The proposal and application of a 2-Dimensional Fuzzy Monte Carlo Frontier analysis for estimating Islamic bank efficiency

Review of Quantitative Finance and Accounting
https://doi.org/10.1007/s11156-024-01340-x

ORIGINAL RESEARCH

The proposal and application of a 2-Dimensional Fuzzy Monte Carlo Frontier analysis for estimating Islamic bank efficiency

Yong Tan[1] · Md Abul Kalam Azad[2] · Antônio Mamede Araújo de Medeiros[3] · Peter Fernandes Wanke[4]

Accepted: 9 August 2024

Abstract
The current study proposes a novel 2-Dimensional Fuzzy Monte-Carlo Frontier Analysis to estimate and compare the level of efficiency for a sample of 49 Islamic Banks across 25 countries worldwide over the period 2013–2021. Additionally, in the second stage, we propose a bootstrapped robust regression approach to comprehensively examine the determinants of efficiency. Our results show that there is heterogeneity in the level of efficiency within the Islamic banking sector. Furthermore, we find that the Islamic banks in the sample experienced an improvement in efficiency over the examined period. Finally, we find that bank size, bank liquidity (measured by the ratio between net loans and gross loans), and bank risk (proxied by the ratio between loan loss reserves and gross loans) have a significant and positive impact on Islamic bank efficiency. Policy implications based on our findings are provided.

Keywords Efficiency analysis · Islamic banking · Monte-Carlo analysis · 2-dimenstional; Robust regression

JEL Classification C14 · C61 · D24 · G21 · G28

✉ Yong Tan
y.tan9@bradford.ac.uk

Md Abul Kalam Azad
kalam@iut-dhaka.edu

Antônio Mamede Araújo de Medeiros
medeiros.antonio@fgv.edu.br

Peter Fernandes Wanke
peter@coppead.ufrj.br

[1] School of Management, University of Bradford, Bradford, West Yorkshire BD7 1DP, UK

[2] Department of Business and Technology Management, Islamic University of Technology, Gazipur, Bangladesh

[3] EBAPE, Fundação Getúlio Vargas, Rio de Janeiro, Brazil

[4] COPPEAD Graduate Business School, Federal University of Rio de Janeiro, Rua Paschoal Lemme 355, Rio de Janeiro 21949-900, Brazil

Published online: 21 September 2024

1 Introduction

Islamic banking plays a crucial role in promoting financial inclusion and social justice in Muslim-majority countries and beyond. In many cases, Islamic banks provide an alternative to conventional banking for individuals who are uncomfortable with interest-based transactions (riba) prevalent in conventional banking. Islamic banking emphasizes ethical and socially responsible practices that align with the principles of Islamic finance, seeking to promote the welfare of society as a whole. Furthermore, Islamic banking has the potential to attract investors seeking ethical and sustainable investment options.

However, like any other banking system, Islamic banking faces various challenges and problems. Some of the significant problems faced by Islamic banking include: 1) Limited Product Diversity: Islamic banks offer a narrower range of products compared to conventional banks. This limitation arises from the Shariah principles governing Islamic finance, which prohibit certain financial practices like charging interest (riba) and engaging in speculative transactions (Maysir). Consequently, Islamic banks have constraints on their product offerings, making it challenging to compete with conventional banks; 2) Lack of Standardization: Another significant challenge faced by Islamic banking is the lack of standardization in Shariah interpretations. Multiple interpretations of Shariah principles exist, leading to confusion and inconsistency in Islamic finance products and services. This lack of standardization can also hinder the expansion of Islamic banks on an international scale; 3) High Cost of Funds: Islamic banks often experience higher costs of funds compared to conventional banks. This situation arises because Islamic finance products are typically asset-based, requiring the bank to purchase and own the underlying assets. Consequently, the bank bears the associated risks and costs of asset ownership, leading to higher costs; 4) Limited Liquidity Management Tools: Islamic banks have a restricted set of liquidity management tools compared to conventional banks. The prohibition on interest-bearing instruments, such as government bonds and interbank deposits commonly used by conventional banks, limits the options available to Islamic banks for managing liquidity; 5) Lack of Skilled Personnel: There is a shortage of skilled personnel in Islamic banking. This shortage stems from the relative novelty of Islamic finance, as there are insufficient qualified professionals with expertise in Shariah principles and Islamic finance to meet the demand; 6) Regulatory Challenges: Islamic banks face regulatory challenges in many countries where Islamic finance is not fully recognized. These challenges can impede the growth of Islamic banking and create difficulties for Islamic banks to operate in certain markets. Additionally, some regulatory frameworks may not be well-suited to Islamic finance, further complicating compliance for Islamic banks with local regulations.

Investigating efficiency in Islamic banking holds great significance for several reasons. Firstly, efficiency is a critical factor that determines a bank's profitability and long-term sustainability. Inefficient banks may struggle to compete with their more efficient counterparts, resulting in lower profits and an increased likelihood of failure. Understanding the factors that contribute to efficiency in Islamic banking can assist policymakers and industry practitioners in identifying areas for improvement and making informed decisions regarding resource allocation.

Secondly, efficiency is closely tied to the social and economic impact of Islamic banking institutions. More efficient banks are better equipped to provide financing and other services to individuals and businesses, thereby contributing to economic growth and development. Additionally, efficient banks are more capable of fulfilling their social responsibilities, such as promoting financial inclusion and supporting small and medium-sized

enterprises (SMEs). Overall, investigating efficiency in Islamic banking is crucial for comprehending the industry's potential for growth and development. By identifying the factors that contribute to efficiency, policymakers and practitioners can take steps to promote a more sustainable and inclusive financial system that aligns with the principles of Islamic finance. Furthermore, a better understanding of efficiency can aid investors in making informed decisions regarding resource allocation and contribute to the development of ethical and sustainable investment options.

Islamic banking is increasingly gaining attention from scholars, policymakers, and investors due to its unique features and growing significance in the global financial landscape. The principles of Islamic finance, which emphasize ethical and socially responsible practices, resonate with a broader audience seeking sustainable and ethical investment options beyond Muslim-majority countries. Islamic banks play a crucial role in promoting financial inclusion and social justice by offering alternatives to conventional banking systems that may not align with the values and beliefs of certain individuals. Moreover, Islamic banking has demonstrated resilience and stability, particularly during times of financial crises, which underscores its potential for contributing to a more sustainable and resilient financial system on a global scale. Additionally, as Islamic banking continues to expand its presence internationally, understanding its operations, challenges, and impact is essential for policymakers, regulators, and industry practitioners worldwide.

Islamic banking serves as a representative of the Islamic market, embodying the principles and values of Islamic finance within the broader financial ecosystem. The Islamic market encompasses not only Muslim-majority countries but also diverse communities and regions where individuals seek financial services aligned with Islamic principles. Islamic banking institutions cater to this market by offering products and services that adhere to Shariah principles, such as the prohibition of interest (riba) and adherence to ethical investment practices. By providing a viable alternative to conventional banking, Islamic banks act as key players in shaping and defining the Islamic market, serving the needs of individuals and businesses seeking financial solutions that align with their religious and ethical beliefs. As such, understanding Islamic banking efficiency offers insights into the dynamics and characteristics of the Islamic market and its potential implications for financial systems worldwide.

Investigating Islamic banking efficiency offers valuable insights not only for enhancing the performance and resilience of Islamic banks but also for advancing the broader understanding of efficiency in the banking industry and its implications for financial stability and inclusion across diverse markets. Therefore, while Islamic banking may have its unique characteristics and challenges, the lessons learned and insights gained from studying Islamic banking efficiency can indeed be generalized and applied to inform policies and practices in other countries and financial systems.

We significantly contribute to the literature on efficiency analysis in the banking industry, particularly in the estimation of efficiency in Islamic banking, by proposing a novel 2-Dimensional Fuzzy Monte-Carlo Frontier Analysis (2DMCFA). Our proposed method offers the following advantages: 1) 2DMCFA incorporates the uncertainty and fuzziness associated with real-world data by utilizing fuzzy sets and Monte Carlo simulations. This renders it a more realistic and robust method for analyzing efficiency and frontiers in real-world situations; 2) Traditional efficiency analysis methods typically focus on a single dimension, analyzing only one input or output at a time. In contrast, 2DMCFA enables the analysis of two-dimensional efficiency, making it a more comprehensive and accurate approach for assessing efficiency; 3) 2DMCFA allows for the examination of nonlinear relationships between inputs and outputs, which often occur in real-world scenarios.

Traditional methods assume linear relationships, which can lead to inaccurate results; 4) 2DMCFA permits the analysis of multiple inputs and outputs, making it more applicable to complex systems with multiple factors influencing efficiency; 5) 2DMCFA is a flexible method that can be applied to a wide range of industries and sectors, making it a versatile tool for efficiency and frontier analysis; 6) In addition to the pioneering proposal of 2DMCFA in the first stage, we also contribute to the literature in the second-stage analysis by utilizing a bootstrapped robust regression approach to examine the determinants of efficiency.

The structure of the current paper is as follows: Sect. 2 provides a literature review on bank efficiency estimation and Islamic bank efficiency analysis. Section 3 presents and explains our innovative methodology, followed by Sect. 4, which offers an analysis and discussion of our results. Finally, concluding remarks are provided in Sect. 5.

