Investigation into the effect of friction decay factor on the modelling and attenuation of stick-slip vibrations of oilwell drilling systems.

Item Type	Article
Authors	Alkaragoolee, M.Y.A.;Bryant, David
Citation	Alkaragoolee, MYA, Bryant D (2021) Investigation into the effect of friction decay factor on the modelling and attenuation of stick-slip vibrations of oilwell drilling systems. Petroleum. 8(3): 344-351.
DOI	https://doi.org/10.1016/j.petlm.2021.06.005
Rights	© 2022 Southwest Petroleum University. This is an Open Access article distributed under the Creative Commons CC-BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/)
Download date	2025-04-23 12:39:40
Link to Item	https://bradscholars.brad.ac.uk/handle/10454/20268

Petroleum 8 (2022) 344–351

Contents lists available at ScienceDirect

Petroleum

journal homepage: www.keaipublishing.com/en/journals/petlm

Investigation into the effect of friction decay factor on the modelling and attenuation of stick-slip vibrations of oilwell drilling systems

Mohammed Y.A. Alkaragoolee [a, *], David Bryant [b]

[a] Technical Institute Baqubah, Middle Technical University, Iraq
[b] Faculty of Engineering & Informatics, University of Bradford, UK

ARTICLE INFO

Article history:
Received 26 August 2020
Received in revised form
2 February 2021
Accepted 24 June 2021

Keywords:
Stribeck effect
Stick-slip vibration
Drillstring
Distributed-lumped parameters model

ABSTRACT

The self-excited stick-slip oscillations of oilwell drillstrings are attributed to the nonlinear interaction between the drill-bit and the rock formation. Development of more accurate models will lead to improved predictions allowing more potential for successful suppression of the drillstring vibrations, thus reducing damage to the drilling system, prevention of expensive failures and increased output from the oilwell. In this paper, the effect of the transition from static friction to Coulomb friction on modelling of stick-slip phenomenon of oil well drill string is investigated through an analysis of the so called 'decay factor'. Based on a distributed-lumped parameter model (DLPM) of the drilling system, the governing equations of motion for the system are obtained. By using different values of decay factor (low, high and medium), the stick-slip vibrations of the drill string are validated against published data from full-scale drill strings. The results from the simulation show that lowering the decay factor increases the critical speed and thus reduces the propensity for stick slip motion. However, a reduction in the decay factor also has the effect of inducing worse stick-slip motion once the critical speed has been reached. The results indicate the wider impact of both correct modelling of the decay factor, but also the importance of correct characterisation of the mud viscosity and drill/well contact for more accurate selection of drilling parameters in the field.

1. Introduction

Drillstring vibrations are considered to be the leading cause of loss of performance in the oil drilling industry and can lead to problems such as premature wear of the drill bit, fracture of the drilling equipment due to fatigue and induced failures such as pipe wash-out and twist-off [1]. A significant waste of drilling energy [2] and induced borehole instabilities which reduce the directional control of the drillstring [3] are additional issues caused by these vibrations.

Many studies have been conducted to identify, characterise and describe the drillstring vibrations during drilling operations and these have led to the identification and classification of vibrations depending upon their direction into three primary modes: torsional (stick-slip), longitudinal (bit bouncing) and lateral (whirling) modes [4,5]. The initiation of these vibration modes can be related to the interaction between the drill-bit and rock formation during cutting, and also the contact between the drillstring (drillpipe, drillcollar, and stabilisers) and the wall of the wellbore. In addition, bent and misaligned drillstrings are also known to cause drillstring vibrations [6].

At lower speeds, stick-slip vibration is considered to be the main reason for torsional vibrations and results in most damage when compared with the bouncing and whirling modes of vibration [7–9]. Stick-slip vibration in an oil drilling operation is defined as the cyclic reduction and corresponding increase of the instantaneous angular velocity of part or parts of the drillstring [10]. This vibration occurs due to the nonlinear interaction between the bit/formation and the drillstring/borehole [11], which leads to the angular velocity of the bottom hole assembly (BHA) falling to zero (sticking) for a finite time interval and then subsequently slipping,

---

* Corresponding author.
E-mail address: mohammedabd84@mtu.edu.iq (M.Y.A. Alkaragoolee).
Peer review under responsibility of Southwest Petroleum University.

Production and Hosting by Elsevier on behalf of KeAi

https://doi.org/10.1016/j.petlm.2021.06.005

Nomenclature

C_{ds}, C_{bh}	Equivalent viscous damping of drive system and BHA (Nms/rad)
d_i,d_p,d_o,d_p	Inner and outer diameter of the drillpipe (mm)
G	Shear modulus of steel (N/m^2)
J_m, J_{rt}	Mass moment of inertia of motor and rotary table kg•m^2
J_{ds}, J_{bh}	Equivalent mass moment of inertia of the drive system and BHA (kg·m^2)
l_{dp}, l_{dc}, l_{hw}	Length of drillpipe, drillcollar and HWDP (m)
n	A combined gear ratio of gearbox and bevel gear
R_b	Radius of the bit (m)
t_{dp}	Delay time at drillpipe (s)
T_m, T_{rt}	Torque of motor and rotary table (Nm)
T_{fb}, T_{ab}	Friction torque and applied torque on the bit (Nm)
T_{sb}, T_{cb}	Static and Coulomb friction torque on the bit (Nm)
W_{Ob}	Weight on bit (WOB) (N)
Dω	Limit velocity interval (rev/min)
γ_b	Decay factor
Γ_{dp}	Propagation constant of the drillpipe (s/m)
ξ_{dp}	Characteristic impedance of drillpipe (Nms)
μ_b	Friction coefficient at the bit
μ_{cb}, μ_{sb}	Coulomb and static friction coefficient
θ_m, θ_{rt}, θ_b	Angular displacement of motor, rotary table and bit (rad)
ρ	Density of steel (kg/m^3)
ω_{rt}, ω_b	Angular velocities of rotary table and bit (rev/min)
$T_{1,dp}$, $T_{2,dp}$	Torque at the inlet and outlet of the drillpipe (Nm)
$\omega_{1,dp}$, $\omega_{2,dp}$	Angular velocities at the inlet and outlet of the drillpipe (rev/min)

at which point the angular velocity of the BHA can exceed the imposed velocity by two to three times. This vibration typically continues for several minutes and in some instances can be significantly longer in duration [12].

