Do compensation plans with performance targets provide better incentives?*

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March 10, 2014

Abstract

Guided by academic literature, industry practice and policy recommendations, we analyze a wide range of option and restricted stock plans with exercise and vesting conditions that may be contingent on stock price performance. To assess the effectiveness of these plans at attracting and providing incentives to executives, we create compensation plans with fixed firm cost and executive valuation and calculate their expected total lifetime incentives. We show that performance vesting targets provide the least cost effective incentives, performance exercise targets provide the largest risk incentives, option plans are generally superior to restricted stock plans, and calendar vesting is only efficient up to a maximum of three years. Performance exercise targets can increase the expected total lifetime incentives provided by compensation plans, but in general, standard options with short vesting periods provide the most cost effective pay-for-performance incentives.

Keywords: Executive compensation, Performance target plans, Incentives, Restricted stock, options.

*Pinto is from the Department of Accounting, Finance, and Economics at the University of Bradford. Widdicks is from the Department of Finance at the University of Illinois at Urbana-Champaign and the Department of Accounting and Finance, Lancaster University. We thank an anonymous reviewer, Martin Conyon, Itay Goldstein, Dirk Hackbarth, Naveen Khanna, Timothy Johnson, Bart Lambrecht, Andrew Marshall, Grzegorz Pawlina, Joshua Pollet, and seminar participants at Michigan State University, Lancaster University, the FMA 2011 European conference, and the MFA 2012 annual meetings for helpful comments.

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1 Introduction

There has been considerable public outrage regarding both the amount paid to executive directors of larger firms and the fact that some of these executives can be rewarded regardless of performance. Some researchers even argue that remuneration policies might have contributed to the recent financial crisis (e.g. Bebchuck, Cohen, and Spaman, 2010). In response, some governments and professional bodies have analyzed existent remuneration policies. Worldwide, the Financial Stability Board designed principles and standards that aim to improve compensation practices in financial institutions (see FSB Principles for sound compensation practices, 2009). Countrywide, policy documents have advised changes to the current remuneration policies of financial institutions. In the UK, the 2009 Walker Review states that Long Term Incentive Plans (LTIPs) might have motivated managers to take short term decisions and suggests that compensation plans should have vesting periods of up to 5 years and be subject to pre-vesting performance target conditions. The use of performance target conditions in compensation plans was suggested previously in the Greenbury (1995) report. Since the Greenbury report, UK firms commonly attach performance targets to their compensation plans. In the USA, although some researchers (e.g. Bebchuck, Fried, and Walker, 2002) have argued in favour of performance target options, there seems to be less pressure from regulators towards the introduction of these target conditions in compensation plans. However Lublin (2006) argues that changes in accounting rules may well result in a widespread usage of performance target options by US firms.

Stock and option plans are typically justified on the grounds of attracting and providing incentives to executives in order to increase firm value (see e.g. Hall and Murphy, 2002). Nevertheless, it is unclear whether stock or options are the most effective at achieving these aims. Moreover, it is unclear how some of the features of stock and option plans can impact on the incentives provided by these plans. Hall and Murphy (2002, p. 5) argue that “incentives may be provided more efficiently through plans of restricted stock rather than options”. Ditmann and Maug (2007) also argue that stock should dominate options in compensation plans and that, optimally, CEOs should be granted no options. Moreover, the Greenbury (1995) report suggested that compensation plans with restricted stock may be more effective than options in linking pay with firm performance. Contrary to these studies, our results show that the use of options, to compensate utility maximising risk averse executives, is consistent with maximizing total expected lifetime pay for performance incentives (and risk increasing incentives). This appears to justify why, in practice, firms do continue to include options in compensation plans.

Guided by industry practice and policy recommendations, and with the objective of assessing the incentives provided by different compensation plans, we develop an adaptable, dynamic option valuation model for restricted stock and option plans. In particular, we incorporate calendar vesting periods, exercise cond-

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1 Similar recommendations also appear in the Association of British Insurers (ABI) (2009) guidelines.
2 This recommendation can also be found in the ABI guidelines (2009) and in the combined code (2010).
tional on stock price performance and vesting conditional on stock price performance. In our multi-period dynamic model a risk-averse executive is granted either at-the-money American options or restricted stock. Consistent with Hall and Murphy (2002) and Carpenter (2002) the executive makes optimal exercise decisions to maximise terminal wealth utility. We also allow the executive to exercise the option or stock holding in stages (see Pollet, White and Widdicks, 2013). The possibility of exercise in stages allows for the analysis of different performance targets for each different option or restricted stock within a single grant.

As is typical in the literature, our analysis of the compensation plans focuses on determining the pay-for-performance sensitivity incentives, or the plan’s delta, and the plan’s incentives to increase the firm’s risk, or the plan’s vega. However, unlike other models that consider only incentives at the issuance date, we calculate the expected total incentives over the lifetime of the plan. In our analysis, the different plans result in the same firm cost and executive valuation, and so the best plan maximizes the incentives over the plan’s lifetime. Naturally, a compensation plan with a larger expected lifetime delta should motivate further the executive to increase the stock price and is the best plan from a shareholder perspective. More contentious is the argument that the best plan should maximise the executive’s incentive to take risk. In our model the executive is risk averse and thus, might want to avoid risky but profitable projects, or to seek risk reduction even if this risk reduction is costly to the firm (Smith and Stulz, 1985 and Tufano, 1986). Dittman and Yu (2011) show that, under optimal contracting, executive compensation practices should aim to provide risk incentives. Thus, plans which lead to a risk increase should better align the interests of risk averse executives with the interests of risk neutral shareholders.

The expected lifetime deltas and vegas are intuitive measures of the incentives and their duration. To provide a simplified example, consider zero interest rates and a risk-neutral executive which can be granted one of two possible contracts with the same cost and value to the executive. One contract has a constant 0.8 delta for one year, giving a lifetime delta of 0.8 and the other contract has a constant delta of 0.5 but an expected lifetime of 2 years, giving a lifetime delta of 1. The contract with the lifetime delta of 0.8 is inferior since, over a two-year period, issuing (sequentially) two contracts of the first type would produce a lifetime delta of 1.6 and issuing two (simultaneously) of the second type of contract would produce a lifetime delta of 2. Thus, the second contract provides larger overall incentives (and has the same cost and value) and is therefore superior.

It is typical (see e.g. Hall and Murphy, 2002, and Ditmann and Maug, 2007) to consider the instantaneous delta at $t = 0$ without any consideration for expected lifetime. However, this could potentially lead to misleading conclusions since a plan can have a very large delta but subsequently be exercised (sold in the case of stocks) very shortly after issuance. As an illustration, for the plans we consider, the restricted stock with sliding scale sale has a very large instantaneous delta (see Table 3) as when $P = 1$ a very small increase in the stock price (to $P > 1$) would lead to an instant payoff of $\$1$. This discontinuous payoff ($0$ if $P \leq 1$)

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4 As a robustness check we also consider compensation plans in which vesting or exercise are conditional on accounting performance targets. Our main findings do not change.
and 1 if $P > 1$) leads to large deltas but the incentive typically lasts only for very short periods of time, because as soon as the stock price moves above 1 the stock will be sold and all incentives will disappear.

If firms were only interested in instantaneous incentives then the issuance of contracts with discontinuous payoffs, such as digital options, would be current practice. Contracts with discontinuous payoffs will always lead to very large incentives to change share price but are often not ideal since those large incentives typically last only for a short period of time. This is because as soon as they move in the money, the executive is so concerned that they may drop back out-of-the money that they are immediately exercised.

Comparing standard restricted stock with vanilla option plans we find that, over the lifetime of the contract, vanilla option plans provide larger expected lifetime pay-for-performance incentives and larger expected lifetime incentives to increase the firm’s risk. This result is an example of the trade-offs between the value of the plan to the executive, the cost to the firm, the pay-for-performance incentives, and the expected lifetime. The results show that when keeping the cost of the contract fixed, the standard restricted stock plan is generally more valuable to the executive, but vanilla options have larger instantaneous pay-for-performance incentives (deltas), vegas and expected lifetimes. Rational executives sell their standard restricted stock plans as soon as possible which results in these instruments having short expected lifetimes. A possible way to increase the expected lifetime is to increase the vesting period, yet increasing the vesting period reduces the value of the compensation plan to the executive. This value reduction results in a restricted stock valuation and pay-for-performance incentive that is below that of vanilla options. Thus, for incentive purposes vanilla option are more effective forms of executive compensation than standard restricted stock.

We find that, for vanilla options, increases in vesting periods beyond three years result in lower expected lifetime pay-for-performance incentives. This is a surprising result, given academic (e.g. Hall and Murphy, 2002) and UK policy recommendations. Hall and Murphy (2002) do show that calendar vesting has undesirable features. Vesting can prevent the executive from exercising optimally, making options less valuable to the executive. Additionally, by forcing the executive to exercise later, vesting can also increase the cost to the firm. These two effects necessarily result in a lower ratio of executive value per dollar of cost. Nevertheless, since calendar vesting may ensure that the incentives are maintained for longer, the authors suggest that calendar vesting of up to 5 years may be beneficial for firms. Our findings show that, such an increase in the vesting period of option plans results in smaller expected lifetime incentives.

Moreover, comparing across all option and restricted stock contracts we find that, at median volatility levels, vanilla options with vesting periods of three years or lower and no performance target features provide the largest expected pay-for-performance incentives over the lifetime of the compensation plan. This result that, generally, options with short vesting periods and no performance targets provide the largest expected incentives to increase share price, supports the extensive use of vanilla options in compensation plans in countries like the USA.

At higher volatility levels and for executives with median and lower liquid wealth there is support for using
options that can only be exercised once a stock price target has been reached (performance exercise) since these plans can lead to larger expected lifetime pay for performance incentives. Risk incentive arguments can also justify the use of performance exercise targets in compensation plans. Performance exercise option plans result in the larger expected lifetime risk taking incentives except for very wealthy executives at median volatility levels. Surprisingly, for wealthy executives (at median volatility levels), restricted stock plans with performance exercise sale features result in the larger expected lifetime risk taking incentives. As expected, calendar vesting can lead to risk avoidance incentives.

In line with policy recommendations, increasing the vesting period does result in larger expected lifetime pay-for-performance incentives for restricted stock plans. However, for restricted stock, performance sale targets always lead to yet larger expected lifetime pay-for-performance incentives and positive risk-taking incentives. This suggests that imposing performance sale targets is a more effective way of increasing the overall incentives of restricted stock plans.

Interestingly, we find no justification for the usage of performance vesting plans. Compensation plans with performance vesting targets provide the smallest expected lifetime pay-for-performance incentives and never provide the largest incentives to increase the firm’s risk at all wealth levels considered. This result goes against UK policy recommendations.

### 1.1 Related literature

Our analysis takes into consideration that the executive values the compensation plan differently from the firm. Lambert Larcker and Verrecchia (1991), Kulatilaka and Marcus (1994), Carpenter (1998), Hall and Murphy (2002) and Ingersoll (2006) are examples of authors which also consider the divergence between the cost of the option to the firm and the executive valuation. Lambert, Larcker and Verrecchia (1991) analyze the risk incentives of non-tradable stock options and show that in-the-money executive option values can be decreasing functions of volatility. Kulatilaka and Marcus (1994) and Carpenter (1998) develop binomial models for the valuation of the firm’s cost of granting executive options considering that the executive’s exercise policy is affected by the non-tradability of executive options. Hall and Murphy (2002) analyze the pay-for-performance incentives of non-tradable compensation options. Ingersoll (2006) derives a model for the marginal value of options which allows for the valuation of options with different maturities.

Our modeling approach is approximately the dual of the first stage of the analysis used in Grossman and Hart (1983) since we search for the plan with the maximum incentives for a set fixed cost (and value). Note that since we do not model how the executives’ actions affect the firm’s production function or the disutility function of those actions, it is beyond the scope of this paper to find the optimal compensation plan through the classical agency models derived by Holmstrom (1979), Grossman and Hart (1983) and Holmstrom and Milgrom (1987). Similar, simplified approaches to ours, have been applied to executive stock options by e.g. Hall and Murphy (2002).
Similar approaches to ours, but modeling also the production function in the first stage of the optimal compensation contract, are presented in Dittmann and Maug (2007), Edmans, Gabaix and Landier (2008), Dittmann, Maug and Spalt (2010) and Dittmann and Yu (2011). Dittman and Maug (2007) calibrate a standard principal-agent model and conclude that stocks should be preferred to options. Edmans, Gabaix and Landier (2008) analyze the optimal pay for performance incentives in a competitive market equilibrium considering multiplicative utility and production functions. Dittmann, Maug and Spalt (2010) consider loss averse executives. Dittmann and Yu (2011) extend the standard agency model to the situation in which shareholders want to grant a compensation plan which provides not only effort incentives but also risk incentives. Both Dittmann, Maug and Spalt (2010) and Dittmann and Yu (2011) show that the principal agent model can explain observed option holdings.

Although it can be argued that the approach followed in Dittmann and Maug (2007), Dittmann, Maug and Spalt (2010), Dittmann and Yu (2010) and Edmans, Gabaix and Landier (2008) is richer, since it is explicit in these authors’ models how the executive actions can affect share prices, these authors do not consider that the executive can exercise the options before maturity. Naturally, the fact that options can be, and typically are, exercised before maturity affects the firm’s cost, the executive’s assessment of the compensation plan and the incentives provided by the plan. The exercise behavior is non trivial even for vanilla options, as in good states of the world the expected lifetimes are shorter, deltas are larger and vegas are smaller, whereas in bad times expected lifetimes are longer, deltas are small and vegas are large. Simply using a set lifetime of the option and at-the-money incentives removes a great deal of the interesting features of executive options.

The literature on the effects of the addition of performance targets to traditional compensation plans is scarce. Bettis, Bizjak, Coles and Kalpathy (2008) and Kuang and Qin (2009) analyse empirically the relation between performance target options and firm performance. Johnson and Tian (2000a and b) analyse in detail the incentive effect of performance target options based on European options in a risk-neutral, Black-Scholes framework. This gives a rough approximation of the cost of the option plans to issuing firms, but not much can be concluded in terms of the incentives provided by these options. Brisley (2006) analyses the risk incentives of performance vesting options when the executive is risk averse. He finds that progressive vesting schedules in which the number of vested options is contingent on stock price appreciation may help ensure adequate risk incentives.

In the next section we report on the usage of performance target options, by large UK and US firms. We also provide some simple descriptive statistics on the type of performance targets used. In section 3 we present our model and in section 4 we analyze our results. Finally, in section 5 we conclude and discuss possible implications of our results for firms and regulators.
2 The adoption and characteristics of performance target plans

The Greenbury (1995) report advised UK firms to condition compensation plans on performance targets. To analyze whether large UK firms commonly use performance target options, we search for details of executive compensation plans in the remuneration section and notes of the accounts of the 2005 and 2008 annual reports of FTSE100 firms.

A significant majority of the FTSE 100 firms use stock vesting awards in their long term incentive plans. Although it has been reported that UK firms make less use of executive options than their US counterparts (Conyon and Murphy, 2000), 89% of the firms had option plans for their executives, and 50% granted options in 2008. Of the firms with option plans, 84% had plans conditional on the achievement of performance targets. Commonly, the performance target impacts on the vesting of the options, which normally occurs after three years. The most frequently used performance target is earnings per share (EPS) growth (71%), with options vesting on a sliding scale conditional on EPS growth outperforming the Retail Price Index.

The second most commonly used performance target is total shareholder return (TSR). 27% of the firms, which grant options conditional on performance targets, condition the vesting of options on TSR or share price appreciation. Commonly, options vest on a sliding scale if the firm’s TSR outperform the TSR of a defined comparator group. However, some firms use TSR or share price appreciation without reference to comparator groups. Below, we present an extract of the remuneration section of the 2008 annual report of Kazakhmys plc.

