70	0.00	6.40	12.49	3.74	8.85	15.78	0.0341	0.0413
Asset Turnover	0.56	0.60	0.45	0.49	0.17	0.49	0.80	0.26	0.50	0.78	0.0474	0.0426
CAGR Revenue 5 years (%)	4.99	6.88	13.57	11.94	0.00	4.51	11.86	0.00	3.87	10.48	0.0437	0.0402
CAGR Profit 5 years (%)	6.06	7.60	17.87	15.41	0.00	0.00	8.68	0.00	1.55	13.80	0.0339	0.0382

Table 13 Descriptive statistics for FIIs, including entropy weights

Symbol	Mean (10^4) FIIs	Std (10^4) FIIs	p .25 (10^4) FIIs	p .50 (10^4) FIIs	p .75 (10^4) FIIs	E.w FIIs
N° investors	2.58	3.94	0.2920	0.76	3.07	0.0536
N° shares	609	1040	69.00	192	640	0.0491
Assets	47,200	65,100	8403	20,215	53,017	0.0610
Net worth	42,400	59,000	7572	18,318	47,977	0.0608
Net asset / shares	0.014	0.0181	0.0082	0.0098	0.11	0.0488
Total liquidity	24.70	78.6	0.2	1.04	4.96	0.0230
Cash	583	1840	0	0	62	0.0240
G bonds	4.77	22.3	0	0	0	0.0096
P bonds	43,900	61,500	7455	18,140	48,540	0.0606
Total invested	9360	29,800	0	0	0	0.0229
Assets and rights	87.20	408	0	0	0	0.0096
Lands	22,000	41,900	0	190	22,698	0.0486
Finished IP	148	787	0	0	0	0.0069
IP under construction	126	665	0	0	0	0.0076
FP for sale	24.1	139	0	0	0	0.0049
Other assets and rights	12.6	66.9	0	0	0	0.0063
Shares	4230	11,400	0	0	1021	0.0297
FII	1480	4770	0	0	0	0.0221
Shares of Companies for FIIs	60.8	338	0	0	0	0.0051
Stocks of Companies for FIIs	8370	23,500	0	0	2931	0.0293

Table 13 (continued)

Symbol	Mean (10^4) FIIs	Std (10^4) FIIs	p .25 (10^4) FIIs	p .50 (10^4) FIIs	p .75 (10^4) FIIs	E.w FIIs
CRI	15.3	84.3	0	0	0	0.0053
LCI	5660	11,100	85	1033	4584	0.0438
Receivables	2070	4360	0	0	1736	0.0431
Rental receivables	557	2430	0	0	0	0.0116
Receivables PS	2370	6140	18	184	1421	0.0316
Other receivables	2850	4890	43	870	2893	0.0525
Payables	2270	4510	170	504	1820	0.0427
Distributable income	40,600	89,900	818	3653	26,429	0.0381
Administrative FP	243	416	30	79	206	0.0498
Performance FP	117	427	0	0	0	0.0173
Obligations PA	8930	28,100	0	0	0	0.0233
Obligations RS	8700	34,400	0	0	0	0.0143
Provisions	46.7	219	0	0	0	0.0109
Other payables	9430	24,600	86	626	4809	0.0306

