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INFLUENCE OF INCLINED WEB REINFORCEMENT ON REINFORCED CONCRETE DEEP BEAMS WITH OPENINGS

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ABSTRACT

This paper reports the testing of fifteen reinforced concrete deep beams with openings. All beams tested had the same overall geometrical dimensions. The main variables considered were the opening size and amount of inclined reinforcement. An effective inclined reinforcement factor combining the influence of the amount of inclined reinforcement and opening size on the structural behaviour of the beams tested is proposed. It was observed that the diagonal crack width and shear strength of beams tested were significantly dependent on the effective inclined reinforcement factor that ranged from 0 to 0.318 for the test specimens. As this factor increased, the diagonal crack width and its development rate decreased, and the shear strength of beams tested improved. Beams having effective inclined reinforcement factor more than 0.15 had higher shear strength than that of the corresponding solid beams. A numerical procedure based on the upper bound analysis of the plasticity theory was proposed to estimate the shear strength and load transfer capacity of reinforcement in deep beams with openings. Predictions obtained from the proposed formulas have a consistent agreement with test results.

Keywords: deep beams, openings, shear strength, inclined reinforcement, upper-bound analysis.

INTRODUCTION

Openings are frequently placed in the web area of reinforced concrete deep beams in order to facilitate essential services, such as conduits, network system access or even movement from one room to another. These openings often interrupt the load transfer by concrete struts and can cause a sharp decrease of strength and serviceability of deep beams[1-4]. Although the strength evaluation and reinforcement details around openings in deep beams are essential consideration, there are very little published data[5, 6] on such members. Furthermore, their design details have not been yet provided by most code provisions[7-10].

Many diagonal cracks can be developed above and below openings in reinforced concrete deep beams due to high stress concentration at corners and the abrupt change of the main load path. These diagonal cracks would accelerate the decreasing rate of the effective strength of concrete because of high transverse tensile strains at the diagonal crack plane as pointed out by Vecchio and Collins[11]. Kong et al.[3] and Tan et al.[5] showed that inclined reinforcement around openings is more effective in improving the ultimate shear strength of deep beams with openings than horizontal or vertical reinforcement. To understand the influence of inclined reinforcement on the structural behavior of deep beams with openings, it is necessary to examine the relation between the amount of inclined reinforcement and geometrical condition of beams such as opening size, opening position, and shear span-to-overall depth ratio. However, experimental data available on the amount of inclined reinforcement required to complement the strength reduced by openings are scarce.

In this paper, fifteen reinforced concrete deep beams with web openings subjected to two point top concentrated loads were tested to failure. The main variables considered were the width and depth of openings, and amount of inclined reinforcement around openings. Four sizes of web openings and three amounts of inclined reinforcement were investigated. To understand the effect of the amount of inclined reinforcement and opening size on the structural behavior of such beams, an effective inclined reinforcement factor was proposed. Also a numerical technique based on the upper bound analysis of the plasticity theory was proposed to estimate the shear strength and load transfer capacity of reinforcement in deep beams with openings

RESEARCH SIGNIFICANCE

There is no published data available on the relation between the amount of inclined reinforcement around openings and the shear behavior of deep beams with openings. Effective inclined reinforcement factor is suggested to account for the influence of inclined reinforcement and opening size on the structural behavior of deep beams with openings. Test results reported in the present investigation show that the structural behavior of deep beams with openings is significantly dependent on this effective inclined reinforcement factor.

EFFECTIVE INCLINED REINFORCEMENT FACTOR

A failure plane in deep beams with rectangular openings is usually formed along the significant diagonal cracks connecting the edges of load plates and opening corners opposite to the load points, lines EA and CF in Fig. 1, as shown in Kong and Sharp's[1] and Yang et al.'s[4] test results. In the following, equilibrium of forces at the failure plane and the opening size are considered to develop a formula to assess the effectiveness of inclined reinforcement crossing the failure plane. The force transferred to inclined reinforcement, F_D, along the failure plane is:

$$F_D = A_{d1} f_s \sin(\beta + \theta) \tag{1}$$

where A_{d1}, and f_s = the area and stress of inclined reinforcement, respectively, θ = the angle of the failure plane to the longitudinal axis of the beam, and β = the angle between inclined reinforcement and the longitudinal axis of the beam as shown in Fig. 1, where subscripts t and b indicate the top chord above openings and bottom chord below openings, respectively. The total capacity produced by inclined reinforcement arranged above and below the opening can then be calculated as follows:

$$\sum F_D = n_t A_{d1} f_{yh} \sin(\beta_t + \theta_t) + n_b A_{d1} f_{yh} \sin(\beta_b + \theta_b) \tag{2}$$

where n = the number of inclined reinforcement, and f_{yh} = yield strength of inclined reinforcement. To effectively control diagonal cracks, the load transfer capacity of inclined reinforcement has to be larger than the total transverse tensile force developed across diagonal crack planes. Therefore, it can be expressed as follows:

$$\sigma_T b_w (k_2 h/\sin\theta_b + (1-k_2-m_2)h/\sin\theta_t) \leq \sum F_{DT} = n_t A_{d1} f_{yh} \sin^2(\beta_t+\theta_t) + n_b A_{d1} f_{yh} \sin^2(\beta_b+\theta_b) \quad (3)$$

where σ_T = transverse tensile stresses in concrete acting on diagonal crack planes, b_w and h = the width and overall depth of section, respectively, k_2 = the ratio of the distance between soffit of the beam and the bottom face of the opening to the overall section depth, m_2 = the ratio of the opening depth to the overall section depth, and F_{DT} = $F_D \sin(\beta+\theta)$ = inclined reinforcement capacity component. Therefore, the ratio of transverse tensile stresses in concrete to the yield strength of inclined reinforcement is calculated by re-arranging Eq. (3) as given below:

$$\frac{\sigma_T}{f_{yh}} \leq \frac{n_t A_{d1} \sin^2(\beta_t+\theta_t) + n_b A_{d1} \sin^2(\beta_b+\theta_b)}{b_w (k_2 h/\sin\theta_b + (1-k_2-m_2)h/\sin\theta_t)} \quad (4)$$

The control of diagonal crack size and confinement effect of concrete provided by shear reinforcement greatly depend on the state of stress in reinforcement which can be determined by the amount of reinforcement and angle of diagonal cracks as pointed out by Vecchio and Collins[12]. The right hand side of Eq. (4) gives the ratio of the area component of inclined reinforcement to the area of the top chord above and bottom chord below opening, ρ_{od}. It would be interpreted as an effective inclined reinforcement ratio to provide enough resistance to transverse tensile stresses.

Fig. 2 presents the effect of the opening area ratio, $\rho_{OA} = m_1 \cdot m_2$, which is the ratio of the opening area, $m_1 a \cdot m_2 h$, to the shear span area, $a \cdot h$, on the normalized shear strength, $V_n/(V_n)_S$, where V_n is the shear strength of deep beams with openings and $(V_n)_S$ is the shear strength of the corresponding solid deep beam. Test results presented in Fig. 2 are those reported by Kong and Sharp[1] and Yang et al[6] for deep beams with openings and without shear reinforcement. Fig. 2 clearly shows that the shear strength of deep beams with openings decreases with the increase of the opening area ratio, ρ_{OA}. The increase of the opening area would cause higher transverse tensile stresses due to the reduction in the section area. Fig. 2 explains that the amount of inclined reinforcement around openings should be proportional to the opening area ratio in order to compensate for the shear strength reduction due to openings. Considering the opening size effect, the effective inclined reinforcement factor, ξ, to assure the serviceability and strength enhancement of deep beams with openings can be suggested as follows:

$$\xi = \frac{\rho_{od}}{\rho_{OA}} = \frac{n_t A_{d1} \sin^2(\beta_t + \theta_t) + n_b A_{d1} \sin^2(\beta_b + \theta_b)}{m_1 m_2 b_w (k_2 h / \sin\theta_b + (1 - k_2 - m_2) h / \sin\theta_t)} \quad (5)$$

EXPERIMENTAL INVESTIGATION

Details of geometrical dimensions and reinforcement used in test specimens are given in Table 1 and Fig. 3. The opening size and amount of inclined reinforcement were selected as the main variables to evaluate the relation of the effective inclined reinforcement factor and shear strength of deep beams with openings. Beams tested were classified into two groups according to the opening width: T-series and F-series for opening widths of $0.25a$ and $0.5a$, respectively, where a indicates the shear span. The opening depth varied between $0.1h$ and $0.3h$, where h indicates the overall depth of the beam tested. When the opening completely interrupts the natural load path joining the edges of load and support plates, the shear strength of beam is significantly reduced as indicated by Kong and Sharp[1] Mansur and Tan[12]. In each beam tested, the opening center was positioned in accordance with that of the shear span area. Inclined shear reinforcement was arranged in layers above and below openings, each consisting of three bars of 10 mm diameter. The angle of all inclined reinforcement was chosen to be 45° to the longitudinal axis of beams and placed symmetrically at the top chord above openings and bottom chord below openings. The effective inclined reinforcement ratio ρ_{od} as calculated using the right hand side of Eq. (4) varied from 0.0 to 0.0152, and the effective inclined reinforcement factor ξ as estimated from Eq. (5) had ranged between 0.0 and 0.318 as given in Table 1.

