SIZE OF FRP LAMINATES TO STRENGTHEN REINFORCED CONCRETE SECTIONS IN FLEXURE

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ABSTRACT

This paper presents an analytical method for estimating the flexural strength of reinforced concrete beams strengthened with externally bonded fibre reinforced polymer (FRP) laminates. The method is developed from the strain compatibility and equilibrium of forces. Based on the size of external FRP laminates, several flexural failure modes may be identified, namely tensile rupture of FRP laminates and concrete crushing before or after yielding of internal steel reinforcement. Upper and lower limits to the size of FRP laminates used are suggested to maintain ductile behaviour of strengthened reinforced concrete sections. Comparisons between the flexural strength obtained from the current method and experiments show good agreement. Design equations for calculating the size of FRP laminates externally bonded to reinforced concrete sections to enhance their flexural strength are proposed.

Key words: Concrete structures, Codes of practice & standards, Buildings structure & design, Stress analysis.

INTRODUCTION

Although external plate bonding to the surface of existing reinforced concrete (RC) structures has been widely accepted as an effective technique of structural upgrading, there are little independent design guidelines[1,2] and related code regulations. For example, there is no British Standard dealing specifically with the structural design of RC beams strengthened with FRP laminates.

The experimental research carried out on RC beams strengthened in flexure by externally bonded FRP laminates[3-16] identified two general mechanisms of failure, namely flexure and premature. The flexural mechanism of failure is usually due to either the tensile rupture of FRP laminates or concrete crushing in compression before or after yielding of internal steel reinforcement. The premature failure is attributed to de-bonding or de-lamination of FRP plate ends and ripping off the concrete cover along the internal steel reinforcement level. Experimental tests[5,8,9,11,14] indicated that increasing the anchorage length of the external sheets or using anchorage systems in the form of bonded U-shaped channels or jackets at the plate ends may inhibit the premature peeling failure.

There have been extensive experimental investigations[3-16] on RC beams strengthened with FRP laminates but very few theoretical studies have focused on such structures[16,17]. This paper presents an analytical method for estimating the bending capacity of RC sections strengthened with externally bonded FRP laminates. The method is based on the same principles as those adopted in the BS8110 provisions[18] for flexural strength of conventional RC but extended here to account for externally bonded FRP laminates.

CONSTITUTIVE MODELLING OF MATERIALS

Concrete

The stress-strain curve for concrete in compression shown in Figure 1(a) is used. This relation is the same as specified in BS8110[18] for concrete. It may be written in the following form:

$$\sigma_c = E_c \varepsilon_c - \frac{E_c}{2\varepsilon_o}\varepsilon_c^2 \qquad \varepsilon_c < \varepsilon_o \qquad (1(a))$$

$$\sigma_c = 0.67 f_{cu} \qquad \varepsilon_o \le \varepsilon_c \le \varepsilon_{cu} \qquad (1(b))$$

where σ_c and ε_c are the stress and strain in concrete, respectively, f_{cu} (N/mm^2) is the cube compressive strength, E_c ($= 5500\sqrt{f_{cu}}$ N/mm^2) is the initial tangent modulus of concrete, ε_o ($= 0.00024\sqrt{f_{cu}}$) is the strain at the end of the parabolic part of the stress-strain diagram and ε_{cu} ($=0.0035$) is the ultimate strain of concrete as shown in Figure 1(a). For the ultimate moment calculation, concrete is cracked in tension, therefore the tensile strength of concrete is ignored.

Steel reinforcement

The steel reinforcing bars in both tension and compression are assumed to be elastic perfectly plastic as given below (see Figure 1(b)):

$$\sigma_s = E_s \varepsilon_s \qquad \varepsilon_s < \varepsilon_y \tag{2(a)}$$

$$\sigma_s = f_y \qquad \varepsilon_s \geq \varepsilon_y \tag{2(b)}$$

where σ_s and ε_s are the stress and strain in the internal steel reinforcement, respectively, E_s is the elastic modulus of steel and f_y and ε_y are the yield stress and strain of steel, respectively.

FRP laminates

The stress-strain relationship for uni-directional fibre laminates is linear elastic up to rupture. It is given by:

$$f_f = E_f \varepsilon_f \qquad \varepsilon_f < \varepsilon_{fu} \tag{3(a)}$$

$$f_f = 0 \qquad \varepsilon_f \geq \varepsilon_{fu} \tag{3(b)}$$

where f_f and ε_f are the stress and strain in FRP laminates, respectively, E_f is the modulus of elasticity of FRP laminates, and f_{fu} and ε_{fu} are the ultimate strength and strain of FRP laminates, respectively as shown in Figure 1(c).