2 Literature review

Efficiency analysis in the banking industry, both in general and specifically in Islamic banking, has undergone continuous development in terms of empirical investigation and methodological innovations. In the broader banking sector, the initial analysis of bank efficiency was facilitated by traditional non-parametric data envelopment analysis and parametric stochastic frontier analysis (Tan and Floros 2013; Fukuyama and Tan 2021; Tan et al. 2017; Nasim et al. 2024; Tan and Walheer 2024), along with the use of relevant accounting indicators (Tan and Floros 2012). Building upon these methods, various methodological contributions have emerged in the estimation of bank efficiency. These include Bayesian stochastic frontier analysis (Galan and Tan 2024), network data envelopment analysis (Fukuyama et al. 2023; Tan et al. 2021; Fukuyama and Tan 2022a, b; Fukuyama and Tan 2024a), multi-criteria decision-making analysis (Maredza et al. 2022; Wanke et al. 2023a), and output distance function (Tan and Tsionas 2022). Few studies also combine the non-parametric data envelopment analysis and multi-criteria decision-making method in estimating bank performance (Antunes et al. 2024a, 2024b). The literature has also witnessed the proposal and utilization of advanced techniques in second-stage analysis to investigate the determinants of bank efficiency. These innovative methodologies include the stochastic structural relationship programming model based on neural networks (Antunes et al. 2024c), robust endogenous neural network analysis (Antunes et al. 2022), and multi-layer perceptron–hidden Markov model (Tan et al. 2021), among others.

In the past decade, numerous studies have examined efficiency in Islamic banking from various perspectives. Initially, accounting indicators such as the ratio between total non-interest expenses and total operating revenue, and the ratio between total overheads and gross income, were used as proxies for inefficiency (Abedifar et al. 2016; Alqahtani et al. 2017a; Khan et al. 2017). With the introduction of operational research in finance, traditional non-parametric data envelopment analysis (DEA) was applied by Ismail et al. (2013) to investigate and compare efficiency levels between Islamic and conventional banks in Malaysia from 2006 to 2009. This was followed by a second-stage Tobit regression analysis examining the determinants of efficiency, focusing on factors such as capital adequacy, bank size, loan quality, expense management, and bank profitability. Similar estimation techniques were employed by Rosman et al. (2014) in the context of Islamic banking in Middle Eastern and Asian countries.

Moving beyond technical, pure technical, and scale efficiencies, Kamarudin et al. (2014) used non-parametric DEA to estimate and compare profit, revenue, and cost efficiencies of Islamic and conventional banks in Gulf Cooperation Council countries. Revenue efficiency of a sample of domestic and foreign Islamic banks was also estimated by Sufian and Kamarudin (2015) using non-parametric DEA, followed by a second-stage analysis investigating the determinants of revenue efficiency through ordinary least squares and generalized least squares methods. Non-parametric DEA techniques were also applied by Belanès et al. (2015), Daly and Frikha (2017), and Bitar et al. (2020).

Johnes et al. (2014) made a significant contribution to the literature on the estimation of Islamic and conventional banking by proposing non-parametric data envelopment analysis (DEA) and meta-frontier analysis. This approach offers the advantage of decomposing overall efficiency into net efficiency and type efficiency. In the second stage, they employed a random effects model to examine the determinants of efficiency. In addition to non-parametric DEA, parametric stochastic frontier analysis has also been proposed and used to estimate efficiency levels in Islamic banking. For example, Gheeraert and Weill (2015) employed the one-stage procedure developed by Battese and Coelli (1995) to estimate efficiency levels and determinants of efficiency for Islamic banks worldwide between 2000 and 2005. Mohanty et al. (2016) extended this approach by proposing a one-step heteroskedastic stochastic cost frontier model and heteroskedastic stochastic profit frontier model. These models allow for the random noise to be heteroskedastic, meaning that it varies across observations and may depend on the values of the explanatory variables. They provide a more flexible modeling of the stochastic frontier by assuming that the variance of the error term is a function of the explanatory variables. Saeed et al. (2020) also conducted a one-stage stochastic frontier analysis.

Safiullah and Shamsuddin (2019) proposed and applied the stochastic profit meta-frontier model to investigate and compare the efficiency levels between Islamic and conventional banks across 28 countries from 2003 to 2014. This method allows for the accounting of heterogeneity across different groups and accounts for the dynamic nature of the production process, providing an advantage over traditional parametric stochastic frontier analysis by allowing for the inclusion of different types of inputs, outputs, and environmental factors. In contrast to the one-stage approaches for estimating efficiency and determinants of efficiency, Hardianto and Wulandari (2016) proposed a two-stage parametric stochastic frontier approach. They investigated and compared efficiency levels between Islamic and conventional banks in Indonesia from 2011 to 2013. In the second stage, they used generalized least squares with fixed effect model, generalized least squares with pooled least squares, and random effect models to investigate the determinants of efficiency. Mobarek and Kalonov (2014) examined efficiency using both non-parametric DEA and parametric stochastic frontier analysis, followed by a second-stage analysis investigating the impact of efficiency on bank stability. They employed OLS cross-sectional and pooled regression, adjusting for robustness by checking heteroscedasticity. Alqahtani et al. (2017a) also applied non-parametric DEA and parametric stochastic frontier analysis, followed by Tobit regression and bootstrapped truncated regression. They examined and compared efficiency levels between Islamic and conventional banks in six Gulf Cooperation Council (GCC) countries from 1999 to 2012.

Wanke et al. (2016a) distinguished themselves from other studies by proposing a multicriteria decision-making technique called Technique for Order Preference by Similarity to the Ideal Solution. They used this technique to assess the level of efficiency of 114 Islamic banks from 24 countries worldwide. In the second-stage analysis, the study utilized neural networks to predict the influencing factors of bank performance.

The findings suggest that the level of efficiency is influenced by the country of origin and cost structure, and Islamic banks would benefit from increased competition in the overall banking market. Wanke et al. (2016b) conducted a similar study focusing on Islamic banks in Malaysia.

Saeed and Izzeldin (2016) made a significant contribution to the literature by being pioneers in investigating the efficiency-risk paradigm in the context of both Islamic and conventional banks. They estimated profit efficiency using parametric stochastic frontier analysis and evaluated default risk using the distance-to-default measure proposed by Merton (1974). The causality between efficiency and default risk was examined using a panel vector autoregression framework. The results suggest that the trade-off between efficiency and default risk is evident for conventional banks but not for Islamic banks. Asmild et al. (2019) proposed and applied the multi-directional efficiency analysis using a sample of Islamic banks from Bangladesh between 2001 and 2015. This method differs from traditional non-parametric DEA as it allows for the investigation of improvement potentials in each input dimension and the identification of a benchmark proportionate to those potential improvements. The findings indicate that Islamic banks outperformed conventional banks during the period of the global financial crisis.

Alqahtani et al. (2017b) investigated and compared the efficiency of Islamic banks and conventional banks in the GCC region during and after the global financial crisis using both a single-stage stochastic frontier analysis and a two-stage data envelopment analysis. This was followed by a second-stage analysis controlling for the impact of bank characteristics and the macroeconomic environment through a bootstrapped truncated regression analysis. Saâdaoui and Khalfi (2024) measured the efficiency of Islamic banks in developed and developing countries between 2007 and 2015 using two proxies: the ratio of non-performing loans to total loans and the ratio of operating expenses to total assets. The innovation of the study lies in the proposal of a Multivariate Adaptive Regression Splines (MARS) method for estimating the relationship between bank efficiency and bank governance. This method offers a higher level of flexibility in modeling high-dimensional data. Considering credit risk, measured by the volume of impaired loans, as one of the inputs, Mirzaei et al. (2024) evaluated and compared the risk-adjusted efficiency of Islamic banks and conventional banks through a risk-adjusted data envelopment analysis. The study further applied multivariate regression analysis to examine the impact of bank efficiency on stock returns.

In addition to estimating and comparing the level of efficiency between Islamic and conventional banks, the literature has also focused on evaluating productivity levels. For example, Bahrini (2015) employed a bootstrapped Malmquist index approach to analyze the productivity change of 33 Islamic banks from 10 MENA countries between 2006 and 2011. In the second stage, a fixed-effects regression analysis was conducted to investigate the determinants of productivity.

While extensive research has been conducted on Islamic banking efficiency using a variety of models, our approach introduces a novel 2-Dimensional Fuzzy Monte-Carlo Frontier Analysis (2DMCFA) that significantly deviates from traditional single dimension efficiency analyses (Nieswand and Seifert 2018; Ahn et al. 2023). This method not only accommodates the inherent uncertainty and fuzziness of real-world data but also enables a more nuanced exploration of the efficiency landscape by considering multiple inputs and outputs simultaneously. Most existing methods, including the widely employed DEA and stochastic frontier approaches, typically do not incorporate fuzziness directly into their efficiency estimation, which can result in overlooking some of the complexities inherent in financial data (Wanke et al. 2016c).