The performance condition will measure growth in share price on an ‘end to end’ basis over a fixed three year period commencing on the date of the grant. The vesting schedule will be as follows: if the performance increase in share price over the performance period is equal or greater than 50% then, 100% of the option becomes exercisable; if the performance increase in share price over the performance period is between 30% to 50% then, between 33% and 100% of the option becomes exercisable on a straight line basis; if the performance increase in share price over the performance period is less than 30%, then none of the option will become exercisable. To the extent that the above targets are not satisfied on the third anniversary of plan, the option will lapse.

Performance target conditions are less common in the US. The differences in executive pay between these two countries are not new to the literature (see e.g. Conyon and Murphy, 2000 and Conyon, Core and Guay, 2010) and might occur from differences in ‘outside’ pressures from regulators and other stakeholders, and/or from differences in the accounting rules of both countries. Concerning differences in accounting rules, until 2005 US firms could use the intrinsic value method to disclose option plans if they were using the fixed accounting method. As a consequence, at-the-money vanilla options with calendar vesting did not

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4 We find no major differences in the characteristics of the compensation plans in both years and so our discussion concentrates on 2008 annual reports.
have to be expensed. This accounting treatment may have penalised the use of certain performance target options. From June 2005 US firms have to expense all option plans under the fair value method. These new accounting rules may yet result in a widespread use of performance target options in the US (Lublin, 2006).

FW Cook & Co. publishes yearly reports on the compensation practices of the 250 largest US firms. According to their 2005 report, 90% of these firms granted options to their executives, 3% used performance accelerated vesting schemes and a further 3% used performance vesting schemes which will lead to forfeiture if the target is not achieved. 6% used reload options, less than 1% used discount options and 2% used premium options.

In 2009 FW Cook & Co. reports a decrease in the number of firms which use option plans (77%). Accelerated vesting was almost nonexistent in 2009 (less than 1% of firms), performance vesting was used by 4%, and premium options were only used by 2%. None of the firms granted reload, discount or indexed linked options. It might be argued that as US firms now use more performance linked stock awards they maintain their plans of options with traditional features. The 2009 FW Cook & Co. report shows an increase in performance stock awards from 44% in 2006 to 63% in 2009.

Bettis et al. (2008) report a potentially higher use of performance target options in the US. Using a sample of 2,055 firms from 1995 to 2001, the authors found that 22% granted awards with performance linked features. The large majority of the plans (46%) used stock price as the sole performance target. Plans with sole accounting based targets accounted for 22% of the grants. Given the policy recommendations we undertake a thorough investigation of these plans, focussing upon performance targets driven by the stock price.

3 Model

3.1 Basic model

Our model is designed to be flexible in order to consider various types of executive stock and option plans. To achieve this, we assume that the firm issues either restricted stock or at-the-money options to one executive. The options have 10 years until expiration and both the options and the restricted stock plans may have vesting periods. Executives are prohibited from short selling the firms stock. Vesting may be automatic, after a designated period of time (calendar vesting), or may be contingent on certain stock price performance criteria (performance vesting). Exercise can occur at any time outside of the vesting period, unless it is also dependent upon stock price performance criteria (performance exercise). In order to accurately model the vesting and performance conditions we also allow for partial exercise. Partial exercise may also be restricted by performance criteria.

Consistent with the standard executive stock option literature (e.g., Kulatilaka and Marcus, 1994; Car-
ponent, 1998; Hall and Murphy, 2000) we use a terminal wealth power utility model. As shown by Lambert, Larcker and Verrecchia (1991) the executive’s value will be different from the firm’s valuation as the executive faces a large non-diversifiable risk. Conversely, the firm’s shareholders are fully diversified. We first determine the executive’s optimal exercise strategy and the executive’s valuation, then separately calculate the firm’s cost under the risk-neutral measure using the executive’s known exercise strategy.

The executive has constant relative risk aversion and is granted an option\(^6\) at time 0. Exercise decisions are taken to maximize the utility of wealth, \(W\), at the maturity of the options \(T\). The utility of wealth at maturity is given by:

\[
U(W) = \frac{W^{1-\gamma}}{1-\gamma}
\]

where \(\gamma\) is the coefficient of risk aversion. The executive invests all of the wealth in the riskless asset. So, that

\[
dW = rWdt
\]

where \(r\) is the risk-free rate. The stock price, \(P\), follows a geometric Brownian motion,

\[
dP = \mu_P P + \sigma_P PdZ
\]

where \(\mu_P\) and \(\sigma_P\) are the mean and volatility of the stock price return respectively and \(dZ\) is a Brownian motion. To make the executive rationally select the riskless portfolio the expected return of the stock price is chosen to be equal to the riskfree rate, \(\mu_P = r\).

Note that although unrealistic, this constraint retains the key features of the full portfolio allocation and option exercise problem, without the increased complexity of including a market asset. If the CAPM holds and the executive was allowed to hold a market asset, then it would not be optimal to allocate any wealth to the stock. Instead, wealth would be optimally allocated to the market asset as it dominates the stock. Without the market asset, imposing the restriction that \(\mu_P = r\) ensures that there is no desire for the executive to hold stock and that the executive’s desire to hold the option is only due to the possibility that the option could produce a large payoff.

In the model, the options granted to the executive are American and are issued at-the-money with exercise price, \(X = P_0 = 1\). We also consider restricted stock in which case \(X = 0\). Options can be issued with a calendar vesting condition in which case the options cannot be exercised until the vesting time \(t = T_V\). The executive optimally exercises the options in stages, denoted by \(M\). Klein (2010) finds that executives typically exercise options in multiples of one fifth, one quarter or one third and we limit the number of stages to four (\(M \leq 4\)), implying that the executive must exercise at least 25% of the plan at any point in time\(^7\).

\(^6\)We consider restricted stock as an option with zero exercise price

\(^7\)There is no difficulty in extending the number of stages beyond four and in unreported results we extend the number of stages to eight and find that our main results still hold. Pollet, White and Widdicks (2013) find that increasing the number of stages from 5 to 10 has a very minor effect on the value to the executive and the cost to the firm
The size of the executive plan holding is given by $H$, where $0 < H < 1$. When there is no exercise then
\[ dH = 0 \]

At an exercise point $(P^e, W^e, H^e, t^e)$ the executive’s liquid wealth increases and the plan holding decreases so that
\[
\begin{align*}
W^e_+ &= W^e + N^e \times \frac{1}{M}(P^e - X) \\
H^e_+ &= H^e - N^e \times \frac{1}{M}
\end{align*}
\]

where $N^e \leq M$ is an integer, which denotes the amount of the option holding being exercised at $t_e$. The model ensures that the minimum amount of options that the executive can exercise is $1/M$, but it does not preclude the executive from exercising more. Indeed, any multiple ($N^e$) of $1/M$ can be exercised at any point in time. This is achieved, numerically, by calculating the value function, $V$, first for $H = 0$, then for $H = 1/M$ and continuing until $H = 1$. The numerical routine checks for exercise at $H = 1$ (point $(P, W, 1, t)$) by comparing the continuation value with the early exercise value $V(P, W + \frac{1}{M}(P - X), 1 - \frac{1}{M}, t)$. If exercise is optimal and the new point, $(P, W + \frac{1}{M}(P - X), 1 - \frac{1}{M}, t)$ is also an exercise point then the executive exercises $2/M$ of the option holding at $(P, W, 1, t)$ and $V(W, P, 1, t) = V(P, W + \frac{2}{M}(P - X), 1 - \frac{2}{M}, t)$, and so on.

Since the model allows exercise in stages, wealth cannot be eliminated as a state variable. Note that once part of the holding has been exercised, terminal wealth can only be determined by knowing the exercise path. Thus, the executive’s value function $V(P, W, H, t)$ depends on the executive’s wealth, the stock price, the plan holding and time and is given by
\[
V(P, W, H, t) = E_t \left[ \frac{W^{1 - \gamma}}{1 - \gamma} \right]
\]
in the absence of exercise the Bellman equation is
\[
0 = rWV_W dt + \mu_P PV_P dt + \frac{1}{2} \sigma_P^2 P^2 V_{PP} dt + V_t dt
\]
and the correspondent partial differential equation is
\[
0 = rWV_W + \mu_P PV_P + \frac{1}{2} \sigma_P^2 P^2 V_{PP} + V_t
\]
(1)

At an exercise point $(P^e, W^e, H^e, t^e)$ exercise is optimal when
\[
V(P^e, W^e, H^e, t^e) < V \left( P, W^e + \frac{N^e}{M}(P^e - X), H^e - \frac{N^e}{M}, t^e \right)
\]
if no vesting or exercise conditions apply.

Naturally, at maturity, if there are unexercised options, the executive exercises all of the remaining options (or the options expire worthless). This is ensured by the terminal condition for $V$: \[
V(P, W, H, T) = V^* (W + H \max \{P - X, 0\}, T)
\]
where $V^*$ is defined in equation 2 below. According to Carpenter, Stanton and Wallace (2010) optimal exercise policy can be characterized by a region of exercise that is not convex. In our numerical method, we check for optimality of exercise at all values of the state variables within the grid. Our solution technique is therefore valid even if exercise regions are not convex. Value matching and smooth pasting conditions arise automatically from the optimality of the exercise decision and are not required as explicit inputs into the numerical exercise, except in the case of the inequality above.

After all options are exercised or after the options maturity ($T$), the executive’s problem is simplified to investing liquid wealth. The executive’s Bellman equation in this case is

$$0 = \max E_t \left[ V^*_t dtV^*_t dW + \frac{1}{2} V^*_W W (dW)^2 \right].$$

Using the explicit process for $W$, the Bellman equation yields the following PDE

$$0 = V^*_t + rWV^*_W.$$

Assuming that the solution is of the form $K(t) \left( W^{1-\gamma} / (1 - \gamma) \right)$ it is possible to solve for $K(t)$ using the PDE. We verify that

$$V^*(W,t) = e^{(1-\gamma)r(T-t)} \frac{W^{1-\gamma}}{1-\gamma}$$

is a solution to the PDE.

### 3.1.1 The executive’s value

Following Lambert, Larcker and Verrecchia (1991) the executive’s option value can be calculated by considering the certainty equivalent. From equation 1 we can solve to find the executive’s value function $V(P,W,H,t)$ and from equation 2 we can determine the amount of money the executive would be willing to exchange for the option, $G(P,W,H,t)$

$$G(P,W,H,t) = \left( (1 - \gamma)e^{-(1-\gamma)r(T-t)}V(P,W,H,t) \right)^{\frac{1}{1-\gamma}} - W$$

### 3.1.2 Firm’s valuation

We assume that the firm’s shareholders are well diversified investors and so it is appropriate to value the option from their perspective by using a risk-neutral approach. Denote this value by $F$. In the absence of exercise, the proportional drift of $F$ under the risk-neutral measure is equal to the risk-free rate,

$$E^Q [dF] = r F dt.$$

As exercise is dependent upon the executive’s wealth, $W$, the firm’s cost will also depend on $W$. This results in the following partial differential equation for $F$,

$$0 = r W F_W + r P F_P + \frac{1}{2} \sigma^2 P^2 F_{PP} + F_t - r F.$$

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8In all cases we evaluate, convexity does appear to hold.
The exercise strategy is determined by the executive’s optimization problem. At an exercise point \((P^e, W^e, H^e, t^e)\)

\[
F(P^e, W^e, H^e, t^e) = \frac{N^e}{M}(P^e - X) + F(P^e_+, W^e_+, H^e_+, t^e) \\
= \frac{N^e}{M}(P^e - X) + F(P^e, W^e + \frac{N^e}{M}(P^e - X), H^e - \frac{N^e}{M}, t^e).
\]

The value of the option before exercise must match the value after exercise, plus the cash flow accruing to the executive at exercise. This cash flow is \(N^e (P^e - X)/M\).

Smooth pasting conditions do not apply here because exercise is chosen to maximize \(V\) (and not to minimize \(F\)). For this reason, \(F\) is not necessarily continuously differentiable in \(W\) or \(P\), although it must be continuous. The terminal condition for this PDE is

\[
F(P, W, H, T) = H \max \{P - X, 0\}.
\]

### 3.1.3 Expected time to exercise

We denote the expected lifetime of the entire option plan by \(\theta\). The expected lifetime can represent the time until the option has expired, been exercised or cancelled due to a failure to meet performance vesting conditions. Time to exercise or cancellation is a stopping time and is a function of the stock price, wealth, time and option holding. Thus, since \(\theta\) is the expectation of a function of \(P, W, H, t\) then \textit{in the absence of early exercise or cancellation} \(\theta\) satisfies the following, Feynman-Kac, partial differential equation (see appendix)

\[
0 = rW\theta_W + \mu_P P\theta_P + \frac{1}{2}\sigma_P^2 P^{2}\theta_{PP} + \theta_t. \quad (3)
\]

At exercise points, \((P^e, W^e, H^e, t^e)\), or at points where options are cancelled, if the option being exercised (or cancelled) is the last option held \((H^e = N^e/M)\) then \(\theta\) is known for certain and is equal to \(t\). If the option exercised (or cancelled) is not the last one then \(\theta\) becomes the expected lifetime of the post-exercise option holding. Thus,

\[
\theta(P^e, W^e + N^e/M(P^e - X), H^e - N^e/M, t) \quad \text{if} \ H^e > N^e/M \\
t \quad \text{if} \ H^e = N^e/M.
\]

At expiry the option lifetime is known with certainty and so the terminal condition for equation 3 is known,

\[
\theta(P, W, H, T) = T.
\]

### 3.1.4 Vesting and performance targets

Options can be issued with exercise restrictions based on the stock price level. The first restriction allows exercise if the stock price is twice the value of the exercise price. Thus, when considering early exercise two
criteria must now be satisfied

\[
V(P^e, W^e, H^e, t^e) < V \left( P^u, W^u + \frac{N^u}{M} \max(P^e - X, 0), H^e - \frac{N^e}{M}, t \right)
\]

\[P^e > 2X.\]

The second exercise restriction allows options to be exercised on a sliding scale. In particular, the first 50% of the option plan can be exercised with no condition. The next 25% can be exercised only if the stock price is twice the value of the exercise price. Lastly, the final 25% can only be exercised if the stock price is three times the value of the exercise price. In this case exercise only occurs if

\[
V(P^e, W^e, H^e, t^e) < V \left( P^u, W^u + \frac{N^u}{M} \max(P^e - X, 0), H^e - \frac{N^e}{M}, t \right)
\]

\[P^e > X \text{ when } H^e - \frac{N^e}{M} = 0.75, 0.5\]

\[P^e > 2X \text{ when } H^e - \frac{N^e}{M} = 0.25\]

\[P > 3X \text{ when } H^e - \frac{N^e}{M} = 0.\]

Alternatively, firms may issue options with stock price performance restrictions on vesting.\(^9\) In this case if the performance target is not met then the option is cancelled and expires worthless. We consider three performance vesting conditions similar to the performance exercise conditions described above.

The first performance vesting condition allows the plan to vest at \(T_{V}\) if the options are in-the-money. Thus at \(t = T_{V}\), if \(P < X\) then \(V(P, W, H, T_{V}) = V^\ast(W, T_{V})\). The second performance condition allows the plan to vest if at the vesting date the stock price is twice the exercise price. Thus at \(t = T_{V}\), if \(P < 2X\) then \(V(P, W, H, T_{V}) = V^\ast(W, T_{V})\). Lastly, in the third performance condition, vesting occurs on a sliding scale. In more detail, the first 50% of the option plan vests at \(T_{V}\) as long as the option is in-the-money, the next 25% vests if \(P \geq 2X\). The final 25% only vests if \(P \geq 3X\). The sliding scale performance vesting is challenging to model. At vesting, the number of sub-options held may be reduced as certain vesting criteria may not have been met. This needs to be factored in at \(T_{V}\).

