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A Simplified QO-STBC Based on Hadamard Matrices

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Abstract – **In this paper, a simplified approach for implementing QO-STBC is presented. It is based on the Hadamard matrix, in which the scheme exploits the Hadamard property to attain full diversity. Hadamard matrix has the characteristic that diagonalizes a quasi-cyclic matrix and decoding matrix that are diagonal matrix permit linear decoding. Using quasi-cyclic matrices in designing QO-STBC systems require that the codes should be rotated to reasonably separate one code from another such that error floor in the design can be minimized. It will be shown that, orthogonalizing the secondary codes and then imposing the Hadamard criteria that the scheme can be well diagonalized. The results of this simplified approach demonstrate full diversity and better performance than the interference-free QO-STBC. Results show about 4 dB gain with respect to the traditional QO-STBC scheme and performs alike with the earlier Hadamard based QO-STBC designed with rotation. These results achieve the consequent mathematical proposition of the Hadamard matrix and its property also shown in this study**.

Keywords – ***Hadamard matrix, QO-STBC, full diversity***

I. Introduction

One of the ways of achieving dependable broadband network in wireless communications systems is by the use of multiple input multiple output (MIMO) technology. MIMO technology is a transmission scheme that is used to transfer high data rate depending on the number of transmission branches (diversities). The commonest of all is the space-time block codes (STBC) for two transmit diversities discussed in [1]. This technique exploits full transmission power for orthogonal codes so long as the transmitter diversity order is no more than two [2, 3]. In transmissions involving more than two antennas, the full rate power is not attainable [1]. Beyond the two transmit diversity transmissions for full rate, the codes are rather formed in a special orthogonal way. The new codes are usually described as the quasi-orthogonal STBC (QO-STBC). Besides performing transmissions of more than two antennas, the QO-STBC also improves the channel capacity and also improves bit error ratio (BER) statistics for MIMO technology [3].

The QO-STBC scheme [2, 4, 5] has been discussed to achieve full transmission rate but not full diversity [6]. The BER curves suggest that the codes outperform the codes of orthogonal design only at low SNRs, but worsen at increased SNRs. This is due to the fact that the slope of the performance curve depends on the diversity order gain, i.e. whether full of partial diversity gain. One of the major problems that limit the BER performance of the QO-STBC system is from interference incurred in the decoding process. These interference terms are off-diagonal terms that violate the possibility of simple linear decoding such that full diversity is not achievable. It is believed that these interfering terms are removable by some methods [5-6]. Examples of common methods of eliminating these interfering terms to improve the QO-STBC codes performance towards full diversity have been discussed in [3, 6-9].

For instance, the minimum distance between different codewords may reduce the likelihood of correctly decoding the right code. Hence, it is discussed that by properly choosing the constellations such that the minimum distance between the STBC codewords is increased, then the QO-STBC performance can be improved towards full diversity [6, 9, 10]. Some other common methods by which full diversity can be attained by interference reduction have been shown in [3, 7, 8]. Most of these techniques have been presented for three and four transmit antennas QO-STBC designs.

The Hadamard matrix has the ability to daigonalize quasi-cyclic matrices [11, 12] and matrices are readily invertible if there are complex [10, 13]. These properties have been exploited in discussing QO-STBC systems based on Hadamard matrices in [7, 10]. In this work, the Hadamard based QO-STBC is extended. The base quasi-cyclic codes are rather formulated according to the space-time block codes design criteria earlier discussed in [1]. This design is shown for frequency flat fading channel using a QPSK mapping scheme. The Hadamard based QO-STBC studied is for three and four transmit diversities. Using the Hadamard matrix, it will be shown that the Hadamard based STBC maintains the orthogonality criteria with no interference. The resulting decoding matrix show perfect diagonal matrix with no off-diagonal terms and results obtained agree with the ones earlier presented in [10] with optimal rotation. This optimal rotation is not used in this study.

In Section II, the system model is described and then in Section III the Hadamard based STBC is presented. The numerical simulation results are compared with that of interference-free QO-STBC and the traditional orthogonal QO-STBC and ended with summarized conclusions.