All tested beams had the same section size, distance between two-point concentrated top loads, longitudinal reinforcement ratio, concrete design strength, and shear span-to-overall depth ratio as follows: section width, b_w, was 160 mm, overall section depth, h, was 600 mm, distance between two-point top loads was 300 mm, longitudinal bottom reinforcement ratio, $\rho_{st} = A_{st} / b_w d$, where A_{st} =the longitudinal bottom reinforcement area and d = the effective section depth, was 0.0097, design concrete strength was 55 MPa, and shear span-to-overall depth ratio, a/h, was 0.5. The longitudinal bottom reinforcement was continuous over the full length of the beam and welded to 160×100×10 mm end plates to provide sufficient anchorage. Horizontal and vertical web reinforcing bars of 6 mm diameter were provided to satisfy the minimum amount recommended in ACI 318-05[7]. Both horizontal and vertical web steel bars were arranged at a spacing of 120 mm apart.

The beam notation given in Table 1 includes three parts except the solid deep beam, N0. The first letter refers to the opening width: T for $0.25a$ and F for $0.5a$, where a =the beam shear span. The second number 1, 2 or 3 indicates an opening depth of $0.1h$, $0.2h$ and $0.3h$, respectively. The third part is used to identify the number of layers of inclined reinforcement around openings. For example, T1-1 is a deep beam having an opening size of $0.25a \times 0.1h$ and one layer of inclined reinforcement (3ϕ10) at the top and bottom chords above and below openings.

Material properties

The ingredients of ready-mixed concrete used in test specimens were ordinary Portland cement, fly-ash, irregular gravel of maximum size of 25 mm, and sand. The water-binder ratio was 0.29 and admixture ratio of fly-ash was 0.15. All beams were cast in a vertical position. Control specimens which were 100 mm diameter × 200 mm high cylinder were cast and cured simultaneously with beams to determine the compressive strength of concrete. The result of the concrete compressive strength was 55.8 MPa for all beams tested. Table 2 shows the mechanical properties of all reinforcement used in the beams tested.

Instrumentation and test set-up

All beams were tested to failure under two-point concentrated top loads with loading rate of 20 kN/min using a 3000 kN capacity universal testing machine (UTM). Each beam tested was supported on a hinge at one end and a roller at the other end. At locations of load or support points, a steel plate of 100 mm wide was provided to prevent premature crushing or bearing failure. Vertical deflections were measured by 50 mm capacity linear variable differential transducers (LVDT) mounted at the bottom face at mid-span. Both beam sides were whitewashed to aid on the observation of crack development during testing. The diagonal crack width at concrete struts connecting the edges of load plates and opening corners (AE, BE, CF, and DF in Fig. 1) was monitored by 5 mm capacity PI type gages. The strains of inclined reinforcement were recorded by 5 mm electrical resistance strain gages (ERS) bonded at the region crossing the line joining the edges of load plates and opening corners. All test data were captured by a data logger and automatically stored.

EXPERIMENTAL RESULTS AND DISCUSSION

Crack propagation and failure mode

Fig. 4 shows the crack propagation against load increase according to the variation of the opening size and the amount of inclined reinforcement. Just before failure, the crack patterns above and below the opening were very similar. A symmetrical crack pattern was also observed for both sides of the deep beams tested before failure. The first crack in all beams tested except the solid beam occurred at opening corners near load points (at B and D in Fig. 1) and propagated toward the load points with the load increase. Bottom flexural cracks at the beam mid-span followed and then diagonal cracks originated at opening corners opposite to the load points (at A and C in Fig. 1). This crack sequence seemed to be independent on the effective inclined reinforcement factor related to the opening size and amount of inclined reinforcement. However, the distribution and propagation of diagonal cracks were strongly influenced by the effective inclined reinforcement factor, ξ. Beams having ξ less than 0.032 (Fig. 4 (a), Fig. 4 (e), and Fig. 4 (f)) failed soon after the occurrence of the diagonal cracks at opening corners opposite to the load points and had very few diagonal cracks at both the top and bottom chords above and below openings. For beams having ξ more than 0.096 (Fig. 4 (b), Fig. 4 (c), Fig. 4 (d), and Fig. 4 (h)), several diagonal cracks developed forming a fan-shaped distribution. Even beam F3-3 having large openings showed a good distribution of diagonal cracks (Fig. 4 (h).

All beams tested except the solid beam failed unsymmetrically along diagonal cracks joining the edges of the load plates and opening corners opposite to the load points, AE and CF, as shown in Fig. 1. These failure planes followed the upper and lower force paths proposed by Kong et al.[3] regardless of the effectiveness inclined reinforcement factor. At failure, each beam was divided into two blocks separated by failure planes. One block had transitional and rotational displacement relative to the other. The observed failure planes reinforce the introduction of the proposed effective inclined reinforcement factor and also suggest the use of mechanism analysis to predict the shear strength of deep beams with openings as presented later in this paper.

Load versus mid-span deflection

Mid-span deflections of different beams tested against the total applied load are given in Fig. 5: Fig. 5 (a) for beams in T-series and Fig. 5 (b) for beams in F-series. The initial stiffness of beams until the occurrence of the first diagonal crack at opening corners was nearly independent on the opening size and amount of inclined reinforcement. After the first diagonal crack appeared, the deflection of beams having no inclined reinforcement sharply increased, but the stiffness of beams having inclined reinforcement nearly maintained the same initial stiffness, which even exhibited in beams T3 and F3 with large openings. The two beams T1-1 and F1-2 having the same ξ value showed similar load-deflection behavior and ultimate load. This indicates that the effective inclined reinforcement factor has a significant influence on the structural behavior and stiffness of deep beams with web openings.

Diagonal crack width

Fig. 6 shows the development of diagonal crack width against the total load in the lower load path connecting the edge of the support plate and opening corner opposite to the support (CF in Fig. 1), but in the case of the solid deep beam measured in the natural load path joining the edges of load and support plates. On the same figure, the limit crack width of 0.4mm specified for serviceability of concrete members in ACI 318-02[13] is also plotted. For beams having no inclined reinforcement, as soon as the first diagonal crack occurred, its width dramatically extended up to 0.1 mm~0.21 mm. However this phenomenon hardly occurred for beams having inclined reinforcement. The diagonal crack width and its development rate decreased with the increase of the effective inclined reinforcement factor. In particular, the diagonal crack width of beams having ξ greater than 0.097 was less than that of solid beam N0. Two beams T1-1 and F1-2 having the same value of ξ (= 0.0158) exhibited similar development of diagonal crack width with the load increase. The strain in inclined reinforcement at diagonal cracks is plotted in Fig. 7 against the total applied load. The strains presented in Fig. 7 are those recorded at the nearest inclined reinforcement to the opening at the region joining the edge of the support plate and opening corner opposite to the support as shown in Fig. 7 (a). The strain in inclined steel reinforcement quickly developed with the occurrence of diagonal cracks as expected. This amount of strain development was strongly dependent on ξ. The smaller the value of ξ, the higher the rate of strain development. It means that the inclined reinforcement with a smaller area developed higher stresses due to the transfer of transverse tensile force across diagonal cracks. Strains of all inclined reinforcement reached the yield strain before the ultimate shear strength. The rate of increase of crack width and strains in inclined reinforcement according to various ξ values confirm that the inclined reinforcement effectively transfer transverse tensile stresses across the diagonal crack surface as assumed in Fig. 1.

Shear strength

The variation of the shear strength, V_n, against the effective inclined reinforcement factor is shown in Fig. 8 and Table 3. The shear strength of deep beams without inclined reinforcement was greatly dropped compared with that of the solid deep beam. For example, the shear strength of beam F3-0 having large openings and ξ=0 was reduced by 40 % of that of the solid deep beam N0. However, the arrangement of inclined reinforcement greatly improved the strength of deep beams with openings. The shear strength increased in nearly linear proportion to ξ. Deep beams with openings having an effective inclined reinforcement factor larger than 0.15 exhibited higher shear strength than the corresponding solid beam, N0.

Shear strength prediction using upper-bound analysis

The failure mode of reinforced concrete deep beams with openings as presented in Fig. 9, which was ascertained in current investigation and else where[1, 4], usually can be idealized as an assemblage of two rigid blocks separated by two yield lines as proposed by Ashour and Rishi[14]. As a result, rigid block *I* undergoes a relative rotation around an instantaneous center (I.C.) as shown in Fig. 9.