The equations above are solved using a numerical method based on Pollet et al. (2011) and a description can be found in the appendix.

3.2 Incentives

To assess the overall effectiveness of the compensation plans our analysis follows from Grossman and Hart (1983), Lambert et al. (1991), Hall and Murphy (2002) and Ditmann and Maug (2007). The executive is granted a number of options or stocks, \(n\), and a cash amount, \(C\), such that the total cost of the plan \(F(P, W + C, H, 0; n) + C\) and executive’s value \(G(P, W + C, H, 0; n) + C\) are fixed to given levels. The cost and value of the plan are set equal to the restricted stock with vesting period \(T_{V} = 3\), but our results are

\(^9\)As discussed in section 2, large UK firms commonly impose performance conditions on the plan’s vesting.
not sensitive to this choice. For each plan \((n, C)\) we then calculate the lifetime pay-for-performance measure (delta) and the lifetime incentive to increase risk (vega). Given that all plans result in the same firm cost and executive valuation, the best plan maximizes the incentives over the plan’s lifetime.

The lifetime incentives are calculated by first estimating the instantaneous deltas and vegas. The instantaneous deltas and vegas, at each point in time, are given by:

\[
\Delta(P, W, H, t) = \frac{G(P + dP, W, H, t) - G(P - dP, W, H, t)}{2dP},
\]
\[
\nu(P, W, H, t) = \frac{G(P, W, H, t; \sigma + d\sigma) - G(P, W, H, t; \sigma - d\sigma)}{2d\sigma}.
\]

\(dP\) represents a small change in the stock price and \(d\sigma\) is a small change in the stock price volatility. The expected lifetime incentives \(TI(P, W, H, t)\) are then calculated by summing, across each of the possible stock paths, the size of the time step \(\Delta t\) multiplied by the instantaneous deltas and vegas. These sums are scaled by the probability of each path occurring to produce an expected total lifetime delta or vega. In practice, this expectation is calculated by solving the partial differential equation 6 backwards in time, until the date of issuance. As an illustration, for the lifetime delta: at maturity there are no incentives, and thus \(TI(P, W, H, T) = 0\). At any other point, we numerically solve the recursive equation,

\[
TI(P, W, H, t) = \begin{cases} 
\Delta(P, W, H, t)\Delta t + \eta(P, W, H, t) & \text{if } (P, W, H, t) \text{ is not an exercise point} \\
TI(P, W + N^c/M(P - X), H - N^c/M, t) & \text{if } (P, W, H, t) \text{ is an exercise point and } H > N^c/M \\
0 & \text{if } (P, W, H, t) \text{ is an exercise point and } H = N^c/M
\end{cases}
\]

where each \(\Delta t\) is the size of the time step, the expected future total incentives \(\eta(P, W, H, t) = E_t[TI(P, W, H, t + \Delta t)]\) is the conditional expectation of total incentives from time \(t + \Delta t\) viewed from time \(t\). Finding \(\eta(P, W, H, t)\) requires determining the numerical solution to the Feynman-Kac PDE,

\[
0 = rW\eta_W + \mu_P P\eta_P + \frac{1}{2}\sigma_P^2 P^2\eta_{PP} + \eta_t.
\]

Note that in the procedure \(\eta\) at \(t + \Delta t\) is known as it is \(TI(P, W, H, t + \Delta t)\) calculated at \(t + \Delta t\). The numerical solution calculates the probability of reaching each of these future states given the time \(t\) values \((P, W, H, t)\). The vega calculations are performed analogously.

Determining the values of \(C\) and \(n\) is challenging. As \(C\) increases the executive’s risk aversion decreases affecting both the exercise decision and the executive’s valuation. This in turn affects the firm cost. As \(n\) increases the executive may feel wealthier but is also more exposed to the risk of the option or stock position. To overcome this we value the plans for a wide range of \(C\) and \(n\) and find the combinations that exactly match the cost and have a value equal to or slightly greater than the value of the benchmark restricted stock. This process is computationally intensive and so we only consider a reduced number of contracts, considering sizes of \(n\) in steps of 0.05 or 0.1 and so if the value is not matched exactly we will take ones that
are slightly greater than the benchmark. Once \( n \) and \( C \) have been determined, the lifetime deltas and vegas are calculated.

To understand the factors which affect the lifetime incentives and the trade-offs between cost and valuation of the compensation plans and the incentives provided by these plans we analyze individually four measures. The first measure calculates how much the executive’s valuation differs from the shareholders issuance cost. The second measure calculates the instantaneous incentive to change stock price (delta) per dollar of firm cost and the third measure the instantaneous incentive to change the firm’s risk (vega) per dollar of firm cost. Lastly, the fourth measure considers the expected lifetime of the plan.

To estimate the first measure we calculate the executive’s value for a range of \( P, W \) and \( t \) values. We then divide this value by the cost of issuing an at-the-money option plan at time 0,

\[
\text{Valuation measure} = \frac{G(P, W, H, t)}{F(1, W, 1, 0)}
\]

Note that the firm cost is calculated for fixed values of \( P, H, \) and \( t \) but we allow the executive to have a range of possible initial wealth levels. This measure follows from Hall and Murphy (2002) and it indicates the divergence between the executive valuation and the firm’s cost of the plan. Therefore, the measure is an indicator of efficiency.

The second measure is calculated using the rate of change of the executive’s plan value with respect to changes in the stock price scaled by the cost of issuing an-at-the-money option plan at time 0. This gives the

\[
\text{Delta measure} = \frac{\Delta(P, W, H, t)}{F(1, W, 1, 0)}.
\]

where \( \Delta(P, W, H, t) \) is defined in equation 4 above.

The third measure is calculated using the rate of change of the executive’s plan value with respect to changes in the stock price volatility scaled by the cost of issuing an-at-the-money option plan at time 0. This gives the

\[
\text{Vega measure} = \frac{\nu(P, W, H, t)}{F(1, W, 1, 0)},
\]

where \( \nu(P, W, H, t) \) is also defined in equation 4 above. Finally, we calculate the expected lifetime as described above.

The vega measure is the most contentious of the four measures since it is not immediately obvious how an increase in the incentive to take risk will affect firm value. Figure 1 shows how the compensation plan can create incentives to change volatility and how these incentives can motivate executives to take actions which can result in lower firm values. The figure presents the vanilla option; and the three performance vesting options presented in section 3.1.4. For different moneyness levels, the figure displays the percentage change in stock price which would leave an executive\(^{10}\) indifferent between maintaining the existing volatility level, 0.3, or decreasing it to 0.25.

\(^{10}\)with current \( W = 1 \) holding one of the above plans.
If the option is out-of-the-money, a risk averse executive lowering the firm’s risk has to significantly increase the share price in order to maintain the value of the option plan. The implication of this is that the executive will find it optimal to leave volatility at its current level, or alternatively increase volatility as, in this scenario, the risk incentive (vega) is positive. As the option approaches the at-the-money and in-the-money regions the required change in stock price becomes increasingly more negative.\textsuperscript{11} For vanilla options there is never a willingness to reduce the stock price associated with a reduction in risk. For options with performance vesting features, vega is often negative implying that for many reasonable moneyness levels ($P > 1$), the executive increases (or maintains) the value of the option by lowering volatility even if that results in a decrease in stock price. For example, at the grant date if $P = 1.5$ the executive would be willing to accept a 3\% drop in the stock price in order to reduce the volatility of the $P > X$ vesting option. If the stock price is twice the exercise price then the executive would be willing to forego up to 5\% of the stock price to reduce the volatility. Thus, for performance vesting options the incentive to decrease risk might result in the executive taking damaging corporate actions, such as costly hedging strategies which can result in a lower stock price.

4 Results

We consider a variety of option and restricted stock plans to analyze how calendar vesting and performance targets affect the incentives of the compensation plans. The comparison begins by analyzing the four measures separately to assess the individual effects of the cost, valuation, instantaneous incentives and the expected lifetime of the compensation plans. Shareholders prefer each of the measures to be as large as possible since for each dollar of cost shareholders would like the plans to result in the largest value to the executive; to provide the largest incentive to increase the stock price; to encourage the executive to take increased risk and, as long as these incentives are positive, to have the longest expected lifetime. We then fix the cost and the value of the compensation plans and we analyze the expected total lifetime incentives. The total incentives (the expected lifetime delta and the expected lifetime vega) are calculated using equations 4, 5 and 6 and can be viewed as the combination of value, cost, expected lifetime and incentives of the compensation plans. Tables 1 to 5 present the unscaled values of $F, G, \Delta, \nu$ and $\theta$ for a range of $P$ and $W$ for all of the options and restricted stock considered below.\textsuperscript{12} Tables 6 and 7 show the expected total lifetime deltas and vegas of each plan.

\textsuperscript{11}For highly in-the-money options (for our parameters when $P = 3.5$) the percentage change in stock price becomes slightly less negative. This happens because the probability of the option moving out-of-the-money becomes negligible.

\textsuperscript{12}The scaled values can easily be calculated by dividing $G, \Delta nd \nu$ by the firm cost $F$. 

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4.1 Parameter choices

We consider a range of starting liquid wealth from .25 to 5 relative to a Black-Scholes at-the-money option plan value of .49. A liquid wealth of .25 is an appropriate benchmark for a young entrepreneur, without substantial outside wealth relative to the value of the compensation plan. Initial liquid wealth of 5 describes an executive possessing substantial outside wealth relative to the value of the compensation plan. We use 2 as the coefficient for relative risk aversion, consistent with Hall and Murphy (2002), as we vary wealth considerably it is not necessary to also consider various levels of risk aversion.

We initially set firm volatility equal to 30%, which is roughly the median volatility of executive stock option issuers from Carpenter (1998) but we also consider volatilities of 45% to verify that the results hold even for high volatility firms. The risk free rate, $r$, is assumed to be equal to 4% and $T$, the time to expiration, equal to 10 years following Carpenter (1998) and Hall and Murphy (2002). Since dividend payments may affect the optimal exercise behaviour we assume that the firm does not pay dividends. As explained in section 3 all option/stock positions consist of four sub-options ($M = 4$) that can be exercised independently. For some contracts each of these sub-options may have different exercise or vesting conditions. Finally, we assume that the expected annualized continuously compounded return on the underlying stock is equal to the riskfree rate, 4%.

4.2 Individual measures analysis

4.2.1 Options

In Figures 2 to 5 we analyze the incentives of vanilla, performance exercise and performance vesting options. In the comparative analysis, we present the vanilla option and also two different types of performance exercise options and three different types of performance vesting options described in Section 3.4.

Figure 2 shows the ratio of executive value to firm cost for performance exercise and performance vesting options. As performance target options force the executive to keep the option for longer and have a higher probability of expiring worthless, the risk-averse executive will value these options lower than vanilla options (see Table 3). Concerning firm cost there are two opposing effects, since the executive keeps the option for longer the firm cost increases, on the other hand, since performance target options have a lower probability of finishing in the money, the firm cost decreases.

The figure shows that a risk averse executive holding at-the-money options discounts performance target options more heavily than vanilla options. Also, the gap between the executive’s assessment of the option’s value and the firm’s cost widens as the executive’s wealth decreases (i.e. the less wealthy executive discounts performance target options further). At-the-money performance vesting options are more heavily discounted than performance exercise options because of the substantial probability that performance vesting options will not vest. As the options become more in-the-money the ratio of value to firm cost of performance vesting options becomes larger than the ratios for other type of options. This occurs since performance
vesting options are significantly cheaper for the firm (see Table 4).

Figure 3 shows the ratio of executive delta to firm cost for varying wealth and moneyness levels. For low wealth levels and when the options are out-of-the-money, vanilla options still provide the best incentives per dollar of cost. Nevertheless, at-the-money performance target options provide larger incentives at still relatively modest levels of wealth (approximately $W = 1$). In the case of performance vesting options, even at $W = 1$ and if the options are slightly out-of-the-money, performance vesting options seem to dominate pure vanilla options for this measure.

Figure 4 shows the ratio of vega to firm cost (performance exercise target options are plotted in the top half of the figure and performance vesting options in the bottom half). Concerning options, from the executive point of view, increases in volatility result in two competing factors: volatility makes the options more likely to become significantly in-the-money but it also increases the risk of the options finishing out-of-the-money. At the grant date, the large majority of the at-the-money performance target options analysed provide the executive with larger incentives to increase risk (per firm cost) than vanilla options. This occurs because volatility increases the probability that the targets will be achieved, thus performance target options result in larger risk incentives. Also, as expected, a wealthier executive has larger incentives to increase the firm’s risk because these executives are less risk averse.

For performance target options at different levels of moneyness the structure of the performance target plays a crucial role. Comparing the two types of performance exercise plans, tougher targets lead to higher risk incentives. Initially the 2X exercise plan results in higher risk incentives, but when the options are more in the money, the sliding scale plan dominates. In particular, as the stock price goes above the target (2X) then vega becomes negative and provides an incentive to reduce risk since the executive becomes more concerned that the stock price will fall below the target. For the sliding scale exercise option the executive is initially concerned with ensuring that the options are in-the-money but as the stock price increases the executive believes that the next target can be achieved and thus has additional incentives to increase stock price volatility (see Figure 4). This result suggests that sliding scale targets can maintain positive risk incentives for longer.

Figure 5 shows the expected lifetime of the plan. At the grant date, at-the-money performance target options have longer expected lifetimes than vanilla options at low wealth levels. However, at high wealth levels vanilla options have the longest expected lifetime. The fact that performance exercise options sometimes result in larger expected lifetimes should not be surprising since performance targets make exercise more challenging. However, at higher wealth levels the early exercise boundary can potentially be larger than the performance target, leading to longer expected lifetimes of vanilla options. This is because for performance target options there is an increased desire to exercise early to avoid the possibility of the stock price falling below the performance target. The short expected lifetimes for most performance vesting options is not surprising given that in many scenarios the options will expire worthless at the vesting date (3 years).

Expected lifetimes vary significantly with moneyness. At low moneyness levels sliding scale performance
target exercise options have longer expected lifetimes than vanilla options. However, since the targets make performance exercise options more difficult to exercise, once they are reached, the executive is more likely to exercise the options. Therefore, for in-the-money options, the expected lifetime of performance exercise options falls below that of the vanilla option. Conversely, as performance vesting options cannot be exercised until the vesting date their expected lifetime remains longer than for vanilla options for in-the-money options. As there is a chance that the options may not vest, out-of-the-money performance vesting options have a shorter expected lifetime than vanilla options.

Overall, although generally less valuable to the executive than vanilla options (for out and at the money options), performance target options can provide larger incentives. In particular, performance vesting can provide very large incentives to increase the stock price, whereas performance exercise options provide large incentives to increase risk. In general, vanilla options have the longest expected lifetime (except for low wealth executives and highly in the money options).

4.2.2 Restricted stock

We can also consider similar performance targets on restricted stock plans. In Figures 6 to 8 we analyze five types of performance target restricted stock. The first two types condition the sale of the stock to performance targets: \( P > 2 \) sale represents restricted stock which can only be sold if \( P > 2 \); stock with sliding scale sale represents a restricted stock plan in which 50% of the stock can only be sold if the stock price goes above 1, 25% if above 2 and the remaining 25% if the stock goes above 3. In the remaining three types of performance target plans, the stock will only vest if a specified condition is attained: in stock with \( P > 1 \) vesting, the restricted stock will only vest, in 3 years’ time, if the stock is larger than 1; in stock with \( P > 2 \) vesting, the restricted stock will vest, in 3 years’ time, if the stock price is larger than 2; in the stock with sliding scale vesting, the restricted stock will vest, in 3 years’ time, conditioned on a sliding scale as defined above. For comparison purposes we also plot in the figures a vanilla option and a standard restricted stock plan with a 3 years vesting period, \( T_V = 3 \).

