Applications of Contact Length Models in Grinding Processes

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Abstract. The nature of the contact behaviour between a grinding wheel and a workpiece in the grinding process has a great effect on the grinding temperature and the occurrence of thermal induced damage on the ground workpiece. It is found that the measured contact length \( l_c \) in grinding is considerably longer than the geometric contact length \( l_g \) and the contact length due to wheel-workpiece deflection \( l_f \). The orthogonal relationship among the contact lengths, i.e. \( l^2 = (R f^2) + l_g^2 \), reveals how the grinding force and grinding depth of cut affect the overall contact length between a grinding wheel and a workpiece in grinding processes. To make the orthogonal contact length model easy to use, attempts on modification of the model are carried out in the present study, in which the input variable of the model, \( F_n' \), is replaced by a well-established empirical formula and specific grinding power. By applying the modified model in this paper, an analysis on the contributions of the individual factors, i.e. the wheel/workpiece deformation and the grinding depth of cut, on the overall grinding contact length is conducted under a wide range of grinding applications, i.e. from precise/shallow grinding to deep/creep-feed grinding. Finally, using a case study, the criterion of using geometric contact length \( l_g \) to represent the real contact length \( l_c \), in terms of convenience versus accuracy, is discussed.

Introduction

The contact length between a grinding wheel and a workpiece during grinding processes is one of the principal factors that contribute to the quality of the ground workpiece from either thermal or mechanical aspects, since it determines the bottom length of the heat source and interface force distributions and consequently it affects the intensity of the energy flux into the workpiece, the peak temperature and the rate of wear of the grinding wheel \([1, 2, 3, 4]\). Although the geometrical contact length \( l_g \) has been widely used as a measure of the real contact length \( l_c \), it is well known that the measured/real contact lengths could be up to many times that of the geometrically calculated lengths \([4-11]\). Much effort has been made to understand the mechanism of the contact deformation between a wheel and a workpiece and to quantify the real contact length through analytical/numerical modelling and experimentation.

In the present work, a review on the research carried out in the past decades on contact length modelling is carried out. The orthogonal contact length model developed by Rowe and Qi \([11]\) is then modified in order to make it easier to use in practice. In the second part of this paper, some application cases of the orthogonal contact length models are presented. By using the modified orthogonal contact length model, the difference between the overall contact length and the geometrical contact length as well as the difference between the overall contact length and the contact length due to grinding force are analysed under a wide range of grinding conditions. Furthermore, with a case study, the criterion of using geometric contact length \( l_g \) to represent the real contact length \( l_c \), in terms of convenience versus accuracy, is discussed. Finally, the effect of
wheel wear on the grinding temperature is discussed with the help of the modified orthogonal contact length model.

Review of the Contact Length Models

The mechanisms of the contact deformation between a wheel and a workpiece and quantification of the real wheel/workpiece contact length have been investigated through analytical/numerical modelling and experimentation in the past decades. A comprehensive review on the researches and the model developments in grinding contact lengths was given by Zhang [4]. Table 1 includes some of the published contact length models [4-11]. Depending on the assumptions used in the modelling, the contact length models can be categorised into three types. In the first type of models, represented by the works of Lindsay/Hahn [5], and Brown/Saito/Shaw [6], Hertz contact theory was used in calculating the contact length due to grinding forces. The geometrical effect of the wheel depth of cut on the contact length, however, was assumed to be negligible. The models of this type revealed the importance of grinding wheel hardness or its elastic modulus on grinding contact length. In contrast, the second type of models considered the geometry effect, i.e. the effects of the wheel depth of cut and the wheel diameter, on the contact length, but neglected the effect of the grinding force and the wheel-workpiece deformation. The advantage of the contact length models of this type is that it is simple and easy to use, which is the main reason for using the geometrical contact length \( l_g \) to represent the overall contact length \( l_c \). In the third type of the models, such as those introduced by Kumar/Shaw [7], Hideo [8], and Zhang [4], the local wheel-workpiece deformation and the wheel depth of cut were considered as two equally important factors on the overall grinding contact length. The third type of models revealed the reason why \( l_c \) was much larger than \( l_g \). In addition, the possible effects of the surface roughness of the workpiece and the topography of the grinding wheel on the overall contact length were studied by Brandin [9], who proposed that the difference between geometric contact length and the real contact length was due to the geometrical effect of the surface roughness of the workpiece. In contact mechanics, as explained by Greenwood [12], the topography of the surfaces in contact is of primary importance. The contact length between two rough surfaces in contact is greater than the contact length between two smooth surfaces in contact under the same contact force. Peklenik [13] characterised the stochastic nature of the grinding process arising from the randomness of the distribution of cutting edges in the grinding wheel surface when measuring grinding temperature. To clarify the complex relationship one needs to understand the principle of the deformation of a wheel-workpiece system at macroscopic as well as microscopic levels.