Both of the upper and lower yield lines seldom have the same displacement rate and angle about I.C., as they are formed discontinuously by the opening.

Concrete is assumed to be a rigid perfectly plastic material with the modified Coulomb failure criteria[15] as yield condition. The tensile strength is ignored and the effective compressive strength, f_c^*, is

$$f_c^* = v_e f_c^{'} \tag{6}$$

where v_e = effectiveness factor and $f_c^{'}$ = cylinder compressive strength of concrete. Tensile and compressive reinforcing bars are assumed to be a rigid perfectly plastic material with yield strength, f_y.

Work Equation

The upper-bound theorem of plasticity theory is based on the energy principle of equating the total internal energy, W_I, to the external work done, W_E. The total internal energy commonly depends on the position of the instantaneous center and internal stresses in both concrete along the yield line and reinforcing bars crossing the yield line. Because the relative displacement rate, δ, equals $\omega \cdot r$ as shown in Fig. 9, the energy, W_c, dissipated in concrete in both the upper and lower yield lines as proposed by Nielsen[15] is

$$W_c = \frac{1}{2} b_w \omega \left[(f_c^*)_t r_t (1-\sin\alpha_t) l_t + (f_c^*)_b r_b (1-\sin\alpha_b) l_b \right] \tag{7}$$

where r = distance between the midpoint of the chord of the yield line and the instantaneous center, ω = relative rotational displacement of rigid block *II* to rigid block *I* about I.C., α = angle between the relative displacement at midpoints of the chord and yield line, and l = the length of the yield line. Also, subscripts t and b indicate the upper yield line formed at top chord above opening and lower yield line formed at bottom chord below opening, respectively. Because the relative displacement of reinforcement, δ_s, can be written as $\omega \cdot r_s$, the energy, W_s, dissipated in all reinforcement crossing the yield line is calculated from

$$W_s = \sum_{i=1}^{m} \omega (A_s)_i (f_y)_i (r_s)_i \cos(\psi_s)_i \tag{8}$$

where m = number of reinforcing bars crossing the yield line, $(A_s)_i$, and $(f_y)_i$ = area and yield strength of the reinforcing bar i crossing the yield line, respectively, $(r_s)_i$ = distance between the intersection point of reinforcing bar i with yield line and I.C., and $(\psi_s)_i$ = angle between the relative displacement about I.C. and the reinforcing bar i crossing the yield line.

The external work done, W_E, can be easily estimated from Fig. 9 by considering the displacement rate of the support reaction in rigid block *I* relative to the center of the upper and lower yield lines.

$$W_E = V_n \omega |X_{ic}| \tag{9}$$

Equating the total internal energy dissipated in concrete and reinforcement to the external work done gives the ultimate shear strength of deep beams with openings as follow:

$$V_n = \frac{b_w h}{2|X_{ic}|} \left[\begin{array}{l} (f_c^*)_t r_t (1 - \sin\alpha_t) \dfrac{(1 - k_2 - m_2)}{\sin\theta_t} + (f_c^*)_b r_b (1 - \sin\alpha_b) \dfrac{k_2}{\sin\theta_b} + \\ 2\sum_{i=1}^{m} (\rho_s)_i (f_y)_i (r_s)_i \cos(\psi_s)_i \end{array} \right] \tag{10}$$

where $(\rho_s)_i$ = the reinforcement ratio i crossing the yield line, which can be calculated from $(A_s)_i / b_w h$. The third term of the right-hand side of Eq. (10) indicates the load transfer capacity of reinforcement.

Effectiveness Factor of concrete

Both top and bottom chords above and below openings in deep beams tested are considered to be in a state of biaxial tension-compression. The presence of transverse tensile strains makes the compressive strength of cracked concrete greatly deteriorated as concluded in panel tests subjected to biaxial tension-compression carried out by Vecchio and Collins[11]. This indicates that the strength of cracked concrete greatly depends on the amount of transverse tensile strains in the yield line as well as the material properties as proposed by Nielsen[15]. There is usually discrepancy in the amounts of transverse tensile strain of both upper and lower yield lines in deep beams with openings, and they are also influenced by the amount and configuration of reinforcement placed around openings. The effectiveness factor of concrete was proposed as a function of the concrete strength and the ratio of the principal strains by Vecchio and Collins[11]. In the present study, this model is adopted to consider the discrepancy of transverse tensile strains in the upper and lower yield lines and also modified to reflect the influence of the size effect as follows:

$$
\begin{aligned}
v_e &= \frac{\zeta}{1.0 + K_c K_f} \\
K_c &= 0.35\left(-\frac{\varepsilon_1}{\varepsilon_3} - 0.28\right)^{0.8} \geq 1.0 \\
K_f &= 0.1825\sqrt{f_c^{'}} \geq 1.0 \\
\zeta &= \frac{1}{\sqrt{1 + \frac{d}{25 d_a}}}
\end{aligned}
\qquad (11)
$$

where ε_1 and ε_3 = the principal tensile and compressive strains in the yield line, respectively. As the principal strains[15] $\varepsilon_{1,3}$ is $\frac{1}{2}\frac{\delta}{\Delta}(\sin\alpha \pm 1)$ in the yield line having a discontinuous width of Δ, $-\frac{\varepsilon_1}{\varepsilon_3}$ in factor K_c can be written as $\frac{1+\sin\alpha}{1-\sin\alpha}$. This indicates that the influence of the transverse tensile strain on the effectiveness factor can be determined by the position of I.C. and the yield line angle to the beam longitudinal axis. The factor ζ proposed by Bažant and Kim[16], which is a function of the effective depth, d, and the maximum size of aggregate, d_a, is to consider the influence of the size effect. Tan and Lu[17], and Yang et al.[18] showed that the size effect has a significant influence on the shear strength of deep beams.

Solution procedure

The shear strength is implicitly expressed as a function of the position, (X_{ic}, Y_{ic}), of the instantaneous center as given in Eq. (10). According to the upper-bound theorem, the collapse occurs at the least strength, which indicates minimum work done to fail the deep beam. The minimum value of the shear strength can be obtained by varying the position of the instantaneous center. All tested beams were reinforced with high longitudinal bottom reinforcement index, $\frac{\rho_{st} f_y}{f_c^{'}}$ =0.139. Therefore, the position of the instantaneous center may be located at the level of the longitudinal bottom reinforcement as it would not yield at failure. The position of the instantaneous center is iteratively tuned until the minimum shear strength is achieved. The process of adjusting the coordinates of the instantaneous center is established by reliable numerical optimization procedures available in Matlab Software.

Comparisons of proposed and experimental results

To examine the validity of the proposed model, comparisons between the predictions and experimental results of the shear strength, V_n , and the load transfer capacity of inclined reinforcement, V_{sd}, are given in Table 3 and Fig. 10. The predictions obtained by Kong et al.'s formula[3] empirically developed from their experimental results are also shown in Table 3 and Fig. 10. The load transfer capacity of the inclined reinforcement, V_{sd}, is obtained from the difference between the shear strength of beams with inclined reinforcement and that of the corresponding beam without inclined reinforcement.

The mean and standard deviation of the ratio between the experimental and predicted shear strengths, $(V_n)_{Exp.}/(V_n)_{Pro.}$, obtained from Kong et al.'s formula are 1.66 and 0.22, respectively, indicating much lower predictions than experimental shear strength. Also, the load transfer capacity of the inclined reinforcement was greatly underestimated using Kong et al.'s formula as the mean and standard deviation of the ratio between the experimental and predicted load transfer capacities, $(V_{sd})_{Exp.}/(V_{sd})_{Pro.}$, are 1.66 and 0.51, respectively. While the predictions obtained from the present mechanism analysis show good agreement with experimental results as the mean and standard deviation of the ratio $(V_n)_{Exp.}/(V_n)_{Pro.}$ are 0.99 and 0.09, respectively. Also, the predictions of the load transfer capacity of inclined reinforcement reasonably agree with the test results except in beam T1-2.

CONCLUSIONS

Fifteen reinforced concrete deep beams with web openings were tested and an effective inclined reinforcement factor on the beams tested combining the effect of the amount of inclined reinforcement and opening size was suggested. A numerical technique based on the upper bound analysis of the plasticity theory was developed to estimate the shear strength of deep beams with openings. The following conclusions may be drawn:

1. The distribution and propagation of diagonal cracks were strongly influenced by the effective inclined reinforcement factor proposed. For beams having an effective inclined reinforcement factor more than 0.096, several diagonal cracks developed forming a fan-shaped distribution before failure.
2. The diagonal crack width and its development rate decreased with the increase of the effective inclined reinforcement factor.
3. The shear strength increased with the increase of the effective inclined reinforcement factor. The shear strength of beams having an effective inclined reinforcement factor above 0.15 was higher than that of the corresponding solid deep beam.