The period of stick-slip oscillation depends on many factors such as the length of the drillpipe, speed of the rotary table and the nature and location of friction, and can appear in up to 50% of the drilling operation duration [13–15]. It has been observed that the stick-slip vibration occurs mostly with low angular velocity and a significant weight on the bit [8,16–18], typically during conditions where the Stribeck effect is most predominant, and can be attributed to the large difference between static and dynamic friction coefficients which leads to a transfer of the stored energy in the drillpipe to inertial energy in the BHA and subsequent increase in the rotational speed of the BHA following the ‘stick’ period [13]. Some researchers have attributed the stick-slip vibration to the mechanical structure of the bit and type of bit, since stick-slip is more predominant with polycrystalline diamond compact (PDC) bits [13], whilst other authors have related stick-slip to the size of the bit because with an increase in bit size leads to an increase in the reactive torque on the bit [19]. Despite the fact that stick-slip is known to be directly associated with the bit/rock interaction, other factors such as the condition and tortuosity of the wellbore, the type of formation and the lubricity of the drilling interface have a significant influence on the occurrence of stick-slip [12].

Various published articles have highlighted the undesirable nature of stick-slip in oil production processes and these can be summarised as follows:

(1) Reduction in the rate of penetration (ROP) in the drilling operation [12,20].
(2) Increased cost of drilling [21–24].
(3) Affect on borehole quality [24].
(4) Fatigue problems in the drillstring connection [15,22].
(5) Failure of the components of the BHA [22].
(6) Wellbore instability, leading to collapse in worst cases [25,26].
(7) Excessive bit wear [27–30].
(8) Decrease in the accuracy of measurement while drilling (MWD) [31].
(9) Decrease in drilling efficiency [30].

Therefore, due to these problems, drilling companies are continuously working to utilize new methods and technologies to better understand the stick-slip mechanism, the causes, and the methods that are used to suppress it. Measurement while drilling (MWD) tools are one of the methods that are used to collect downhole data which are used in the real-time adjustment of the drilling parameters (rotary table speed, weight on bit and torque) to eliminate stick-slip vibration. However, due to the limitations of these tools in that they are largely reactive rather than proactive, the industry has become more motivated to investigate and develop more sophisticated models of the drillstring to tackle the problem of vibrations, especially stick-slip vibration [32].

The development of accurate dynamic models of a drillstring is considered the first step required to develop control strategies to reduce or eliminate the drillstring vibration in order to prevent premature component failures. Moreover, such models can be used to study the effect of the main drilling parameters such as rotary speed, torque, and weight on bit (WOB) on the behaviour of the drillstring. These unwanted vibrations can subsequently be reduced or eliminated using MWD in conjunction with data derived from the predictive models.

Different types of models have been used to study and analyse the stick-slip oscillation of drillstrings such as lumped-parameter models (LPM) [20,33–35] and distributed-lumped parameter models (DLPM – also known as ‘hybrid’ models) [36]. In the LPM the drillpipe is considered to be inertia-less when compared to the rotary table and the drillcollar, whilst in DLPM the drillpipe is considered to be a distributed mass. One of the problems of modelling the stick-slip vibration is the friction appearing between the BHA and the rock formation which, as has previously been mentioned, is considered to have an important influence on stick-slip vibration. Typically a dry friction model has been used to model the friction between the BHA and the formation.

One of the parameters of a dry friction model is the transition from static friction to Coulomb (or dynamic) friction, [37]. Most of the past studies focussing on stick-slip vibration have paid no attention to the influence of the decay factor on the behaviour of stick-slip vibration, and in general they have used a constant value for the decay factor (typically 0.9). Thus far, there has been no published study to show the effect of this value on the stick slip behaviour of long slender drill strings. Therefore, this paper focusses on the effect of the decay factor on the behaviour of stick-slip vibration by performing a parametric study to understand the interactions between different attenuation factors (decay constant, static friction, coulomb friction and weight on bit) under the same drilling conditions.

2. Dynamic model of the drillstring

The basic components of a rotary drilling system for the exploration and production of oil and gas are the derrick and hoist, swivel, kelly and drive system (rotary table and electric motor with a mechanical transmission box) as shown in Fig. 1 (a). The drillstring mainly consists of the drillpipe and bottom hole assembly (BHA). The drillpipe is a slender tube screwed end to end to each other to form a long pipe reaching more than 8 km in length. The BHA consists of drill collar, heavy-weight drillpipe (HWDP), specialised subs and a rotary drill bit. The drill collar, which is a thick-walled tube several hundred meters in length, is responsible for providing the weight on bit (WOB) to generate sufficient cutting force. The HWDP is used as a transition section between the drillpipe and drill collar to reduce the stress between the two and to prevent failure in the area of connection between the two pipes [38].

In this paper, a distributed-lumped parameter model (DLPM) [36] will be used to model the stick-slip vibration of the drillstring. This type of model has been selected due to its ability to more accurately model the delay time between BHA and rotary table which is created by the 'twist' of the drillpipe. The accuracy and validity of this model compared to lumped models have been studied in detail in Ref. [36]. A schematic of the idealised drilling system represented as a DLPM is shown in Fig. 1 (b) where the rotary table is driven by the torque T_{rt} coming from the gearbox which is driven by motor torque T_m. The torque is transmitted from the rotary table to the BHA by the drillpipe where the inertia and stiffness are continuous functions of drillpipe length. The BHA and rotary table are modelled as lumped elements, whereas the drillpipe is a distributed element.

For the sense of brevity, the relationship between the input and output torques and velocities of the drillpipe are given in Refs. [36,39] and can be represented in matrix form as follows.

$$\begin{bmatrix} T_{1,dp}(s) \\ T_{2,dp}(s) \end{bmatrix} = \begin{bmatrix} \xi_{dp} w_{dp}(s) & -\xi_{dp}\sqrt{\left(w_{dp}^2(s)-1\right)} \\ \xi_{dp}\sqrt{\left(w_{dp}^2(s)-1\right)} & -\xi_{dp} w_{dp}(s) \end{bmatrix} \begin{bmatrix} \omega_{1,dp}(s) \\ \omega_{2,dp}(s) \end{bmatrix} \tag{1}$$

where $T_{1,dp}$ and $T_{2,dp}$ are the torque at inlet and outlet of drillpipe, $\omega_{1,dp}$ and $\omega_{2,dp}$ are the angular velocities at the inlet and outlet of drillpipe and ξ_{dp} is the characteristic impedance of the drillpipe ($\xi_{dp} = J_{dp}\sqrt{G_s \rho}$).