Figure 6 shows the ratio of executive value to firm cost. At the grant date with \( P = 1 \), across all wealth levels, whilst the standard restricted stock is the most valuable to the executive (relative to the firm cost), the sliding scale sale plan is almost as valuable. Also, the executive discounts the remaining stock plans even more than the vanilla option. When \( P = 1 \) restricted stock plans in which vesting is conditional on a performance target are considerably less valuable than the other restricted stock packages. However, as the stock price increases restricted stock with conditional vesting becomes significantly more valuable as a result of their very low issuance cost.

Figure 7 shows the incentive to increase the stock price. When \( P = 1 \) the incentives provided by the sliding scale sale restricted stock are considerably larger than the price incentives provided by standard restricted stock plan. In other words, although the executive discounts performance restricted stock further than he discounts standard restricted stock, the former results in better stock price incentives. When \( P = 1 \)
the sliding scale restricted stock also provides better incentives than the vanilla option. The latter is also true for all low stock price levels. As \( P \) approaches 2 the plan in which exercise is only possible when \( P > 2 \) provides the largest incentives per firm cost. Thus, as with options, per dollar of cost, performance exercise conditions can motivate the executive to increase the stock price further than standard contracts.

When \( P = 1 \), performance vesting restricted stock can have their targets designed such that they provide larger incentives per dollar cost than the standard restricted stock package. For reasonably wealthy executives \((W > 1.3)\), the incentives provided by performance vesting restricted stock are even higher than the incentives provided by the option plan. As \( P \) increases the sliding scale and \( P > 2 \) vesting plans provide larger incentives than the option, with the sliding scale vesting plan providing the largest incentives of all of the restricted stock packages. However, for smaller stock prices the incentives to increase the share price for performance vesting plans are smaller than those provided by the performance exercise plans. For very low stock price levels the incentive is even smaller than the incentive provided by the standard restricted stock.

Figure 8 shows the incentive to increase risk. The addition of performance exercise features results in risk incentives which are almost always positive. This occurs since the executive knows that by increasing volatility the likelihood of the stock price hitting the target also increases. As soon as this target is reached the executive is free to take the optimal decision and sell the stock. Moreover, it is often the case that the restricted stock with performance exercise features provides even larger incentives to increase risk (per dollar cost) than the vanilla option. This result is unexpected since the literature commonly associates holdings of stock, by executives, with corporate risk avoidance actions.

Performance vesting restricted stock can result in positive vegas if the stock price is lower than the performance target. As the stock price moves above the target level then the executive wants the price to move as little as possible to ensure that the plan will vest. Vega then becomes large and negative encouraging extreme risk avoidance.

Finally Table 5 shows the expected lifetime of the plan. As expected, the restricted stock with sliding scale sale has the longest expected lifetime of all the restricted stock plans. The exception is for large stock price levels since the vesting feature prevents the sale of the stock in performance vesting plans. It is interesting to note that for low wealth levels, the restricted stock with sliding scale exercise has a longer expected lifetime than the vanilla option.

Overall, performance exercise plans (particularly with sliding scale sale) can be almost as valuable as standard restricted stock plans but provide larger incentives and longer expected lifetimes (except for very high stock prices). If at the grant date we were to increase the vesting period of standard restricted stock so that the expected lifetime matched that of the sliding scale exercise contract, then, it is likely that the sliding scale exercise plan would be more valuable and have larger incentives, making it preferred to standard restricted stock. Additionally, performance exercise plans can provide larger incentives than the vanilla option whilst also resulting in a larger value to the executive. Performance vesting restricted stock plans are less valuable to the executive (except at very high stock prices). Also, although these plans can
provide very large incentives to increase the stock price, they can often create counterproductive risk reducing incentives.

4.3 Comparison across all contracts

Our analysis in this section considers the trade-offs between the incentives, the valuation of the executive, the firm cost and the expected lifetime of the compensation plan. We assume that market competition forces the firm to grant to the executive a specific compensation plan value; otherwise the executive joins a competitor. Also, the firm sets the cost which is allocated to the plan. Both the cost and the value of the compensation plan are assumed to be fixed.

We also assume that the compensation plan contains at-the-money options or stock plus cash, and that the firm can choose the proportion of the elements in the compensation plan which results in the pre-defined (fixed) value and cost. The firm can also choose which type of option (or stock) is included in the compensation plan. We can then calculate for all different compensation plans our measures of expected total lifetime delta and expected total lifetime vega from equations 4, 5 and 6 above. If the firm wants to grant a compensation plan which maximizes the incentives to increase share price for longer then, the best plan will have the larger expected lifetime delta. If the firm wants to grant a compensation plan which maximizes the incentives to increase the firm’s risk for longer then, the best plan will have the larger expected lifetime vega.

Table 6 reports the values of the lifetime incentives \( TI(P, H, W, t) \) for delta and vega for each of the plans across a range of wealth levels and at two levels of firm volatility. The cost and value of the plans is set equal to those of the restricted stock with \( T_V = 3^{13} \).

4.3.1 Lifetime delta

Comparing standard restricted stock plans to vanilla option plans in Table 6, we can see that the vanilla option clearly dominates the restricted stock for both the expected lifetime pay-for-performance measure (delta) and the expected lifetime risk incentives (vega), across all wealth levels. From Tables 1 to 5 we see that although restricted stock has a larger value to the executive, vanilla options have larger instantaneous delta to cost ratio, instantaneous vega to cost ratio and expected lifetime, at all wealth levels. Since there is no advantage in holding the restricted stock for longer than its vesting period (this is not the case for options since a rational executive will hold the option if the exercise boundary has not been attained), generally, restricted stock have shorter lifetimes than options. Also, the cost of issuing an option is considerably smaller than the cost of issuing restricted stock which results in smaller pay for performance incentives (per firm cost). This combination leads to smaller expected lifetime pay for performance incentives. Of course, the expected lifetime of restricted stock could be increased by increasing the vesting period, but this, in turn will

\(^{13}\)Our results are not sensitive to this choice.
lead to a reduction in the executive valuation of the plan. For example, the restricted stock with a vesting period of five years has a lower lifetime delta (and vega) than the vanilla option at all wealth levels and both volatility levels.

This result supports why many firms (in the US) still prefer to grant options over stocks. Frederick W. Cook & Co. (2009) report that in the 250 largest US firms, the shift from option plans to performance stocks and performance units seems to have stabilized in the majority of the firms, with options continuing to be the preferred long term incentive instrument.

Comparing standard compensation plans to those with performance features in Table 6 we see that the performance exercise/sale features can result in compensation plans with larger expected lifetime deltas. However, in Table 6, we see that performance vesting plans always result in the lowest lifetime deltas for both stock and options. This result may seem unexpected as we reported that performance vesting options have the largest instantaneous delta (per firm cost). However, recall that executives heavily discount performance vesting options and stock as there is a substantial chance that these plans will expire worthless. This large discount means that to keep the cost and value fixed, the firm cannot include many of these instruments in the compensation plan. The resulting plan has a large proportion of cash and thus, overall low expected lifetime pay-for-performance incentives. Thus, there seems to be little justification for using performance vesting features in compensation plans. Note that this is even the case for the plans that vest as long as $P > X$, so this is not a consequence of too demanding vesting conditions but any restriction upon vesting.

The impact of calendar vesting can also be analyzed from Tables 6 and 7. Calendar vesting periods are commonly used by firms in the design of their compensation plans. The main objective of calendar vesting is to prevent the executive from exercising the options, or selling the stock, immediately after the grant date. Thus, longer vesting periods can help to ensure that the incentives provided by compensation plans are present for longer. Nevertheless, Hall and Murphy (2002) show that calendar vesting has certain undesirable features. Vesting can prevent the executive from exercising the options optimally, making these instruments less valuable to the executive. Additionally, by forcing the executive to exercise later, vesting can also increase the cost to the firm as the later the exercise the closer the cost gets to the risk-neutral, Black-Scholes value. These two effects necessarily result in a larger gap between the firm cost and the executive valuation, and thus decrease the plan’s efficiency. Regardless of these undesirable features, Hall and Murphy (2002) argue that as long as the firm maintains a ‘short’ vesting period (less than 5 years), increases in the vesting period will not result in significant decreases in efficiency. Moreover, since calendar vesting may ensure that the incentives are present for longer, the authors suggest that these benefits may rebalance the undesirable features and that calendar vesting of up to 5 years may be beneficial for firms.

We initially consider two vesting periods: 3 years, which is the most common practice (Bettis et al., 2005), and 5 years as suggested by the Walker (2009) report. Table 6 shows that for options the largest lifetime pay for performance measure is provided by either no calendar vesting or by calendar vesting of up to 3 years. In Tables 1 to 5 we saw that calendar vesting leads to lower valuation, delta, and vega per dollar
of firm cost but results in a longer expected lifetime. Nevertheless, the increase in the expected lifetime is not large enough to lead to larger incentives over the lifetime of the plan when vesting increases beyond three years.

For restricted stock, increasing the vesting period does lead to an increase in the lifetime delta. This is because in this case an increase in the vesting period leads to a substantial increase in the expected lifetime of the plan. This more than compensates the decrease in the delta measure (see Table 5).

Overall, table 6 shows that, for most executive wealth levels, when $\sigma = 0.3$, the vanilla option with a three year vesting period results in the largest lifetime delta value compensation plan. The only exceptions are for wealth levels of 0.5 and 1 in which cases vanilla options with no vesting result in the largest lifetime delta. This potentially explains the extensive use of standard options with short vesting periods in executive compensation. When volatility increases to $\sigma = 0.45$ the trends become less clear. For $W = 0.25$ the option with a three year vesting period has the largest lifetime delta but for $W = 0.5, 1$, the largest lifetime pay for performance incentives are provided by the option in which exercise is only allowed once $P > 2X$. At higher wealth levels ($W = 2, 5$) vanilla options have the largest lifetime pay for performance incentives. Interestingly, the best plan always consists of options and cash justifying compensation plans heavily weighted towards options instead of stock.

Across different wealth and volatility levels there appears to be no clear best compensation plan. However, in all cases the best expected lifetime delta values are provided by vanilla options with or without calendar vesting and options with performance exercise targets. Since the plans selected in Table 6 may be considered too restrictive, in Table 7, we analyze both calendar vesting and performance exercise options in more detail. The Table shows the lifetime deltas and vegas for varying calendar vesting periods from 1 to 5 and performance exercise targets from $1.2X$ to $2X$ for both volatility levels. Figures 9 and 10 show the expected lifetime deltas for an individual option with different vesting periods and performance targets.

Table 7 confirms that option plans with short vesting periods are preferable to plans with longer vesting periods. At all wealth levels and for both volatility levels we find that the lifetime incentives for $T_V = 4$ and $T_V = 5$ are always dominated by options with shorter or no vesting period. We also find that as volatility increases, shorter vesting periods ($T_V = 0, 1, 2$) become preferable to the standard $T_V = 3$. These results can be explained by the effect of calendar vesting.

In figure 9 we see that as the vesting period increases the expected lifetime delta of an individual option increases, and so the decrease in the compensation plan lifetime deltas is due to value and cost considerations. Vesting delays exercise by forcing executives to hold their options for longer. As volatility increases it is more likely that executives will want to exercise the option, resulting in a larger discount in their valuation of the option. This results in fewer options being issued to match the cost and value constraints and thus a compensation plan with lower expected lifetime delta. The executive’s wealth also has an impact. When wealth is low, the executive would typically exercise at lower stock prices. Thus increasing the vesting period can substantially delay otherwise optimal exercise (see Table 5), leading to longer expected lifetimes.
Increasing the vesting period also leads to a reduction in the executive’s valuation and an increase in firm cost. It appears, however, that increasing the vesting period has a positive effect on the expected lifetime deltas of the plans at very low wealth levels, if the vesting period is not too long ($T_V = 2$ is optimal when $\sigma = 0.3$). Thus, at very low wealth levels the effect on expected lifetime dominates.

At moderate wealth levels, vesting periods have a smaller effect on expected lifetime and options with $T_V = 0$ or 1 are preferred. At highest wealth levels, the vesting period has little effect since the stock prices that would lead to exercise before $T_V = 3$ have a very small probability of being reached. Thus, although for high wealth levels, calendar periods of 3 years, $T_V = 3$, result in the best pay for performance incentives vanilla options, $T_V = 0$, result in similar incentives’ level. At higher volatility levels we see a similar trend with $T_V = 1$ producing higher lifetime deltas for low wealth ($W = 0.25, 0.5$) and high wealth ($W = 5$) but $T_V = 2$ producing higher lifetime deltas at moderate wealth levels ($W = 1, 2$). Overall, we find no support for vesting periods longer than 3 years. As wealth increases, the effect of vesting on expected lifetimes diminishes, as executives would naturally hold their options for longer, at this point the effect on value is more prominent leading to long vesting periods being ineffective.

Table 7 confirms that performance exercise targets can result in larger pay for performance incentives especially when volatility is high. When $\sigma = 0.45$, for wealth levels of 0.25 to 1 the largest lifetime delta incentives result from performance target options. For wealth of 0.25 the target is $1.4X$, increasing to $1.8X$ when $W = 1$.

Figure 10 plots the expected lifetime delta for an individual performance target option at both volatility levels. For a given target, as wealth increases we initially see a sharp increase in the lifetime delta, followed by a sudden decrease (at a trigger wealth level), and then by a gradual increase. As the target increases the critical wealth level which triggers the sudden drop in the lifetime delta also increases but the level of the expected lifetime delta after the drop decreases.

This can be explained by the effect of the performance exercise target. For high wealth executives, the performance target has little effect since their exercise boundary is generally above the performance target. In fact, the only effect the performance target has is forcing earlier exercise (to prevent the possibility that the plan will expire worthless if the stock price later falls below the target). As executive wealth decreases, the exercise boundary decreases and for a critical wealth level, at certain stages in the option lifetime, the performance target will move above the exercise boundary. This has three effects. First, it makes the option slightly less valuable to the executive and slightly more costly to the firm. Second, it increases the instantaneous delta substantially, as now an increase in the stock price could potentially lead to both a larger payoff and a reduction in the overall risk of the portfolio. Finally, it also increases the expected lifetime. Overall, the increase in instantaneous delta and expected lifetime seems to dominate and for any given wealth level an appropriate performance exercise target can result in compensation plans with larger expected lifetime pay for performance incentives. Note, that these effects are more pronounced when volatility is high, since there is a greater incentive for the executive to exercise and reduce the risk of
the portfolio, increasing the instantaneous deltas. The performance target also means that executives with wealth above the critical level will also be more inclined to exercise early leading to a larger drop in the total delta at higher wealth levels.

This analysis appears to support the use of an appropriate performance target but the effect of the target is very sensitive to the wealth level. If the executive’s wealth is known then it is possible to create an appropriate target. However, in practice it is hard to ascertain precise wealth levels and so the practical use of these contracts is limited.

Overall, unless executive characteristics are well known, compensation plans with standard options with short vesting periods of up to $T_V \leq 3$ generally provide the largest lifetime deltas. However, if the executive’s wealth and risk aversion are known, it is possible to create a performance exercise target which results in a compensation plan that provides very large lifetime incentives.