4. The mechanism analysis developed to predict the shear strength of deep beams with openings and load transfer capacity of inclined reinforcement showed a good agreement with experimental results.

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NOTATION

A_{d1} area of inclined reinforcement

A_{st} area of longitudinal bottom reinforcement

a shear span

b_w width of deep beam

d effective depth of deep beam

d_a maximum size of aggregate

E_s elastic modulus of reinforcement

F_D transferred force to inclined reinforcement

$f_c^{'}$ cylinder compressive strength of concrete

f_c^* effective strength of concrete

f_{su} tensile strength of reinforcement

f_y yield strength of reinforcement

h overall depth of deep beam

k_1, k_2 coefficient of opening position

l length of yield line

l_p width of loading plate

m_1, m_2 coefficient of opening size

r distance between the midpoint of the chord of the yield line and the instantaneous center

V_{sd} load transfer capacity of inclined reinforcement

V_n ultimate shear strength

α angle between the relative displacement and yield line

β angle between the inclined reinforcement and the longitudinal axis of member

θ angle of a failure plane to the longitudinal axis of member

ρ_{OA} ratio of opening area to shear span area

ρ_{od} effective inclined reinforcement ratio defined by Eq. (4)

ρ_{st} longitudinal bottom reinforcement ratio

σ_T transverse tensile stress in diagonal crack planes

ν_e effectiveness factor

ξ effective inclined reinforcement factor defined by Eq. (5)

ψ_s angle between the relative displacement about I.C. and the reinforcing bar crossing yield line

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TABLES AND FIGURES

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Table 1–Details of test specimens

Specimen	$f_c^{'}$, MPa	Size of opening			Inclined reinforcement		
		m_1	m_2	ρ_{OA}	No. & diameter*	ρ_{od}	ξ
N0	55.8	-	-	-	-	-	-
T1-0		0.25	0.1	0.025	-	-	-
T1-1					3ϕ10	0.0039	0.159
T1-2					6ϕ 10	0.0079	0.318
T2-0			0.2	0.05	-	-	-
T2-3					9ϕ 10	0.0134	0.269
T3-0			0.3	0.075	-	-	-
T3-3					9ϕ 10	0.0152	0.203
F1-0		0.5	0.1	0.05	-	-	-
F1-2					6ϕ 10	0.0079	0.158
F1-3					9ϕ 10	0.0119	0.237
F3-0			0.3	0.15	-	-	-
F3-1					3ϕ 10	0.0048	0.032
F3-2					6ϕ 10	0.0097	0.064
F3-3					9ϕ 10	0.0145	0.097

* The same inclined reinforcement in both chords above and below openings

Table 2–Mechanical properties of reinforcement

Diameter, mm	f_y, MPa	ε_y	f_{su}, MPa	E_s, GPa
6*	483	0.0044	549	199
10	408	0.0021	548	195
19	803	0.0041	898	194

* The yield strength of 6 mm diameter reinforcement was obtained by 0.2 % offset method.

TABLE 3-Details of test results and predictions

Specimen	ξ	V_n, kN			V_{sd}, kN			$(V_n)_{Exp.}/(V_n)_{Pro.}$		$(V_{sd})_{Exp.}/(V_{sd})_{Pro.}$	
		Exp.	Kong	This study	Exp.	Kong	This study	Kong	This study	Kong	This study
N0	-	817	734	818	-	-	-	1.113	0.999	-	-
T1-0	0.0	726	407	741	-	-	-	1.782	0.980	-	-
T1-1	0.159	864	494	861	138	87	185	1.748	1.003	1.592	0.746
T1-2	0.318	1177	581	987	451	173	251	2.026	1.193	2.601	1.797
T2-0	0.0	627	370	671	-	-	-	1.695	0.934	-	-
T2-3	0.269	1107	638	1082	480	268	376	1.736	1.023	1.792	1.277
T3-0	0.0	565	329	610	-	-	-	1.716	0.926	-	-
T3-3	0.203	998	605	985	433	276	345	1.650	1.013	1.571	1.255
F1-0	0.0	644	358	679	-	-	-	1.797	0.948	-	-
F1-2	0.158	910	545	984	266	187	316	1.668	0.925	1.422	0.842
F1-3	0.237	1133	639	1110	489	281	376	1.773	1.021	1.742	1.301
F3-0	0.0	480	289	478	-	-	-	1.664	1.004	-	-
F3-1	0.032	688	384	628	208	95	164	1.794	1.096	2.188	1.268
F3-2	0.064	697	479	772	217	190	250	1.456	0.903	1.141	0.868
F3-3	0.097	740	574	897	260	285	316	1.290	0.825	0.912	0.823
Mean								1.660	0.986	1.66	1.13
Standard deviation								0.223	0.086	0.51	0.34

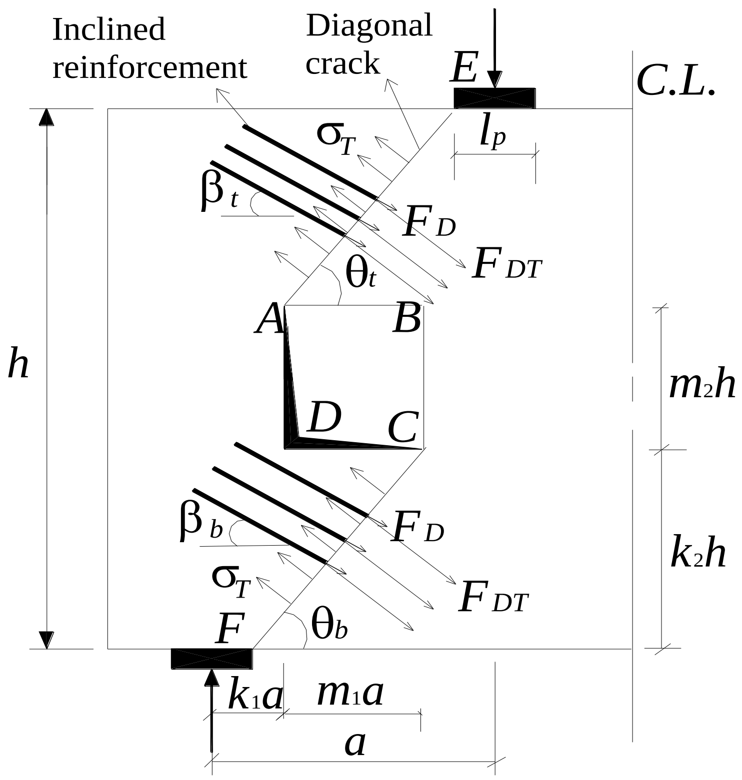


Fig. 1-Symbol identification in effective inclined reinforcement factor.

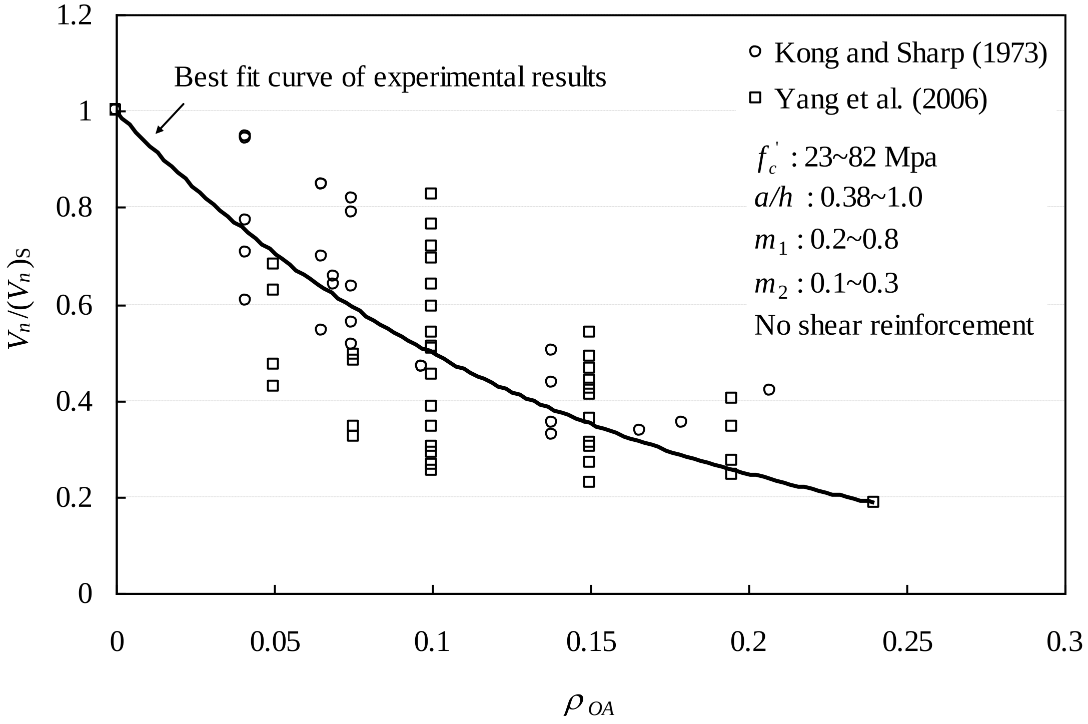


Fig. 2-Relative shear strength against opening area ratio.