FLEXURAL CAPACITY AND FAILURE MODES

Figure 2(a) shows a concrete section having a width *b* and an overall depth *h*, reinforced with:

- internal longitudinal tension steel bars of an area A_s at an effective depth *d* from the top face;
- internal longitudinal compression steel bars of an area A_s' at a depth d' from the top face;
- externally bonded FRP laminates having an area A_f at a depth d_f from the top face.

It is assumed that premature failure, such as peeling or separation of FRP laminates, is prevented and only flexural failure modes are studied.

Compatibility conditions

Provided that plane section before bending remains plane after bending, the strain at any point across the section is linearly proportional to its distance from the neutral axis as shown in Figure 2(b) in which x represents the neutral axis depth. Considering similar triangles on the strain diagram shown in Figure 2(b) and assuming perfect bond between concrete and both internal steel reinforcement and external FRP laminates, strains ε_s in the tension reinforcement, ε_s' in the compression reinforcement and ε_f in the FRP laminates are calculated in terms of ε_c as:

$$\varepsilon_s = \frac{d-x}{x}\varepsilon_c \tag{4(a)}$$

$$\varepsilon_s' = \frac{x-d'}{x}\varepsilon_c \tag{4(b)}$$

$$\varepsilon_f = \frac{d_f - x}{x}\varepsilon_c \tag{4(c)}$$

At the instant of failure, either the concrete strain ε_c at the extreme compression fibre or the FRP composite strain at the extreme tension fibre reaches the respective ultimate strain; i.e. $\varepsilon_c = \varepsilon_{cu} = 0.0035$ or $\varepsilon_f = \varepsilon_{fu}$.

Equilibrium conditions

Having determined strains in concrete, steel and FRP laminates, stresses σ_c in concrete, σ_s in tension reinforcement, σ_s' in compression reinforcement and f_f in FRP laminates are calculated using the respective stress-strain relationships (Eqs. (1, 2 and 3)) for different materials. The concrete compressive stress distribution shown in Figure 2(c) may be replaced by an equivalent rectangular stress block. This idealised rectangular block is expressed in terms of two parameters k_1 and k_2, where k_1 is the ratio of the average compressive stress to the concrete cube strength f_{cu} and k_2 is the ratio of the depth of the idealised rectangular stress block to the neutral axis depth as shown in Figure 2(c). The values of k_1 and k_2 depend on the strain ε_c at the extreme compression fibre and the concrete compressive strength f_{cu} as given in Appendix I. The internal forces on the cross section can be calculated as (see Figure 2(c)):

- Compressive force C in concrete = $k_1 k_2 f_{cu} bx$
- Compressive force C_s in steel bars above the neutral axis = $A_s^{'} \sigma_s^{'}$
- Tensile force T_s in steel bars below the neutral axis = $A_s \sigma_s$
- Tensile force T_f in FRP laminates = $A_f f_f$

Considering the equilibrium of forces, the following equation is obtained:

$$C + C_s = T_s + T_f$$

$$k_1 k_2 f_{cu} bx + A_s^{'} \sigma_s^{'} = A_s \sigma_s + A_f f_f \quad (5)$$

In the above Eqs. (1 to 5), the neutral axis depth x is in fact the only unknown. An iterative trail and error procedure is usually adopted to find the correct value. An initial value for x is assumed and the strains and hence stresses are then determined. If Eq. (5) is not satisfied, the value of x is adjusted and the procedure is repeated until sufficient accuracy is attained. However, the neutral axis depth x may be explicitly estimated in some special cases as explained below. The moment capacity M_u of the section is then calculated by taking moments of forces about any horizontal axis in the section; for instance, about the centroid of the FRP laminates:

$$M_u = k_1 k_2 f_{cu} bx\left(d_f - \frac{k_2 x}{2} \right) + A_s^{'} \sigma_s^{'} (d_f - d^{'}) - A_s \sigma_s (d_f - d) \quad (6)$$