Furthermore, our application of a bootstrapped robust regression in the second-stage analysis to explore the determinants of Islamic banking efficiency distinguishes our study from previous research that predominantly relies on more traditional regression techniques (Mobarek and Kalonov 2014; Hardianto and Wulandari 2016). This approach allows for a more robust inference by mitigating the influence of outliers and leveraging the data's inherent variability. This methodology not only contributes a novel perspective to the empirical literature on Islamic banking but also provides actionable insights that could help enhance the operational strategies of these institutions. By addressing both methodological innovation and practical application, our research offers a comprehensive enhancement to the existing body of knowledge on banking efficiency.

3 Methodology

3.1 2DFMCA and extensions in efficient frontier and envelopment problems

2-Dimensional Fuzzy Monte-Carlo Analysis (2DFMCA) is a computational approach used for solving multi-objective decision-making problems under uncertainty (Wanke et al. 2023b). It combines the Monte Carlo simulation method with fuzzy set theory to model the uncertain input parameters and produce a fuzzy Pareto front, which represents the set of optimal solutions for the problem (Nasr et al. 2021). 2DFMCA has been used in several applications (Arunraj et al. 2013).

Departing and differently from previous studies, the general mathematical model for 2-Dimensional Fuzzy Monte-Carlo Analysis (2DFMCA) developed in this research involves the following steps:

- Define the input matrix X and output matrix Y, where each row represents a single observation and each column represents a different input or output variable.
- Apply fuzzy logic to the output matrix Y to model the vagueness and uncertainty in the maximal achievable outputs. This involves defining fuzzy sets for each output variable that represent the possible range of values it can take on.
- Apply Monte Carlo simulation to the input matrix X to model the uncertainty in the characteristic distribution of the inputs. This involves generating random samples from the input distribution for each input variable.
- Use the fuzzy output matrix and the Monte Carlo simulated input matrix to calculate the output values for each observation using a mathematical model or function. This generates a set of output values for each input sample.

3.2 Background in COLS

Corrected Ordinary Least Squares (COLS) is a linear regression-based approach used in frontier analysis to estimate the production frontier, which represents the maximum output that can be produced with a given set of inputs (Kumbhakar and Lovell 2000). The COLS approach corrects for the bias in the ordinary least squares (OLS) estimator due to the presence of inefficiencies in the data.

One advantage of the COLS approach over other frontier models such as Data Envelopment Analysis (DEA) and Stochastic Frontier Analysis (SFA) is that it is computationally simple and requires less assumptions about the distribution of the inefficiency

term (Kumbhakar and Lovell 2000). However, one limitation of the COLS approach is that it assumes a constant returns to scale (CRS) technology, which may not be valid in some cases where there are increasing or decreasing returns to scale (Kumbhakar and Tsionas 2011). Also, it does not allow for the estimation of technical efficiency scores, which are important for benchmarking and performance evaluation. The standard COLS model assumes that the output vector Y is a linear function of the input matrix X, plus a random error term e:

$$\mathrm{Y} = X\beta + \mathrm{e} \tag{1}$$

3.3 Handling multiple outputs in COLS

One possible approach to dealing with the single output limitation in COLS modelling is to use a multiple-input single-output model, where the output is a composite of multiple measures of performance. The weights assigned to each performance measure can be based on the relative importance of each measure, which can be determined using either a fuzzy preference relation (Tripathi et al. 2021) or maximal entropy econometrics (Golan et al. 1996a).

Differently in this research, we propose Minimum Entropy Regression onto the Mean (MEROM), which is a regression-based method that seeks to estimate the mean of a response variable (the mean single output vector) given a set of predictor variables (the output matrix). The method is based on the principle of information theory, which seeks to find the probability distribution that is consistent with the available information and has the lowest noise level.

The concepts of information theory has been applied to regression problems in various forms. One of the earliest papers on this topic is by Fisher, who proposed the use of entropy methods for the estimation of parameters in linear models (Fisher 1921). Later, entropy regularization has been used as a method for dealing with ill-posed linear inverse problems (Tikhonov 1963). In the context of linear regression, entropy regularization has been studied by various authors (Jaynes 1957; Denzau et al. 1989; Golan et al. 1996a, 1996b; Tan et al. 2021). Given a matrix of predictors Y (output matrix) and a response vector y (mean output vector), the objective is to find the coefficients β that minimize the entropy of the mean deviation of the residuals $e = y - \mathrm{Y}\beta$. This can be expressed mathematically as:

$$\begin{aligned} &\text{minimize} : \left(\sum e_i \log\left(e_i\right)\right)/n \\ &\text{subject to} : e = y - \mathrm{Y}\beta;\ \beta \neq 0 \end{aligned} \tag{2}$$

where e_i are the residuals, n is the number of observations, and $\beta \neq 0$ indicates that the coefficients are non-zero. Considering that log of negative values cannot be computed, the vector of the residuals have been scaled-up (translation) by the modulus of min(e_i). Nelder-Mead algorithm is used to solve the programming problem in Eq. (2).

While, to the best of our knowledge, there is no literature specifically discussing the incorporation of fuzzy logic elements into COLS modelling, there are some studies that have applied fuzzy regression models to efficiency analysis. These models extend the standard regression model to handle fuzzy inputs and outputs and incorporate the uncertainty and imprecision associated with these data (Gong et al. 2018; Khayum et al. 2020).

3.4 Representing the maximal achievable output by HT2FS

To incorporate fuzzy logic into the model, we can represent the output matrix Y as a Hesitant Type 2 Fuzzy Set. A Hesitant Type 2 Fuzzy Set (HT2FS) is a fuzzy set that allows for uncertainty not only in the membership degree of an element, but also in the degree of uncertainty associated with the membership degree. The use of hesitant fuzzy sets in decision-making has been gaining attention in recent years (insert citations). Yet, we can represent the Hesitant Type 2 Fuzzy Set as a nested set of Type 1 Fuzzy Sets (Herrera et al. 2014):

$$Y = \left\{\left\{\mu L(y),\ \mu L'(y)\right\},\ \left\{\mu M(y),\ \mu M'(y)\right\},\ \left\{\mu U(y),\ \mu U'(y)\right\}\right\} \tag{3}$$

where μL, μM, and μU are the membership functions for the lower, middle, and upper bounds of the fuzzy set, respectively, and $\mu L^{'}$, $\mu M^{'}$, and $\mu U^{'}$ are the membership functions for the degrees of uncertainty associated with the membership functions.

In this research, we use the concept of vector parallelism to make the application of HT2FS operational with respect to the maximal output achievable. Vector parallelism is a concept used in fuzzy set theory to compare two fuzzy vectors and determine their degree of similarity (Manacero et al. 2022). It measures the degree to which the two vectors have similar shapes, directions, and magnitudes. A vector parallelism measure can be used to compare the fuzzy sets used in HT2FS (Ali and Al-kenani 2023).

Precisely, the following HT2FS model is proposed to capture the vagueness wrt to the maximal achievable output as regards the output matrix Y, which is populated by crisp values, and alpha, which is a scalar parameter that determines the degree of hesitation or indeterminacy in the resulting fuzzy sets. Besides Y, which is an n-by-m matrix with n observations and m outputs, there are two additional matrices of equal dimensions, U and L, which represents the upper and lower bounds of the HT2FS, respectively. The mathematical description of the HT2FS function for the maximal achievable outputs is given as follows:

Let Y = [y_{ij}] be an n-by-m matrix of crisp values, where y_{ij} is the j-th variable for the i-th observation. The Hesitant Type 2 Fuzzy Set of Y is defined as:

$$Y\_HT2FS = \{U, L\} \tag{4}$$

where U = [u_{ij}] and L = [l_{ij}] are n-by-m matrices representing the upper and lower bounds of the HT2FS for each variable, respectively. The elements of U and L are given by:

$$u\_\{ij\} = \max\left(y\_\{ij\} + \text{alpha} \times y\_\{ij\},\ y\_\{ij\}\right) \tag{5}$$

$$l\_\{ij\} = \min\left(y\_\{ij\} - \text{alpha} \times y\_\{ij\},\ y\_\{ij\}\right) \tag{6}$$

for all i = 1, …, n and j = 1, …, m, where alpha is a scalar parameter that determines the degree of hesitation or indeterminacy in the resulting fuzzy sets.

3.5 Representing the maximal achievable output by HT2FS

The COLS model can then be estimated using the least squares method to minimize the sum of squared errors between the observed output vector Y – obtained by MEROM—and the predicted output Xβ:

$$\text{argmin}\,\beta\,\Sigma(\mathrm{Y} - X\beta)^2 \tag{7}$$

To account for the fuzzy output, we can use a fuzzy regression approach, which involves minimizing a fuzzy objective function that captures the uncertainty and imprecision associated with the fuzzy output. One possible fuzzy objective function is the Fuzzy Mean Squared Error (FMSE):

$$\text{FMSE} = \sum \left(\mathrm{Y} - X\tilde{\beta}\right)^2 \tag{8}$$

where $\left(\mathrm{Y} - X\tilde{\beta}\right)$ is the Hesitant Type 2 Fuzzy Set representing the difference between the observed output Y and the predicted output Xβ. The FMSE can be minimized using the least squares method, which involves solving a system of linear equations:

$$\mathrm{X}'X\hat{\beta} = \mathrm{X}'\tilde{\mathrm{Y}} \tag{9}$$

where $\hat{\beta}$ is the estimated parameter vector, X' is the transpose of the input matrix X, and $\tilde{\mathrm{Y}}$ is the Hesitant Type 2 Fuzzy Set representing the observed output vector Y. The estimated production frontier can then be obtained by solving for the maximum achievable output vector Y, given a set of input vectors X and the estimated parameter vector β.