II. System Model

In this section, the applied system model is presented. Assume that there are N_T transmit antennas and N_R receive antennas. In addition, QO-STBC that encodes a vector of input symbols $[g_1\ g_2\ g_3 \dots g_{2L}]$ into G where $G \in C^{L\times N_T}$ is considered, such that $C^{L\times N_T}$ is a complex matrix, L is the block size. Let the channel impulse response be correlated such that multipath with θ_k phase that influences signal of α_k amplitude exist. If the transmission channel is flat, then α_k will be uniform for all paths, *K*, where *K* is the number of all paths traversed by the signal. Consequently, the channel impulse response will be $h(k)=\alpha(k)e^{j\theta_k}$. For more than one transmit antenna such as $i \in N_T$ whose output is received by each *j*-antenna (*j* is equivalently 1 in this study), then;

$$h_{i,j}(k)=\sum_{n=1}^{N_T}\alpha_{i,j}(k)\delta(\tau-\tau_k(t))\,e^{j\theta_k},\ \forall n\in\{1,\cdots,4\} \qquad (2)$$

The frequency response of Equation 2 becomes:

$$H_{i,j}(k)=\sum_{n=1}^{N_T}\alpha_{i,j}(k)\,\delta(t-\tau_k).e^{-j\theta_k},\ \forall k\in\{0,1,\cdots K-1\} \qquad (3)$$
$$\forall n\in\{1,2,3,4\}$$

If *H* describes the channel matrix, the transmit signal is *G*, then the received symbol will be [14];

$$\boldsymbol{Y}=\sqrt{\frac{\rho}{N_T}}\,\boldsymbol{G}H+\boldsymbol{Z} \qquad (4)$$

where $\boldsymbol{Z}$ is the Gaussian noise term and $\boldsymbol{G}$ is the QO-STBC codeword matrix formed according to the approaches in [2, 5] also discussed in [7, 10];

$$\boldsymbol{G}=\begin{bmatrix} g_1 & g_2 & g_3 & g_4 \\ -g_2^* & g_1^* & -g_4^* & g_3^* \\ g_3 & g_4 & g_1 & g_2 \\ -g_4^* & g_3^* & -g_2^* & g_1^* \end{bmatrix} \qquad (5a)$$

where N_T is the maximum number of transmit antenna and *N* is the length of the input symbols. Notice that $\boldsymbol{G}\in C^{N\times N_T}$ matrix from code-word in Equation 5a. Notice that *G* is formed from the Alamouti space time coding method [1] as:

$$G_{12}=\begin{bmatrix} g_1 & g_2 \\ -g_2^* & g_1^* \end{bmatrix},\ G_{34}=\begin{bmatrix} g_3 & g_4 \\ -g_4^* & g_3^* \end{bmatrix} \qquad (5b)$$

Such that in QO-STBC scheme,

$$G=\begin{bmatrix} G_{12} & G_{34} \\ G_{34} & G_{12} \end{bmatrix}$$

It is possible to decompose *G* into two such as G_1 and G_2 to permit maximum-likelihood decoding. This can be expressed in the following way [9]:

$$\boldsymbol{G}=G_1(g_1,0,g_3,0)+G_2(0,g_2,0,g_4)$$

This is because, $G_1^H G_2+G_2^H G_1=0$.

The equivalent channel matrix of the above QO-STBC code (*G*) can be represented as:

$$H=\begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ h_2^* & -h_1^* & h_4^* & -h_3^* \\ h_3 & h_4 & h_1 & h_2 \\ h_4^* & -h_3^* & h_2^* & -h_1^* \end{bmatrix} \qquad (6)$$