Mode I: Tensile rupture of FRP laminates

If the area of FRP laminates externally bonded to the RC section is below a certain limit to be defined later, the FRP strain ε_f reaches the ultimate strain value ε_{fu} while the concrete strain ε_c at the extreme compression fibre is still below the ultimate strain ε_{cu} as shown in Figure 3(a). In such cases the failure is due to the tensile rupture of FRP laminates. When $\varepsilon_f = \varepsilon_{fu}$ and $\varepsilon_c = \varepsilon_{cu}$ simultaneously, the strain distribution is unique and the neutral axis depth x_l is calculated using Eq. (4(c)) with ε_f and ε_c replaced by ε_{fu} and *0.0035*, respectively:

$$x_l = d_f \frac{0.0035}{0.0035 + \varepsilon_{fu}} \quad (7)$$

The limiting area A_{fl} of FRP laminates can readily be calculated from Eq. (5) with σ_s and f_f replaced by f_y and f_{fu}, respectively:

$$A_{fl} = \frac{k_1 k_2 f_{cu} b x_l + A_s' \sigma_s' - A_s f_y}{f_{fu}} \tag{8}$$

The stress σ_s' in the compression reinforcement is obtained from Eq. (2), where ε_s' is calculated using Eq. 4(b) with x and ε_c replaced by x_l and 0.0035, respectively. The area A_{fl} of FRP laminates (given by Eq. (8)) forms a lower limit to the size of FRP laminates in order to avoid tensile plate rupture. Obviously, if the neutral axis depth of the unstrengthened RC section is greater than or equal to x_l, the limiting area A_{fl} of FRP laminates calculated from Eq. (8) is negative and this failure mode would not occur whatever the area of FRP laminates provided. The moment capacity M_{ul} of a RC section strengthened by externally bonded FRP laminates of an area A_{fl} is calculated using Eq. (6) as follows:

$$M_{ul} = k_1 k_2 f_{cu} b x_l \left(d_f - \frac{k_2 x_l}{2} \right) + A_s' \sigma_s' (d_f - d') - A_s f_y (d_f - d) \tag{9}$$

Mode II: Yielding of steel reinforcement followed by crushing of concrete (under-reinforced case)

This mode of failure is characterised by yielding of steel reinforcement followed by crushing of concrete (see Figure 3(b) for strain distribution). In this case, the area of FRP laminates is greater than A_{fl}, therefore tensile rupture of FRP laminates would not occur. This case is similar to the under reinforced section of conventional RC beams[18]. The upper limit to the area of FRP laminates, A_{fu}, is reached when strains ε_c in the extreme compression fibre of concrete and ε_s in the tension reinforcement simultaneously reach ε_{cu} and ε_y, respectively. By following similar calculations to those presented above, the neutral axis depth x_u, the area A_{fu} of FRP laminates and the moment capacity M_{uu} corresponding to this upper limit of FRP size are calculated from:

$$x_u = d \frac{0.0035}{0.0035 + \varepsilon_y} \tag{10}$$

$$A_{fu} = \frac{k_1 k_2 f_{cu} b x_u + A_s' \sigma_s' - A_s f_y}{f_f} \tag{11}$$

$$M_{uu} = k_1 k_2 f_{cu} b x_u \left(d_f - \frac{k_2 x_u}{2} \right) + A_s' \sigma_s' (d_f - d') - A_s f_y (d_f - d) \quad (12)$$

The neutral axis depth x_u given by Eq. (10) is the same as that defines the balanced section for conventional RC beams without externally bonded FRP laminates[18]. The stress f_f in the FRP laminates is calculated using Eq. 3(a) where ε_f is determined from Eq. 4(c) with x and ε_c replaced by x_u and 0.0035, respectively. To ensure ductile flexural behaviour, the area of FRP laminates provided should be smaller than A_{fu} given in Eq. (11) and the moment M_{uu} is the maximum permissible moment capacity of a RC section with externally bonded FRP laminates.

Mode III: Crushing of concrete before yielding of steel reinforcement (Over-reinforced case)

If the area of FRP laminates used is greater than A_{fu}, the concrete strain ε_c reaches the ultimate value ε_{cu} before any yielding of tension reinforcement as shown in Figure 3(c). In this case, there is a large amount of internal and external reinforcements and the section is over reinforced. Such failure, often explosive, occurs with little warning, similar to that of conventional over RC beams. Table 1 summarises the range of different parameters for the three flexural failure modes presented.