3.6 Bootstrapping X and $\tilde{\mathrm{Y}}$ and computation of inefficiency scores

Bootstrapping is a resampling technique used to estimate the sampling distribution of a statistic (Efron 1979), such as the coefficients in a COLS regression model. In bootstrapping, multiple random samples are drawn with replacement from the original dataset – X and Y matrices—to create a simulated dataset. By repeating this process many times, we can generate a distribution of statistics that can be used to estimate confidence intervals, *p* values, and other inferential statistics (Davison and Hinkley 1997), such as the residuals, which is of the utmost interest of this research, cornerstones for the computation of inefficiency scores. In the context of frontier analysis using the corrected ordinary least squares (COLS) method, the inefficiency term represents the unobserved heterogeneity or inefficiency in the production process. As previously mentioned, the COLS method is a modification of the OLS method that accounts for the presence of inefficiency by adding an additional error term to the model. This error term captures the unobserved heterogeneity and is assumed to be a non-negative random variable that affects the output but not the inputs. In our case, this is the error vector *e* computed obtained from MEROM. Once the residuals are estimated from COLS they can be used to compute the inefficient frontier by applying $\exp(-\mathrm{u})$, where u is the vector of COLS residuals.

The proposed methodology combines advanced statistical and computational techniques to address the complexity and uncertainty inherent in banking data. For example, traditional efficiency analysis methods often assume precise and deterministic data. However, in reality, banking data, particularly regarding Islamic banking, are subject to uncertainty and fuzziness due to factors like varying interpretations of Shariah law, diverse banking

practices across countries, and the unpredictable nature of financial markets. The 2DMCFA method acknowledges and integrates this uncertainty directly into the efficiency analysis, providing a more accurate and realistic assessment of bank performance.

Unlike conventional methods that might analyze inputs and outputs in isolation or assume a simple, linear relationship between them, 2DMCFA allows for a comprehensive examination of how multiple inputs and outputs interact in complex ways. This holistic approach is crucial for understanding the multifaceted operations of Islamic banks, which may not adhere strictly to the financial models designed for conventional banking systems. The banking environment, especially within the Islamic banking sector, involves nonlinear relationships between various factors affecting efficiency. For example, the impact of loan loss reserves on bank efficiency might not be linear, with different levels having varying effects on performance. Similarly, the relationships between bank size, liquidity, and efficiency are multifaceted and may not be adequately captured by linear models. 2DMCFA accommodates these complexities, providing a tool for analyzing nonlinear relationships and multifactor interactions.

In the second stage of the analysis, where the determinants of efficiency are examined, the paper employs a bootstrapped robust regression approach. This method is chosen to ensure that the findings are not overly sensitive to specific model assumptions. Bootstrapping, a resampling technique, allows for the estimation of the sampling distribution of a statistic and is useful for assessing the stability and reliability of regression results. This is important because Islamic banking data can exhibit peculiarities such as outliers, heteroskedasticity, or non-normality that might violate the assumptions of conventional regression models. A robust regression technique mitigates these issues, ensuring that the analysis of efficiency determinants remains valid even in the presence of data anomalies. By utilizing a robust regression approach, the research method allows for a more nuanced examination of the factors influencing Islamic bank efficiency. It enables the identification of significant determinants across different conditions and scenarios, providing insights that are critical for policymakers and practitioners in the Islamic banking sector.

4 Analysis and discussion of results

The dataset used in this research was obtained from FitchConnect and contains information for 49 different banks from 2013 to 2021, spanning across 25 countries. Table 1 presents the descriptive statistics for the inputs, outputs, and contextual variables commonly used to assess efficiency in Islamic Banks. Similar to conventional banking, Islamic banking aims to optimize its efficiency to maximize profits and meet stakeholders' expectations. Building on the work of Johnes et al. (2014) and Mirzaei et al. (2024), we consider the following inputs and outputs for our analysis, following Fukuyama and Tan (2024b):

- Total Deposits (Input): Banks utilize deposits to finance their operations. In Islamic banking, deposits are managed through Shari'ah-compliant accounts like Mudarabah and Musharakah.
- Fixed Assets (Input): Fixed assets, including buildings, equipment, and tangible assets, can influence the efficiency of Islamic banks. These assets are used to provide services to customers and generate revenue.
- Total Equity (Input): Total equity represents the capital invested by the bank's owners, playing a crucial role in generating profits and maintaining solvency.

Table 1 Descriptive stats of inputs outputs and contextual variables

Variable	Type	Min	Median	Max	Mean	SD	CV	Skewness	Kurtosis	Information entropy
Total deposits	Input	0	237.760	939.370	281.715	227.127	0.806	0.542	−0.643	0.322
Fixed assets	Input	0	4.265	73.560	8.765	11.213	1.279	2.329	6.654	0.188
Total equity	Input	0.07	39.800	532.450	74.439	95.030	1.277	2.680	7.405	0.194
Expenses	Input	0.09	9.055	136.050	12.911	14.275	1.106	4.078	24.175	0.143
Gross loans	Output	0.04	181.450	895.220	226.760	185.679	0.819	0.886	0.248	0.300
Total securities	Output	0	21.780	476.390	49.284	70.937	1.439	2.343	6.010	0.171
Year	Contextual	2013	2017	2022	2016.893	2.631	0.001	0.140	−1.151	0.357
LLRGL	Contextual	0	0.020	1	0.064	0.131	2.045	3.745	15.793	0.108
Net loans/Total assets	Contextual	0	0.573	0.985	0.536	0.228	0.425	-0.523	−0.598	0.342
Total assets (Log)	Contextual	0.262	5.854	6.904	5.590	1.084	0.194	-1.809	4.443	0.257

- Expenses (Input): Expenses encompass the costs associated with operating the bank, such as salaries, rent, utilities, and other overhead expenses.
- Gross Loans (Output): Gross loans denote the amount of money lent out by the bank to its customers.
- Total Securities (Output): Total securities refer to the bank's investment portfolio, which includes bonds, Sukuk, and other types of securities.

On the other hand, the contextual variables presented in Table 1 serve as endogenous drivers for Islamic banking efficiency, and their rationale is elaborated below:

The year variable in our analysis represents the timeframe during which the bank's performance is evaluated, and it serves as an endogenous driver for efficiency. Changes in economic conditions and regulatory environments over time can influence the bank's operations and performance. For example, fluctuations in interest rates, inflation rates, and government policies can impact profitability and efficiency. Additionally, the year variable may capture the impact of a learning curve on efficiency (Tan 2016). LLRGL, or the loan loss reserves to gross loans ratio, reflects a bank's ability to absorb potential credit losses. In the context of Islamic banks operating under Shariah law principles, this ratio significantly affects efficiency. A higher LLRGL ratio indicates greater resource allocation towards risk management and compliance, enhancing efficiency by reducing the likelihood of losses and regulatory fines. However, an excessively high ratio may signal an overly conservative approach, potentially limiting profit generation and market competitiveness (Sun and Chang 2011). Another endogenous driver for efficiency is the Net Loans/Total Assets ratio, representing the proportion of loans to total assets. Changes in lending policies and portfolio composition can affect profitability and efficiency. For instance, increasing lending to high-risk borrowers may boost revenue but heighten credit risk, potentially reducing efficiency (Tan and Anchor 2017). Finally, Total Assets signify the scale of the bank's operations. Larger banks may benefit from economies of scale, leading to improved efficiency and profitability. However, expansion may introduce additional regulatory and operational challenges, impacting efficiency. Taking the logarithm of total assets accounts for non-linear relationships between size and efficiency (Tan and Floros 2018).

Results from the bootstrapped MEROM regression, aimed at reducing the number of outputs to their aggregate mean, are presented in Fig. 1. Significant results were found for both gross loans and total securities, based on the lower and upper output crisp values inferred using HT2FS. However, the intercept did not show significant results. The MEROM regression effectively captured the isotonicity between the two outputs, as they both had positive signs in predicting their overall aggregated mean. Subsequently, after reducing the two outputs to a common aggregate mean value (Y), COLS regressions were conducted considering the upper and lower crisp values for the inputs and outputs generated through HT2FS inference. The resulting densities for the corresponding efficiency score vectors are illustrated in Fig. 2. The efficiency scores demonstrated a good discriminatory power, with the lower and upper estimates being respectively distant from the boundaries of 0 and 1, as expected from the HT2FS modeling. However, there was some overlap between the distribution estimates due to the inherent characteristics of the HT2FS approach. The scalar parameter alpha, which determines the degree of hesitation or indeterminacy in the resulting upper and lower fuzzy sets, was set to the compromise value of 0.5. Lower values of alpha imply higher juxtaposing of densities, while higher values of alpha result in lower juxtaposing.