4.3.2 Lifetime vega

In Table 6 we also present the expected lifetime vega of the different compensation plans. The introduction of long vesting periods ($T_V$ larger than 3), results in lower expected lifetime vegas, especially for low wealth executives. As the executive becomes wealthier, risk aversion decreases, and the executive is less worried about downside risk and thus is more willing to increase volatility. Brisley (2006) showed that in-the-money calendar vesting may generate counterproductive incentives since the executive will want to avoid the risk of the options falling out-of-the-money. Our results show that not only the moneyness (as in Brisley, 2006) but also the executive’s wealth may result in these counterproductive incentives as early as the plan’s issuance date.

In table 6 we can also see that, across all wealth levels, performance exercise target plans have the largest lifetime vega values. At $\sigma = 0.3$, for low and median wealth levels, the largest lifetime vega value is produced by the option in which exercise is only permitted once $P > 2X$. At $W = 5$, the largest total vega value is produced by restricted stock with sales permitted if $P > 2$. At higher volatility levels ($\sigma = 0.45$) the largest lifetime vegas are again for the $P > 2X$ options when $W = 0.25, 0.5, 1$. For larger wealth, $W = 2, 5$ the largest lifetime vegas are for the option with sliding scale exercise. This result is expected, as toughest targets should increase the executive’s willingness to increase volatility.

Considering a broader range of plans in Table 7, we find a similar pattern to the lifetime deltas. At low wealth levels, when $\sigma = 0.3$ changing the performance target to a value lower than 2 simply decreases the lifetime vega. When $\sigma = 0.45$ then at $W = 0.25$, the option with exercise only permitted when $P > 1.4X$ has the largest vega, then $P > 1.8X$ at $W = 0.5$ and $W = 1$, lastly, the sliding scale exercise when $W = 2, 5$ (Table 6). The justification for the effects of wealth and firm volatility on the plan’s lifetime vega is similar to the justification for lifetime delta. Low wealth executives obtain a large payoff and risk reduction if they reach a tough performance target. This leads to large incentives to reach this tough target which can be
done by either increasing the stock price itself or, in this case, its volatility.

Table 6 also shows that, as expected, negative vega values are observed in restricted stock plans with calendar vesting. Performance vesting options can also have very small lifetime vegas and thus provide very small incentives to increase the firm’s risk. This occurs since performance vesting options prevent the executive from exercising the options before the end of the vesting period and the executive will want to avoid the risk of the options failing to vest. Thus, vesting periods, even if associated with performance vesting plans, can easily induce risk incentives which may result in underinvestment problems. This problem is far less pronounced in performance exercise options and almost non-existent for the sliding scale performance exercise option we consider, since these plans do not impose a calendar vesting constraint (see Table 4).

Our analysis so far, as is typical in the classic agency theory framework, assumes that executives are the risk averse agents of risk neutral shareholders and also that compensation contracts are designed to maximise shareholder’s wealth. Wiseman and Gomez-Mejia (1998), Edmans and Liu(2010), Bolton and Mehran and Shapiro (2010) and Bebchuck and Spammann (2010) argue that executives may want to increase risk and may even have incentives to take excessive risks. This is particularly true in leveraged firms since, in these firms, shareholders share the downside of investment outcomes with debtholders but not the upside. Thus, as shareholders’ agents, executives of leveraged firms have an incentive to shift risk from shareholders to bondholders which can result in negative present value investments being undertaken. This risk shifting problem is particularly severe in banks and other financial institutions since they have highly leveraged capital structures. Moreover, shareholders of banks share the downside risk not only with debtholders but also with governments who insure deposits up to a certain amount. Bebchuck and Spammann (2010) argued that this risk shifting incentive was a contributing factor to the most recent financial crisis.

An interesting research question is, therefore, how do compensation plans impact on the risk incentives of an already risk loving executive? In our model the executive wants to avoid risk and we do not address this issue directly, but the analysis of the effect of increases in the executive’s wealth gives an indication of the potential impact of risk incentives for a less risk averse or risk loving executive. Note that as the executive becomes wealthier, risk aversion decreases, and the executive is less worried about downside risk and thus is more willing to increase volatility.

As above, when wealth increases (and risk aversion decreases) the executive finds the compensation plan more valuable, options are held for longer, they are more costly to the firm and give larger incentives to take risk. Restricted stock plans also gives larger incentives to take risk but they have the same cost and expected lifetime.

Keeping the value and cost of the plan fixed, we see from table 6 that, at-the-money option plans always result in positive risk incentives for less risk averse executives. Note that these incentives are increasing as risk aversion decreases. This is true for all option plans even if calendar vesting, performance vesting or exercise conditions are attached to the plan. For less risk averse executives, restricted stock with exercise and performance vesting conditions still generally result in positive incentives to take risk. The exception
is for restricted stock with an already attained vesting target since the less risk averse executive will want to avoid the risk of the stock price falling leading to a loss. Calendar vesting also results in negative risk incentives for restricted stock.

Firms in which executives have larger incentives to take excessive risk should therefore avoid option plans and use standard restricted stock with calendar vesting. Although, performance vesting conditions (in restricted stock) can also reduce the executive willingness to take risk they result in smaller deltas than standard calendar vesting plans and thus the calendar vesting plans dominate.

4.4 Accounting performance targets

Many executive compensation plans have exercise or vesting targets that are based upon accounting measures such as earnings or earnings per share. Although our models are designed to analyse compensation plans with stock price performance targets, it is possible to make small adjustments to consider the effect of having accounting performance targets. Using the framework of Duffie and Lando (2001) we consider the stock price, \( P(t) \), to be a noisy proxy for earnings, \( E(t) \), or another accounting number. We then write

\[
\log E(t) = \log P(t) + U(t)
\]

where \( U(t) \) is normally distributed and independent of \( S(t) \), and we assume that \( E(0) = P(0) = 1 \). The standard deviation of accounting noise is \( a \) and for simplicity we also assume that \( U(t) \) has drift \(-a^2/2\) so that \( E[e^{U(t)}] = 1 \) and the stock price is an unbiased estimator of the earnings level. As before, for performance target plans, vesting or exercise is conditional on the achievement of a performance target. The performance target is now defined as a ‘set’ level of earnings. Finally, we assume that the earnings level is infrequently observed. In the appendix, we show how to adapt the performance vesting and exercise plans to earnings rather than stock price performance targets.

In Table 8 we report the lifetime deltas and vegas for several\(^{14}\) compensation plans. We set the standard deviation of accounting noise to the same level as Duffie and Lando (2001), \( a = 0.1 \), and assume that earnings are observed every three months. Table 8 shows that our main findings still hold when targets are based upon accounting performance rather than stock price performance. In particular, the largest lifetime deltas are provided either by the option with a vesting period of three years or by the option in which exercise is only allowed once \( P > 2X \). Performance vesting options have the smallest lifetime delta, and restricted stock lifetime deltas are smaller than those from equivalent option plans. Total lifetime vegas are again larger for options and restricted stock plans with performance exercise features. Performance and calendar vesting can result in negative risk incentives.

\(^{14}\)We omit the sliding scale exercise option and restricted stock plans as the adaptation to the standard models is far more complex in these cases. A more detailed explanation is found in the appendix.
5 Conclusion and Policy Implications

Our results have important implications for firms and regulators concerning the effect of calendar vesting and performance targets on the incentives provided by compensation plans. Our results also have strong implications regarding the usage of restricted stock in place of options. Hall and Murphy (2002) suggest that a firm choosing to grant options rather than stocks constitutes a puzzle. Dittmann, Maug and Spalt (2010) and Dittmann and Yu (2011) show that the principal agent model can explain observed option holdings, under either the assumption of loss aversion or risk increasing incentives. Our results support the use of vanilla options or options with vesting periods of up to 3 years for rational averse executives when firms wish to maximise expected lifetime pay-for-performance incentives. In recent years, firms appear to have been substituting options with restricted stock plans but in the US the trend seems to have stabilized, with firms still preferring to use vanilla options. We show that vanilla options dominate the standard restricted stock plans for executives. In particular, when keeping the firm cost and executive value fixed the expected lifetime delta and the expected lifetime vega of the vanilla option are substantially larger than those for restricted stock, at all wealth levels.

Concerning calendar vesting, the Walker (2009) review advised financial institutions to use longer vesting periods. Our results show that, keeping the cost and value fixed, calendar vesting of more than three years decreases the expected lifetime delta and expected lifetime vega of the option. Our results do show some support for using longer calendar vesting periods for restricted stock plans since vesting periods significantly increase the expected lifetime of these plans.

Murphy (2002) argues that performance target options, like market indexed options, will be heavily discounted by the executive thus, vanilla options might result in the optimal incentive instrument. Our results confirm that performance target options are more heavily discounted by the executive than vanilla options. Also, at median volatility levels we confirm that when keeping the cost and value fixed, options with calendar vesting of up to three years typically produce the largest expected lifetime delta value, supporting this claim. However, at higher volatility levels it is possible to design a performance target that results in a compensation plan with larger expected lifetime delta incentives. Additionally, performance exercise targets always result in compensation plans with larger expected lifetime risk incentives. Consequently, the addition of carefully calibrated exercise performance targets to standard compensation plans may lead to a better alignment between executives and shareholders.

Regarding the definition of the performance targets the ABI guidelines (2009, p. 13) state that ‘sliding scales are a useful way of ensuring that performance conditions are genuinely stretching. They generally provide a better motivator for improving corporate performance than a ‘single target”. This recommendation is not fully supported by our findings since performance exercise plans with a fixed hurdle are typically preferred.

The Greenbury (1995) and Walker (2009) reports recommend the use of performance vesting features. We
find that performance vesting features produce the lowest lifetime delta values of all of the plans considered and, sometimes, these plans create counterproductive risk incentives. It is, therefore, difficult to find any justification for the use of performance vesting features in compensation plans.

Finally, the choice of the optimal incentives depends not only on the characteristics of the executive, but also on the aims of firms and policymakers. The analysis in this paper assumes that firms wish to maximize lifetime delta and lifetime vega. Nevertheless, there are good reasons why this may not always be the case. At the firm level, when choosing compensation plans for low level employees the incentives may be less important than the plan’s value and expected lifetime. For example Oyer and Schaefer (2005) reject incentives over a retention explanation for the use of option plans. On the policy level, incentives to increase risk may not always be optimal. For example, the Troubled Assets Relief Program (TARP) wishes to discourage risk taking by financial institutions. For institutions covered by TARP, the only possible long term compensation plan consists of calendar vesting restricted stock. This is consistent with our results as the plan with the best incentive to reduce risk is restricted stock with long calendar vesting periods.

Our results also have implications for empirical work on executive compensation and the model offers some testable propositions. The model shows that standard restricted stock has smaller expected lifetime than vanilla options. Empirical work could test if executives do hold on to their options for longer than they hold on to their restricted stock. Historical data on executives’ exercise of options and sale of stock is difficult to collect but the result of this test would show which compensation instrument provides longer lasting incentives.

Guay (1999), Knopf, Nam, and Thornton (2002), Rajgopal and Shevlin (2002) and more recently Coles, Daniel, and Naveen (2006) show that positive managerial risk incentives result in riskier investment decisions. Following Guay (1999) those authors concentrate on the risk incentives provided by options’ plans and generally assume that stocks will not provide such incentives. Our results show that restricted stock with exercise features can provide large risk incentives and therefore, empirical researchers should include these restricted stock plans in the computation of the risk incentive proxies.

Our model also shows that, exercise features result in compensation plans with larger risk incentives. As a consequence we expect that riskier companies; or companies aiming at a risk increase, are more likely to use compensation plans with exercise features. To test this hypothesis empirical researchers could investigate if, for example, young firms with large growth opportunities, or start-ups, are more likely to include exercise features in their compensation plans. Or if, for example, firms in which executives are granted such compensation plans are more likely to take risky investments like M&As, or to have larger R&D investments or more leveraged capital structures.

The most recent empirical compensation literature typically uses Black-Scholes estimates of the portfolio

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15Datta, Iskandar-Datta, and Raman, 2001, and Hagendorff and Vallascas, 2011 showed an increase in firms’ risk following an acquisition.

16Coles, Daniel, and Naveen (2006) showed that R&D investment and leverage are positively associated with managerial risk incentives.
of vegas and deltas of executives, as proxies of CEO risk and pay for performance incentives. These estimations neglect important characteristics of the compensation plans and of the incentives provided by the plans. For example, these proxies do not consider the American feature of the plans, the potential performance target features of the plans, or the fact that the executive is risk averse. Coles and Daniel and Naveen (2006) are one of the few which do acknowledge these drawbacks. Those authors test the robustness of their results to, for example, an adjustment (reduction) in the options’ time to maturity in order to consider the possibility of early exercise. It is nevertheless, unlikely that such adjustments will truly reflect the plan’s characteristics. We show that the characteristics of the executive and of the compensation plan can have significant impact on the incentives provided. Future empirical work should consider these characteristics in the estimation of incentive proxies.

Lastly, our results show surprisingly little relation between instantaneous and lifetime deltas. To illustrate the potential differences of the two measures, we calculated the coefficient of determination between the instantaneous deltas and the lifetime deltas for all of the compensation plans included in Table 6, when \( P = 1, W = 1 \) and the number of options or restricted stock plans, \( n = 1 \). Our results show that only 24% of the variation in instantaneous deltas can be explained by the variation in lifetime deltas. These results suggest that empirical researchers should also be careful about equating instantaneous deltas with lifetime deltas.
6 References


Dittmann, Ingolf and Yu, Ko-Chia, 2011, How important are risk-taking incentives in executive compensation, Working paper Erasmus University.


7 Appendix

7.1 Expected lifetime and total incentive partial differential equations

For many of the calculations in this paper we rely on the Feynmann-Kac partial differential equation. We explain our reasoning below by focussing on the expected lifetime case, providing an analogy with the standard American option scenario and then extend it to our case. In our model the exercise/cancellation time (call it \(\tau\)) is a stopping time which is \(\mathcal{F}_T\) measurable. Note that, in the American option case, in which early exercise is possible, the stopping time can be different from \(T\), and it is not required that \(\tau(T) = f(P_T, W_T)\) as in the case of an European option. After obtaining the exercise times, which we use to determine the boundary conditions, we are able to calculate the expected lifetime.

In more detail, consider the case of a standard American option where the stock price, \(P_t\) is log normally distributed and the expected return equal to the risk-free rate (\(E_0[P_T] = P_0e^{rT}\)). We are interested in the lifetime of the option, meaning the time points at which the option is optimally exercised, or expires. We will denote the lifetime (or stopping time) by \(f(P_t, t)\) and the expected lifetime, meaning the current (time \(t\)) expectation of the lifetime of the option by \(\theta(P_t, t)\).

For standard American options at maturity, \(f(P_T, T) = T\), but the lifetime can be shorter if the option is exercised before maturity, which provides a (free) boundary condition. At an exercise point \((P^e_t, t^e)\) the lifetime of the option is \(t^e\), thus \(f(P^e_t, t^e) = t^e\), and by definition \(\theta(P^e_t, t^e) = t^e\). The expected lifetime can be obtained moving backwards through time,

\[
\theta(P_t, t) = \begin{cases} 
E_t[\theta(P_{t+dt}, t + dt)] & \text{if } (P_t, t) \text{ is not an exercise point} \\
\quad t & \text{if } (P_t, t) \text{ is an exercise point}
\end{cases}
\]  

and so in the no exercise region \(\theta(P_t, t)\) satisfies the Feymann-Kac partial differential equation, and in the exercise region \(\theta(P_t, t)\) is equal to \(t_e\).