COMPARISONS WITH EXPERIMENTS

Test results of 48 reinforced concrete beams strengthened with externally bonded FRP laminates published by other researchers are used to validate the proposed method. Table 2 compares the bending capacities from experiments against those from the current method. All the 48 beams were reported to have failed because of flexure, not peeling or debonding of the FRP laminates as given in Table 2. The average and standard deviation of the ratio between predicted and experimental bending capacities are 0.99 and 8.3%, respectively. In all beams considered, the predicted failure mode agrees with that observed in experiments. The predictions obtained from the current analysis are in good agreement with the experimental results.

EFFECT OF SIZE OF FRP LAMINATES ON BENDING CAPACITY

Figure 4 presents the effect of the internal and external reinforcements on the normalised moment capacity η ($= M_f / bd^2$): Figure 4(a) shows the variation of the normalised moment capacity η against the internal steel reinforcement ratio ρ_s ($=100A_s/bd$) and Figure 4(b) gives the variation of the normalised moment capacity η against the external FRP laminates ratio ρ_f ($=100A_f/bd$). The dotted lines in Figure 4 represent the boundaries of different flexural failure modes. However, the use of externally bonded FRP laminates enhances the moment capacity of reinforced concrete sections, their effect is more pronounced for RC sections having less area of internal steel reinforcement. The increase in the normalised moment capacity η is insignificant when mode III dominates the flexural failure. Therefore, it is recommended that the size A_{fu} (given by Eq. (11)) of FRP laminates forms the upper limit to the area of FRP laminates selected. The higher the size of FRP laminates, the less the rate of increase of the normalised moment capacity η for the same internal steel reinforcement ratio ρ_s.

DESIGN GUIDELINES FOR FRP LAMINATE SIZE IN FLEXURE

In the following, the area A_f of FRP laminates to be externally bonded to a RC section is calculated in order to increase its moment capacity to M_f. It is always desirable that the internal reinforcing steel bars yield before crushing of concrete and tensile rupture of FRP laminates is avoided. In order to achieve this ductile behaviour (under reinforced case), the target moment capacity M_f of the RC section strengthened with externally bonded FRP laminates must satisfy the following constraints:

$$M_{ul} < M_f \leq M_{uu} \tag{13}$$

where M_{ul} and M_{uu} are the lower and upper limits to the moment capacity as given by Eqs. (9) and (12), respectively. In this case, the strain distribution shown in Figure 3(b) is valid. Substituting for σ_s and σ_s' into Eq. (6) with the yield strength of the tension and compression reinforcements produces:

$$M_f = k_1 k_2 f_{cu} bx\left(d_f - \frac{k_2 x}{2}\right) + A_s' f_y (d_f - d') - A_s f_y (d_f - d) \tag{14}$$

In the above equation, the compression steel reinforcement is assumed yielded, otherwise, the size of FRP laminates has to be iteratively determined. The neutral axis depth x may be expressed as:

$$x = \frac{2\left(d_f - Z_f\right)}{k_2} \tag{15}$$

where Z_f is the lever arm between the concrete compressive force C and tensile force T_f in the FRP laminates as shown in Figure 2(c). Substituting for x into Eq. (14) and re-arranging:

$$M_f - A_s^{'} f_y\left(d_f - d^{'}\right) + A_s f_y\left(d_f - d\right) = 2k_1 f_{cu} b Z_f\left(d_f - Z_f\right) \tag{16}$$

Assuming:

$$M_1^{mod} = M_f - A_s^{'} f_y\left(d_f - d^{'}\right) + A_s f_y\left(d_f - d\right) \tag{17}$$

and

$$k_{mod} = \frac{M_1^{mod}}{b d_f^2 f_{cu}} \tag{18}$$

Substituting for M_1^{mod} and k_{mod} in Eq. (16) yields:

$$\left(\frac{Z_f}{d_f}\right)^2 - \frac{Z_f}{d_f} + \frac{k_{mod}}{2k_1} = 0 \tag{19}$$

Solving the above quadratic equation gives:

$$Z_f = d_f\left[0.5 + \sqrt{0.25 - \frac{k_{mod}}{2k_1}}\right] \tag{20}$$

The above equation is similar to that given in BS8110[18] to calculate the lever arm between the concrete compressive and steel tensile forces for conventional RC sections. Taking moments of forces shown in Figure 2(c) about the concrete compressive force C gives:

$$M_f = A_s f_y(d - 0.5k_2 x) + A_f f_f Z_f + A_s^{'} f_y(0.5k_2 x - d^{'}) \tag{21}$$