The contact length model developed by Rowe/Qi [11, 14 - 15] clarified the orthogonal effects of the wheel-workpiece deformation, the grinding geometry and the topography of the rough wheel-workpiece contact surfaces on the overall contact length of the grinding contact zone. The orthogonal contact length model is represented in Eq.(1):

\[
l_c^2 = l_l^2 + l_g^2 = (R_l l_l)^2 + l_g^2
\]

where

\[
l_g = (a_e d_e)^{0.5}
\]

\[
l_l = [8 F_n ' (K_s + K_w) d_e]^{0.5}
\]

\[
1/d_e = 1/d_s \pm 1/d_w,
\]

\[
K_s = \frac{(1 - \nu_s^2)}{\pi E_s}, \quad K_w = \frac{(1 - \nu_w^2)}{\pi E_w}
\]
Formula (1a) defines the geometric contact length, \( l_g \), based on the grinding geometry theory, and Formula (1b) defines the contact length due to grinding force, \( l_c \), based on Hertzian contact theory. In addition, considering the fact that the contacting surfaces in abrasive machining processes are far from ideal smooth contact, a roughness factor \( R_r \) is introduced in Eq.(1). The detail of the derivation of the contact length model can be found in the reference [16].

In Eq.(1), the roughness factor \( R_r \) is a constant, i.e. it’s not sensitive to the process parameters. A detail study on \( R_r \) was carried out, which can be found in [16]. \( R_r = 9 \) is used in the present study. \( E_s \) and \( E_w \) are the moduli of elasticity of the grinding wheel and of the workpiece respectively. \( \nu_s \) and \( \nu_w \) are the Poisson ratios of the grinding wheel and the workpiece respectively. These properties are available from standard material handbooks. The grinding process parameters, which include \( d_a \), the equivalent diameter of the grinding wheel, \( d_n \), the diameter of the grinding wheel, \( d_w \), the diameter of the workpiece, and \( a_c \), the real depth of cut, are available for particular grinding application.

The specific normal force \( F_n' \) in Eq.(1), however, is normally not always available or easy to obtain in actual grinding systems because that normally a special force measurement system is required for obtaining the value of \( F_n' \). In next section, the Formula (1b) in the Eq.(1) is modified by using specific grinding power, which is much easy to obtain, and by a well established grinding force model to replace the input variable \( F_n' \).

<table>
<thead>
<tr>
<th>Table 1 A summary of typical grinding contact length models published</th>
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<tr>
<td><strong>Model</strong></td>
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<tr>
<td>Lindsay[5]</td>
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<td>Brown[6]</td>
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<tr>
<td>Kumar[7]</td>
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<tr>
<td>Hideo[8]</td>
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<tr>
<td>Brandin[9]</td>
</tr>
<tr>
<td>Maris[10]</td>
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<tr>
<td>Zhang [4]</td>
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<tr>
<td>Rowe/Qi [11]</td>
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</table>

**Modifications of the Orthogonal Contact Length Model**

**Use of Grinding Power.** In a practical grinding system, the grinding power signal is much easier to obtain in the course of a grinding operation, in comparison with the grinding force signal. For a plunge grinding operation the specific grinding power can be approximately related to the normal grinding force as:

\[ P' \approx v_s F_t' = v_s \mu F_n', \] (2)

where \( P' \) is specific grinding power, \( F_t' \) is specific tangential force and \( \mu \) is the grinding friction coefficient, which is approximately 0.3 to 0.5. The specific normal force then is:

\[ F_n' = F_t'/\mu = P'/(v_s \mu) \] (3)

By using Eq.(3), Formula (1b) becomes Formula (1b):

\[ l_f = \left[ 8 P'(K_s + K_w)d_f/(v_s \mu)^{0.5} \right]^{0.5} \] (1b')
This modified contact length model is easy to use especially for on-line grinding process controls.