The detection matrix can be formed as:

$$D=H^H H=\begin{bmatrix} \lambda & 0 & \beta & 0 \\ 0 & \lambda & 0 & \beta \\ \beta & 0 & \lambda & 0 \\ 0 & \beta & 0 & \lambda \end{bmatrix} \qquad (7)$$

where λ is the diagonal of the (4 x 4)I_4 matrix which is the sum of the channel power (or the path gains) and represented as $\lambda=\sum_{n=1}^{N_T}\|h_n\|^2,\ \forall n=0,1,2,3,4$. Also, the second terms in the detection matrix, β, represents the interfering terms that deplete the full diversity performance expected of the 4-transmit antenna elements and is computed as: $\beta=h_1 h_3^*+h_2 h_4^*+h_1^* h_3+h_2^* h_4$. Thus, β will degrade the BER (the pairwise error probability) performance of the system so long as the following decoding approach is followed and the full diversity will not be attained.

III. HADAMARD BASED QO-STBC

In this section, the traditional Hadamard based QO-STBC system is reviewed, then with the simplified Hadamard based QO-STBC approach following. The Hadamard matrices are described as matrices of 1's and -1's entries whose columns are orthogonal. It has the property that [13];

$$H_n H_n^H=H_n^H H_n=n\boldsymbol{I}_n \qquad (8)$$

where $\boldsymbol{I}_n$ is an identity matrix for an *n* x *n* order which belongs to the channel gain. Equation 8 has the property that the channel gain is amplified *n*-times. Since the Hadamard matrix is defined for *rem*(*n*,4)=0, then in our case where *n*=4, the channel gain is amplified four times.

Let the Hadamard matrix be thought of as being formed from the traditional orthogonal STBC codes. Then, recall the orthogonal codes of the channel matrix for a two transmitter system discussed in [15] as;

$$H_2=\begin{bmatrix} h_1 & h_2 \\ h_2 & -h_1 \end{bmatrix} \qquad (9)$$

From Equation 9, the eigenvectors of the matrix can be given as;

$$V_2=\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad (10)$$

By Equation 10 tradition, we further define eigenvectors for a 4 transmit element (with one receiver) system as in [16] following Equation 10 as follows:

$$V_4 = \begin{bmatrix} V_2 & V_2 \\ V_2 & -V_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \quad (11a)$$

It is well-known that the codes that construct QO-STBC are not orthogonal, instead quasi-orthogonal. However, only the codes that construct the orthogonal STBC are orthogonal. We shall henceforth refer to Equation 11a as:

$$V_{Had} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \quad (11b)$$

Meanwhile, the columns of the QO-STBC system hold orthogonal characteristics. Then by constructing an encoding matrix according to the Hadamard matrix yields QO-STBC systems whose decoding matrix is a diagonal matrix provided a proper quasi-cyclic Hadamard design is maintained. This explicitly eliminates any interfering terms (by default) so that exact full diversity will be achieved.

A. Traditional Hadamard based QO-STBC

The earlier design of QO-STBC based on Hadamard matrix like in [7] stemmed on quasi-cyclic matrix discussed in [11, 12]. [7] described the QO-STBC for the quasi-cyclic Hadamard matrix as:

$$\boldsymbol{S} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ s_2 & s_1 & s_4 & s_3 \\ s_3 & s_4 & s_1 & s_2 \\ s_4 & s_3 & s_2 & s_1 \end{bmatrix} \quad (12)$$

Equation 12 can be decomposed into two to permit maximum likelihood decoding according to [9] as follows:

$$\boldsymbol{S} = S_1(s_1,0,s_3,0) + S_2(0,s_2,0,s_4)$$

This is because, $S_1^H S_2 + S_2^H S_1 = 0$ provided the orthogonal space-time block coding criteria discussed in [1] is satisfied. In the case of a Hadamard QO-STBC, the codewords belong to [10];

$$\boldsymbol{S} = \begin{bmatrix} S(s_1,s_2) & S(s_3,s_4) \\ S(s_3,s_4) & S(s_1,s_2) \end{bmatrix} \quad (13)$$

Where,

$$S(s_1,s_2) = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix}$$

Sometimes, the constellation $S(s_3,s_4)$ are rotated (see [10] and then [9]). This rotation is to increase the minimum separation between different received codewords which minimizes the error floor of received symbols and improves the BER performance statistics of the QO-STBC code. The channel matrix of Equation 12 is constructed similarly to the encoding matrix of Equation 12 except for changing symbols s_i to h_i $\forall i \in 1 \cdots 4$ in that case.