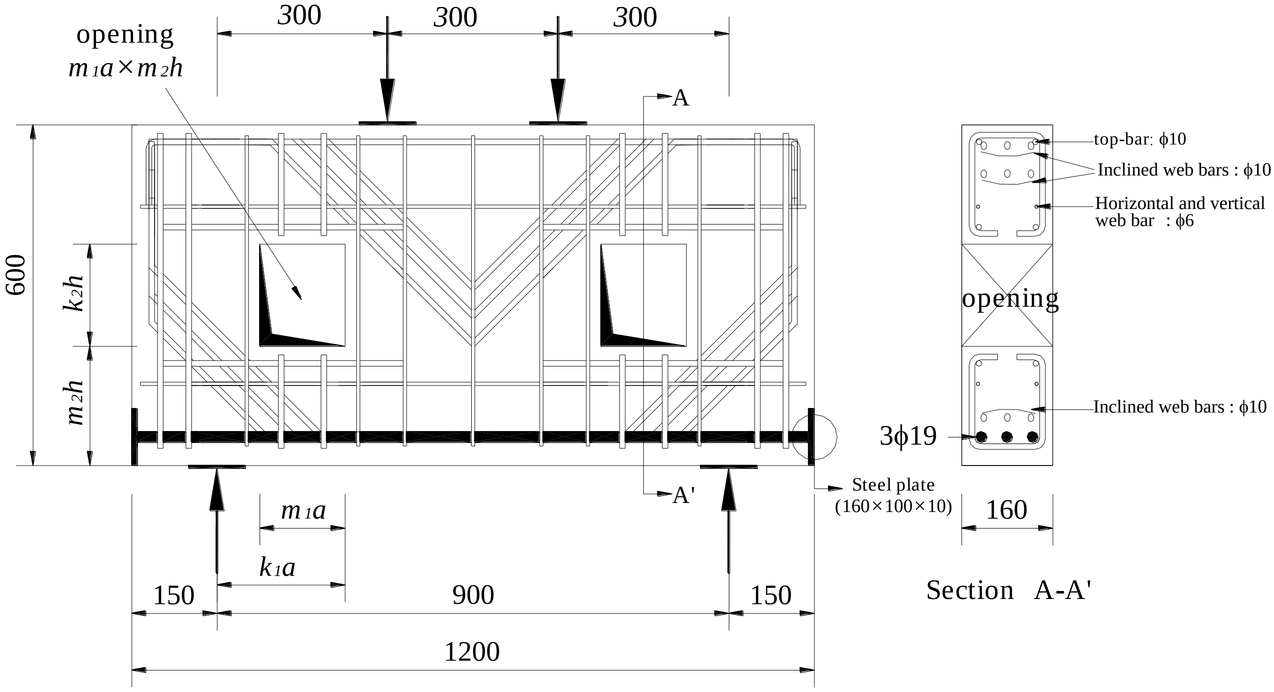
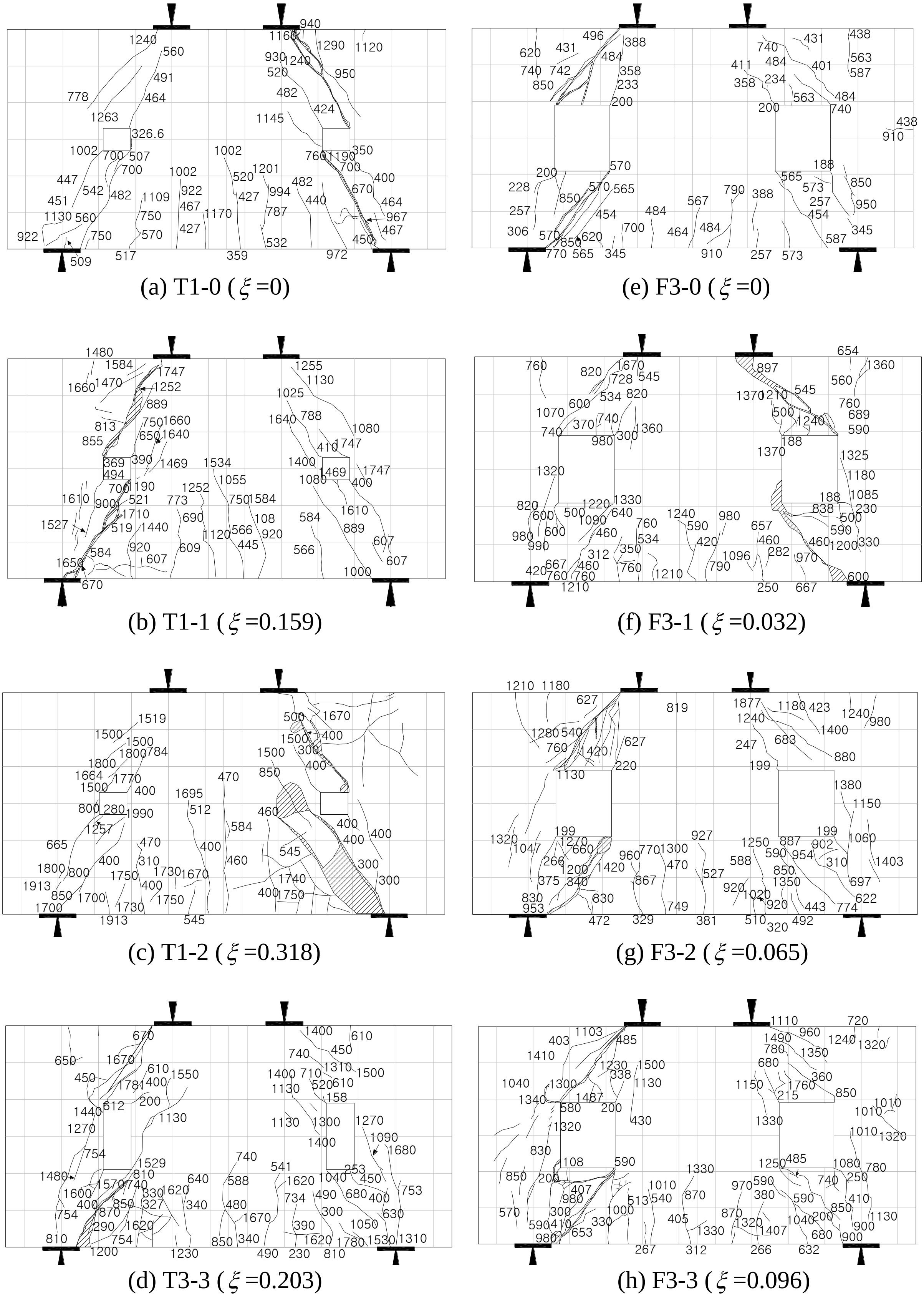


Fig. 3-Specimen details and arrangement of reinforcement (all dimensions are in mm)



(a) T1-0 (ξ =0)

(e) F3-0 (ξ =0)

(b) T1-1 (ξ =0.159)

(f) F3-1 (ξ =0.032)

(c) T1-2 (ξ =0.318)

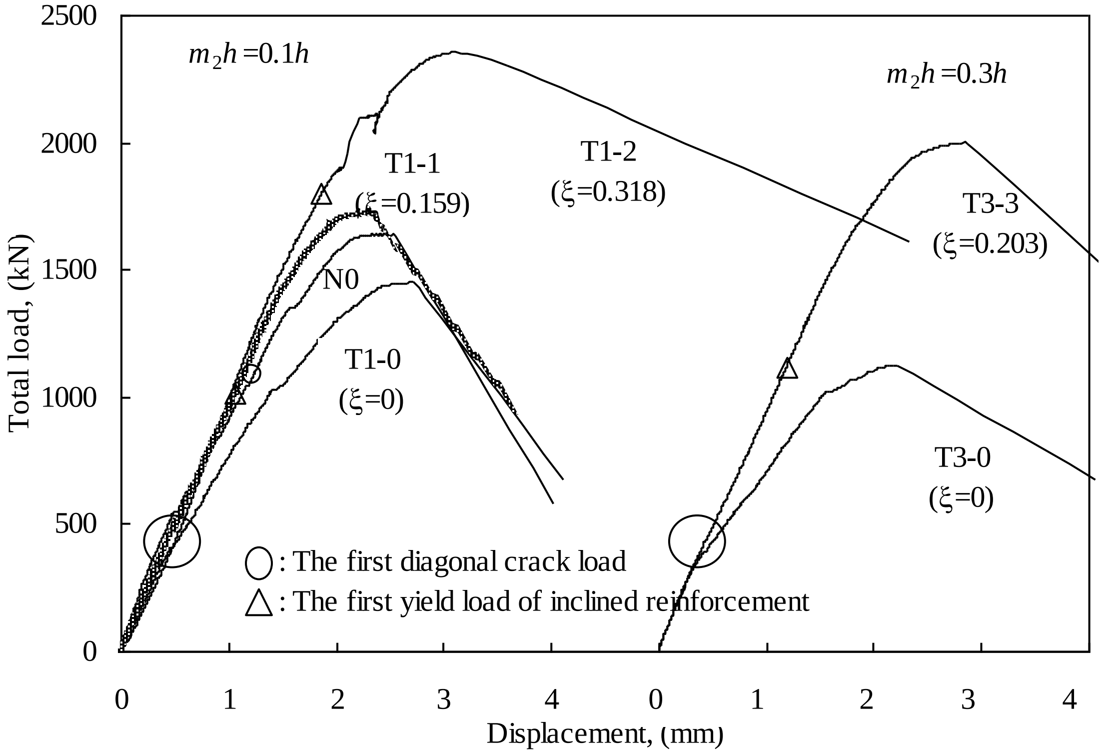
(g) F3-2 (ξ =0.065)