Also

$$w_{dp\ (s)} = \frac{e^{2\Gamma_{dp} l_{dp}} + 1}{e^{2\Gamma_{dp} l_{dp}} - 1} = \frac{1 + e^{-2t_{dp}s}}{1 - e^{-2t_{dp}s}} \tag{2}$$

$$\sqrt{\left(w_{dp}^2(s) - 1\right)} = \frac{2e^{-2\Gamma_{dp} l_{dp}}}{1 - e^{-2\Gamma_{dp} l_{dp}}} = \frac{2e^{-t_{dp}s}}{1 - e^{-2t_{dp}s}} \tag{3}$$

where $\Gamma_{dp} = s\sqrt{\rho/G}$ is the propagation constant of the drillpipe, l_{dp} is the length of drillpipe and $\Gamma_{dp} l_{dp} = s l_{dp}\sqrt{\rho/G} = s t_{dp}$, where t_{dp} represent the delay time of the drillpipe.

The Eqs. of motion of the drive system and BHA are:

$$J_{ds}\ddot{\theta}_{rt} + C_{ds}\dot{\theta}_{rt} = T_{rt} - T_{1,dp} \tag{4}$$

$$J_{bh}\ddot{\theta}_{b} + C_{bh}\dot{\theta}_{b} = T_{2,dp} - T_{fb} \tag{5}$$

where: J_{ds} and C_{ds} represent the equivalent inertia and viscous damping of the drive system respectively; $\dot{\theta}_{rt}$ is the angular velocity of the rotary table and it is equal to $\omega_{1,dp}$; T_{rt} is the applied torque on the rotary table; J_{bh} is the equivalent mass moment inertia of the

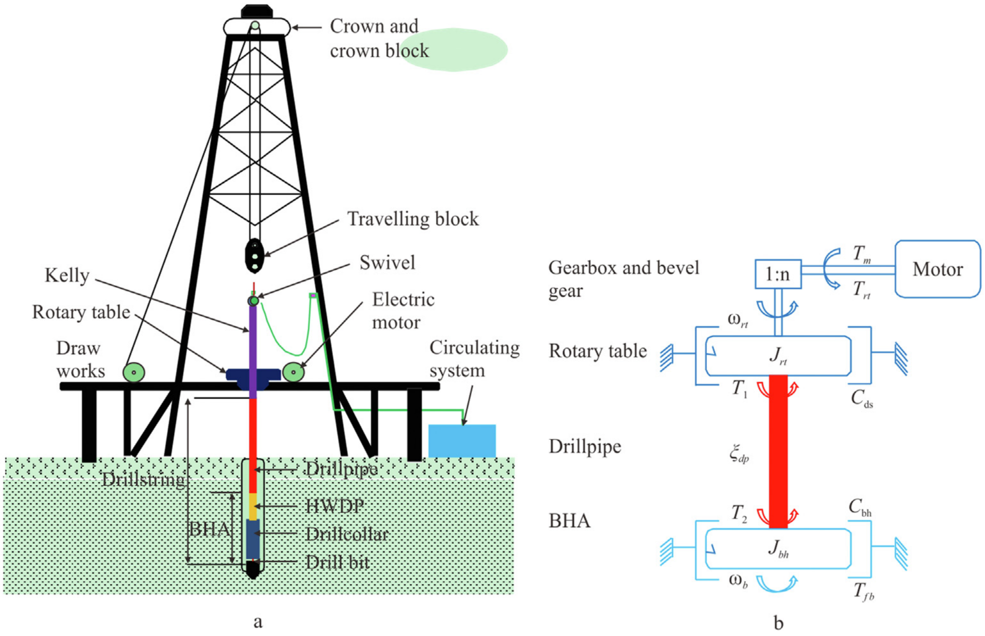

Fig. 1. (a) The rotary drilling rig; (b) representation of a drilling system as a DLPM.

BHA (drillcollar and HWDP); C_{bh} is the equivalent damping of the BHA; $\dot{\theta}_b$ is the angular velocity of the bit and it is equal to $\omega_{2,dp}$.

Eq. (6) gives the inertia of the BHA:

$$J_{bh} = G_s\,\rho\,\frac{\pi}{32}\left[l_c\left(d^4_{o,dc} - d^4_{i,dc}\right) + l_h\left(d^4_{o,hw} - d^4_{i,hw}\right)\right] \tag{6}$$

where T_{fb} is the reactive friction torque at the bit and is responsible for reproducing stick-slip torsional vibrations. Since the nonlinear reactive friction torque is directly proportional to the WOB, the coefficient of friction and the radius of the bit the equation of $(T_{fb}(\dot{\theta}_b))$ can be written as:

$$T_{fb} = W_{Ob}R_b\mu_b\left(\dot{\theta}_b\right) \tag{7}$$

where W_{Ob} is the weight on the bit (WOB) and is related to the 'hook-on-load' applied at the surface, R_b is the radius of the bit and $\mu_b(\dot{\theta}_b)$ is the friction coefficient at the bit. From Eqs. (1), (4) and (5) the overall DLPM of a drilling system for the purposes of simulation can be shown as a block diagram as Fig. 2.

3. The nonlinear bit-rock interaction

The stick-slip torsional vibration in the oil well drillstring is reproduced by the friction forces and the reactive friction torque at the bit. The non-linear reactive friction torque is proportional to the vertical force applied on the tool, type of rock and radius of the bit [38]. This friction torque, as a function of the bit speed, can be calculated by the following function:

$$T_{fb} = W_{Ob}R_b\mu_b\left(\dot{\theta}_b\right) \tag{8}$$

where W_{Ob} is the weight on the bit, R_b is the radius of the bit and $\mu_b(\dot{\theta}_b)$ is the friction coefficient at the bit.

The reactive friction torque is combined with two types of torque: First, static friction torque (T_{sb}) at zero velocity which is responsible for clamping the bit at zero velocity; and secondly the dynamic friction torque, or Coulomb friction torque (T_{cb}), where the bit rotates with steady state velocity. The transition between static and the Coulomb friction and the discontinuities in the equation of friction torque make the model of stick-slip motion a challenge for researchers [40].

Many models have been used to describe the friction torque on the bit. In this study the friction torque is a combination of a dry friction model together with Stribeck effect to model the friction torque on the bit [37]. The advantage of this type of modelling is that it can be used to efficiently describe the parameters, including decay factor, that lead to the occurrence of stick-slip vibration in a drill pipe and thus can be used to model and predict the response of various strategies that are proposed by other researchers to mitigate stick-slip vibration [41]. A Karnopp friction model [42] is introduced with a limit velocity interval $D\omega$ to overcome the problem of switching between static and dynamic friction torque by considering the bit velocity to be zero at this interval. The model was proposed by Leine [43] to overcome the discontinuity at zero velocity as shown in equation (9) and Fig. 3.