The required area A_f of FRP laminates is:

$$A_f = \frac{M_2^{mod}}{f_f Z_f} \tag{22}$$

where f_f is the stress in the FRP laminates calculated using Eq. 3(a) where the strain ε_f is determined by substituting for x (Eq. (15)) and ε_c (=0.0035) in Eq. 4(c) and

$$M_2^{mod} = M_f - A_s^{'} f_y (0.5k_2 x - d^{'}) - A_s f_y (d - 0.5k_2 x) \tag{23}$$

The area A_f calculated above using Eq. (22) should satisfy the following condition:

$$A_{fl} < A_f \le A_{fu} \tag{24}$$

An example showing how to calculate the area of FRP laminates to enhance the moment capacity of a reinforced concrete section is given in Appendix II.

CONCLUSIONS

A simplified analytical method for predicting the bending capacity of RC sections with externally bonded FRP laminates has been introduced. Although the technique described in this paper was developed for rectangular sections, it is of general validity and could be extended for other section shapes. Comparisons between the flexural capacity and failure mode obtained from the current analysis and experiments show good agreement.

Strengthening RC sections with externally bonded FRP laminates is particularly effective in case of a relatively low tensile steel reinforcement. The flexural failure mode is controlled by the size of FRP laminates. Minimum and maximum amount of FRP laminates are proposed in order to ensure ductile behaviour of the strengthened sections. Design equations for the area of FRP laminates used to increase the moment capacity of RC sections are developed.

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APPENDIX I: FORMULAE FOR K_1 AND K_2

In the previous analysis, the concrete compressive stress distribution was replaced with a fictitious rectangular block. The properties of the idealised block may be expressed in terms of two parameters k_1 and k_2. Comparing the idealised rectangular and actual concrete compressive stress blocks, the following formulae for k_1 and k_2 may be driven:

Where $\varepsilon_c < \varepsilon_o$:

$$k_1 = \frac{E_c \varepsilon_c}{3 f_{cu}} \frac{(3-\alpha)^2}{(4-\alpha)} \tag{25}$$

$$k_2 = \frac{4-\alpha}{2(3-\alpha)} \tag{26}$$

and where $\varepsilon_c \geq \varepsilon_o$:

$$k_1 = \frac{4(3\alpha - 1)^2}{9(6\alpha^2 - 4\alpha + 1)} \tag{27}$$

$$k_2 = \frac{6\alpha^2 - 4\alpha + 1}{2\alpha(3\alpha - 1)} \tag{28}$$

where α (=ε_c / ε_o) is the ratio of the extreme compressive concrete strain ε_c (see Figure 2(b)) to the strain ε_o at the end of the parabolic part of the stress-strain diagram (see

Figure 1(a)). E_c, ε_c, ε_o and f_{cu} have the same definition as given above. For design purposes, BS8110[18] gives fixed values for k_1 (= 0.67) and k_2 (= 0.9), when $\varepsilon_c = \varepsilon_{cu} = 0.0035$.

APPENDIX II: NUMERICAL EXAMPLE

The moment capacity of the unstrengthened reinforced concrete section shown in Figure 5 is *225.0 kNm*, calculated according to BS8110[18] with all safety factors removed. It is required to determine the size A_f of externally bonded FRP laminates to increase the moment capacity to *300 kNm* (33% increase in the moment capacity). The ultimate strength f_{fu} and modulus of elasticity E_f of the externally bonded FRP laminates are assumed to be $400\ N/mm^2$ and $37000\ N/mm^2$, respectively. Values of different parameters are estimated and presented in Table 3, below.

Table 1 Effect of size of FRP laminates on different parameters.