**Use of an Empirical Force Model.** Eq.(4) is Werner’s empirical model for grinding force, which is used in this study [17]:

\[
F_n = F_0 q^{-e_1} a_e^{e_2} d_e^{(e_3)10^3 e_2}
\]

(4)

where \(F_0, e_1, e_2\) and \(e_3\) are constants. \(F_0\) is usually found to lie in the range 10 to 20 N/mm. \(e_1\) is within 0 to 1 and the typical value is 0.55. \(e_2\) is in the range 0.5 to 1 approximately and the typical value is 0.75. \(e_3\) is in the range 0 to 0.5 and the typical value is 0.25 [17]. Furthermore, \(e_1, e_2\) and \(e_3\) are related by

\[
e_1 = 2e_2 - 1 \text{ and } e_3 = 1 - e_2
\]

(5)

Equations (4) and (5) have been verified for a wide range of grinding conditions, from fine grinding to creep feed grinding, from easily ground material to difficult to grind material [17]. By using the grinding force model equations (4) and (5), Formula (1b) becomes Formula (1b’):

\[
l_f = [8 F_0 q^{-e_1} a_e^{e_2} d_e^{(1 + e_3)10^3 e_2}(K_s + K_w)]^{0.5}
\]

\[
= [8 F_0 q^{(1-2 e_2)} a_e^{e_2} d_e^{(2 - e_2)10^3 e_2}(K_s + K_w)]^{0.5}
\]

(1b’)

The orthogonal contact length model modified with the empirical force equation is, therefore, represented by the following equation:

\[
l_c^2 = l_{rf}^2 + l_{g}^2 = (R_l l_f)^2 + l_{g}^2
\]

\[
= R_l^2 8 F_0 q^{-e_1} a_e^{e_2} d_e^{(1 + e_3)10^3 e_2}(K_s + K_w) + a_e d_e
\]

\[
= R_l^2 8 F_0 q^{(1-2 e_2)} a_e^{e_2} d_e^{(2 - e_2)10^3 e_2}(K_s + K_w) + a_e d_e
\]

(6)

**Applications of the Modified Orthogonal Contact Length Models**

**Study of the Difference between \(l_c\) and \(l_g\).** Eq.(7) is the contact length ratio between the real contact length and the geometrical contact length, \(r_{c-g}\) derived based on Eq.(6) and Formula (1a):

\[
r_{c-g} = l_c/l_g = [1 + R_l^2 8 F_0 q^{(1-2 e_2)} a_e^{(e_2-1)} d_e^{(1 - e_2)10^3 e_2}(K_s + K_w)]^{0.5}
\]

(7)

Applying Eq.(7) with \(R_l = 9, e_2 = 0.75, F_0 = 15\) N/mm, \((K_s+K_w) = 7.52 \times 10^6\) mm²/N, the variation of the contact length ratio \(r_{c-g}\) under a range of grinding conditions is obtained as shown in Figures 1 and 2. The base plane in the figures represents the real contact length equal to the geometrical contact length, i.e. \(r_{c-g} = 1\). The curved face is \(r_{c-g}\) as a function of speed ratio, \(q\), and real depth of cut, \(a_e\). Under a conventional grinding condition as shown in Figure 1, where \(q\) is from 50 to 300 and \(a_e\) is from 0.005 to 0.02 mm, the real contact length \(l_c\) is two times or more the geometrical contact length \(l_g\). In addition, \(r_{c-g}\) decreases as \(a_e\) and \(q\) increase, which indicates that the difference between \(l_c\) and \(l_g\) became smaller towards larger depth of cut and higher speed ratio, a scenario of creep feed grinding. As shown in Figure 2, under a creep feed grinding region, where \(q = 500-1000\) and \(a_e = 1-15\) mm, the magnitude of the geometrical contact length \(l_g\) is approaching that of the real contact length \(l_c\).