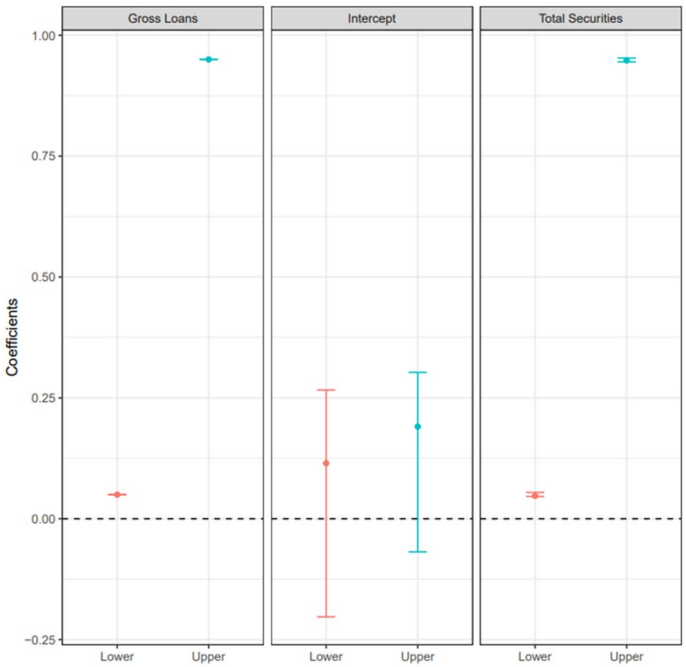

Fig. 1 MEROM regression results

A comparison of the final COLS scores (upper and lower averaging) with the results obtained from classic DEA (VRS output orientation specification) and SFA models is presented in Fig. 3 and Table 2. The COLS method demonstrates better discriminatory power compared to the classic frontier models. While the DEA results appear to be biased towards 1, the SFA model tends to exhibit a flat density across a broader range of scores compared to COLS estimates. Considering the principle of maximal information entropy, which suggests that the most appropriate efficiency model is the one with higher entropy to mitigate epistemic uncertainty in model selection, COLS appears to be the most suitable choice as its efficiency score vector has maximal information entropy (refer to Table 2). Lastly, results for a robust regression approach of COLS aggregate scores onto contextual variables were obtained using a bootstrapped robust regression approach. Assumptions for Tobit, Simplex, and Beta distributed scores were simultaneously tested through differential optimization and minimization of the covariance matrix of their residuals. For a comprehensive discussion on this approach, readers are encouraged to refer to the following papers: Wanke et al. (2017), Wanke et al. (2019), Wanke et al. (2022), among others.

While Fig. 4 indicates that the Tobit assumption received a median of 87.5% of the weights, Beta received 12.5%, and Simplex received 0% across all bootstrapped replications, Fig. 5 reveals that efficiency in Islamic banks exhibits geographic heterogeneity and is dependent on risk parsimony and reporting standards. The results demonstrate that certain countries, such as Jordan, UAE, Oman, Iraq, West Bank and Gaza, as well as the Syrian Arab Republic, have significantly higher levels of efficiency in their Islamic banking systems. On the other hand, countries like Pakistan, Maldives, Tunisia, Malaysia, Thailand, and Kuwait exhibit significantly lower levels of efficiency. Multiple factors contribute to this variation. One possible factor is the regulatory environment in each country.

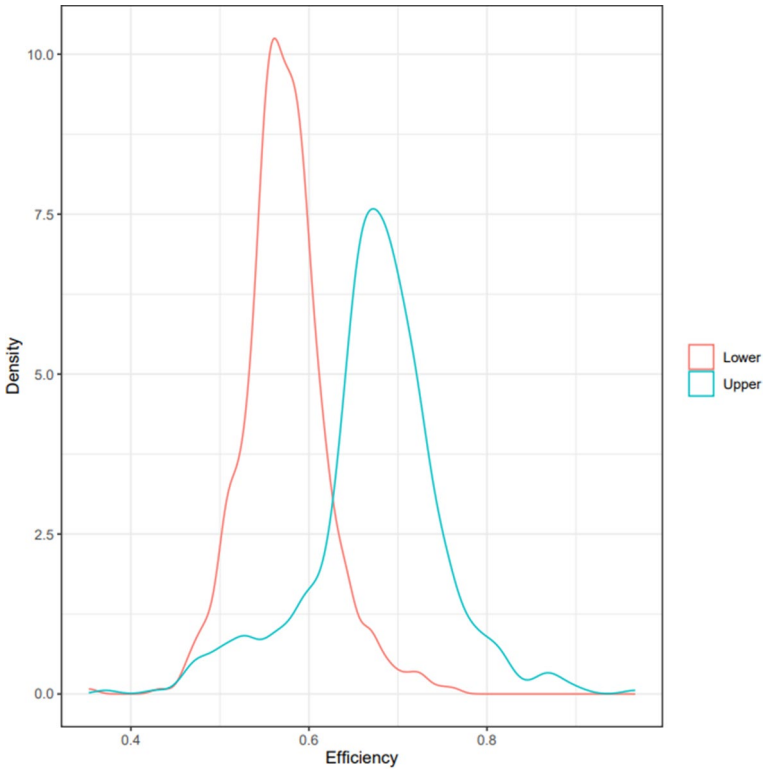

Fig. 2 Efficiency density plots for COLS regressions using HT2FS upper and lower estimates

Effective regulatory frameworks can promote greater efficiency in Islamic banking systems by establishing standards, fostering competition, and ensuring compliance with best practices. Another factor is the level of competition in the market. Increased competition can drive banks to become more innovative and efficient, resulting in higher efficiency levels. Additionally, disparities in the development of the financial sector, legal systems, and overall economic environment may influence the efficiency of Islamic banking systems. Countries with more developed financial sectors, supportive legal systems, and stable macroeconomic environments tend to promote efficiency in their Islamic banking systems. It is essential to note that lower efficiency levels in Islamic banking systems do not necessarily indicate poor overall performance in those countries. Instead, these findings highlight areas for improvement and suggest opportunities to enhance efficiency in Islamic banking. Policymakers and managers can utilize these insights to develop strategies and policies aimed at improving efficiency, which can lead to better financial performance, increased competitiveness, and sustainable economic growth. Moreover, the observed geographic heterogeneity in Islamic bank efficiency underscores the importance of context-specific factors in shaping performance outcomes. For instance, socio-cultural factors, such as consumer preferences and attitudes towards Islamic finance, may influence the demand for Islamic banking products and services, thereby impacting the efficiency of Islamic banks in different countries. Furthermore, differences in regulatory enforcement, political stability, and the quality of governance can further contribute to the variation in efficiency levels across

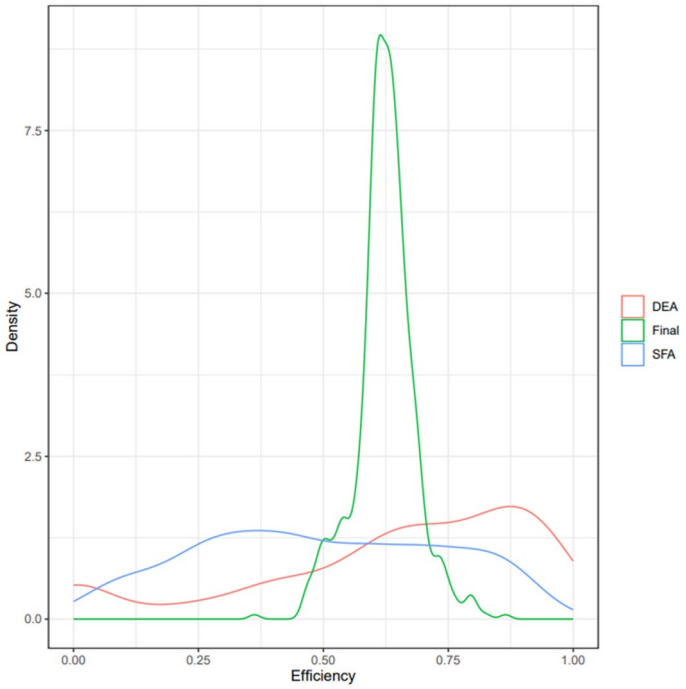

Fig. 3 Density comparison between classic frontier models (DEA and SFA) and COLS averaged upper and lower scores

Table 2 Information entropy and KL divergence results

		KL divergence		
	Entropy	Final	DEA	SFA
Final	6.294	0	1.122	0.211
DEA	6.153	0.150	0	0.068
SFA	6.160	0.146	0.423	0

jurisdictions. Therefore, a nuanced understanding of the local context is essential for devising targeted interventions and policies to improve efficiency in Islamic banking systems, ultimately fostering sustainable economic development and financial stability.