$$T_{fb}\left(\dot{\theta}_b\right) = \begin{cases} T_{ab} \quad \text{if}\left|\dot{\theta}_b\right| < D\omega\; T_{ab} \le T_{sb} \quad \text{stick} \\ T_{sb}\,sign(T_{ab})\;\text{if}\left|\dot{\theta}_b\right| < D\omega\; T_{ab} > T_{sb}\;\text{stick to slop transition} \\ T_{cb}\,sign\left(\dot{\theta}_b\right)\;\text{if}\left|\dot{\theta}_b\right| > D\omega \end{cases} \tag{9}$$

where T_{ab} is the external torque applied by the drillstring on the bit which must overcome the static friction torque, T_{sb}, to move the bit. T_{sb} is the static friction torque associated with J_{bh} and T_{cb} is the sliding friction torque (cutting torque). A limit velocity interval ($D\omega$ >0) specifies a small enough neighbourhood around $\dot{\theta}_b = 0$. The static and dynamic friction torque can be calculated as follows:

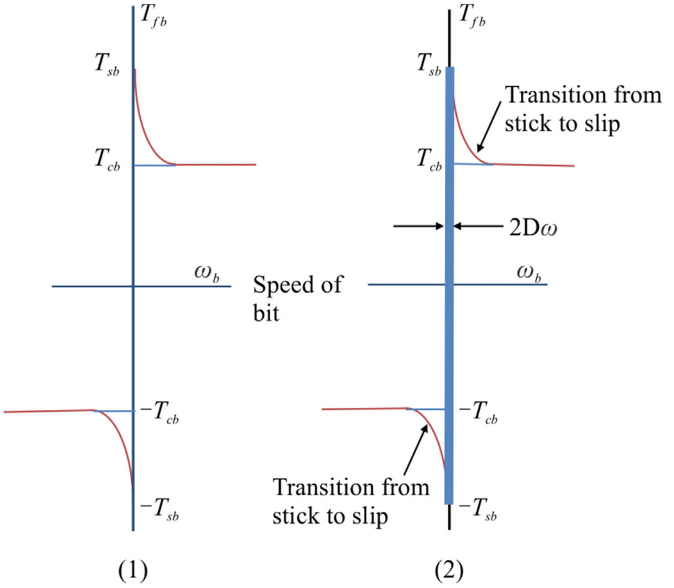

Fig. 3. Friction torque at the bit: (1) dry friction with exponential-decaying law at the sliding phase; (2) switch, friction model with a variation of Karnopp's friction model.

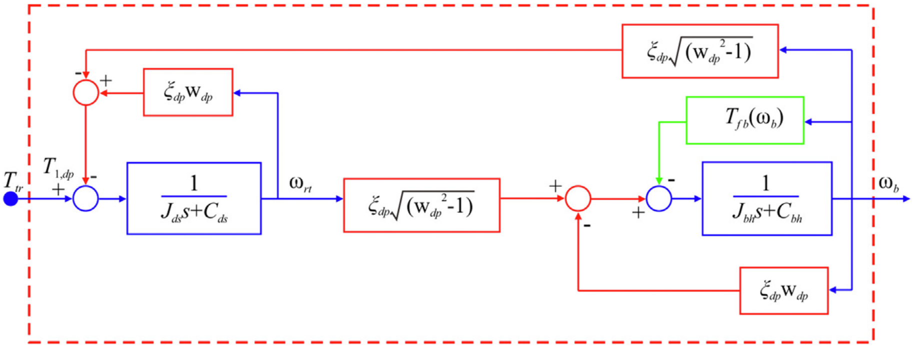

Fig. 2. Block diagram of the drilling system as a distributed-lumped parameter model.

$$T_{sb} = R_b W_{Ob} \mu_{sb} \tag{10}$$

$$T_{cb} = R_b W_{Ob} \mu_b \left(\dot{\theta}_b \right) \tag{11}$$

where W_{ob} is the weight on the bit, R_b is the radius of the bit and μ_b, μ_{sb}, are the velocity-dependent dry friction coefficients at the bit and static friction coefficients associated with J_{eb} respectively. The friction at the bit can be calculated as follows:

$$\mu_b \left(\dot{\theta}_b \right) = \left[\mu_{cb} + (\mu_{sb} - \mu_{cb}) \right] e^{-\gamma_b \left| \dot{\theta}_b \right|} \tag{12}$$

where γ_b is a positive constant (the ‘decay factor’) defining the transition from static to dynamic friction.

4. Simulation results and discussion

The effect of decay factor on the behaviour of the drillstring has not been studied very extensively during the past decades, therefore, in this section different values of decay factor have been studied in order to investigate the effect of these values on the behaviour the drillstring vibration. The simulations are carried out using the DLPM of the drillstring presented in the previous section in conjunction with Matlab/Simulink to investigate the effect of the decay constant on the stick-slip oscillation.

Table 1 shown below gives the parameters which are used for the purposes of simulation in this paper; the data is typical of many drill strings and has been compiled from various sources [5,44,45].

Most of the past studies which have focussed on the modelling of oil well drillstrings have used a value of 0.9 [34,40,41] for the decay factor when representing the dry friction torque, with little or no analysis of the effect of this factor. Therefore, to show the effect of this value on the torsional vibration of the drillstring, three values of decay constant have been used in this study, 0.2 (lower), 1.1 (baseline) and 2.0 (upper). The principle of choosing the decay factors depends on many parameters such as, the type of surfaces in contact, the temperature, the type of oil lubricant and so on [37] and theses values represent the time required to transition from static to dynamic friction. In this study for oil drilling system for selecting these values was based upon past studies which concentrated on the modelling of stick-slip vibration. The upper limit of 2.0 was the highest value found in available literature [38], whilst the values of 1.1 and 0.2 were selected to give uniform spacing of the values whilst maintaining the baseline value close to the nominal value found in other published studies.

The first investigation is the effect of these values on the transition from static friction torque to Coulomb (dynamic) friction torque. Simulation results show that decreasing values of decay factor (γ_b) lead to an increase in the time of transition from static friction torque to Coulomb friction torque as shown in Fig. 4. In addition, it can be seen that despite the upper and lower decay factors being numerically equidistant from the baseline value, the transition time from stick to slip for the upper and baseline decay factors is significantly less than that of the lower decay factor. This increment in the time of transition for the lower decay factor shows that the velocity of the bit will reach a comparatively high value before completely transitioning from stick phase to full sliding phase as shown in Fig. 5.

Table 1
Parameters used in the simulation.