(d) T3-3 (ξ =0.203)

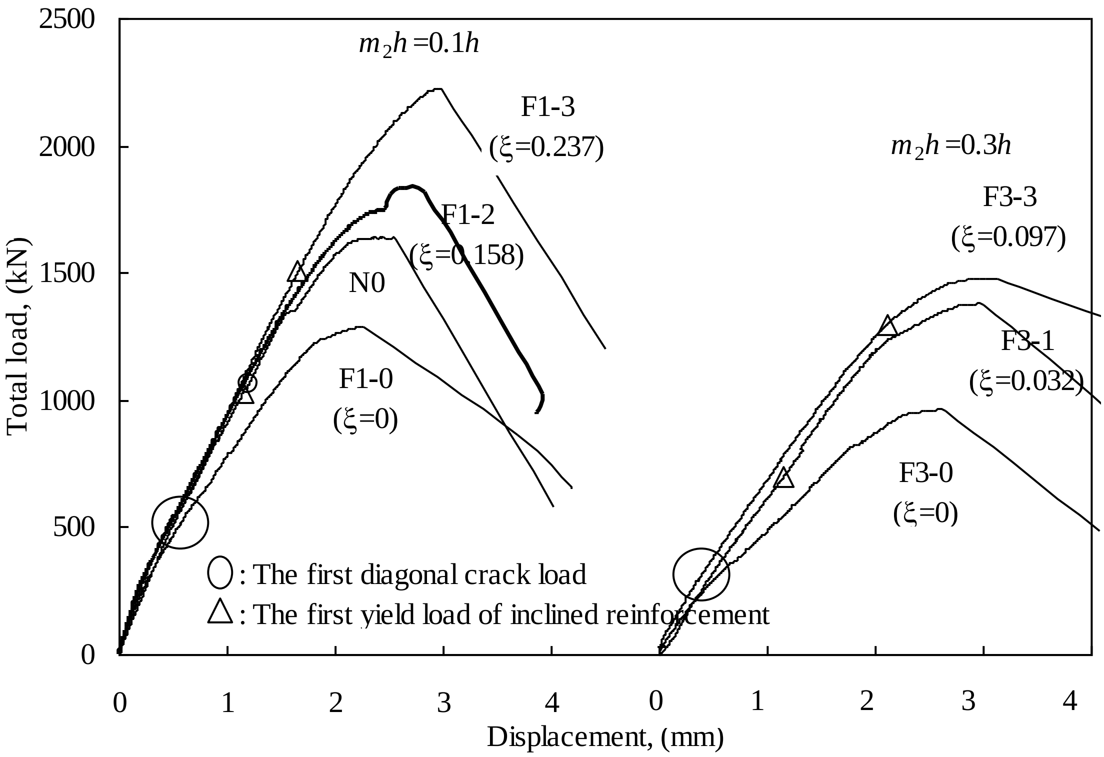
(h) F3-3 (ξ =0.096)

Fig. 4–Crack patterns and failure of beams tested

(Numbers indicate the total load in kN at which crack occurred.)