In this work, we describe a Hadamard-based formulation of QO-STBC system whose decoding matrix is a diagonal matrix which would lead to linear decoding and has the form:

$$C_4 C_4^H = \begin{Bmatrix} 4(\lambda+\beta) & 0 & 0 & 0 \\ 0 & 4(\lambda+\beta) & 0 & 0 \\ 0 & 0 & 4(\lambda-\beta) & 0 \\ 0 & 0 & 0 & 4(\lambda-\beta) \end{Bmatrix} \quad (14)$$

If properly formulated, the encoding matrix which is usually complex and invertible must satisfy the condition $C_n C_n^H = C_n^H C_n = n\boldsymbol{I}_n$ where $\boldsymbol{I}_n$ is the identity of $n \times n$ matrix and the superscript, $(.)^H$ is the Hermitian transpose operator. Notice that $n = 4$ in this study.

B. Simplified Hadamard based QO-STBC

Now, let the formulation of the quasi-cyclic Hadamard codes for the channel matrix proceed in the following way:

$$h_{12} = \begin{bmatrix} h_1 & h_2 \\ h_2 & h_1 \end{bmatrix} \quad (15a)$$

And,

$$h_{34} = \begin{bmatrix} h_3 & h_4 \\ h_4 & h_3 \end{bmatrix} \quad (15b)$$

Instead of the combination that yielded Equation 12, let the orthogonal space time block coding reported in [1] be invoked such that:

$$H_4 = \begin{bmatrix} h_{12} & h_{34} \\ -h_{34}^* & h_{12}^* \end{bmatrix} \quad (16)$$

Then, by multiplying the resulting codes of Equation 16 according to the Hadamard matrix to form the new channel matrix, we obtain that:

$$H_v = \begin{bmatrix} V_2 & V_2 \\ -V_2 & V_2 \end{bmatrix} \times \begin{bmatrix} h_{12} & h_{34} \\ -h_{34}^* & h_{12}^* \end{bmatrix} \quad (17)$$

This is equivalent to:

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$$Hv=\begin{bmatrix} h_1+h_2+h_3^*+h_4^* & h_1+h_2+h_3^*+h_4^* & h_3+h_4-h_1^*-h_2^* & h_3+h_4-h_1^*-h_2^* \\ h_1-h_2+h_3^*-h_4^* & h_2-h_1-h_3^*+h_4^* & h_3-h_4-h_1^*+h_2^* & h_4-h_3+h_1^*-h_2^* \\ h_3^*-h_2-h_1+h_4^* & h_3^*-h_2-h_1+h_4^* & -h_3-h_4-h_1^*-h_2^* & -h_3-h_4-h_1^*-h_2^* \\ h_2-h_1+h_3^*-h_4^* & h_1-h_2-h_3^*+h_4^* & h_4-h_3-h_1^*+h_2^* & h_3-h_4+h_1^*-h_2^* \end{bmatrix}$$

By applying Equation 8, it can be found that (remembering that $H_v(H_v)^H = n\boldsymbol{I}_n$):

$$Q=(H_v H_v^H)=\begin{bmatrix} 4(\lambda+\beta) & 0 & 0 & 0 \\ 0 & 4(\lambda+\beta) & 0 & 0 \\ 0 & 0 & 4(\lambda-\beta) & 0 \\ 0 & 0 & 0 & 4(\lambda-\beta) \end{bmatrix} \quad (18)$$

where $\lambda = \sum_1^N \|h_n\|^2 \forall n = 0,1,2,3,4$ and $\beta = h_1h_3^* + h_2h_4^* + h_1^*h_3 + h_2^*h_4$
Notice that Equation 18 yields full diversity and is 4-times louder in amplitude compared to the interference-free QO-STBC which is a diagonal matrix. Notice also that the Hadamard property of Equation 8 defined as $H_v(H_v)^H = n\boldsymbol{I}_n$ is well satisfied. The equivalent encoding matrix can as well be easily formed.