Name	Symbol	Value	Unit
Inertia of motor and rotary table	J_m, J_{rt}	23,930	kg·m^2
Inertia of BHA	J_{bh}	528	kg·m^2
Damping of drive system	C_{ds}	425	Nms/rad
Damping of BHA	C_{bh}	50	Nms/rad
Length of drillpipe	l_{dp}	2000	m
Static and Coulomb friction coefficient	μ_{sb}, μ_{cb}	0.8, 0.5	

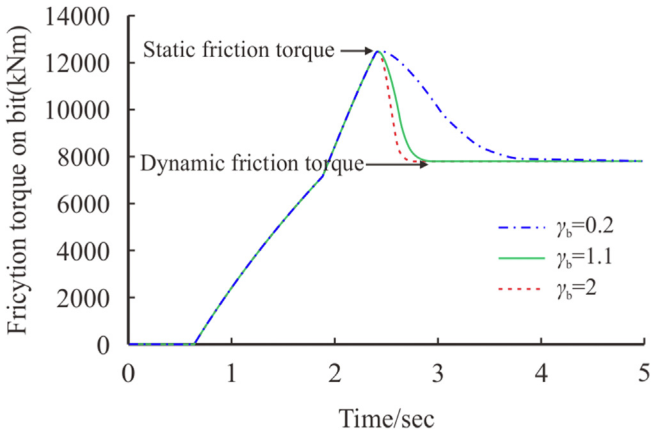

Fig. 4. Effect of decay factor on the transition from static friction torque to Coulomb friction torque.

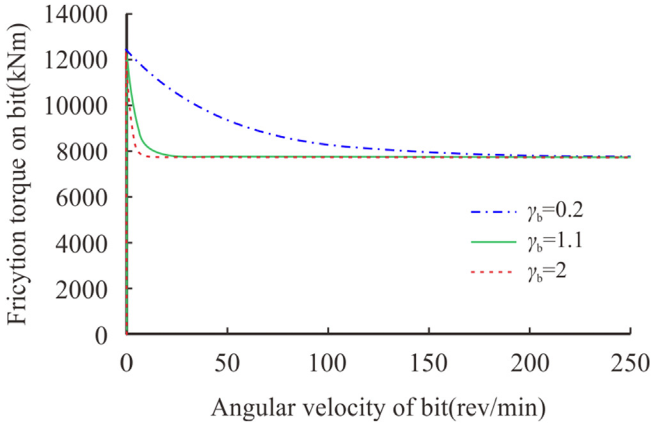

Fig. 5. Effect of decay factor on the transition from static friction torque to Coulomb friction torque (friction torque vs angular velocity of bit).

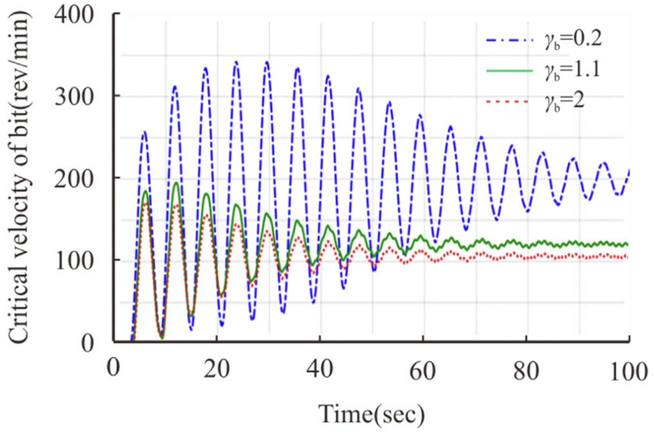

Fig. 6. Comparison between decay factor values at the critical speed of the bit.

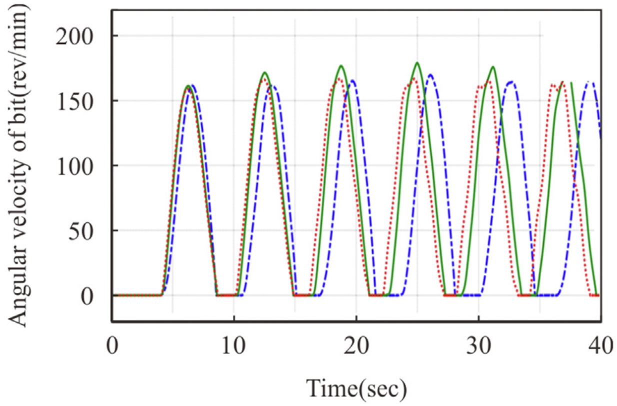

Fig. 7. Comparison between decay factor values in the stick-slip phase.

Table 2
Parameters for the parametric study.

No.	Parameters	Min.	Mid.	Max.
1	Decay constant (γ_b)	0.2	1.1	2
2	Static friction constant (μ_s)	0.6	0.75	0.9
3	Column friction constant (μ_c)	0.1	0.3	0.5
4	Weight on bit (kN)	10	80	150

The stick-slip phenomenon occurs when the velocity of the rotary table falls below the critical rotational velocity [46], at which point it is possible for the bit to rapidly transition from 'slip' towards 'stick' as its velocity falls into the Stribeck range. The effect of the different drilling parameters, such as the surface speed, WOB, drillstring stiffness and inertia, on the critical speed have been studied by many researchers [18,34,47], however the effect of decay factor has not received as much attention. It can be seen from Fig. 6 that the critical speed of the lower value of decay factor is higher than the two other values, indicating a higher propensity towards stick slip as supported by Figs. 4 and 5. The results presented in Fig. 6 also show that the lower decay factor has a larger effect on the transient response with significantly higher amplitude and correspondingly longer time to reach steady state.

Analysis of Fig. 7 shows the effect of decay factor on the behaviour of stick-slip vibration during the 'stick' phase (zero velocity). It can be seen that low values of decay factor promote longer periods of 'stick' with conversely shorter periods of 'slip'.

In order to have a complete view of the effect of decay factor on the critical speed of the drillstring, a parametric study has been conducted by studying the interaction between the static friction constant (μ_s), Coulomb friction constant (μ_c), weight on bit (W_{ob}) and decay factor and their effect on critical speed. The parameters used in the study are presented in Table 2, whilst the resultant interaction plot is shown in Fig. 8 where −1, 0 and 1 stand for the low, medium and high values of these parameters. Interactions between parameters are subsequently presented as surface plots to aid interpretation and can be found in Fig. 9 − 12.

The interaction plot presented in Fig. 8 shows that the relationship between the decay factor and critical speed is influenced to some degree by all of the other parameters as evidenced by the non-parallel lines. The greatest interaction with decay factor is with the weight on the bit (W_{ob}), as seen in Figs. 8 and 12, and indeed the weight on the bit shows greatest interaction with all parameters studied here.

Figs. 9 and 10 show that an increase in the weight on bit and static friction coefficient and a decrease of decay factor leads to an increase in the critical drilling speed, especially with low decay factor and high weight on bit. The other parameters which show an interaction with the decay factor are the Coulomb friction constant (μ_c) as shown in interaction plot (Fig. 8) and surface plot (Fig. 11). Fig. 11 shows that a decrease of both the Coulomb friction constant (μ_c) and decay constant (γ_b) leads to a corresponding increase in the critical speed, this would result in early onset of stick-slip vibration. In addition, it can be seen that with an increase in the difference between the static friction constant (μ_s) and Coulomb friction constant (μ_c) the critical speed also increases, the effect being more pronounced with low values of decay factor as shown in Fig. 12.