In our case since partial exercise is allowed the lifetime of the option depends on \(P, W, H\) and \(t\), and

\[
\begin{align*}
    dP_t &= \mu_P P_t dt + \sigma_P P_t dZ_t \\
    dW_t &= rW_t dt \\
    dH_t &= 0
\end{align*}
\]

and in the absence of exercise \(dH = 0\). Note that we know the terminal condition:

\[
\theta(P_T, W_T, H_T, T) = T.
\]

and the exercise points as these have been determined from the executive’s value function calculation. Thus we have our boundary conditions. For the PDE we need to separately consider the exercise and no exercise regions:

\[
\theta(P_t, W_t, H_t, t) = \begin{cases} 
E_t[\theta(P_{t+dt}, W_{t+dt}, H_{t+dt}, t + dt)] & \text{if } (P_t, W_t, H_t, t) \text{ is not an exercise point} \\
\theta(P_t, W_t + N^c/M(P_t - X), H_t - N^c/M, t) & \text{if } (P_t, W_t, H_t, t) \text{ is an exercise point and } H_t > N^c/M \\
t & \text{if } (P_t, W_t, H_t, t) \text{ is an exercise point and } H_t = N^c/M
\end{cases}
\]

\(^{17}\) \(f(P_T, W_T)\) denotes the lifetime (or stopping time) at maturity.
Thus, as for the standard American option case above, at points in which the option is not exercised the expected lifetime at \( t \) is equal to the \( t + dt \) expected lifetimes weighted by the probability of moving from \( P_t \) to \( P_{t+dt} \). At points at which the option is exercised one of two things may happen. If the sub option being exercised is the last option held (\( H_t = N^c/M \)) then the expected lifetime is known (since the total option ceases to exist) and is equal to \( t \). If the executive still holds options after the exercise then the expected lifetime becomes the expected lifetime of the post-exercise option holding.

Equivalently, in the no exercise region \( E_t[d\theta(P_t,W_t,H_t,t)] = 0 \) and by Ito’s lemma (dropping time scripts),

\[
E[d\theta(P,W,H,t)] = \left( \theta_t + rW\theta_W + \mu_P P\theta_P + \frac{1}{2} \sigma_P^2 P^2 \theta_{PP} \right) dt
\]

(noting that \( dH = 0 \) in the no exercise region) and so if \( E[d\theta(P,W,H,t)] = 0 \) then we have our no exercise region PDE that is equation 3, namely

\[
\theta_t + rW\theta_W + \mu_P P\theta_P + \frac{1}{2} \sigma_P^2 P^2 \theta_{PP} = 0
\]

with exercise conditions:

\[
\theta(P,W + N^c/M(P - X), H - N^c/M,t) \quad \text{if } (P,W,H,t) \text{ is an exercise point and } H > N^c/M
\]

\[
t \quad \text{if } P,W,H,t \text{ is an exercise point and } H = N^c/M.
\]

### 7.2 Numerical method

Since the PDEs derived in section 3 describe the solution to a free-boundary problem with a finite horizon, they have to be solved by numerical methods. We solve these equations using a modified implicit finite difference method with projected Gauss-Seidel iteration. The modifications arise from the difficulty in determining the exercise conditions for each of the \( M \) sub-options. The implicit finite difference scheme is chosen because it is generally the most stable. The method is a simplification of the model developed in Pollet et al. (2013) but with extra considerations for vesting.

The grid is defined by the following variables: wealth \( W \in [W_{\min}, W_{\max}] \), the underlying stock price \( P \in [0, P_{\max}] \), and time \( t \in [0, T] \). \( W \) is transformed using the natural log function to provide enough resolution for low values of liquid wealth. There are \( i_{\max} \) time steps, \( j_{\max} \) stock price steps and \( 2k_{\max} \) wealth steps. The values of \( t, P \) and \( W \) are

\[
t = i dt, \quad 0 \leq i \leq i_{\max}, \quad dt = \frac{T}{i_{\max}}
\]

\[
P = j dP, \quad 0 \leq j \leq j_{\max}, \quad dP = \frac{P_{\max}}{j_{\max}}
\]

\[
W = W_0 e^{k dy}, \quad -k_{\max} \leq k \leq k_{\max}, \quad dy = \frac{\ln (W_{\max}/W_{\min})}{2k_{\max}}, \quad W_0 = W_{\max} e^{-k_{\max} * dy}.
\]

The exercise conditions create a challenge for standard numerical techniques. First, for each valuation problem there are \( M \) simultaneous partial differential equations that need to be solved. The exercise condition
requires replacing the level of the value function in the $H = 1$ equation with values from the $H = 0.75$ equation and those in the $H = 0.75$ equation with values from $H = 0.5$ equation. This continues until we replace values in the $H = 0.25$ equation with the analytical solution $V^* (W, t)$ for $H = 0$. To implement this approach, at each point in time we calculate the value function for increasing values of $H$ from 0 to 1. For each level of $H$, we can then check the relevant exercise condition.

The grid size is set carefully to accommodate the switch between PDEs when each sub-option is exercised. At exercise, $H$ reduces to $H_{+} = H - 1/M$ and there is a substantial increase in the executive’s liquid wealth from $W$ to $W_{+} = W + (P - X)/M$. To overcome this problem, the range for $W$ increases as $H$ decreases from 1 to 0. In particular, $W \in [W_{\text{min}}, W_{\text{max}} + (1 - H)(P_{\text{max}} - X)]$. This procedure guarantees that, upon exercising a given sub-option, the new value $V(W_{+}, P_{+}, H_{+}, t_{+})$ has already been calculated. For these exercise calculations, linear interpolation between the two closest wealth points is used where necessary. Note that there are also corresponding exercise conditions for $F$, and $\theta$ which also require the grid for liquid wealth to be adapted as described above.

The performance exercise conditions can be easily adapted into the model as exercise is only allowed to occur if the stock price is at the appropriate level for the value of $H$. The vesting conditions are more challenging. Even calendar vesting requires a change in the boundary conditions, as for certain values of $P$ and $W$ exercise would be optimal if there was not a vesting constraint. The new boundary conditions are listed below. For performance vesting, from $T$ back to $T_{V}$ the option is valued as usual, then at $T_{V}$ the vesting conditions are applied and $V(P, W, H, T_{V})$ is set to $V^*(W, T_{V})$ where necessary. Note that the boundary conditions are not fully specified for $W = W_{\text{min}}$ here sometimes boundary conditions are not imposed. Instead, we estimate the PDE directly using forward difference approximations rather than the standard central difference approximations.
For vanilla options:

\[
\begin{align*}
G(0, W, H, t) &= V^*(W) \\
G(P_{\text{max}}, W, H, t) &= V^*(W + H(P - X)) \\
G(P, W_{\text{min}}, H, t) &= G\left(P, W_{\text{min}} + \frac{1}{M}(P - X), H - \frac{1}{M}, t\right) \text{ if } P > X \\
G(P, W_{\text{max}}, H, t) &= V^*(W_{\text{max}} + H \times BS(P, t)) \\
F(0, W, H, t) &= 0 \\
F(P_{\text{max}}, W, H, t) &= H(P - X) \\
F(P, W_{\text{min}}, H, t) &= F\left(P, W_{\text{min}} + \frac{1}{M}(P - X), H - \frac{1}{M}, t\right) \text{ if } P > X \\
F(P, W_{\text{max}}, H, t) &= H \times BS(P, t) \\
\theta(0, W, H, t) &= T \\
\theta(P_{\text{max}}, W, H, t) &= T \\
\theta(P, W_{\text{min}}, H, t) &= \theta\left(P, W_{\text{min}} + \frac{1}{M}(P - X), H - \frac{1}{M}, t\right) \text{ if } P > X \text{ if } P > X \\
\theta(P, W_{\text{max}}, H, t) &= T \\
\end{align*}
\]

where BS\((P, T)\) denotes the risk-neutral valuation of the vanilla option with stock price \(P\) at time \(t\). For options with a vesting period, the conditions are the same as for vanilla options when \(T > T_V\). For \(t < T_V\) they are:

\[
\begin{align*}
G(0, W, H, t) &= V^*(W) \\
G(P_{\text{max}}, W, H, t) &= V^*(W + H(P - e^{-r(T_V-t)}X)) \\
G(P, W_{\text{min}}, H, t) &= V^*(W + H(P - e^{-r(T_V-t)}X)) \text{ if } P > X \\
G(P, W_{\text{max}}, H, t) &= V^*(W_{\text{max}} + H \times BS(P, t)) \\
F(0, W, H, t) &= 0 \\
F(P_{\text{max}}, W, H, t) &= H(P - e^{-r(T_V-t)}X) \\
F(P, W_{\text{min}}, H, t) &= H(P - e^{-r(T_V-t)}X) \text{ if } P > X \\
F(P, W_{\text{max}}, H, t) &= H \times BS(P, t) \\
\theta(0, W, H, t) &= T \\
\theta(P_{\text{max}}, W, H, t) &= T_V \\
\theta(P, W_{\text{min}}, H, t) &= T_V \text{ if } P > X \\
\theta(P, W_{\text{max}}, H, t) &= T \\
\end{align*}
\]

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For performance exercise features:

\[
G(0, W, H, t) = V^*(W)
\]
\[
G(P_{max}, W, H, t) = V^*(W + H(P - e^{-r(T_V - t)}X))
\]
\[
G(P, W_{min}, H, t) = G(P, W_{min} + \frac{1}{M}(P - X), H - \frac{1}{M}, t) \text{ if } P > target_H
\]
\[
G(P, W_{max}, H, t) = V^*(W_{max} + H \times BS_{adj}(P, t, H))
\]
\[
F(0, W, H, t) = 0
\]
\[
F(P_{max}, W, H, t) = H(P - e^{-r(T_V - t)}X)
\]
\[
F(P, W_{min}, H, t) = F(P, W_{min} + \frac{1}{M}(P - X), H - \frac{1}{M}, t) \text{ if } P > target_H
\]
\[
F(P, W_{max}, H, t) = H \times BS_{adj}(P, t, H)
\]
\[
\theta(0, W, H, t) = T
\]
\[
\theta(P_{max}, W, H, t) = t
\]
\[
\theta(P, W_{min}, H, t) = \theta(P, W_{min} + \frac{1}{M}(P - X), H - \frac{1}{M}, t) \text{ if } X \leq P > target_H
\]
\[
\theta(P, W_{max}, H, t) = T
\]

where \(target_H\) represents the exercise target for that particular option holding, and \(BS_{adj}(P, t, H)\) represents the risk-neutral valuation of an option with these exercise constraints. \(BS_{adj}(P, t, H)\) can now be dependent on the option holding \(H\) as the exercise target can vary as the option holding varies.

For performance vesting after \(T_V\) the vanilla option boundary conditions can be used, before \(T_V\) we can use
the vesting conditions below

\[ G(0, W, H, t) = V^*(W) \]
\[ G(P_{max}, W, H, t) = V^*(W + H(P - e^{-r(T_V - t)}X)) \]
\[ G(P, W_{min}, H, t) = V^*(W + H(P - e^{-r(T_V - t)}X)) \text{ if } P > \text{largest target} \]
\[ G(P, W_{max}, H, t) = V^*(W_{max} + H \times BS_{adj}(P, t, H)) \]
\[ F(0, W, H, t) = 0 \]
\[ F(P_{max}, W, H, t) = H(P - e^{-r(T_V - t)}X) \]
\[ F(P, W_{min}, H, t) = H(P - e^{-r(T_V - t)}X) \text{ if } P > \text{largest target} \]
\[ F(P, W_{max}, H, t) = H \times BS_{adj}(P, t, H) \]
\[ \theta(0, W, H, t) = T_V \]
\[ \theta(P_{max}, W, H, t) = T_V \]
\[ \theta(P, W_{min}, H, t) = T_V \text{ if } P > \text{largest target} \]
\[ \theta(P, W_{max}, H, t) = T \]

where 'largest target 'represents the most demanding target at \( t = T_V \) required to exercise the entire option holding. Extra difficulty comes at the vesting date for the modelling of the sliding scale performance vesting. At vesting, the number of sub-options held may be reduced as certain vesting criteria may not have been met. This needs to be factored in at \( T_V \). If \( P \geq 3 \) there is no adjustment, if \( 2 \leq P < 3 \) then 25% of the plan (or one suboption) is cancelled and so \( V(P, W, H, T_V) = V(P, W, H - 1/M, T_V) \) but, unlike with exercise, the wealth level does not increase. If \( 1 \leq P < 2 \) then 50% of the plan (or two suboptions) is cancelled and so \( V(P, W, H, T_V) = V(P, W, H - 2/M, T_V) \). Finally if \( P < 1 \) the entire plan is cancelled and \( V(P, W, H, T_V) = V^*(W, T_V) \). The other performance vesting options are simpler as the option expires worthless if the criteria is not met and the expected lifetime is equal to the vesting date.

The conditions are largely the same for restricted stock, as it can be considered an option with zero exercise price. The main difference arises from the fact that the large wealth valuation of the stock is not the same as it is optimal to sell the stock before expiry. Thus the large wealth conditions become:

For restricted with a vesting period, when \( T > T_V \)

\[ G(P, W_{max}, H, t) = V^*(W_{max} + H \times P) \]
\[ F(P, W_{max}, H, t) = H \times P \]
\[ \theta(P, W_{max}, H, t) = t \]
whereas for \( t < T_V \) they are

\[
\begin{align*}
G(P,W_{\text{max}},H,t) & = V^*(W_{\text{max}} + H \times P) \\
F(P,W_{\text{max}},H,t) & = H \times P \\
\theta(P,W_{\text{max}},H,t) & = T_V
\end{align*}
\]

For performance exercise features:

\[
\begin{align*}
G(P,W_{\text{max}},H,t) & = V^*(W_{\text{max}} + H \times BS_{\text{adj}}(P,t,H)) \\
F(P,W_{\text{max}},H,t) & = H \times BS_{\text{adj}}(P,t,H) \\
\theta(P,W_{\text{max}},H,t) & = T_E(P,t)
\end{align*}
\]

where here \( BS_{\text{adj}}(P,t,H) \) represents the risk-neutral valuation of the stock with these exercise constraints. \( BS_{\text{adj}}(P,t,H) \) is calculated as the probability of the target being reached at any time before expiry, multiplied by the holding, \( H \), and multiplied by the stock price, \( P \). \( T_E(P,t) \) is the expected time required to reach the performance target given the current stock price, if the stock price is above the target then \( \theta(P,W_{\text{max}},H,t) = t \). Since \( P \) follows a geomeasure Brownian motion, these probabilities can be calculated.

For performance vesting after \( T_V \) the above boundary conditions can be used, before \( T_V \) we can use the vesting conditions below

\[
\begin{align*}
G(P,W_{\text{max}},H,t) & = V^*(W_{\text{max}} + H \times BS_{\text{adj}}(P,t,H)) \\
F(P,W_{\text{max}},H,t) & = H \times BS_{\text{adj}}(P,t,H) \\
\theta(P,W_{\text{max}},H,t) & = T_V
\end{align*}
\]

where again \( BS_{\text{adj}}(P,t,H) \) represents the risk-neutral valuation of the stock. The risk neutral valuation is calculated as probability of having reached the target at \( T_V \) multiplied by the holding, \( H \), and multiplied by the stock price, \( P \). This is simpler than for the performance exercise stock since only one date needs to be considered.

### 7.3 Accounting performance plans

It is relatively straightforward to adapt the performance vesting models to have vesting conditional on an earnings level rather than a stock price level. On the vesting date, \( T_V \) the option or restricted stock will vest if \( E \) is greater than the required level. However, from the PDE the value function \( V(P,W,H,T_V) \) is dependent upon \( P \) and so it is necessary to calculate probability of the earnings level being larger than the
target, \( B \) say, conditional on \( P \). As the relationship between \( E \) and \( P \) is straightforward then so is the vesting condition:

\[
V(P,W,H,T_V) = \text{Prob} \times V_+(P,W,H,T_V) + (1 - \text{Prob}) \times V^*(W,T_V)
\]

where

\[
\text{Prob} = N(d), \quad d = \frac{\log(P/B) - \frac{1}{2}a^2}{a}
\]

and \( N(\cdot) \) denotes a cumulative normal distribution.