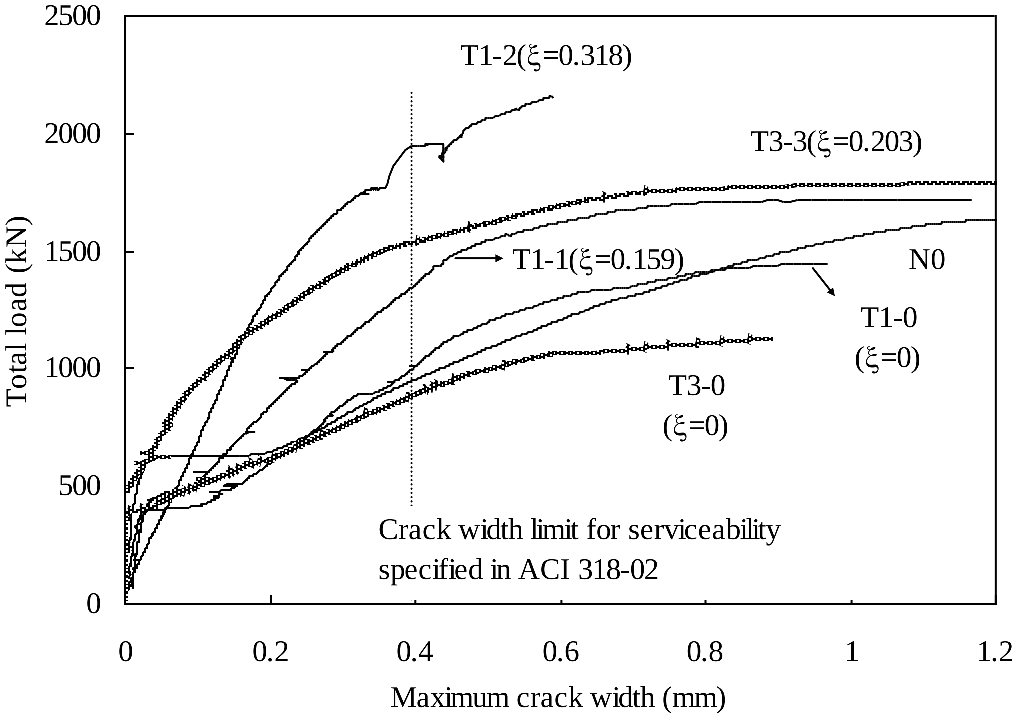


(a) T-series

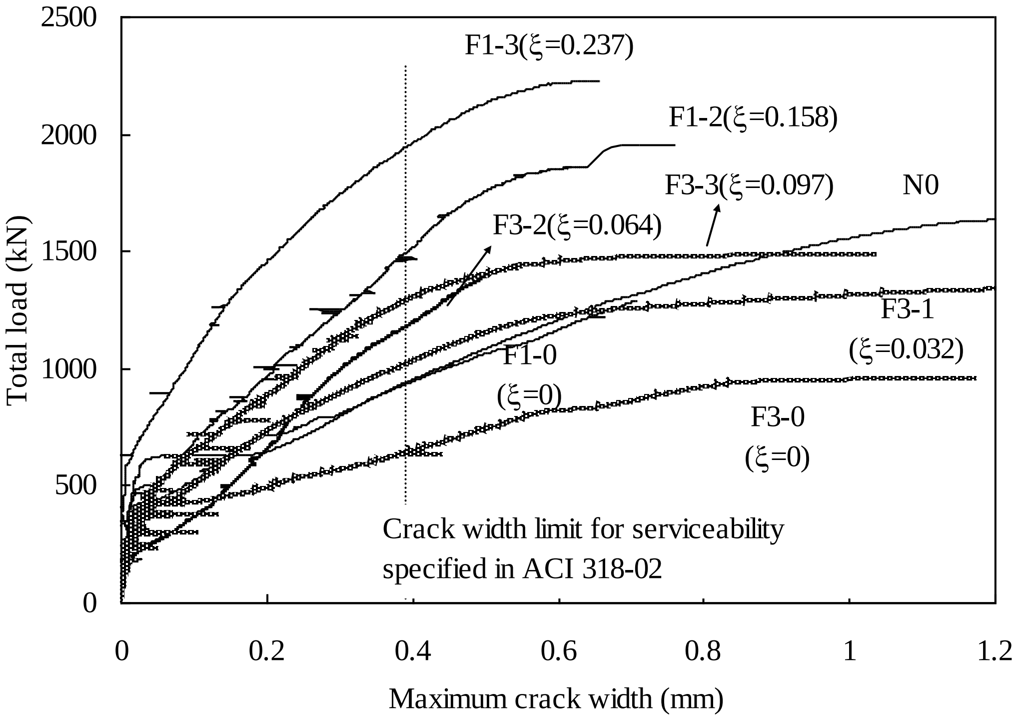


(b) F-series

Fig. 5–Mid-span deflection against total load

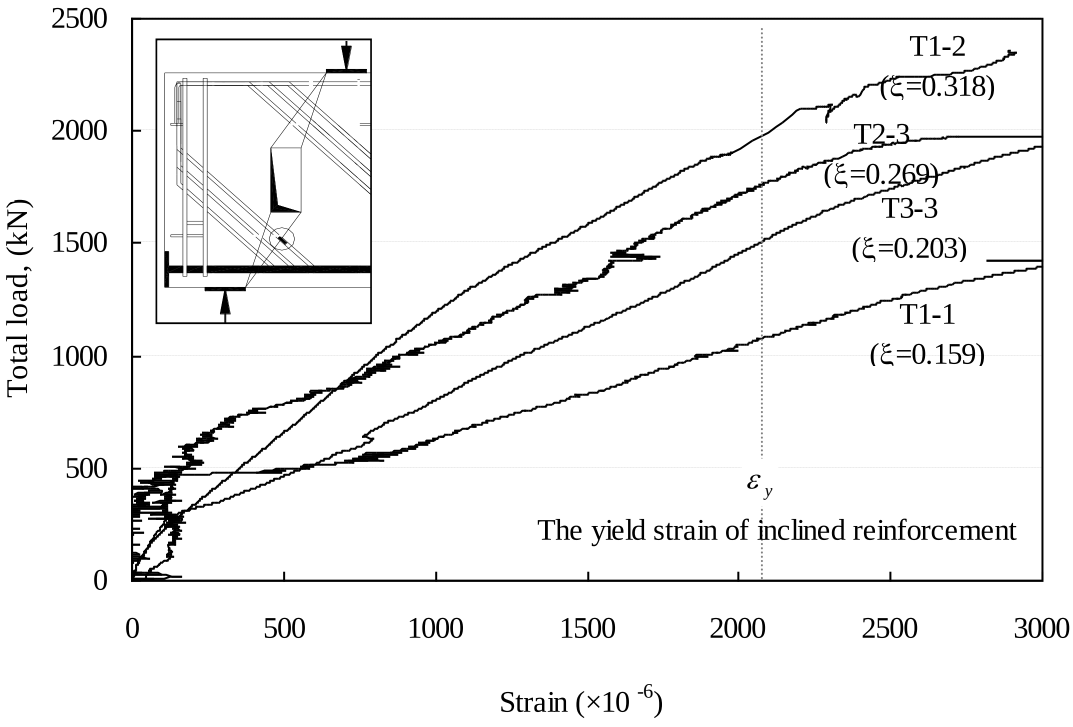


(a) T-series

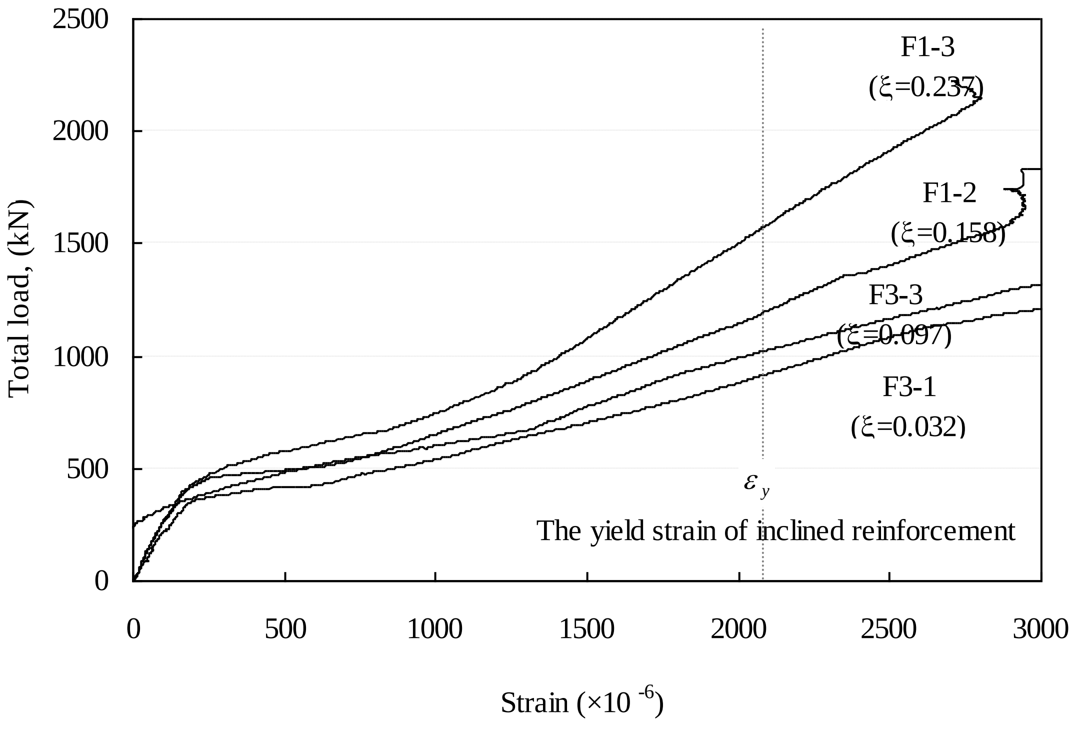


(b) F-series

Fig. 6–Maximum crack width against total load



(a) T-series



(b) F-series