Appendix B

Ticker	Md1	Md2	20n 3DY	20n 3.5DY	20n 4DY	30n 3DY	30n 3.5DY	30n 4DY	MV	C_j^+
REGN34	0	0	0,01	0	0	0,01	0	0	0,014	0,615
GOGL35	0	0	0	0	0	0	0	0	0	0,604
GOGL34	0	0	0	0	0	0	0	0	0	0,604
L1RC34	0	0	0	0	0	0	0	0,01	0	0,601
A1TH34	0	0	0	0	0	0	0	0	0	0,593
NETE34	0	0	0	0	0,01	0	0,01	0,01	0	0,593
A1NE34	0,015	0	0,01	0,01	0	0,01	0,01	0	0,039	0,590
GSGI34	0	0	0	0	0,01	0	0,01	0,01	0	0,588
M1NS34	0	0	0	0	0	0	0	0	0	0,587
N1VO34	0	0	0,035	0,01	0,01	0,01	0,01	0,01	0,094	0,585
M1UF34	0,005	0	0,087	0,084	0,038	0,08	0,066	0,021	0,076	0,576
BIDU34	0	0	0	0	0	0	0	0	0	0,574
TEXA34	0	0	0	0	0	0	0	0	0	0,573
NVDC34	0	0	0	0	0	0	0	0	0	0,573
I1SR34	0	0	0	0	0	0	0	0	0	0,573
BABA34	0	0	0	0	0	0	0	0	0	0,572
BONY34	0	0	0	0	0	0	0	0	0	0,571
U1LT34	0,08	0	0,035	0,01	0	0,029	0,01	0,01	0,073	0,571
BOAC34	0	0	0	0	0	0	0	0	0	0,569
WFCO34	0	0	0	0	0	0	0	0	0	0,566
BKNG34	0	0	0	0	0	0	0	0	0	0,563
K1LA34	0	0	0	0	0	0	0	0	0	0,563
INTU34	0	0	0	0	0	0	0,01	0,01	0	0,563
MSCD34	0	0	0	0	0	0	0	0	0	0,560
C1GP34	0	0	0	0	0	0	0	0	0	0,560
TJXC34	0	0	0,01	0	0	0,01	0,01	0	0	0,559
BCSA34	0	0	0	0	0	0	0	0	0	0,556
BLAK34	0	0	0	0	0	0	0	0	0	0,556
MUTC34	0	0	0	0	0	0	0	0	0	0,554
JPMC34	0	0	0	0	0	0	0	0	0	0,553
USBC34	0	0	0	0	0	0	0	0	0	0,553
SPGI34	0	0	0	0	0	0	0,01	0,01	0	0,552
SCHW34	0	0	0	0	0	0	0	0	0	0,552
TXSA34	0	0	0	0	0,01	0,01	0,01	0,01	0	0,551
GILD34	0	0	0,016	0,01	0,01	0,01	0,01	0,01	0,003	0,549
T1AL34	0	0	0	0	0	0,01	0	0	0,002	0,549
N1UE34	0,152	0	0,161	0,194	0,244	0,154	0,188	0,241	0,102	0,548
A1MT34	0	0	0	0	0	0	0	0,01	0	0,547
P1KX34	0	0	0	0	0	0	0	0	0	0,547

Ticker	Md1	Md2	20n 3DY	20n 3.5DY	20n 4DY	30n 3DY	30n 3.5DY	30n 4DY	MV	C_j^+
EAIN34	0	0	0	0	0	0	0	0	0	0,547
V1IP34	0	0	0	0	0	0	0	0	0	0,546
ASML34	0	0	0	0	0	0	0	0,01	0	0,546
TSMC34	0	0	0	0	0	0	0	0	0	0,545
RIOT34	0	0	0	0	0	0	0	0,01	0	0,545
PAGS34	0	0	0	0	0	0	0	0	0	0,542
MSFT34	0	0	0	0	0	0	0	0	0	0,542
ITLC34	0	0	0	0	0	0	0	0	0	0,542
AVGO34	0,03	0	0,039	0,042	0,039	0,034	0,034	0,028	0,022	0,541
A1VB34	0	0	0	0	0	0	0	0	0	0,541
DGCO34	0	0	0,01	0,01	0,01	0,01	0,01	0,01	0	0,540
ADBE34	0	0	0	0	0	0	0	0	0	0,540
AMGN34	0	0	0	0	0	0	0	0	0	0,540
C1NS34	0	0	0	0	0	0	0	0	0	0,539
VRTX34	0	0	0,01	0,01	0	0,01	0,01	0	0,104	0,539
E1DU34	0,009	0	0	0	0	0	0	0	0	0,539
AAPL34	0	0	0	0	0	0	0	0	0	0,539
STZB34	0	0	0	0	0	0	0	0	0	0,539
S1KM34	0	0	0,027	0,031	0,032	0,024	0,027	0,026	0,009	0,538
L1CA34	0	0	0	0	0	0	0	0	0	0,537
ROST34	0	0	0	0	0	0	0	0	0	0,537
VISA34	0	0	0	0	0	0	0	0	0	0,536
ATVI34	0	0	0	0	0	0	0	0	0	0,535
B1TI34	0	0	0,01	0,01	0,01	0,01	0,01	0,01	0	0,534
CTGP34	0	0	0	0	0	0	0	0	0	0,534
ULEV34	0	0	0	0	0	0,01	0,01	0,01	0	0,532
C1BS34	0	0	0	0	0	0	0	0	0	0,531
E1OG34	0,132	0,279	0,114	0,15	0,235	0,11	0,153	0,232	0,049	0,531
F1TN34	0	0	0	0	0	0,01	0	0	0	0,530
S1HW34	0	0	0	0	0	0	0	0	0	0,529
UNHH34	0	0	0	0	0	0,01	0,01	0	0	0,529
H1SB34	0	0	0,068	0,076	0,08	0,048	0,051	0,047	0,018	0,528
A1AP34	0	0	0	0	0	0	0	0	0	0,527
CSCO34	0	0	0	0	0	0	0	0	0	0,526
MMMC34	0	0	0	0	0	0	0	0	0	0,525
QCOM34	0	0	0	0	0	0	0	0	0	0,521
TSNF34	0	0	0	0	0	0	0	0	0	0,520
VLOE34	0,207	0,721	0,146	0,135	0,053	0,148	0,119	0,055	0,177	0,520
ORCL34	0	0	0	0	0	0	0	0	0	0,520
JNJB34	0	0	0	0	0	0	0	0	0	0,520
T1TW34	0	0	0	0	0	0	0	0	0	0,519
E1QR34	0	0	0	0	0	0	0	0	0	0,519