It is more challenging to adapt our model for performance exercise options. This is because exercise can occur at any point in time and vesting occurs at a single point in time. In our extended model we assume that earnings are only observed every three months, so if \( T = 10 \) we have forty earnings dates \( TE_i, i = 1, \ldots, 40 \). For performance exercise options exercise can only occur if the most recent earnings level was above the performance target, \( B \). Thus, at a particular time \( t \), the option can only be exercised if \( E(TE_i) > B \) where \( TE_i \leq t < TE_{i+1} \). This creates a problem for our finite difference method as it calculates the exercise strategy working backward in time. Thus we need to modify our valuation methodology. Working backward in time from maturity, \( T \), for \( (T =)TE_{40} > t > TE_{39} \), we calculate two scenarios for the value function, \( V(s,P,W,H,t) \), firm cost, \( F(s,P,W,H,t) \), and total delta and vega, \( TI(s,P,W,H,t) \) where \( s = 1,2 \) denotes the scenario. In the first scenario exercise is possible and, as in the vanilla option case, will occur if

\[
V(1,W,P,H,TE_{39}) < V\left(1, W + \frac{N^c}{M} \max(P - X, 0), P, H - \frac{N^c}{M}, t\right)
\]

and \( F \) and \( TI \) are determined based upon this exercise strategy. In the second scenario exercise is not permitted and we proceed as in the standard calendar vesting cases. At \( t = TE_{39} \) then the earnings level is known and the values of \( V \), for a given \( P \), can be determined based upon the known probability of the two scenarios occurring, thus,

\[
V(P,W,H,TE_{39}) = \text{Prob}V(1,P,W,H,TE_{39}) + (1 - \text{Prob})V(2,P,W,H,TE_{39})
\]

\[
F(P,W,H,TE_{39}) = \text{Prob}F(1,P,W,H,TE_{39}) + (1 - \text{Prob})F(2,P,W,H,TE_{39})
\]

\[
TI(P,W,H,TE_{39}) = \text{Prob}TI(1,P,W,H,TE_{39}) + (1 - \text{Prob})TI(2,P,W,H,TE_{39})
\]

We then proceed back through time until \( TE_1 \). Between \( TE_1 \) and \( t = 0 \) we do not need to consider two scenarios as the earnings level at \( t = 0 \) is known to be \( E(0)(= P(0)) \). It is far more complex to calculate \( V, F, \) and \( TI \) in the sliding scale exercise case as this involves a far larger set of possible scenarios as we need one for each of the possible ranges of exercise levels, thus we do not report these results in Table 8.
Figure 1: This figure show how much the stock price must change by to keep the executive's utility constant when the volatility decreases by 0.05 for a range of different stock prices, $P$, levels and is presented for four different types of options: a vanilla option, the sliding scale vesting option, an option that only vests if $P > 2X$ and an option that only vests if $P > X$. Other parameters are: $M = 4, \gamma = 2, H = 1, T = 10, \mu_P = 0.04, r = 0.04, \sigma_P = 0.3, X = 1, \ell_{\max} = 2500, j_{\max} = 100, k_{\max} = 50, W = 1$. 


Wealth

The ratio of executive's option value \((G)\) to cost of issuing one stock/option \((F)\) with \(P = 1\) at \(t = 0\)

Vanilla Option
P > 2X exercise
Sliding scale exercise
P > X vesting
P > 2X vesting
Sliding scale vesting

Figure 2: These figures show the ratio of executive value \((G(P, W, H, t))\) to the firm cost \((F(1, W, 1, 0))\) for a range of different wealth, \(W\), and moneyness, \(P/X\), levels and is presented for six different types of options: a vanilla option, the sliding scale exercise option, an option that can only be exercised if \(P > 2X\), the sliding scale vesting option, an option that only vests if \(P > 2X\) and an option that only vests if \(P > X\). Other parameters are: \(M = 4\), \(\gamma = 2\), \(H = 1\), \(T = 10\), \(T_v = 3\), \(\mu_p = 0.04\), \(r = 0.04\), \(\sigma_p = 0.3\), \(X = 1\), \(i_{\text{max}} = 2500\), \(j_{\text{max}} = 100\), \(k_{\text{max}} = 50\). For the second figure \(W = 1\).

Wealth

The ratio of executive's delta \((\Delta)\) to cost of issuing one stock/option \((F)\) with \(P = 1\) at \(t = 0\)

Vanilla Option
P > 2X exercise
Sliding scale exercise
P > X vesting
P > 2X vesting
Sliding scale vesting

Figure 3: These figures show the ratio of the executive's delta \((\Delta(P, W, H, t))\) to the firm cost \((F(1, W, 1, 0))\) for a range of different wealth, \(W\), and moneyness, \(P/X\), levels and is presented for six different types of options: a vanilla option, the sliding scale exercise option, an option that can only be exercised if \(P > 2X\), the sliding scale vesting option, an option that only vests if \(P > 2X\) and an option that only vests if \(P > X\). Other parameters are: \(M = 4\), \(\gamma = 2\), \(H = 1\), \(T = 10\), \(T_v = 3\), \(\mu_p = 0.04\), \(dP = 0.1\), \(\sigma_p = 0.3\), \(X = 1\), \(i_{\text{max}} = 2500\), \(j_{\text{max}} = 100\), \(k_{\text{max}} = 50\). For the second figure \(W = 1\).
Wealth

The ratio of executive’s vega (\(G\)) to the cost of issuing one at-the-money option (\(F\)) at \(t = 0\)

Vanilla option
P > 2X exercise
Sliding scale exercise

Figure 4: These figures show the ratio of the executive’s vega \((G(P, W, H, t))\) to the firm cost \((F(1, W, 1, 0))\) for a range of different wealth, \(W\) and moneyness, \(P/X\), levels and is presented for six different types of options: a vanilla option, the sliding scale exercise option, an option that can only be exercised if \(P > 2X\), the sliding scale vesting option, an option that only vests if \(P > 2X\) and an option that only vests if \(P > X\). Other parameters are: \(M = 4, \gamma = 2, T = 10, \mu_P = 0.04, r = 0.04, d\sigma = 0.1, \sigma_P = 0.3, X = 1, i_{max} = 2500, j_{max} = 100, k_{max} = 50\). For the second figure \(W = 1\).
Figure 5: These figures show the expected lifetime of the option ($\theta(P,W,H,t)$) for a range of different wealth, $W$, and moneyness, $P/X$, levels and is presented for six different types of options: a vanilla option, the sliding scale exercise option, an option that can only be exercised if $P > 2X$, the sliding scale vesting option, an option that only vests if $P > 2X$ and an option that only vests if $P > X$. Other parameters are: $M = 4$, $\gamma = 2$, $T = 10$, $\mu_P = 0.04$, $r = 0.04$, $\sigma_P = 0.3$, $X = 1$, $i_{\text{max}} = 2500$, $j_{\text{max}} = 100$, $k_{\text{max}} = 50$. For the second figure $W = 1$.

Figure 6: These figures show the ratio of executive value ($G(P,W,H,t)$) to the firm cost ($F(1,W,1,0)$) for a range of different wealth, $W$, and moneyness, $P/X$, levels and is presented for seven different types of options and restricted stocks: a vanilla option, restricted stock, sliding scale sale restricted stock, restricted stock that can only be sold if $P > 2X$, the sliding scale vesting restricted stock, restricted stock that only vests if $P > 2X$ and restricted stock that only vests if $P > X$. Other parameters are: $M = 4$, $\gamma = 2$, $H = 1$, $T = 10$, $TV = 3$, $\mu_P = 0.04$, $r = 0.04$, $\sigma_P = 0.3$, $X = 1$, $i_{\text{max}} = 2500$, $j_{\text{max}} = 100$, $k_{\text{max}} = 50$. For the second figure $W = 1$. 
Figure 7: These figures show the ratio of the executive’s delta (Δ(P, W, H, t)) to the firm cost (F(1, W, 1, 0)) for a range of different wealth, W, and moneyness, P/X, levels and is presented for seven different types of options and restricted stocks: a vanilla option, restricted stock, sliding scale sale restricted stock, restricted stock that can only be sold if P > 2X, the sliding scale vesting restricted stock, restricted stock that only vests if P > 2X and restricted stock that only vests if P > X. Other parameters are: M = 4, γ = 2, H = 1, T = 10, μP = 0.04, r = 0.04, dP = 0.1, σP = 0.3, X = 1, i_{max} = 2500, j_{max} = 100, k_{max} = 50. For the second figure W = 1.
Figure 8: These figures show the ratio of the executive’s vega \( G(P,W,H,t) \) to the firm cost \( F(1,W,1,0) \) for a range of different wealth, \( W \), and moneyness, \( P/X \), levels and is presented for seven different types of options and restricted stocks: a vanilla option, restricted stock, sliding scale sale restricted stock, restricted stock that can only be sold if \( P > 2X \), the sliding scale vesting restricted stock, restricted stock that only vests if \( P > 2X \) and restricted stock that only vests if \( P > X \). Other parameters are: \( M = 4, \gamma = 2, H = 1, T = 10, \mu_P = 0.04, r = 0.04, d\sigma = 0.1, \sigma_P = 0.3, X = 1, t_{\text{max}} = 2500, j_{\text{max}} = 100, k_{\text{max}} = 50 \). For the second figure \( W = 1 \).
Figure 9: These figures show the lifetime delta of one option with different vesting periods \( T_V = 0, \ldots, 5 \) for an executive with liquid wealth \( W \), with different volatilities, \( \sigma_P = 0.3, 0.45 \). Other parameters are: \( M = 4, \gamma = 2, H = 1, T = 10, T_V = 3, \mu_P = 0.04, r = 0.04, \sigma_P = 0.3, X = 1, i_{\text{max}} = 2500, j_{\text{max}} = 100, k_{\text{max}} = 50 \).

Figure 10: These figures show the lifetime delta of one option with different performance exercise targets, \( P > X, P > 1.2X, \ldots, P > 2X \), for an executive with liquid wealth \( W \), with different volatilities, \( \sigma_P = 0.3, 0.45 \). Other parameters are: \( M = 4, \gamma = 2, H = 1, T = 10, T_V = 3, \mu_P = 0.04, r = 0.04, X = 1, i_{\text{max}} = 2500, j_{\text{max}} = 100, k_{\text{max}} = 50 \).
Table 1

Executive option valuation, G

This table presents the executive's option/stock valuation for all of the option and restricted stock packages considered. Values are reported for varying levels of executive's liquid wealth, W, and stock price, P. Other parameters are: T = 10, T_v = 3 (unless otherwise specified), γ = 2, M = 4, H = 1, μ_P = 0.04, r = 0.04, σ_P = 0.3, X = 1, i_{\text{max}} = 2500, j_{\text{max}} = 100, k_{\text{max}} = 50.

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<td><strong>Sliding Scale exercise</strong></td>
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</table>
This table presents the firm's option/stock valuation for all of the option and restricted stock packages considered. This is performed using risk-neutral valuation. The values in bold are used to calculate the key metrics. Values are reported for varying levels of executive's liquid wealth, $W$, and stock price, $P$. Other parameters are: $T = 10$, $T_v = 3$ (unless otherwise specified), $\gamma = 2$, $M = 4$, $H = 1$, $\mu_p = 0.04$, $r = 0.04$, $\sigma_p = 0.3$, $X = 1$, $i_{\max} = 2500$, $j_{\max} = 100$, $k_{\max} = 50$.

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</table>
Table 3
Executive's delta, $\Delta$

This table presents the executive's delta for all of the option and restricted stock packages considered. Values are reported for varying levels of executive's liquid wealth, $W$, and stock price, $P$. Other parameters are: $T = 10$, $T_v = 3$ (unless otherwise specified), $\gamma = 2$, $M = 4$, $H = 1$, $\mu_P = 0.04$, $r = 0.04$, $\sigma_P = 0.3$, $X = 1$, $i_{\text{max}} = 2500$, $j_{\text{max}} = 100$, $k_{\text{max}} = 50$.

<table>
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<tr>
<th>Stock Price, $P = 0.5$</th>
<th>Stock Price, $P = 1$</th>
<th>Stock Price, $P = 2$</th>
<th>Stock Price, $P = 5$</th>
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<tr>
<td><strong>Option grants</strong></td>
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<tr>
<td>Vanilla Option</td>
<td>0.198 0.259 0.322 0.383 0.451</td>
<td>0.450 0.481 0.525 0.577 0.648</td>
<td>0.971 0.957 0.935 0.891 0.829</td>
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<tr>
<td>Vesting $T_v = 3$</td>
<td>0.192 0.255 0.320 0.382 0.450</td>
<td>0.332 0.408 0.485 0.558 0.643</td>
<td>0.546 0.589 0.638 0.691 0.763</td>
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<tr>
<td>Vesting $T_v = 5$</td>
<td>0.178 0.242 0.311 0.376 0.449</td>
<td>0.274 0.355 0.441 0.527 0.628</td>
<td>0.404 0.469 0.540 0.615 0.717</td>
</tr>
<tr>
<td>Sliding Scale exercise</td>
<td>0.175 0.236 0.301 0.365 0.436</td>
<td>0.386 0.438 0.494 0.554 0.632</td>
<td>1.020 0.985 0.941 0.879 0.818</td>
</tr>
<tr>
<td>$P &gt; 2X$ exercise</td>
<td>0.116 0.183 0.258 0.324 0.385</td>
<td>0.264 0.378 0.480 0.554 0.625</td>
<td>2.113 1.495 1.124 0.931 0.847</td>
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<tr>
<td>Sliding Scale vesting</td>
<td>0.008 0.014 0.021 0.030 0.040</td>
<td>0.069 0.119 0.186 0.261 0.352</td>
<td>0.246 0.386 0.543 0.689 0.843</td>
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<tr>
<td>$P &gt; 2X$ vesting</td>
<td>0.010 0.017 0.027 0.037 0.049</td>
<td>0.080 0.136 0.210 0.291 0.388</td>
<td>0.274 0.419 0.574 0.713 0.856</td>
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<tr>
<td>$P &gt; X$ vesting</td>
<td>0.104 0.152 0.205 0.258 0.317</td>
<td>0.303 0.405 0.506 0.600 0.704</td>
<td>0.567 0.619 0.669 0.722 0.792</td>
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<tr>
<td><strong>Restricted Stock</strong></td>
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<td>Vesting $T_v = 3$</td>
<td>0.798 0.828 0.868 0.909 0.952</td>
<td>0.779 0.798 0.828 0.868 0.921</td>
<td>0.769 0.779 0.798 0.829 0.882</td>
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<tr>
<td>Vesting $T_v = 5$</td>
<td>0.696 0.740 0.796 0.855 0.919</td>
<td>0.666 0.696 0.740 0.796 0.872</td>
<td>0.651 0.668 0.698 0.743 0.819</td>
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<tr>
<td>Sliding Scale sale</td>
<td>0.426 0.569 0.693 0.787 0.864</td>
<td>2.004 1.642 1.417 1.292 1.215</td>
<td>1.375 1.335 1.286 1.231 1.167</td>
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<tr>
<td>$P &gt; 2X$ sale</td>
<td>0.134 0.230 0.360 0.502 0.657</td>
<td>0.326 0.519 0.735 0.921 1.075</td>
<td>5.556 3.885 2.730 2.029 1.554</td>
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<tr>
<td>Sliding Scale vesting</td>
<td>0.009 0.015 0.026 0.040 0.058</td>
<td>0.074 0.134 0.223 0.336 0.485</td>
<td>0.273 0.453 0.675 0.894 1.113</td>
</tr>
<tr>
<td>$P &gt; 2X$ vesting</td>
<td>0.011 0.020 0.033 0.050 0.072</td>
<td>0.087 0.155 0.257 0.382 0.544</td>
<td>0.309 0.503 0.730 0.942 1.142</td>
</tr>
<tr>
<td>$P &gt; X$ vesting</td>
<td>0.167 0.274 0.404 0.533 0.661</td>
<td>0.532 0.761 0.966 1.117 1.241</td>
<td>1.056 1.053 1.018 0.997 1.008</td>
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</table>
### Table 4

**Executive's vega, $v$**

This table presents the executive's vega for all of the option and restricted stock packages considered. Values are reported for varying levels of executive's liquid wealth, $W$, and stock price, $P$. Other parameters are: $T = 10$, $T_v = 3$ (unless otherwise specified), $\gamma = 2$, $M = 4$, $H = 1$, $\mu_P = 0.04$, $r = 0.04$, $\sigma_P = 0.3$, $X = 1$, $i_{max} = 2500$, $j_{max} = 100$, $k_{max} = 50$.