5. Conclusions

In this paper, a distributed-lumped parameter model has been used to study the effect of decay factor on the behaviour of stick-slip vibration of a typical oil well drillstring. Results have shown that the decay factor has an important influence on the stick-slip type behaviour of the drillstring. Selection of the correct value for the decay factor has shown to have an influence on the accurate prediction of the critical speed, therefore this value should have careful

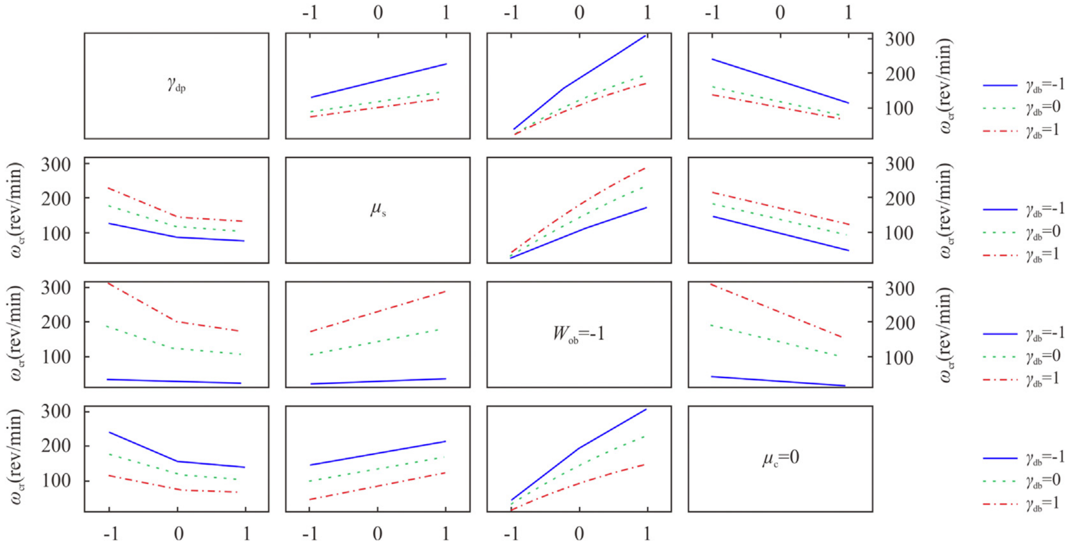

Fig. 8. The interaction plot of modelling parameters with critical speed of drilling.

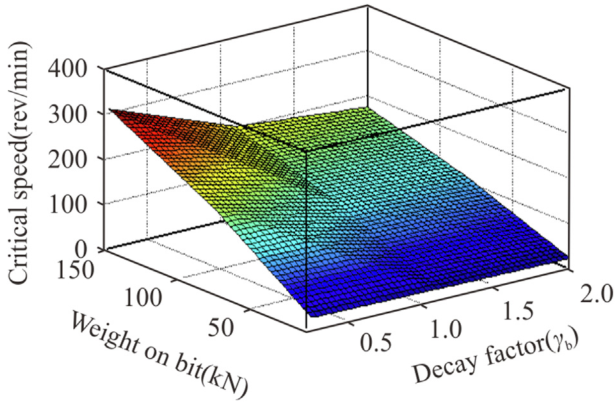

Fig. 9. Surface plot of critical speed vs decay constant and weight on bit.

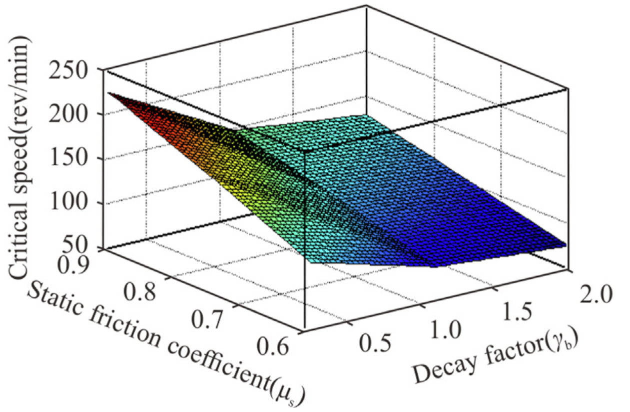

Fig. 10. Surface plot of critical speed vs decay constant and static friction coefficient.

consideration when both modelling the drillstring, and during development of suitable control methodologies to help mitigate stick-slip vibrations. Inaccurate selection could, at best, reduce drilling efficiency by preventing the drill from operating at optimum speeds and loads, whilst in the worst-case scenario stick slip motion would be generated at speeds where it was not expected resulting in damage and premature failure. Taking this into consideration, it is recommended that future work should focus on the accurate characterisation of the decay factor for a range of different cutting interfaces. Indeed, additional work by the author in the area of hybrid drillstring models [39] has shown good correlation with published measured data when considering more detailed models.

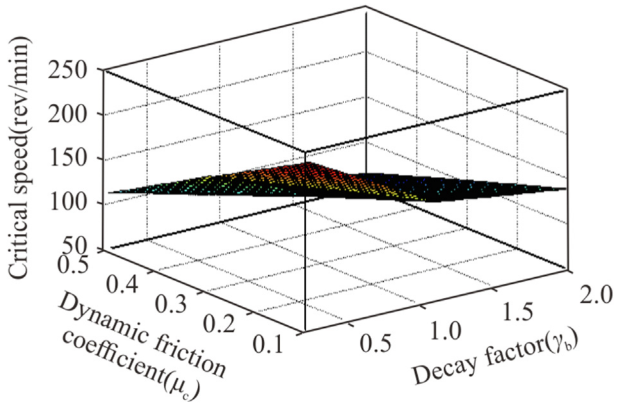

Fig. 11. Surface plot of critical speed vs decay constant and column friction coefficient.

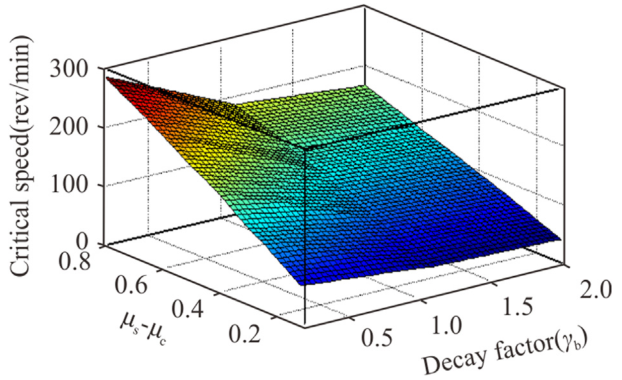

Fig. 12. Surface plot of critical speed vs decay constant and net difference in the friction coefficient ($\boldsymbol{\mu_s} - \boldsymbol{\mu_c}$).