We also find that the ratio between net loans and total assets, as well as the ratio between loan loss reserves and gross loans, have a significant and positive impact on Islamic bank efficiency. The net loans to total assets ratio represents the percentage of a bank's assets that are allocated to loans, which is a crucial source of income for Islamic banks. A higher ratio indicates that the bank is investing more in loans, potentially leading to higher profits. The positive impact of this ratio on Islamic bank efficiency suggests that banks maintaining a higher proportion of net loans to total assets operate more efficiently. On the other hand, the loan loss reserves to gross loans ratio reflects the bank's ability to absorb potential credit losses. It represents the percentage of gross loans set aside as reserves to cover potential losses. A higher ratio indicates a more conservative approach to risk management,

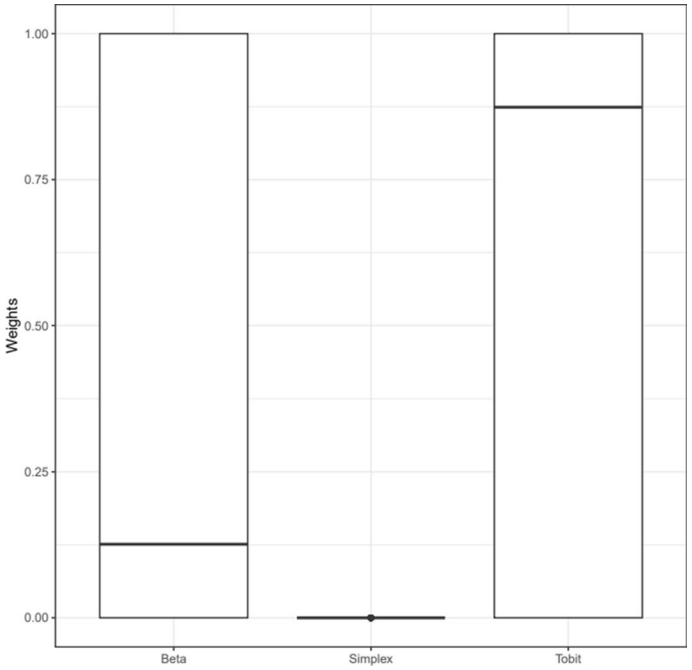

Fig. 4 Distributional assumption weights for COLS scores in the robust regression

enabling the bank to absorb losses without compromising its financial position. The positive impact of this ratio on Islamic bank efficiency suggests that banks maintaining higher levels of loan loss reserves operate more efficiently and effectively manage risks. These findings underscore the critical role of loan management and risk mitigation strategies in driving efficiency within Islamic banking systems. The positive relationship between the net loans to total assets ratio and Islamic bank efficiency highlights the importance of prudent lending practices and effective allocation of resources. Banks that prioritize lending activities and maintain a higher proportion of net loans relative to their total assets demonstrate a greater capacity to generate income and optimize their operations. Similarly, the positive impact of the loan loss reserves to gross loans ratio on Islamic bank efficiency underscores the significance of robust risk management frameworks. By setting aside adequate reserves to cover potential credit losses, banks can enhance their resilience and safeguard their financial stability. These results emphasize the need for Islamic banks to strike a balance between profit maximization and risk mitigation, thereby ensuring sustainable growth and long-term viability.

Lastly, although not highly significant, we find that total assets have a positive and significant impact on Islamic bank efficiency, indicating that larger banks tend to be more efficient. This can be attributed to factors such as economies of scale, greater access to funding, and risk diversification. Larger banks can spread their fixed costs over a larger asset base, reducing their average cost per unit and improving efficiency. Moreover, they may have better access to funding, providing them with a competitive advantage over smaller banks. Additionally, the significant and positive impact of the Trend variable suggests that the Islamic banks in our sample experienced an improvement in efficiency between 2013 and 2021. This improvement could be attributed to

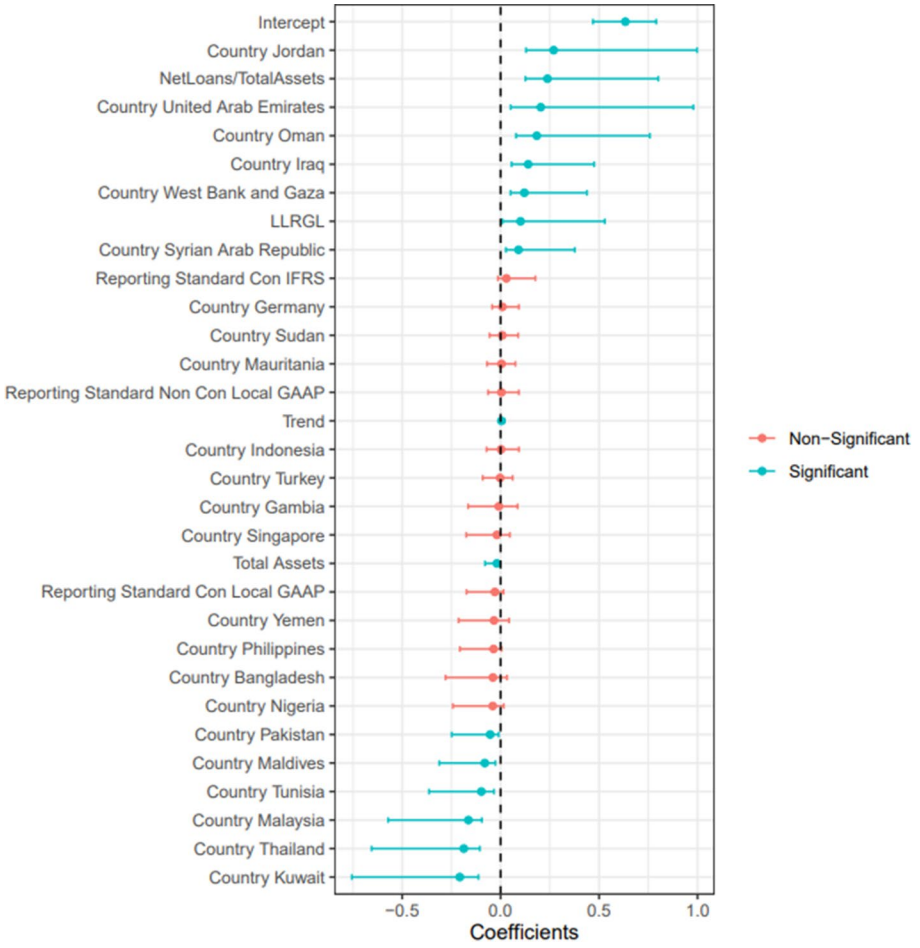

Fig. 5 Significance results for the contextual variables

various factors, including technological advancements, changes in management practices, and increased market competition. In an increasingly competitive banking industry, banks are compelled to enhance their efficiency to remain competitive. Furthermore, technological advancements have allowed banks to streamline operations and reduce costs, contributing to improved efficiency. In addition to the factors discussed, it's worth noting the role of technological advancements in driving efficiency improvements in Islamic banking. The positive impact of the Trend variable underscores the ongoing efforts within the industry to adopt and leverage technology for operational enhancements. Specifically, advancements in digital banking, blockchain technology, and artificial intelligence have revolutionized traditional banking processes, leading to streamlined operations, reduced costs, and improved customer experiences. By embracing these technological innovations, Islamic banks can not only enhance their efficiency but also stay competitive in an ever-evolving financial landscape. Furthermore, the increasing digitization of banking services has opened up new avenues for reaching customers, expanding market reach, and diversifying revenue streams. Therefore, technological

innovation remains a critical factor in shaping the future trajectory of Islamic banking efficiency and competitiveness.

5 Conclusion

The current study makes a significant contribution to the efficiency analysis in the banking industry by proposing a 2-Dimensional Fuzzy Monte-Carlo Frontier Analysis to estimate and compare the efficiency levels of a sample of 49 Islamic banks from 25 countries worldwide between 2013 and 2021. The existing literature has limited development in terms of the methods adopted for estimating Islamic bank efficiency. As reviewed in the literature, the methods are confined to just a few, including stochastic frontier analysis, data envelopment analysis, multi-criteria decision analysis, meta-frontier analysis, multi-directional efficiency analysis, among others.

In this study, a novel 2-Dimensional Fuzzy Monte-Carlo Frontier Analysis is proposed for the first time. This approach offers the advantages of incorporating uncertainty and fuzziness associated with real-world data. Furthermore, the consideration of two-dimensional analysis, compared to one dimension, makes it a more comprehensive and accurate method for assessing efficiency. Additionally, the proposed method addresses the issue of nonlinearity between inputs and outputs, which has not yet been adequately addressed in previous literature studies. In the second stage, a bootstrapped robust regression approach is proposed to comprehensively investigate the determinants of efficiency.

The results of the study reveal several key findings: 1) At the country level, Jordan, UAE, Oman, Iraq, West Bank and Gaza, as well as the Syrian Arab Republic, have significantly higher levels of efficiency in their Islamic banking systems. In comparison, the efficiency levels of Islamic banking in Pakistan, Maldives, Tunisia, Malaysia, Thailand, and Kuwait are significantly lower; 2) The ratio between net loans and total assets, as well as the ratio between loan loss reserves and gross loans, have a significant and positive impact on Islamic bank efficiency; 3) Total assets have a positive and significant impact on Islamic bank efficiency; 4) The Islamic banks included in our sample experienced an improvement in the level of efficiency over the examined period between 2013 and 2021.