Ticker	Md1	Md2	20n 3DY	20n 3.5DY	20n 4DY	30n 3DY	30n 3.5DY	30n 4DY	MV	C_j^+
HOND34	0	0	0	0	0	0	0	0	0	0,518
ELCI34	0	0	0	0	0	0	0	0	0	0,518
ARMT34	0	0	0	0	0	0	0	0	0	0,517
P1LD34	0	0	0	0,01	0,01	0,01	0,01	0,01	0	0,517
A1LB34	0,077	0	0,061	0,09	0,122	0,057	0,081	0,109	0,002	0,516
AXPB34	0	0	0	0	0	0	0	0	0	0,516
O1MC34	0	0	0	0	0	0,01	0,01	0,01	0	0,516
FSLR34	0,23	0	0,051	0,013	0,01	0,042	0,01	0,01	0,102	0,516
L1YG34	0	0	0	0	0,01	0,01	0,01	0,01	0	0,515
NIKE34	0	0	0	0	0	0	0	0	0	0,513
S2HO34	0	0	0	0	0	0	0	0	0	0,513
PHMO34	0	0	0	0	0	0	0	0	0	0,512
UBSG34	0	0	0,01	0,01	0	0,01	0,01	0	0,033	0,512
COWC34	0	0	0	0	0	0,01	0	0	0	0,512
E1QN34	0,063	0	0,09	0,084	0,047	0,084	0,07	0,031	0,08	0,512
VERZ34	0	0	0	0	0	0	0	0,01	0	0,511
S1BS34	0	0	0	0	0	0	0	0	0	0,511
B1SA34	0	0	0	0,01	0,01	0,01	0,01	0,01	0	0,511
Z1TS34	0	0	0	0	0	0	0	0	0	0,510
Total assets:	11	2	20	20	20	30	30	30	18	–

Appendix B

We conducted a comprehensive benchmark analysis to assess the scalability and solver runtime of the dividend-yield-constrained portfolio (Model 3) against the baseline Markowitz model.

In the benchmark procedure, we evaluated both methods across 96 logarithmically spaced portfolio sizes ranging from 10 to 1,000 assets, using synthetic return matrices with 1,000 observations. Each portfolio size was tested 100 times to ensure statistical reliability, with both models using identical return matrices for each configuration. Figure 8 presents median execution times versus portfolio size on a log-log scale, with reference lines indicating O(n), $O(n^2)$, and $O(n^3)$ complexity.

The results reveal three key findings. First, both methods exhibit parallel scaling behavior across all tested configurations, confirming identical asymptotic complexity. Second, for portfolios exceeding 300 assets, both methods approach $O(n^2)$ scaling as expected for quadratic programming solvers. Third, for smaller portfolios ($n < 100$), the observed complexity approaches O(n), reflecting the dominance of fixed computational overhead including matrix allocation and solver initialization.