Fig. 7–Strain of inclined reinforcement against total load

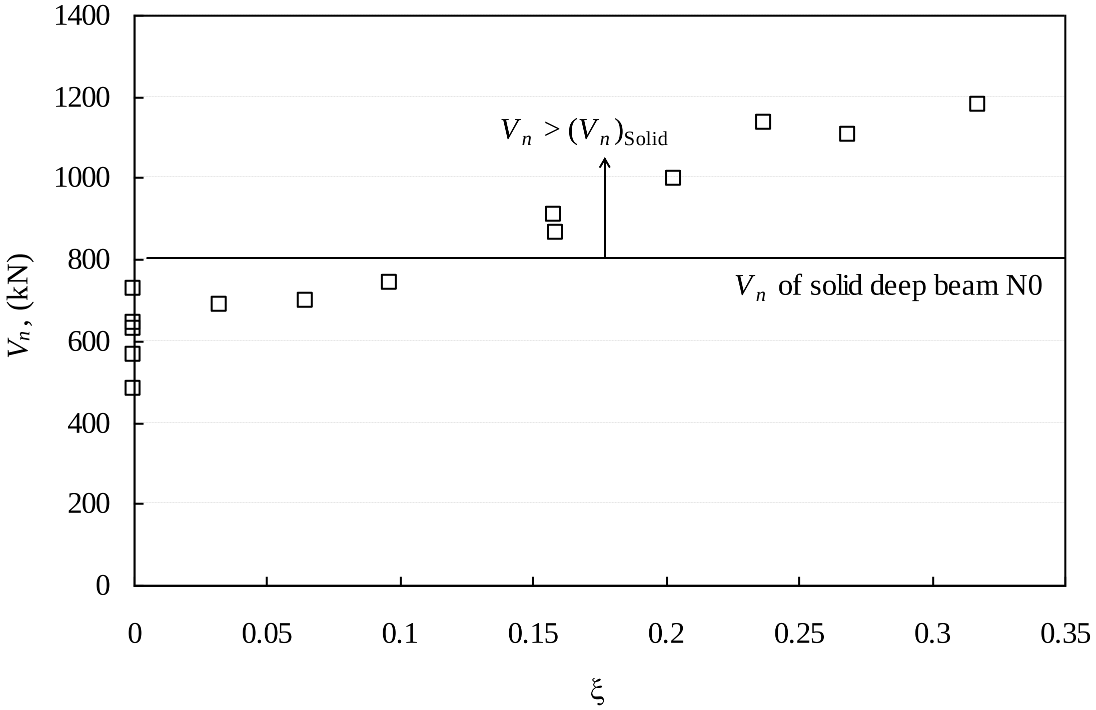


Fig. 8–Relationship between ξ and V_n

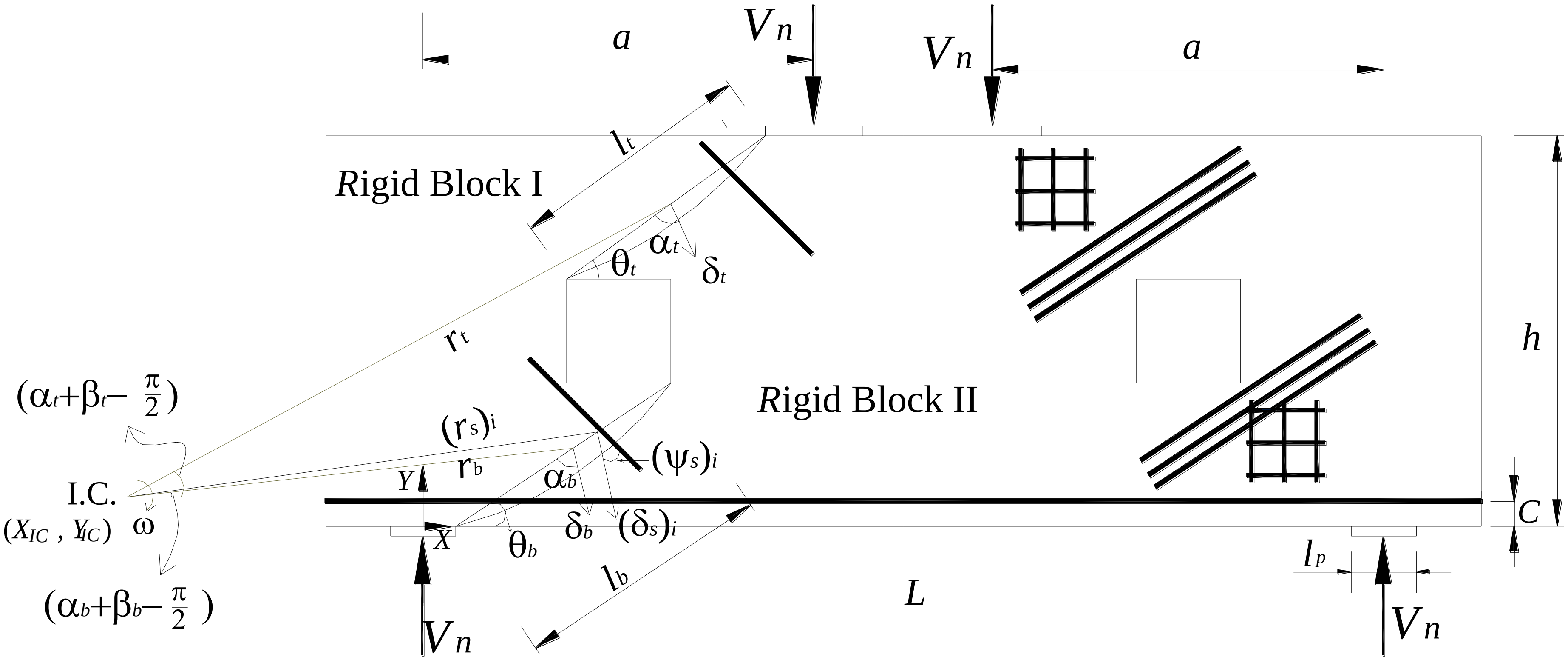
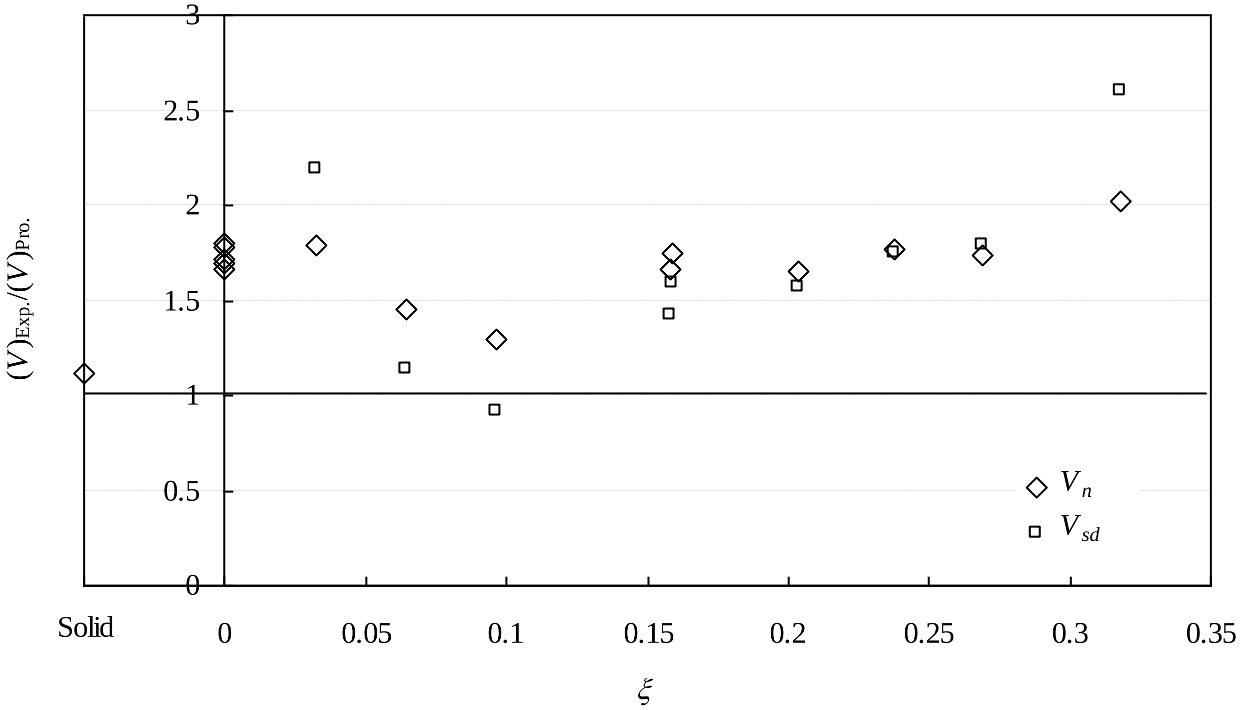
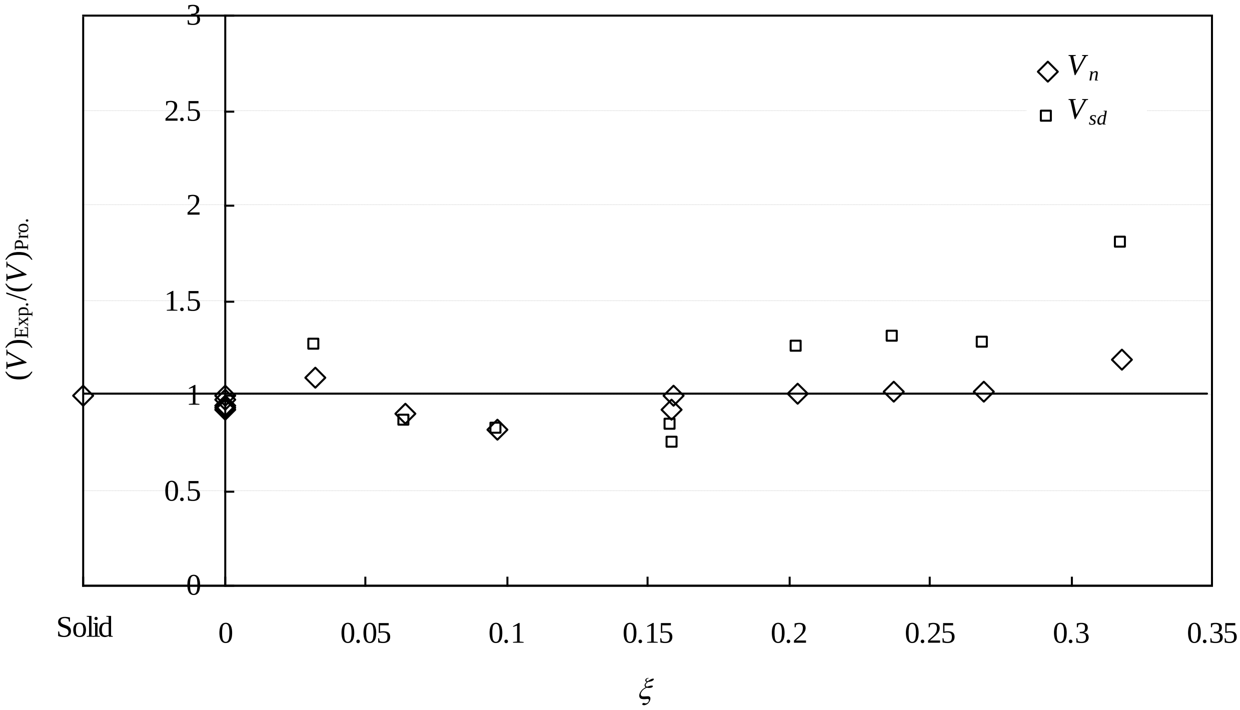


Fig. 9-Idealized failure mode of deep beam with openings



(a) Kong et al.



(b) This study

Fig. 10–Comparison of test results and predictions