Parameters	Range of different parameters for the three failure modes		
	Mode I	**Mode II**	**Mode III**
A_f	$A_f \leq A_{f1}$	$A_{f1} < A_f \leq A_{fu}$	$A_f > A_{fu}$
x	$x \leq x_l$	$x_l < x \leq x_u$	$x > x_u$
ε_s	$\varepsilon_s > \varepsilon_y$	$\varepsilon_s \geq \varepsilon_y$	$\varepsilon_s < \varepsilon_y$
ε_c'	increases as the size of external FRP increases		
ε_f	$\varepsilon_f = \varepsilon_{fu}$	$\varepsilon_f < \varepsilon_{fu}$	$\varepsilon_f < \varepsilon_{fu}$
ε_c	$\varepsilon_c < 0.0035$	$\varepsilon_c = 0.0035$	$\varepsilon_c = 0.0035$
C^*	$C \leq 0.603 f_{cu} bx$	$C = 0.603 f_{cu} bx$	$C = 0.603 f_{cu} bx$
T	$T = A_s f_y$	$T = A_s f_y$	$T < A_s f_y$
T_f	$T_f = A_f f_{fu}$	$T_f < A_f f_{fu}$	$T_f < A_f f_{fu}$
M_u	$M_u \leq M_{ul}$	$M_{ul} < M_u \leq M_{uu}$	$M_{uu} < M_u$

* The compressive force C is calculated based on fixed values for k_1 ($=0.67$) and k_2 ($=0.9$) as given by BS8110[18].

Table 2 Comparisons between the moment capacity from the current method and experiments

Reference	**Beam No.**	A_s (mm^2)	A_f (mm^2)	**Bending capacity (kNm)**		**B/A**	Failure mode
				A*	**B****		
Andreou et al. (2000)	Beam110	157.08	29.00	20.24	19.71	0.97	Concrete crushing
	Beam111	157.08	29.00	19.95	19.71	0.99	Concrete crushing
	Beam109	157.08	58.00	24.50	22.84	0.93	Concrete crushing
Arduini et al. (1997)	B2	398.00	51.00	96.30	101.66	1.06	FRP rupture
Chajes et al. (1994)	A2	71.00	132.08	3.00	2.70	0.90	Concrete crushing
	A3	71.00	132.08	3.43	2.70	0.79	Concrete crushing
	E1	71.00	180.34	3.11	3.01	0.97	FRP rupture
	E2	71.00	180.34	3.11	3.01	0.97	FRP rupture
	E3	71.00	180.34	3.13	3.01	0.96	FRP rupture
	G1	71.00	154.94	3.06	3.28	1.07	FRP rupture
	G2	71.00	154.94	3.46	3.28	0.95	FRP rupture
	G3	71.00	154.94	3.94	3.28	0.83	FRP rupture
EL-Refaie et al. (2000)	H2	100.50	25.74	32.82	33.06	1.01	FRP rupture
	H6	100.50	25.74	30.30	33.10	1.09	FRP rupture

Table 2 (Cont.) Comparisons between the moment capacity from the current method and experiments

Reference	**Beam No.**	A_s *(mm²)*	A_f *(mm²)*	**Bending capacity (kNm)**		**B/A**	Failure mode
				A*	**B****		
Lamanna et al. (2001)	F-21-S-102-1	258.00	326.40	11.90	13.32	1.12	Concrete crushing
	F-21-F-102-1	258.00	326.40	12.40	13.96	1.13	Concrete crushing
	F-21-H-102-1	258.00	326.40	13.30	15.69	1.18	Concrete crushing
	F-21-S-102-2	258.00	326.40	13.30	13.32	1.00	Concrete crushing
	F-21-S-102-2R	258.00	326.40	12.70	13.32	1.05	Concrete crushing
	F-21-S-51-1	258.00	163.20	11.90	11.71	0.98	Concrete crushing
Mukhopadhyaya et al. (1998)	FS2	942.00	525.00	91.85	89.31	0.97	Concrete crushing
Nguyen et al. (2001)	A1500	236.00	96.00	25.96	22.36	0.86	Concrete crushing
Rahimi and Hutchinson (2001)	C3	402.10	60.00	28.10	30.98	1.10	Concrete crushing
	C4	402.10	60.00	28.95	30.98	1.07	Concrete crushing
	C5	402.10	180.00	38.70	38.97	1.01	Concrete crushing
	C6	402.10	180.00	38.00	38.97	1.03	Concrete crushing
	C7	402.10	270.00	32.70	32.48	0.99	Concrete crushing
	C8	402.10	270.00	32.50	32.48	1.00	Concrete crushing

Table 2 (Cont.) Comparisons between the moment capacity from the current method and experiments