Based on these results, the study offers the following policy implications: 1) Countries with low efficiency levels in their Islamic banking systems should examine the factors contributing to this, such as regulatory constraints, inadequate infrastructure, or insufficient technology adoption. Policy makers in these countries could learn from more efficient countries in the sample to identify best practices and strategies for improving efficiency; 2) Islamic banks should focus on maintaining an optimal level of net loans to total assets and loan loss reserves to gross loans ratios to enhance their efficiency. Strategies such as improving credit risk management practices, diversifying loan portfolios, and adopting more effective loan recovery strategies could be considered; 3) Encouraging growth in the Islamic banking sector may lead to increased efficiency. Policy makers could explore ways to promote the growth of Islamic banks, such as through incentives for market entry or increased access to financing; 4) The finding that Islamic banks experienced an improvement in efficiency over the examined period suggests that policy makers should continue to monitor and support the growth of the Islamic banking sector. Regular assessments of efficiency and other performance metrics can help identify areas for improvement and ensure that the sector continues to evolve and meet the needs of its customers.

Appendix

Country	Frequency
United Kingdom	143
Pakistan	92
Indonesia	43
Nigeria	34
Iraq	30
Philippines	30
Sudan	30
Maldives	18
Tunisia	18
Oman	17
Singapore	12
Yemen	11
Bangladesh	9
Thailand	9
Turkiye	9
Kuwait	8
West Bank and Gaza	8
Mauritania	6
Malaysia	5
United Arab Emirates	4
Gambia	2
Germany	2
Jordan	2
Syrian Arab Republic	2
Accounting standards	**Frequency**
NonCon/IFRS	245
NonCon/LOCAL GAAP	219
Con/IFRS	60
Con/LOCAL GAAP	21

Declarations

Conflict of interest The authors have no relevant financial or non-financial interests to disclose.

References

Abedifar P, Hassan I, Tarazi A (2016) Finance-growth nexus and dual-banking systems: relative importance of Islamic banks. J Econ Behav Organ 132:198–215. https://doi.org/10.1016/j.jebo.2016.03.005

Ahn H, Clermont M, Langner J (2023) Comparative performance analysis of frontier-based efficiency measurement methods–a Monte Carlo simulation. Eur J Oper Res 307:294–312. https://doi.org/10.1016/j.ejor.2022.09.039

Ali J, Al-kenani AN (2023) Vector similarity measures of dual hesitant fuzzy linguistic term sets and their applications. Symmetry 15:471. https://doi.org/10.3390/sym15020471

Alqahtani F, Mayes DG, Brown K (2017a) Reprint of economic turmoil and islamic banking: evidence from the Gulf Cooperation Council. Pac Basin Financ J 42:113–125. https://doi.org/10.1016/j.pacfin.2016.06.013

Alqahtani F, Mayes DG, Brown K (2017b) Islamic bank efficiency compared to conventional banks during the global crisis in the GCC region. J Int Financ Mark Inst Money 51:58–74. https://doi.org/10.1016/j.intfin.2017.08.010

Antunes J, Hadi-Vencheh A, Jamshidi A, Tan Y, Wanke P (2022) Bank efficiency estimation in China: DEA-RENNA approach. Ann Oper Res 315:1373–1398. https://doi.org/10.1007/s10479-021-04111-2

Antunes J, Tan Y, Wanke P (2024a) Analyzing Chinese banking performance with a trigonometric envelopment analysis for ideal solutions model. IMA J Manag Math 35:279–401. https://doi.org/10.1093/imaman/dpad026

Antunes J, Wanke P, Azad Md AK, Tan Y, Faria JR, Mamede A (2024b) Estimating Japanese bank performance: Stochastic entropic analysis on the basis of ideal solutions. Expert Syst Appl 242:122762. https://doi.org/10.1016/j.eswa.2023.122762

Antunes J, Hadi-Vencheh A, Jamshidi A, Tan Y, Wanke P (2024c) Cost efficiency of Chinese banks: evidence from DEA and MLP-SSRP analysis. Expert Syst Appl 237:121432. https://doi.org/10.1016/j.eswa.2023.121432

Arunraj NS, Mandal S, Maiti J (2013) Modeling uncertainty in risk assessment: an integrated approach with fuzzy set theory and Monte Carlo simulation. Accid Anal Prev 55:242–255

Asmild M, Kronborg D, Mahbub T, Matthews K (2019) The efficiency patterns of Islamic banks during the global financial crisis: the case of Bangladesh. Q Rev Econ Financ 74:67–74. https://doi.org/10.1016/j.qref.2018.04.004

Bahrini R (2015) Productivity of MENA Islamic banks: a bootstrapped Malmquist index approach. Int J Islam Middle East Finance Manag 8:508–528. https://doi.org/10.1108/IMEFM-11-2014-0114

Belanès A, Ftiti Z, Regaïeg R (2015) What can we learn about Islamic banks efficiency under the subprime crisis? Evidence from GCC Region. Pac Basin Financ J 33:81–92. https://doi.org/10.1016/j.pacfin.2015.02.012

Battese GE, Coelli TJ (1995) A model for technical inefficiency effects in a stochastic frontier production function for panel data. Empir Econ 20:325–332

Bitar M, Pukthuanthong K, Walker T (2020) Efficiency in Islamic vs. conventional banking: the role of capital and liquidity. Glob Finac J 46:100487. https://doi.org/10.1016/j.gfj.2019.100487

Daly S, Frikha M (2017) Determinants of bank performance: comparative study between conventional and Islamic Banking in Bahrain. J Knowl Econ 8:471–488. https://doi.org/10.1007/s13132-015-0261-8

Davison AC, Hinkley DV (1997) Bootstrap methods and their application. Cambridge University Press, Cambridge

Denzau AT, Gibbons PC, Greenberg E (1989) Bayesian estimation of proportions with a crossentropy prior. Commun Stat- Theory Methods 18:1843–1861. https://doi.org/10.1080/03610928908830005

Efron B (1979) Bootstrap methods: another look at the jackknife. Ann Stat 7:1–26. https://doi.org/10.1214/aos/1176344552

Fisher RA (1921) On the mathematical foundations of theoretical statistics. Philos Transact A Math Phys Eng Sci 222:309–368

Fukuyama H, Tan Y (2021) Corporate social behaviour: is it good for efficiency in the Chinese banking industry? Ann Oper Res 306:383–413. https://doi.org/10.1007/s10479-021-03995-4

Fukuyama H, Tan Y (2022a) Implementing strategic disposability for performance evaluation: innovation, stability, profitability and corporate social responsibility in Chinese banking. Eur J Oper Res 296:652–668. https://doi.org/10.1016/j.ejor.2021.04.022

Fukuyama H, Tan Y (2022b) Deconstructing three-stage overall efficiency into input, output and stability efficiency components with consideration of market power and loan loss provision: an application to Chinese banks. Int J Financ Econ 27:953–974. https://doi.org/10.1002/ijfe.2185

Fukuyama H, Tan Y (2024a) Investigating into the dual role of loan loss reserves in banking production process. Ann Oper Res 334:423–444. https://doi.org/10.1007/s10479-021-04365-w
Fukuyama H, Tan Y (2024b) The impacts of innovation and trade openness on bank market power: the proposal of a minimum distance cost function approach and a causal structure analysis. Eur J Oper Res 312:1178–1194. https://doi.org/10.1016/j.ejor.2023.08.016
Fukuyama H, Tsionas M, Tan Y (2023) Dynamic network data envelopment analysis with a sequential structure and behavioural-causal analysis: application to the Chinese banking industry. Eur J Oper Res 307:1360–1373. https://doi.org/10.1016/j.ejor.2022.09.028
Galan J, Tan Y (2024) Green light for green credit? Evidence from its impact on bank efficiency. Int J Financ Econ 29:531–550. https://doi.org/10.1002/ijfe.2697
Gheeraert L, Weill L (2015) Does Islamic banking development favor macroeconomic efficiency? Evidence on the Islamic finance-growth nexus. Econ Model 47:32–39. https://doi.org/10.1016/j.econmod.2015.02.012
Golan A, Judge GG, Miller D (1996a) Maximum entropy econometrics: robust estimation with limited data. Wiley, New Jersey
Golan A, Judge G, Perloff JM (1996b) A maximum entropy approach to recovering information horn multinomial response data. J Am Stat Assoc 91:841–853. https://doi.org/10.1080/01621459.1996.10476952
Gong Y, Yang S, Ma H, Ge M (2018) Fuzzy Regression Model Based on Incentre Distance and Application to Employee Performance Evaluation. Int J Fuzzy Syst 20:2632–2639. https://doi.org/10.1007/s40815-018-0536-x
Hardianto DS, Wulandari P (2016) Islamic bank vs conventional bank: intermediation, fee-based service activity and efficiency. Int J Islam Middle East Financ Manag 9:296–311. https://doi.org/10.1108/IMEFM-01-2015-0003
Herrera E, Cabrerizo FJ, Kacprzyk J, Pedrycz W (2014) A review of soft consensus models in a fuzzy environment. Inf Fusion 17:4–13. https://doi.org/10.1016/j.inffus.2013.04.002
Ismail F, Majid MSA, Rahim RA (2013) Efficiency of Islamic and conventional banks in Malaysia. J Financ Report Account 11:92–107. https://doi.org/10.1108/JFRA-03-2013-0011
Jaynes ET (1957) Information theory and statistical mechanics. Phys Rev 106:620–630. https://doi.org/10.1103/PhysRev.106.620
Johnes J, Izzeldin M, Pappas V (2014) A comparison of performance of Islamic and conventional banks 2004–2009. J Econ Behav Organ 103:S93–S107. https://doi.org/10.1016/j.jebo.2013.07.016
Kamarudin F, Nordin BAA, Muhammad J, Hamid MAA (2014) Cost, revenue and profit efficiency of islamic and conventional banking sector: empirical evidence from Gulf Cooperative Council Countries. Glob Bus Rev 15:1–24. https://doi.org/10.1177/0972150913515579
Khan I, Khan M, Tahir M (2017) Performance comparison of Islamic and conventional banks: empirical evidence from Pakistan. Int J Islam Middle East Financ Manag 10:419–433. https://doi.org/10.1108/IMEFM-05-2016-0077
Khayum N, Rout A, Deepak BBVL, Anbarusu S, Murugan S (2020) Application of fuzzy regression analysis in predicting the performance of the anaerobic reactor co-digesting spent tea waste with cow manure. Waste Biomass Valoriz 11:5665–5678. https://doi.org/10.1007/s12649-019-00874-9
Kumbhakar SC, Lovell CA (2000) Stochastic frontier analysis. Cambridge University Press, Cambridge
Kumbhakar SC, Tsionas EG (2011) Measuring technical and allocative inefficiency in the translog cost system: a correction. J Econom 163:203–210. https://doi.org/10.1016/j.jeconom.2004.05.006
Manacero A, Guariglia E, de Souza TA, Lobato RS, Spolon R (2022) Parallel fuzzy minimals on GPU. Appl Sci 12:2385. https://doi.org/10.3390/app12052385
Maredza A, Wanke P, Antunes J, Pimenta R, Tan Y (2022) Social welfare and bank performance: evidence from a stochastic neural hybrid MCDM approach. J Econ Stud 49:1137–1158. https://doi.org/10.1108/JES-05-2021-0236
Merton RC (1974) On the pricing of corporate debt: the risk structure of interest rates. J Finance 29(2):449–470
Mirzaei A, Saad M, Emrouznejad A (2024) Bank stock performance during the COVID-19 crisis: does efficiency explain why Islamic banks fared relatively better? Ann Oper Res 334:317–355. https://doi.org/10.1007/s10479-022-04600-y
Mobarek A, Kalonov A (2014) Comparative performance analysis between conventional and Islamic banks: empirical evidence from OIC countries. Appl Econ 46:253–270. https://doi.org/10.1080/00036846.2013.839863
Mohanty SK, Lin H-J, Aljuhani EA, Bardesi HJ (2016) Banking efficiency in Gulf Cooperation Council (GCC) countries: a comparative study. Rev Financ Econ 31:99–107. https://doi.org/10.1016/j.rfe.2016.06.004