Based upon the results, the following more specific observations can be made:

(1) A decrease in the value of the decay factor leads to an increase in the time of transition from static to dynamic friction torque.
(2) Lower values of decay factor leads to the stick-slip oscillation starting at higher critical speeds indicating a higher propensity towards stick slip motion.
(3) The time of sticking is higher with lower values of decay factor, but the time of slipping is smaller indicating more aggressive stick-slip motion once the oscillations commence.
(4) Lower values of decay factor in combination with variation in other drilling properties (e.g. WOB, static friction coefficient etc.) can lead to an increase in the critical speed; therefore careful selection of both the drilling parameters based upon the local drilling conditions at the interface is essential.
(5) The difference between static and dynamic friction coefficient has significant effect on the critical speed.

Declaration of competing interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

This research is supported by the Ministry of Higher Education and Scientific Research of Iraq and Technical Institute Baqubah, Middle Technical University, Iraq.

References

[1] J.S. Mason, B.M. Sprawls, Addressing BHA whirl: the culprit in mobile bay, SPE Drilling and Completion 13 (4) (1998) 231–236.
[2] J.D. Macpherson, J.S. Mason, J.E.E. Kingman, Surface measurement and analysis of drillstring vibrations while drilling, in: Proceedings of the 1993 SPE/IADC Drilling Conference, February 23, 1993 - February 25, 1993, Publ by Soc of Petroleum Engineers of AIME, in Drilling Conference - Proceedings, Amsterdam, Neth, 1993, pp. 953–963.
[3] V.A. Dunayevsky, F. Abbassian, A. Judzis, Dynamic stability of drillstrings under fluctuating weight on bit, SPE Drilling and Completion 8 (2) (1993) 84–92.
[4] Y. Kovalyshen, A simple model of bit whirl for deep drilling applications, Journal of Sound and Vibration 332 (24) (2013) 6321–6334, https://doi.org/10.1016/j.jsv.2013.07.001. (Accessed 25 November 2013).
[5] A.P. Christoforou, A.S. Yigit, Fully coupled vibrations of actively controlled drillstrings, Journal of Sound and Vibration 267 (5) (11/6/2003) 1029–1045,

https://doi.org/10.1016/S0022-460X(03)00359-6.

[6] Y.A. Khulief, F.A. Al-Sulaiman, S. Bashmal, Vibration analysis of drillstrings with self-excitedstick–slip oscillations, Journal of Sound and Vibration 299 (3) (2007) 540–558, https://doi.org/10.1016/j.jsv.2006.06.065. (Accessed 23 January 2007).

[7] G. Mensa-Wilmot, M. Booth, A. Mottram, New PDC bit technology and improved Operational practices, in: Presented at the SPE/IADC Drilling Conference, 2000. New Orleans.

[8] F. Abdulgalil, H. Siguerdidjane, PID based on sliding mode control for rotary drilling system, in: Presented at the EUROCON 2005 - the International Conference on Computer as a Tool, November 21, 2005 - November 24, 2005, 2005. Belgrade, Yugoslavia.

[9] Y.A. Khulief, F.A. Al-Sulaiman, S. Bashmal, Modeling of stick-slip in multibody drilling systems, Long Beach, CA, United states, in: r 24, 2005 - September 28, 2005, 6 BAmerican Society of Mechanical Engineers, 2005, pp. 1237–1244. Proceedings of the ASME International Design Engineering Technical Conferences and Computers and Information in Engineering Conference - DETC2005.

[10] R. Duff, An experimental and computational investigation of rotating flexible shaft system dynamics in rotary drilling assemblies for down hole drilling vibration mitigation, PhD thesis, in: The Craft and Hawkins Department of Petroleum Engineering, Graduate Faculty of the Louisiana State University and Agricultural and Mechanical College, 2013.

[11] R.I. Leine, D.H. Van Campen, Stick-slip whirl interaction in drillstring dynamics, in: IUTAM Symposium on Chaotic Dynamics and Control of Systems and Processes in Mechanics: Proceedings of the IUTAM Symposium Held in Rome, Italy, 8-13 June 2003, vol. 122, Springer Verlag, in Solid Mechanics and its Applications, 2005, pp. 287–296, https://doi.org/10.1007/1-4020-3268-4-27.

[12] A. Sassan, B. Halimberdi, Design of a controller for suppressing the stick-slip oscillations in oil well drillstring, Research Journal of Recent Science 2 (6) (2013) 78–82.

[13] J.F. Brett, The genesis of torsional drillstring vibrations, SPE Drilling Engineering 7 (1992) 169–174.

[14] S. Jardine, D. Malone, M. Sheppard, Putting dampers on drilling's bad vibrations, Oilfield Rev 6 (1) (1994) 15–20.

[15] A.P. Christoforou, A.S. Yigit, Active control of stick-slip vibrations: the role of fully coupled dynamics, in: Presented at the SPE Middle East Oil Show, March 17, 2001 - March 20, 2001, Bahrain, Bahrain, 2001.

[16] A.S. Yigit, A.P. Christoforou, Coupled torsional and bending vibrations of actively controlled drillstrings, Journal of Sound and Vibration 234 (1) (2000) 67–83, https://doi.org/10.1006/jsvi.1999.2854.

[17] L. Tang, X. Zhu, X. Qian, C. Shi, Effects of weight on bit on torsional stick-slip vibration of oilwell drill string, Iranian Journal of science and Technology, Transactions of Mechanical Engineering 31 (2017) 4589–4597.

[18] L. Tang, X. Zhu, C. Shi, J.T. Tang, D. Xu, Study of the influences of rotary table speed on stick-slip vibration of the drilling system, Petroleum 1 (2015) 382–387.

[19] J.R. Jain, L.W. Ledgerwood, O.J. Hoffmann, T. Schwefe, D.M. Fuselier, Mitigating torsional stick/slip vibrations in oilwell drilling through PDC-bit design: putting theories to the test, JPT, Journal of Petroleum Technology 63 (12) (2011) 79–80.

[20] G.W. Halsey, A. Kyllingstad, A. Kylling, Torque feedback used to cure slip-stick motion, in: Presented at the Proceedings: 1988 SPE Annual Technical Conference and Exhibition, October 2, 1988 - October 5, 1988, 1988. Houston, TX, USA.