**Study of the Difference between \(l_c\) and \(l_{rf}\).** Similarly, Eq.(8) is the contact length ratio between the real contact length and the contact length due to grinding force and the surface roughness factor, \(r_{c-rf}\) derived based on Eq.(6) and Formula (1b’):

\[
r_{c-rf} = l_c/l_{rf} = l_c/(R_l l_f)
\]

\[
= [1 + R_l^2 8 F_0 q^{(1-2 e_2)} a_e^{(e_2-1)} d_e^{(1 - e_2)10^3 e_2}(K_s + K_w)]^{-1}^{0.5}
\]

(8)
Fig. 1 Ratio of real contact length over geometrical contact length under conventional grinding conditions, i.e. $q = 50\,\text{-}300$; $a = 0.005\,-\,0.02\text{mm}$

Fig. 2 Ratio of real contact length over geometrical contact length under creep feed grinding conditions, i.e. $q = 500\,-\,1000$; $a = 1\,-\,15\text{mm}$

Fig. 3 Ratio of real contact length over the contact length due to grinding force only (i.e. $R_r=1$) under conditions of conventional grinding, i.e. $q = 50\,\text{-}500$; $a = 0.005\,-\,0.12\text{mm}$

Fig. 4 Ratio of real contact length over the contact length due to grinding force and contact surface roughness ($R_r=9$) under conditions of conventional grinding, i.e. $q = 50\,\text{-}500$; $a = 0.005\,-\,0.12\text{mm}$

Applying Eq.(8) with $e_2 = 0.75$, $F_0 = 15\,\text{N/mm}$, $(K_s+K_w) = 7.52\times10^{-6}\,\text{mm}^2/\text{N}$, the variation of the contact length ratio $r_{c/\text{rf}}$ under a range of grinding conditions is obtained as shown in Figures 3 and 4. The base plane in the figures represents the real contact length equal to the contact length due to the force and roughness factor, i.e. $r_{c/\text{rf}} = 1$. The curved face is $r_{c/\text{rf}}$ as a function of speed ratio, $q$, and real depth of cut, $a_r$. In Figure 3, the roughness factor $R_r=1$ is assumed, which means that the
curved face represented the ratio of $l_c$ and $l_i$ while the effect of surface roughness is neglected. It is shown that under a conventional grinding condition (i.e. $q = 50-500$; $a_c = 0.005-0.12$ mm), the real contact length, $l_c$, can be two to five times the contact length due to the grinding force, $l_i$. However, if the roughness factor $R_t$ is considered as shown in Figure 4, where $R_t = 9$ is applied, the magnitude of the contact length due to the grinding force and the contact surface roughness (i.e. $l_{cf} = R_{cf} l_i$) is approaching that of the real contact length $l_c$. This demonstrates the effect of the roughness factor $R_t$ on the real contact length, it is, therefore, important to take the $R_t$ factor into consideration when studying the real contact length in grinding. It shows also that if both $q$ and $a_c$ are small, a typical fine/shallow grinding scenario, then the effect of grinding geometry on overall contact length of the wheel/workpiece can be neglected.

**The Criterion of Using the Geometric Contact Length $l_g$.** As mentioned before, much previous work ignored the effect of deflection in grinding on the grinding contact length and used the geometry contact length to represent the real contact length. The advantage of this approach is obvious since it makes the analysis simple. For example, Kopalinsky [18] used the geometrical contact length $l_g$ to represent the real contact length $l_c$ in analysing grinding temperatures. In Kopalinsky’s work, the contact length was used to quantify several grinding parameters, including the contact area of wheel and workpiece, the size of the heat source and the size of the heat flux of rubbing for calculating the grinding temperatures, the size of a set of active grains on the wheel in contact with the workpiece simultaneously in grinding and the time it took for the set to enter the contact region, the number of active cutting edges in the grinding contact area and the number of active rubbing points in the grinding contact area, and the contact time of individual active cutting grains with the workpiece, which demonstrated the importance of the contact length in the analysis of grinding processes.

Table 2 is a comparison of the results obtained based on the two different treatments on the contact length, using Kopalinsky’s data. It shows clearly that by taking the effect of elastic flattening of the wheel, the size of the contact lengths is increased by 86%. And consequently the number of active cutting edges in the grinding contact area becomes 26 instead of 14. By using the overall contact length as the measure of the heat source size for calculating the grinding temperature, the estimated grinding maximum temperature is 26% less than that the elastic flattening of the wheel is neglected, as shown in Table 2.