Nasim A, Nazir MA, Downing G (2024) Determinants of bank efficiency in developed (G7) and developing (E7) countries: role of regulatory and economic environment. Rev Quant Financ Acc. https://doi.org/10.1007/s11156-024-01272-6

Nasr AK, Tavana M, Alavi B, Mina H (2021) A novel fuzzy multi-objective circular supplier selection and order allocation model for sustainable closed-loop supply chains. J Clean Prod 287:124994. https://doi.org/10.1016/j.jclepro.2020.124994

Nieswand M, Seifert S (2018) Environmental factors in frontier estimation–a Monte Carlo analysis. Eur J Oper Res 265:133–148. https://doi.org/10.1016/j.ejor.2017.07.047

Rosman R, Wahab NA, Zainol Z (2014) Efficiency of Islamic banks during the financial crisis: an analysis of Middle Eastern and Asian countries. Pac Basin Finance J 28:76–90. https://doi.org/10.1016/j.pacfin.2013.11.001

Saâdaoui F, Khalfi M (2024) Revisiting Islamic banking efficiency using multivariate adaptive regression splines. Ann Oper Res 334:287–315. https://doi.org/10.1007/s10479-022-04545-2

Saeed M, Izzeldin M (2016) Examining the relationship between default risk and efficiency in Islamic and conventional banks. J Econ Behav Organ 132:127–154. https://doi.org/10.1016/j.jebo.2014.02.014

Saeed M, Izzeldin M, Hassan MK, Pappas V (2020) The inter-temporal relationship between risk, capital and efficiency: the case of Islamic and conventional banks. Pac Basin Finance J 62:101328. https://doi.org/10.1016/j.pacfin.2020.101328

Safiullah M, Shamsuddin A (2019) Risk-adjusted efficiency and corporate governance: evidence from Islamic and conventional banks. J Corp Fin 55:105–140. https://doi.org/10.1016/j.jcorpfin.2018.08.009

Sufian F, Kamarudin F (2015) Determinants of revenue efficiency of Islamic banks: empirical evidence from the Southeast Asian countries. Int J Islam Middle East Financ Manag 8:36–63. https://doi.org/10.1108/IMEFM-12-2012-0114

Sun L, Chang T-P (2011) A comprehensive analysis of the effects of risk measures on bank efficiency: evidence from emerging Asian countries. J Bank Financ 35:1727–1735. https://doi.org/10.1016/j.jbankfin.2010.11.017

Tan Y (2016) The impacts of risk and competition on bank profitability in China. J Int Financ Mark Inst Money 40:85–110. https://doi.org/10.1016/j.intfin.2015.09.003

Tan Y, Anchor J (2017) The impacts of risk-taking behaviour and competition on technical efficiency: evidence from the Chinese banking industry. Res Int Bus Financ 41:90–104. https://doi.org/10.1016/j.ribaf.2017.04.026

Tan Y, Floros C (2012) Bank profitability and inflation: the case of China. J Econ Stud 39:675–696. https://doi.org/10.1108/01443581211274610

Tan Y, Floros C (2013) Risk, capital and efficiency in Chinese banking. J Int Financ Mark Inst Money 26:378–393. https://doi.org/10.1016/j.intfin.2013.07.009

Tan Y, Floros C (2018) Risk, competition and efficiency in banking: evidence from China. Glob Financ J 35:223–236. https://doi.org/10.1016/j.gfj.2017.12.001

Tan Y, Tsionas M (2022) Modelling sustainability efficiency in banking. Int J Financ Econ 27:3754–3772. https://doi.org/10.1002/ijfe.2349

Tan Y, Walheer B (2024) Stability and economic performances in the banking industry: the case of China. Q J Econ Financ 95:326–345. https://doi.org/10.1016/j.qref.2024.04.009

Tan Y, Floros C, Anchor J (2017) The profitability of Chinese banks: impacts of risk, competition and efficiency. Rev Account Financ 16:86–105. https://doi.org/10.1108/RAF-05-2015-0072

Tan Y, Wanke P, Antunes J, Emrouznejad A (2021) Unveiling endogeneity between competition and efficiency in Chinese banks: a two-stage network DEA and regression analysis. Ann Oper Res 306:131–171. https://doi.org/10.1007/s10479-021-04104-1

Tikhonov AN (1963) On the solution of incorrectly formulated problems and the regularization method. Sov Math Dokl 4:1035–1038

Tripathi KK, Hasan A, Jha KN (2021) Evaluating performance of construction organizations using fuzzy preference relation technique. Int J Constr Manag 21(12):1287–1300

Wanke P, Azad AK, Barros CP, Hassan MK (2016a) Predicting efficiency in Islamic banks: an integrated multicriteria decision making (MCDM) approach. J Int Financ Mark Inst Money 45:126–141. https://doi.org/10.1016/j.intfin.2016.07.004

Wanke P, Azad AD, Barros CP (2016b) Predicting efficiency in Malaysian Islamic banks: a two-stage TOPSIS and neural networks approach. Res Int Bus Financ 36:485–498. https://doi.org/10.1016/j.ribaf.2015.10.002

Wanke P, Barros CP, Emrouznejad A (2016c) Assessing productive efficiency of banks using integrated Fuzzy-DEA and bootstrapping: a case of Mozambican banks. Eur J Oper Res 249:378–389. https://doi.org/10.1016/j.ejor.2015.10.018

Wanke P, Maredza A, Gupta R (2017) Merger and acquisitions in South African banking: a network DEA model. Res Int Bus Financ 41:362–376. https://doi.org/10.1016/j.ribaf.2017.04.055

Wanke P, Azad AK, Emrouznejad A, Antunes J (2019) A dynamic network DEA model for accounting and financial indicators: a case of efficiency in MENA banking. Int Rev Econ Financ 61:52–68. https://doi.org/10.1016/j.iref.2019.01.004

Wanke P, Antunes JJM, Correa HL, Tan Y (2022) Strategic fit of mergers and acquisitions in Latin American airlines: a two-stage DEA approach. Benchmarking 29:1513–1545. https://doi.org/10.1108/BIJ-11-2020-0588

Wanke PF, Pimenta R, Antunes J, Tan Y, Hadi-Vencheh A (2023a) Endogenous performance of rail sections in Brazil: a novel two-dimensional Fuzzy-Monte Carlo approach. Appl Sci 13:2618. https://doi.org/10.3390/app13042618

Wanke P, Azad AK, Antunes J, Tan Y, Pimenta R (2023b) Endogenous and exogenous performance sources in Asian Banking: a hybrid stochastic Multi-Criteria Decision-Making approach based on sign decomposition and transfer entropy. Expert Syst Appl 225:120180. https://doi.org/10.1016/j.eswa.2023.120180