[21] V.S. Dubinsky, D.R. Baecker, Interactive drilling dynamics simulator for drilling optimization and training, in: Presented at the Proceedings of the 1998 SPE Technical Conference and Exhibition. Part Omega, September 27, 1998 - September 30, 1998, 1998. New Orleans, LA, USA.

[22] P.C. Kriesels, et al., Cost savings through an integrated approach to drillstring vibration control, in: Presented at the in the SPE/IADC Middle East Drilling Technology Conference, 1999. Abu Dhabi.

[23] C.A. Guerrero, B.J. Kuli, Deployment of an SeROP predictor tool for real-time bit optimization, in: Presented at the SPE/IADC Drilling Conference and Exhibition 2007, February 20, 2007 - February 22, 2007, 2007. Amsterdam, Netherlands.

[24] X. Zhu, L. Tang, Q. Yang, A literature review of approaches for stick-slip vibration suppression in oilwell drillstring, Advances in Mechanical Engineering 6 (2015) 967952, https://doi.org/10.1155/2014/967952, 967952.

[25] J.C.R. Placido, H.M.R. Santos, Y.D. Galeano, Drillstring vibration and wellbore instability, Journal of Energy Resources Technology, Transactions of the ASME 124 (4) (2002) 217–222, https://doi.org/10.1115/1.1501302.

[26] B. Paic, N. Gaurina-Medimurec, D. Matanovic, Wellbore instability: causes and consequences, Rudarsko Geolosko Naftni Zbornik 19 (2007) 87–98.

[27] H. Henneuse, Surface detection of vibrations and drilling optimization: field experience, in: Presented at the Drilling Conference - Proceedings, February 18, 1992 - February 21, 1992, 1992. New Orleans, LA, USA.

[28] T.M. Warren, J.H. Oster, Torsional resonance of drill collars with PDC bits in hard rock, in: Presented at the Proceedings of the 1998 SPE Technical Conference and Exhibition. Part Omega, September 27, 1998 - September 30, 1998, 1998. New Orleans, LA, USA.

[29] J.D. Macpherson, P.N. Jogi, J.E.E. Kingman, Application and analysis of simultaneous near bit and surface dynamics measurements, SPE Drilling and Completion 16 (4) (2001) 230–238.

[30] B. Besselink, N. Van De Wouw, H. Nijmeijer, A semi-analytical study of stick-slip oscillations in drilling systems, Journal of Computational and Nonlinear Dynamics 6 (2) (2011), https://doi.org/10.1115/1.4002386.

[31] J.R. Bailey, E. Biedigner, S. Sundararaman, A.D. Carson, W.C. Elks, F.E. Dupriest, Development and application of BHA vibrations model, in: Presented at the in: Proceedings of the International Petroleum Technology Conference, 2008. Kuala Lumpur, Malaysia.

[32] A. Ghasemloonia, D. Geoff Rideout, S.D. Butt, A review of drillstring vibration modeling and suppression methods, Journal of Petroleum Science and Engineering 131 (2015) 150–164, https://doi.org/10.1016/j.petrol.2015.04.030.

[33] S.L. Chen, K. Blackwood, E. Lamine, Field investigation of the effects of stick-slip, lateral, and whirl vibrations on roller-cone bit performance, SPE Drilling and Completion 17 (1) (2002) 15–20.

[34] P.A. Patil, C. Teodoriu, Model development of torsional drillstring and investigating parametrically the stick slips influencing factors, ASME J. Energy Resour. Technol. 135 (1) (2013), 0131031-0131037.

[35] Y.-Q. Lin, Y.-H. Wang, Stick-slip vibration of drill strings, Journal of engineering for industry 113 (1) (1991) 38–43.

[36] M.Y.A. Alkaragoolee, K.M. Ebrahimi, R. Whalley, A hybrid model for a drilling process for hydrocarbon well-boring operations, Proceedings of the Institution of Mechanical Engineers, Part K: Journal of Multi-body Dynamics (2017) 1–13, vol. 0, no. 0.

[37] B. Armstrong-Helouvry, P. Dupont, C. Canudas De Wit, Survey of models, analysis tools and compensation methods for the control of machines with friction, Automatica 30 (7) (1994) 1083–1138, https://doi.org/10.1016/0005-1098(94)90209-7.

[38] D. Pavković, J. Deur, A. Lisac, A torque estimator-based control strategy for oil-well drill-string torsional vibrations active damping including an auto-tuning algorithm, Control Engineering Practice 19 (8) (8//2011) 836–850, https://doi.org/10.1016/j.conengprac.2011.04.012.

[39] M.Y.A. Alkaragoolee, Hybrid Modelling and Optimisation of Oil Well Drillstrings, Phd, in: Faculty of Engineering and Informatics, University of Bradford, Bradford, UK, 2018.

[40] V.S. Tikhonov, A.I. Safronov, Analysis of postbuckling drillstring vibrations in rotary drilling of extended-reach wells, Journal of Energy Resources Technology, Transactions of the ASME 133 (4) (2011), https://doi.org/10.1115/1.4005241.

[41] E.M. Navarro-Lopez, a.R. Suarez-Cortez, Modelling and analysis of stick-slip behaviour in a drillstring under dry friction, in: Presented at the Congress of the Mexican Association of Automatic Control, 2004. Mexican.

[42] D. Karnopp, Computer simulation of stick-slip friction in mechanical dynamic systems, Journal of Dynamic Systems, Measurement and Control, Transactions of the ASME 107 (1) (1985) 100–103.

[43] R.I. Leine, Bifurcations in Discontinuous Mechanical Systems of Filippov-type, Ph.D. Thesis, Technical University of Eindhoven, Netherlands, 2000.

[44] E.M. Navarro-López, E. Licéaga-Castro, Non-desired transitions and sliding-mode control of a multi-DOF mechanical system with stick-slip oscillations, Chaos, Solitons and Fractals 41 (4) (2009) 2035–2044, https://doi.org/10.1016/j.chaos.2008.08.008.

[45] J.D. Jansen, L. van den Steen, Active damping of self-excited torsional vibrations in oil well drillstrings, Journal of Sound and Vibration 179 (4) (1995) 647–668, https://doi.org/10.1006/jsvi.1995.0042.

[46] M. Dufeyte, H. Henneuse, Detection and monitoring of the slip-stick motion: field experiments, in: Presented at the in Proceedings of the SPE/IADC Drilling Conference, 1991. Amsterdam, The Netherlands, March, 1991.

[47] L. Tang, X. Zhu, J. Li, Effects of the synchronous variation of the static and the kinetic FrictionCoefficients on stick–slip vibration of drillstring, Iranian Journal of science Technology, Transactions of Mechanical Engineering 43 (2019) S275–S283.