Model 3 demonstrates median execution times consistently 2− 5% faster than the baseline Markowitz across all tested configurations. This result occurs because the dividend yield

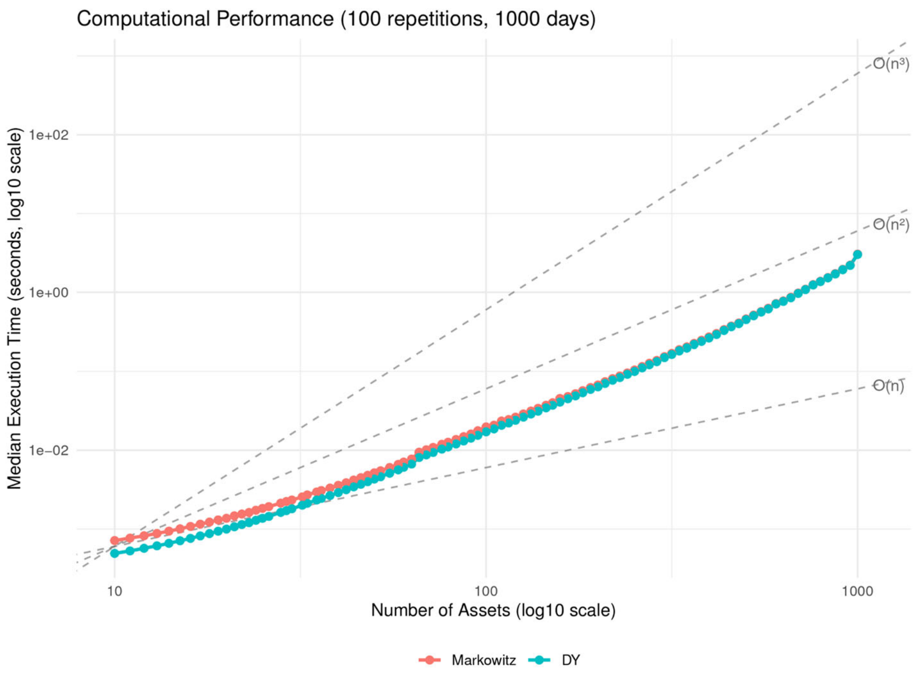

Fig. 8 Solver execution time versus portfolio size for Markowitz and Model 3. Both methods exhibit O(n^2) complexity for large portfolios. Model 3 shows 2–5% faster convergence due to reduced feasible region from DY constraints

equality constraint $\left(\sum w_i dy_i = dy_{obj}\right)$ reduces the feasible region, occasionally enabling faster solver convergence without altering the fundamental optimization complexity.

Appendix C

To validate the robustness of Model 3 across different asset selection methodologies, we conducted a comparative analysis using four MCDM techniques: TOPSIS, COPRAS (Complex Proportional Assessment), VIKOR (VlseKriterijumska Optimizacija I Kompromisno Resenje), and WASPAS (Weighted Aggregated Sum Product Assessment) (Opricovic & Tzeng, 2004; Zavadskas et al., 1994, 2012). Following the same data collection and preprocessing procedures described in Sect. 4, we applied each method to select the top 25 assets for stocks, BDRs, and FIIs based on their financial indicators. The selected tickers for each method and asset type are presented in Table 14. A simplified Model 3 portfolio with a minimum of 25 assets and a 1% dividend yield constraint was then optimized for each selection method, alongside traditional portfolio models (minimum variance, mean-entropy, mean-variance-skewness-kurtosis, and naive) for comparison. A 30-day out-of-sample backtest was performed covering the period from 2022 to October 3, 2025.

Table 15 presents the returns across different portfolio optimization models and asset selection methods. The results demonstrate that Model 3 with 1% DY constraint consistently delivers competitive performance across all three asset types and all four MCDM selection methods. For stocks, Model 3 achieves returns ranging from 0.53% to 5.70% depending

Table 14 Selected tickers by MCDM method for comparative analysis (25 assets per product type)