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<tr>
<th>Stock Price, $P = 0.5$</th>
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<th>Stock Price, $P = 5$</th>
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<tr>
<td><strong>Option grants</strong></td>
<td><strong>Option grants</strong></td>
<td><strong>Option grants</strong></td>
<td><strong>Option grants</strong></td>
</tr>
<tr>
<td><strong>Vanilla Option</strong></td>
<td>0.142 0.204 0.275 0.350 0.439</td>
<td>0.104 0.159 0.232 0.319 0.441</td>
<td>0.002 0.001 0.007 0.014 0.051</td>
</tr>
<tr>
<td><strong>Vesting $T_v = 3$</strong></td>
<td>0.133 0.197 0.272 0.349 0.439</td>
<td>-0.074 0.012 0.128 0.259 0.421</td>
<td>-1.717 -1.415 -1.080 -0.731 -0.301</td>
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<tr>
<td><strong>Vesting $T_v = 5$</strong></td>
<td>0.110 0.174 0.252 0.335 0.434</td>
<td>-0.166 -0.105 0.004 0.149 0.354</td>
<td>-1.801 -1.669 -1.433 -1.111 -0.610</td>
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<tr>
<td><strong>Sliding Scale exercise</strong></td>
<td>0.131 0.188 0.257 0.330 0.428</td>
<td>0.156 0.200 0.248 0.299 0.421</td>
<td>0.180 0.185 0.179 0.080 0.050</td>
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<tr>
<td><strong>P &gt; 2X exercise</strong></td>
<td>0.159 0.252 0.357 0.452 0.548</td>
<td>0.252 0.369 0.484 0.584 0.720</td>
<td>0.034 0.062 0.097 0.146 0.256</td>
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<tr>
<td>Sliding Scale vesting</td>
<td>0.013 0.023 0.036 0.050 0.068</td>
<td>0.111 0.195 0.313 0.454 0.635</td>
<td>-0.327 -0.443 -0.481 -0.390 -0.125</td>
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<tr>
<td><strong>P &gt; 2X vesting</strong></td>
<td>0.016 0.028 0.043 0.060 0.081</td>
<td>0.117 0.203 0.323 0.466 0.648</td>
<td>-0.448 -0.597 -0.645 -0.548 -0.271</td>
</tr>
<tr>
<td><strong>P &gt; X vesting</strong></td>
<td>0.085 0.130 0.183 0.240 0.307</td>
<td>-0.119 -0.076 0.006 0.114 0.257</td>
<td>-2.145 -1.812 -1.422 -1.023 -0.551</td>
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</tr>
<tr>
<td><strong>Restricted Stock</strong></td>
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<tr>
<td><strong>Vesting $T_v = 3$</strong></td>
<td>-0.445 -0.351 -0.257 -0.174 -0.087</td>
<td>-1.040 -0.879 -0.698 -0.512 -0.300</td>
<td>-2.320 -2.070 -1.752 -1.391 -0.907</td>
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<tr>
<td><strong>Vesting $T_v = 5$</strong></td>
<td>-0.622 -0.509 -0.392 -0.281 -0.154</td>
<td>-1.427 -1.229 -1.011 -0.782 -0.492</td>
<td>-3.160 -2.828 -2.434 -1.991 -1.372</td>
</tr>
<tr>
<td><strong>Sliding Scale sale</strong></td>
<td>0.247 0.369 0.502 0.626 0.748</td>
<td>0.407 0.495 0.583 0.664 0.746</td>
<td>0.274 0.279 0.285 0.291 0.298</td>
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<td><strong>P &gt; 2X sale</strong></td>
<td>0.182 0.314 0.494 0.692 0.911</td>
<td>0.304 0.491 0.708 0.906 1.083</td>
<td>-0.035 0.006 0.034 0.051 0.060</td>
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<td>Sliding Scale vesting</td>
<td>0.014 0.026 0.043 0.066 0.097</td>
<td>0.118 0.214 0.363 0.559 0.833</td>
<td>-0.394 -0.609 -0.796 -0.845 -0.650</td>
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<tr>
<td><strong>P &gt; 2X vesting</strong></td>
<td>0.018 0.032 0.053 0.081 0.117</td>
<td>0.124 0.225 0.379 0.579 0.855</td>
<td>-0.549 -0.835 -1.075 -1.139 -0.926</td>
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<tr>
<td><strong>P &gt; X vesting</strong></td>
<td>0.115 0.193 0.294 0.400 0.515</td>
<td>-0.485 -0.617 -0.650 -0.574 -0.401</td>
<td>-5.767 -4.825 -3.702 -2.747 -1.824</td>
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<tr>
<td>Stock Price, P = 0.5</td>
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</tr>
</tbody>
</table>

**Option grants**

- **Vanilla Option**

- **Sliding Scale exercise**

- **P > 2X exercise**

- **Sliding Scale vesting**
  - 3.008, 3.009, 3.012, 3.015, 3.019

- **P > 2X vesting**
  - 3.011, 3.013, 3.017, 3.021, 3.025

- **P > X vesting**
  - 3.524, 3.570, 3.617, 3.658, 3.691

**Restricted Stock**

- **Vesting T = 3**
  - 3.000, 3.000, 3.000, 3.000, 3.000

- **Vesting T = 5**
  - 5.000, 5.000, 5.000, 5.000, 5.000

- **Sliding Scale sale**

- **P > 2X sale**

- **Sliding Scale vesting**
  - 3.000, 3.000, 3.000, 3.000, 3.000

- **P > 2X vesting**
  - 3.000, 3.000, 3.000, 3.000, 3.000

- **P > X vesting**
  - 3.000, 3.000, 3.000, 3.000, 3.000

This table presents the expected lifetime of the option and restricted stock packages considered. Values are reported for varying levels of executive’s liquid wealth, W, and stock price, P. Other parameters are: T = 10, T_v = 3 (unless otherwise specified), γ = 2, M = 4, H = 1, µ_r = 0.04, r = 0.04, σ_p = 0.3, X = 1, i_max = 2500, j_max = 100, k_max = 50.
This table presents the contract of the option and stock plans that have a fixed cost equal to the value of one stock and provide the same expected utility to the executive as the restricted stock with $T_v = 3$. The total delta and total vega over the lifetime of the contract are also reported. Values are reported when $P = 1$, for varying levels of executive’s liquid wealth, $W$. Other parameters are: $T = 10$, $T_v = 3$ (unless otherwise specified), $\gamma = 2$, $M = 4$, $H = 1$, $\mu_p = 0.04$, $r = 0.04$, $\alpha_p = 0.3$, $X = 1$, $i_{max} = 2500$, $j_{max} = 100$, $k_{max} = 50$.

### SIGMA = 0.3

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<th>Total Vega of Contract</th>
<th>Total Delta of Contract</th>
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<td>Wealth</td>
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<td>0.25 0.5 1 2 5</td>
<td>0.25 0.5 1 2 5</td>
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<td>0.689 1.056 1.403 1.739 2.050</td>
<td>3.157 3.338 3.439 3.362 3.727</td>
<td>-0.056 0.173 0.681 1.125 1.793</td>
</tr>
<tr>
<td>Sliding Scale vesting</td>
<td>3.086 3.048 3.109 3.007 3.091</td>
<td>0.602 0.754 0.998 1.146 1.390</td>
<td>2.438 2.307 2.558 2.451 2.581</td>
<td>0.269 0.317 0.644 0.821 1.194</td>
</tr>
<tr>
<td>P &gt; 2X vesting</td>
<td>1.787 1.597 1.784 1.707 1.532</td>
<td>0.756 0.825 1.037 1.228 1.295</td>
<td>1.922 1.853 1.868 1.940 1.795</td>
<td>0.606 0.603 0.766 0.959 1.066</td>
</tr>
<tr>
<td>P &gt; X vesting</td>
<td>3.727 3.759 3.618 3.467 3.518</td>
<td>0.821 0.896 0.944 1.054 1.230</td>
<td>3.198 2.708 3.046 3.046 2.960</td>
<td>0.586 0.309 0.696 0.881 1.096</td>
</tr>
</tbody>
</table>

### SIGMA = 0.45

<table>
<thead>
<tr>
<th>Restricted Stock</th>
<th>Total Delta of Contract</th>
<th>Total Vega of Contract</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vesting $T_v = 3$</td>
<td>2.652 2.683 2.733 2.797 2.884</td>
<td>-1.817 -1.533 -1.203 -0.867 -0.493</td>
</tr>
<tr>
<td>Sliding Scale sale</td>
<td>3.567 3.426 3.456 3.368 3.129</td>
<td>2.325 2.266 2.328 2.330 2.216</td>
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<tr>
<td>Sliding Scale vesting</td>
<td>2.123 2.154 2.198 2.297 2.416</td>
<td>-0.805 -0.617 -0.377 -0.124 0.210</td>
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<tr>
<td>P &gt; 2X vesting</td>
<td>0.879 0.950 1.044 1.146 1.278</td>
<td>0.376 0.511 0.704 0.932 1.224</td>
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<tr>
<td>P &gt; X vesting</td>
<td>2.227 2.296 2.378 2.543 2.719</td>
<td>-1.144 -1.011 -0.830 -0.664 -0.432</td>
</tr>
</tbody>
</table>
Table 7

Total delta and total vega of plans when cost and value are fixed

This table presents the contract of the option and stock plans that have a fixed cost equal to the value of one stock and provide the same expected utility to the executive as the restricted stock with $T_v = 3$. The total delta and total vega over the lifetime of the contract are also reported. Values are reported when $P = 1$, for varying levels of executive's liquid wealth, $W$. Other parameters are: $T = 10$, $T_v = 3$ (unless otherwise specified), $\gamma = 2$, $M = 4$, $H = 1$, $\mu_P = 0.04$, $r = 0.04$, $\sigma_P = 0.3$, $X = 1$, $i_{max} = 2500$, $j_{max} = 100$, $k_{max} = 50$.

<table>
<thead>
<tr>
<th>Wealth</th>
<th>Total Delta of Contract</th>
<th>Total Vega of Contract</th>
<th>Total Delta of Contract</th>
<th>Total Vega of Contract</th>
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</thead>
<tbody>
<tr>
<td>Exercise</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</table>
Table 8

Total delta and total vega of plans with earnings targets

This table presents the contract of the option and stock plans that have a fixed cost equal to the value of one stock and provide the same expected utility to the executive as the restricted stock with $T_v = 3$. The total delta and total vega over the lifetime of the contract are also reported. Values are reported when $P = 1$, and earnings volatility $a = 0.1$ for varying levels of executive's liquid wealth, $W$. Other parameters are: $T = 10$, $T_v = 3$ (unless otherwise specified), $\gamma = 2$, $M = 4$, $H = 1$, $\mu_p = 0.04$, $r = 0.04$, $\sigma_p = 0.3$, $X = 1$, $i_{\text{max}} = 2500$, $j_{\text{max}} = 100$, $k_{\text{max}} = 50$.

<table>
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<tr>
<th>Option grants</th>
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</thead>
<tbody>
<tr>
<td>Vesting $T_v = 5$</td>
<td></td>
<td>3.852</td>
<td>4.032</td>
<td>4.199</td>
<td>4.136</td>
<td>4.078</td>
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<tr>
<td>E &gt; 2 exercise</td>
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<td>4.013</td>
<td>4.200</td>
<td>4.355</td>
<td>4.048</td>
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<tr>
<td>Sliding Scale vesting</td>
<td></td>
<td>3.057</td>
<td>3.082</td>
<td>2.997</td>
<td>3.148</td>
<td>3.090</td>
</tr>
<tr>
<td>E &gt; 2 vesting</td>
<td></td>
<td>1.656</td>
<td>1.716</td>
<td>1.699</td>
<td>1.508</td>
<td>1.602</td>
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</table>

<table>
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<th>Restricted Stock</th>
<th>Wealth</th>
<th>0.25</th>
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<th>1</th>
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</thead>
<tbody>
<tr>
<td>Vesting $T_v = 3$</td>
<td></td>
<td>2.652</td>
<td>2.683</td>
<td>2.733</td>
<td>2.797</td>
<td>2.884</td>
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<tr>
<td>Vesting $T_v = 5$</td>
<td></td>
<td>3.366</td>
<td>3.237</td>
<td>3.323</td>
<td>3.428</td>
<td>3.559</td>
</tr>
<tr>
<td>E &gt; 2 sale</td>
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<td>2.777</td>
<td>2.932</td>
<td>3.117</td>
<td>3.302</td>
<td>3.751</td>
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<tr>
<td>Sliding Scale vesting</td>
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<td>2.073</td>
<td>2.124</td>
<td>2.227</td>
<td>2.351</td>
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<tr>
<td>E &gt; 2 vesting</td>
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<td>0.872</td>
<td>0.964</td>
<td>1.065</td>
<td>1.196</td>
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<tr>
<td>E &gt; 1 vesting</td>
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<td>2.140</td>
<td>2.216</td>
<td>2.307</td>
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<table>
<thead>
<tr>
<th>Total Vega of Contract</th>
<th>Wealth</th>
<th>0.25</th>
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<tbody>
<tr>
<td>Option grants</td>
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<td>Vanilla Option</td>
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<td>E &gt; 2 exercise</td>
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<td>0.714</td>
<td>0.888</td>
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<tr>
<td>E &gt; 2 vesting</td>
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<td>0.779</td>
<td>0.860</td>
<td>1.043</td>
<td>1.145</td>
<td>1.318</td>
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<tr>
<td>E &gt; 1 vesting</td>
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<td>0.861</td>
<td>0.870</td>
<td>0.824</td>
<td>0.987</td>
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</table>

<table>
<thead>
<tr>
<th>Restricted Stock</th>
<th>Wealth</th>
<th>0.25</th>
<th>0.5</th>
<th>1</th>
<th>2</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vesting $T_v = 3$</td>
<td></td>
<td>-1.817</td>
<td>-1.533</td>
<td>-1.203</td>
<td>-0.867</td>
<td>-0.493</td>
</tr>
<tr>
<td>Vesting $T_v = 5$</td>
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<td>-1.932</td>
<td>-1.448</td>
<td>-0.852</td>
</tr>
<tr>
<td>E &gt; 2 sale</td>
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<td>2.483</td>
<td>2.835</td>
<td>3.455</td>
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<tr>
<td>Sliding Scale vesting</td>
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<td>-0.528</td>
<td>-0.302</td>
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</table>