By nulling the fourth antenna element, a three-antenna scheme can be shown as:

$$h_{34} = \begin{bmatrix} h_3 & 0 \\ 0 & h_3 \end{bmatrix} \quad (19)$$

Now, substituting Equation 19 into Equation 17:

$$H_{3v} = \begin{bmatrix} V_2 & V_2 \\ -V_2 & V_2 \end{bmatrix} \times \begin{bmatrix} h_{12} & h_{34} \\ -h_{34}^* & h_{12}^* \end{bmatrix} \quad (20)$$

$$Hv=\begin{bmatrix} h_1+h_2+h_3^* & h_1+h_2+h_3^* & h_3-h_1^*-h_2^* & h_3-h_1^*-h_2^* \\ h_1-h_2+h_3^* & h_2-h_1-h_3^* & h_3-h_1^*+h_2^* & h_1^*-h_2^*-h_3 \\ h_3^*-h_2-h_1 & h_3^*-h_2-h_1 & -h_3-h_1^*-h_2^* & -h_3-h_1^*-h_2^* \\ h_2-h_1+h_3^* & h_1-h_2-h_3^* & h_2^*-h_3-h_1^* & h_3+h_1^*-h_2^* \end{bmatrix}$$

Equivalent encoding matrix is easy to be formed.

IV. Numerical Simulation Results and Discussion

Using the Rayleigh fading channel model for a frequency non-selective fading, we evaluate the performance of the proposed method with respected to the traditional QO-STBC along with the interference-free QO-STBC.

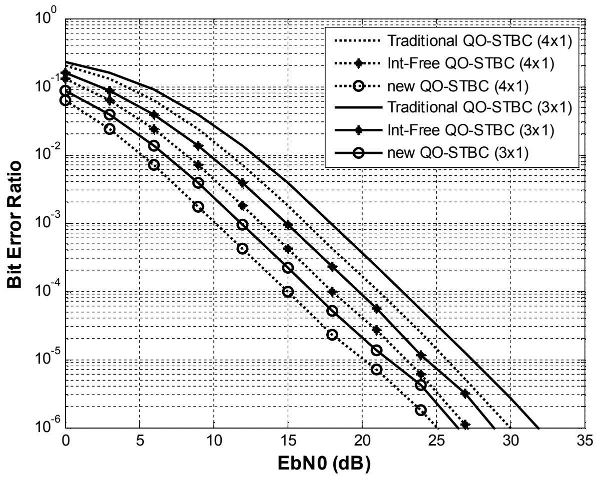

Figure 1: Comparison of new QO-STBC with traditional QO-STBC and interference-free QO-STBC

This performance is carried out for a QPSK system. In the study, it is assumed that the signal total transmit power for the respective three-transmit antenna and four transmit antenna systems were shared uniformly over the respective transmission branches for each case. During the transmission process, it is also assumed that, for three-transmit antennas that the scheme was quasi-static for three time slots and for four-transmit antennas, it was supposed that the system was constant for four-time slots. In Figure 1, the results are shown in terms of bit error ratio (BER). It is seen from the simulation results that the proposed method agrees with the mathematical proposition consequent on the Hadamard matrix property and is four-time louder than the interference-free QO-STBC. Also, the results are similar to the ones reported in [10]. This scheme provides the advantage of improved performance in comparison to interference reduction approach and reduced computation when compared to the rotation method discussed in [10].

V. Conclusion

A simplified method for implementing quasi-orthogonal space –time block codes has been presented. It followed from the earlier proposition from referred authors using quasi-cyclic Hadamard matrices. In the study, it was shown that Hadamard matrix diagonalizes the QO-STBC codes which permit linear decoding. This property paves way for achieving full diversity. The results obtained are consistent with the mathematical property and fully exploits full diversity advantage of the QO-STBC scheme. Consequently, the design of a QO-STBC system using the Hadamard matrix provides useful design advantage.

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