stock				bdr				fii			
copras	topsis	vikor	waspas	copras	topsis	vikor	waspas	copras	topsis	vikor	waspas
alld3	alos3	bmgb4	alos3	a1mt34	a1mt34	a1md34	a1md34	brco11	brco11	brco11	brco11
amer3	arml3	cash3	arml3	a1ne34	a1ne34	a1mt34	a1mt34	btlg11	brcr11	brcr11	brcr11
bbas3	bbas3	csud3	bbas3	b1nt34	aapl34	a1ne34	a1ne34	cpts11	btlg11	btlg11	btlg11
bbdc4	bmgb4	cyre3	bees4	baba34	adbe34	adbe34	adbe34	fiip11	cpts11	cpts11	cpts11
bmgb4	cash3	embr3	bhia3	bcsa34	asml34	airb34	avgo34	hgbs11	hctr11	hctr11	hctr11
cash3	cyre3	flry3	brap4	berk34	bchi39	asml34	b1nt34	hglg11	hgbs11	hgbs11	hgbs11
cyre3	desk3	g2di33	cash3	biau39	bdvy39	avgo34	bdvy39	hgru11	hglg11	hglg11	hglg11
desk3	eter3	gfsa3	cbav3	bivb39	berk34	b1nt34	berk34	hsml11	hgre11	hgre11	hgru11
embr3	eztc3	ggbr4	desk3	boac34	biau39	bcsa34	biau39	itip11	hgru11	hgru11	hsml11
eter3	fict3	hype3	ecor3	chcm34	catp34	biau39	deec34	kncr11	hsml11	hsml11	irdm11
fict3	jhsf3	jhsf3	fict3	chvx34	chcm34	deec34	dher34	knhy11	irdm11	irdm11	kncr11
g2di33	lavv3	lvtc3	ggbr4	coph34	deec34	itlc34	itlc34	knip11	kncr11	kncr11	knhy11
gfsa3	orvr3	meal3	guar3	deec34	dher34	l1mn34	lily34	knri11	knip11	knip11	knip11
jhsf3	pdgr3	neoe3	jfen3	dher34	fdmo34	m1ns34	m1ns34	mxrf11	knri11	knri11	knri11
logn3	petz3	petz3	jhsf3	geoo34	geoo34	mrck34	meli34	rbva11	mxrf11	lvbi11	mxrf11
meal3	plpl3	show3	lavv3	itlc34	itlc34	msbr34	mrck34	rztr11	pqdp11	mxrf11	pvbi11
mult3	rapt4	smft3	orvr3	jpmc34	jpmc34	mscd34	mscd34	scpf11	pvbi11	pvbi11	rbrf11
pdgr3	romi3	tasa4	plpl3	msbr34	m1ns34	n1da34	n1da34	tgar11	recr11	recr11	recr11
rani3	smft3	tpis3	rani3	n1da34	meli34	orcl34	orcl34	trxf11	shph11	rztr11	rztr11
smft3	tgma3	trad3	simh3	orcl34	mrck34	pypl34	pypl34	urpr11	tgar11	tgar11	tgar11

Table 14 (continued)

stock				bdr				fii			
copras	topsis	vikor	waspas	copras	topsis	vikor	waspas	copras	topsis	vikor	waspas
tpis3	tpis3	tupy3	smft3	pags34	n1da34	sbub34	riot34	vghf11	trxf11	trxf11	trxf11
tupy3	tupy3	ucas3	taee11	simn34	orcl34	snec34	sbub34	vilg11	vghf11	vilg11	vghf11
viva3	ugpa3	ugpa3	tpis3	snec34	t1al34	t1al34	t1al34	visc11	visc11	visc11	visc11
vivt3	vamo3	vamo3	tupy3	t1al34	tlnc34	tlnc34	tlnc34	xplg11	xplg11	xplg11	xplg11
vveo3	vveo3	vitt3	vivr3	u1be34	u1be34	u1be34	u1be34	xpml11	xpml11	xpml11	xpml11

Table 15 Backtest returns (30 days) comparing portfolio models across MCDM selection methods. Model 3 demonstrates robust performance regardless of asset selection technique

asset type	model	copras	topsis	vikor	waspas
stock	min_var	0.53%	1.65%	2.07%	5.70%
	mi	– 57.14%	– 57.96%	– 3.98%	6.51%
	dy_0.01	0.53%	1.65%	2.07%	5.70%
	mvsk	1.08%	4.01%	4.13%	6.59%
	naive	1.14%	2.41%	– 1.72%	9.50%
bdr	min_var	4.87%	3.25%	2.84%	3.31%
	mi	7.40%	5.68%	8.52%	3.56%
	dy_0.01	4.06%	0.73%	0.11%	2.60%
	mvsk	10.26%	9.58%	10.35%	9.90%
	naive	6.24%	6.12%	5.94%	3.46%
fii	min_var	2.59%	2.50%	2.77%	2.46%
	mi	3.93%	1.06%	3.81%	3.13%
	dy_0.01	2.59%	2.50%	2.77%	2.46%
	mvsk	3.13%	1.42%	3.11%	3.11%
	naive	3.83%	2.92%	3.78%	2.94%

on the selection method, outperforming the mean-entropy model in all cases and showing comparable or superior performance to minimum variance portfolios. For BDRs, Model 3 demonstrates robust performance with returns between 0.11% and 4.06%, maintaining lower risk profiles compared to more complex models like MVSK. In FIIs, Model 3 consistently achieves positive returns (2.46% to 2.77%) across all selection methods, demonstrating stability in income-focused portfolios. Notably, Model 3 performs well regardless of the MCDM method used for asset selection, indicating that the dividend yield constraint and minimum asset diversification requirements provide robust portfolio construction principles that are not overly dependent on the specific ranking methodology