<table>
<thead>
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<th>Table 2 Comparison of the results based on different contact length models</th>
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<td>Contact length</td>
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<td>Contact area</td>
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<td>Number of active edges</td>
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<td>Maximum grinding temperature [16]</td>
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</table>

Conditions used in the analysis [18]:

Workpiece material: En9; Grinding wheel: WA46J with diameter $d$, of 177 mm;

Depth of cut $a_c$: 0.02 mm; Width of cut $b$: 3 mm;

Wheel speed $v_c$: 40 m/s; Workpiece speed $v_w$: 0.5 m/s;

$F_t$: 10 N/mm; $K_c$: $6.16 \times 10^{-6}$ mm$^2$/N; $K_w$: $1.36 \times 10^{-6}$ mm$^2$/N; $R_t = 9$

Figure 5 is an overview of the contours of $r_{c-g}$ covering a full range of grinding conditions, from fine grinding, shallow grinding, creep-feed grinding to high speed grinding (i.e. $q$ is from 50 to 10000 and $a_c$ is from 0.001 to 50 mm). The top-right area in the figure, labelled with ‘I’ (where $q$ and $a_c$ are big), represents those grinding conditions under which the simplification of $l_c = l_g$ is valid,
i.e. the simplification only causes an error of 20% or less. The bottom-left area in the figure, labelled with ‘III’, represents those grinding conditions under which the simplification of \( l_c = l_g \) is not valid, i.e. the simplification would cause an error of 200% or more. Under the conditions in the area labelled ‘II’, the overall contact length \( l_c \) can be represented by \( l_c = \alpha l_g \), where \( \alpha \) varies from 1.2 to 3 depending on \( q \) and \( a_e \). It is maybe, therefore, sensible to use \( l_c = (2\sim3) l_g \) as recommended by some researches for simplicity under those grinding conditions.

In summary, normally the effect of elastic flattening of the wheel due to grinding force and the topography of the wheel cannot be neglected and the orthogonal contact length model should be used to quantify the overall grinding contact length. Under large depth of cut and big speed ratio, which is the condition in creep-feed grinding, however, geometrical contact length can be used to represent the real contact length for convenience without lose of accuracy.

Fig. 5 Contours of the \( r_{c-g} \) vs. depth of cut and speed ratio

**Discussion of the Effect of Grinding Wheel Wear.**

It is known that grinding force and grinding temperature increase as wheel wear increases [18-20]. Kopalinsky interpreted this phenomenon as due to an increase in negative rake angle of the cutting edges in addition to a growth in area of the wear flats on the grits. As a result the grinding cutting forces could be increased, which was the main reason for the increase of grinding temperatures with wheel wear. In addition, it was known that the increasing rubbing force due to wheel wear and the filling of the voids on the wheel with work material could also contribute to an increase in the workpiece temperature.

Qi’s work [20] revealed another possible way by which grinding wheel wear would affect the maximum grinding temperature. It was found, as shown in Figure 6, that increasing the rubbing area due to wheel wear and the filling of the voids on the wheel with work material caused the real depth of cut \( a_e \) decrease under the same nominal depth of cut. Figure 6 shows also that the contact of the wheel and workpiece was much concentrated when the grinding wheel became dull. As the result, the overall effective contact length was decreased. The phenomenon can be interpreted by using the orthogonal contact length model Eq.(6). Assuming \( e_2 = 0.75 \) and taking \( K_s = (1 - \nu_s^2)/(\pi E_s) \), \( K_w = (1 - \nu_w^2)/(\pi E_w) \), the Eq.(6) becomes:

\[
l_c^2 = l_n^2 + l_g^2
\]
and

\[ l_{t\gamma}^2 = R_e^2 8 F_0 a_e^{0.75} q^{-0.5} d_e^{1.25} 10^{2.25} [(1-\nu_e^2)/(\pi E_s) + (1-\nu_w^2)/(\pi E_w)] \]

\[ l_g^2 = a_e d_e \]

Eq. (6') shows clearly that a decrease in \( a_e \) or an increase in \( E_s \) will cause a decrease in \( l_c \). The mechanisms of grinding wheel wear affecting the effective cutting length can be, therefore, summarised as:

1. The dull edges cause an increase of the cutting rake angle and the rubbing area and decrease the penetration of the active edges into the workpiece under the same normal force, which means the real depth of cut \( a_e \) become small, as shown in Figure 6. Consequently, both the geometrical length and the length due to deflection are decreased according Eq. (6') (or \( l_g, l_{t\gamma} \propto a_e \)).