Appendix D

To demonstrate the international applicability of the proposed portfolio optimization framework, we conducted a case study using the Magnificent Seven technology companies: Apple (AAPL), Microsoft (MSFT), Alphabet (GOOGL), Amazon (AMZN), NVIDIA (NVDA), Meta Platforms (META), and Tesla (TSLA). Historical price and dividend data were obtained from Yahoo Finance covering the period from January 2022 to October 2025, with out-of-sample testing with 30 trading days. Given the characteristically low dividend yields of technology growth stocks (mean DY < 0.5%), the dividend yield constraint for the Model 3 approach was set at 0.1%. Figure 9 presents the cumulative returns for all portfolio models during both in-sample and out-of-sample periods. The results demonstrate that the dividend yield-constrained model (dy_0.001) achieves competitive performance, with approximately 9.5% cumulative return in the out-of-sample period, comparable to the mutual information (mi), minimum variance (min_var), MVSK, and naive portfolios. Notably, all models

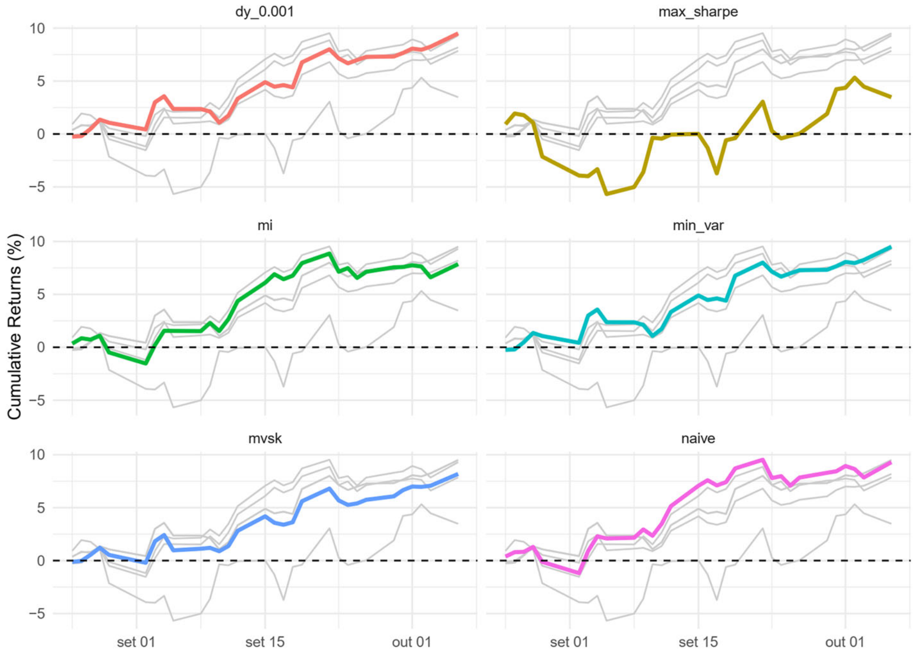

Fig. 9 Out-of-sample cumulative returns for Magnificent Seven case study. Model 3 achieves competitive performance (approximately 9.5% cumulative return) despite the low dividend yield environment, confirming international applicability. All models except maximum Sharpe show positive returns

except maximum Sharpe exhibited stable positive returns, with the dividend yield-constrained approach showing consistent growth and lower volatility compared to the maximum Sharpe strategy. These findings validate that the proposed methodology is not limited to the Brazilian market context and can be effectively applied to international equity portfolios, even in low-dividend-yield environments such as U.S. technology stocks

Funding The authors did not receive support from any organization for the submitted work.

Declarations

Conflict of interest Guilherme Pires declares that he has no conflict of interest. Peter Wanke declares that he has no conflict of interest. Jorge Antunes declares that he has no conflict of interest. Yong Tan declares that he has no conflict of interest. Jorge Antunes declares that he has no conflict of interest. Nickolaos Tzeremes declares that he has no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

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