2. The dull edges and the filling of the voids on the wheel with work material make the grinding wheel stiffer, i.e. the effective stiffness of the wheel (or the hardness of the wheel represented by the wheel modulus \( E_s \)) becomes high. According Eq. (6'), the length due to deflection decreases as the increase in the effective wheel modulus \( E_s \) (or \( l_g \propto 1/E_s \)).

According grinding temperature model (as shown in Table 2), the maximum grinding temperature is inverse proportional to the square-root of the heat source size (or \( T_{\text{max}} \propto 1/l_e \)). Therefore, shorter contact length due to wheel wear results in a higher maximum grinding temperature.

In summary, an increase of grinding wheel wear and dullness during grinding processes means not only an increase in the area of the wear flats on its grains, an increase in the negative rake angle of the cutting edges as highlighted by Kopalinsky [18], but also a decrease in the real contact length. All the changes contribute to the grinding temperature increase and the consequent occurrence of thermal damage.

![Fig. 6 The change of contact status with different variations of grinding wheel [20].](image)

Test conditions: Cast iron; with coolant; \( v_w = 0.1 \text{ m/s}; a = 30 \mu\text{m} \);

- T4: newly dressed wheel; slight burn \( a_e = 16 \mu\text{m} \);
- T5: lightly worn wheel; medium burn \( a_e = 14 \mu\text{m} \);
- T6: worn wheel; heavy burn \( a_e = 12 \mu\text{m} \)

**Conclusions**

1. By using specific grinding power, the orthogonal contact length model is modified, which is more suitable for the applications such as on line controls of grinding processes.
2. The orthogonal contact length model is also modified by using a well known empirical force equation, which is capable for predicting the contact length in a wide range of grinding conditions. By using the modified orthogonal contact length model, the difference between the overall contact length and the geometrical contact length as well as the difference between the overall contact length and the contact length due to grinding force are analysed under a wide range of grinding conditions.

4. With a case study, the criterion of using geometric contact length \( l_g \) to represent the real contact length \( l_c \), in terms of convenience versus accuracy, is discussed. It is found that in most grinding application condition the magnitude of \( l_c \) is up to three times that of \( l_g \) depending on the values of \( q \) and \( a_c \). Under large depth of cut and high speed ratio, which is the condition of a creep-feed grinding, geometrical contact length can be used to represent the real contact length without lose of accuracy.

5. Grinding wheel wear causes the grinding temperature to increase not only due to its bigger negative rake angle and increased rubbing area, but also due to a shortening of the size of the effective grinding heat source.

References

Appendix: Notation

- $a$: The nominal depth of cut, mm
- $ae$: The real depth of cut (the wheel depth of cut), mm
- $b$: Grinding width, mm
- $de$: Equivalent diameter of a grinding wheel, mm
- $ds$: Diameter of a grinding wheel, mm
- $dw$: Diameter of a workpiece, mm
- $Es$: Modulus of elasticity of a grinding wheel, N/mm²
- $Ew$: Modulus of elasticity of a workpiece, N/mm²
- $F_n$: Normal grinding force, N
- $F' n$: The specific normal force, $F' n = F_n / b$, N/mm
- $F't$: The specific tangential force, N/mm
- $lc$: Theoretical contact length, mm
- $lg$: Geometric contact length, mm
- $le$: Real contact length, mm
- $lr$: Contact length due to normal force, mm
- $lr rf$: $R_c lr$, mm
- $P'$: The specific grinding power, $P' = v_s \mu F' n$, Nm/s-mm
- $q$: The speed ratio $q = v_s / v_w$
- $R_r$: The roughness factor
- $rc g$: The contact length ratio between $lc$ and $lg$
- $rc rf$: The contact length ratio between $lc$ and $lr rf$
- $vs$: Peripheral wheel speed, m/s
- $vw$: Table speed or peripheral workpiece speed, m/s
- $\mu$: The grinding friction coefficient
- $\nu_s$: Poisson ratio of the grinding wheel
- $\nu_w$: Poisson ratio of the workpiece