Reference	**Beam No.**	A_s (mm^2)	A_f (mm^2)	**Bending capacity (kNm)**		**B/A**	Failure mode
				A*	**B****		
Ritchie et al. (1991)	E	258.06	732.00	57.00	57.14	1.00	FRP rupture
	F	258.06	732.00	61.00	56.90	0.93	FRP rupture
	L	258.06	193.00	56.10	56.69	1.01	FRP rupture
Ross et al. (1999)	4B	568.00	90.30	49.20	53.17	1.08	Concrete crushing
	4C	568.00	90.30	47.80	53.17	1.11	Concrete crushing
	4D	568.00	90.30	50.80	53.17	1.05	Concrete crushing
	5B	774.00	90.30	67.10	58.80	0.88	Concrete crushing
	5C	774.00	90.30	67.10	58.80	0.88	Concrete crushing
	5D	774.00	90.30	66.50	58.80	0.88	Concrete crushing
	6B1	19.00	90.30	77.30	65.68	0.85	Concrete crushing
	6C1	19.00	90.30	70.00	65.68	0.94	Concrete crushing
	6D1	19.00	90.30	70.00	65.68	0.94	Concrete crushing
Saadatmanesh & Ehsani (1991)	A1	529.00	912.00	317.20	313.31	0.99	Concrete crushing

Table 2 (Cont.) Comparisons between the moment capacity from the current method and experiments

Reference	**Beam No.**	A_s (mm^2)	A_f (mm^2)	**Bending capacity (kNm)**		**B/A**	Failure mode
				A*	**B****		
Sharif et al. (1994)	P1	157.10	150.00	13.20	13.15	1.00	FRP rupture
	P2BW	157.10	300.00	15.33	16.14	1.05	Concrete crushing
	P3J	157.10	300.00	16.11	16.18	1.00	Concrete crushing
Swamy & Mukhopadhyaya (1999)	B1	603.00	225.00	75.71	74.38	0.98	Concrete crushing
	B2	603.00	225.00	76.09	74.98	0.99	Concrete crushing
Triantafillou & Plevris (1992)	2	33.24	8.52	3.01	3.27	1.09	FRP rupture
	3	33.24	12.10	3.95	3.87	0.98	FRP rupture
* **A = Bending capacity from experiments** ** **B = Bending capacity from current method**					Average	0.99	
					Standard Deviation	8.3%	

Table 3 Values of different parameters for the example given in Appendix II.

Parameters	Value and Equation	Notes
• Calculations of the lower limit to the FRP, A_{f1}, and the corresponding moment M_{ul}		
x_l	*111.3 mm* (Eq. (7))	Assuming $d_f \simeq h$
$\varepsilon_s^{'}$	*0.0018* (Eq. (4(b)))	Compression reinforcement does not yield
$\sigma_s^{'}$	*360* N/mm^2 (Eq. (2(a)))	
A_{f1}	*-340.9* mm^2 (Eq. (8))	FRP rupture would not occur whatever the size of FRP laminates; no need to calculate M_{ul}
M_{ul}	- (Eq. (9))	
• Calculations of the upper limit to the FRP, A_{fu}, and the corresponding moment M_{uu}		
x_u	*242.2 mm* (Eq. (10))	
$\varepsilon_s^{'}$	*0.0027* (Eq. (4(b)))	Compression reinforcement yielded
$\sigma_s^{'}$	*456* N/mm^2(Eq. (2(b)))	
A_{fu}	*3070.0* mm^2(Eq. (11))	
M_{uu}	*322.0 kNm* (Eq. (12))	M_f (=300kNm) < M_{uu}; under-reinforced case
• Calculations of the lever arm Z_f between the concrete compressive force and tensile force in FRP and neutral axis depth x		
M_1^{mod}	*288.35 kNm* (Eq. (17))	
k_{mod}	*0.226* (Eq. (18))	
Z_f	*357.0mm* (Eq. (20))	
x	*217.8mm* (Eq. (15))	
• Calculations of stresses f_f in FRP and required area A_f of FRP to increase the moment capacity		
$\varepsilon_s^{'}$	*0.0026* (Eq. (4(b)))	Compression steel yielded
M_2^{mod}	*92.36 kNm* (Eq. (23))	
ε_f	*0.00381* (Eq. (4(c)))	
f_f	*141* N/mm^2 (Eq. (3(a)))	
A_f	*1834.5*mm^2 (Eq. (22))	A_{f1} (=-340.9 mm^2)< A_f (=1834.5mm^2)< A_{fu}(=3070.0 mm^2); under reinforced case
In this example, the compressive force C is calculated based on fixed values for k_1 (=0.67) and k_2 (=0.9) as given by BS8110[18].		

List of Captions:

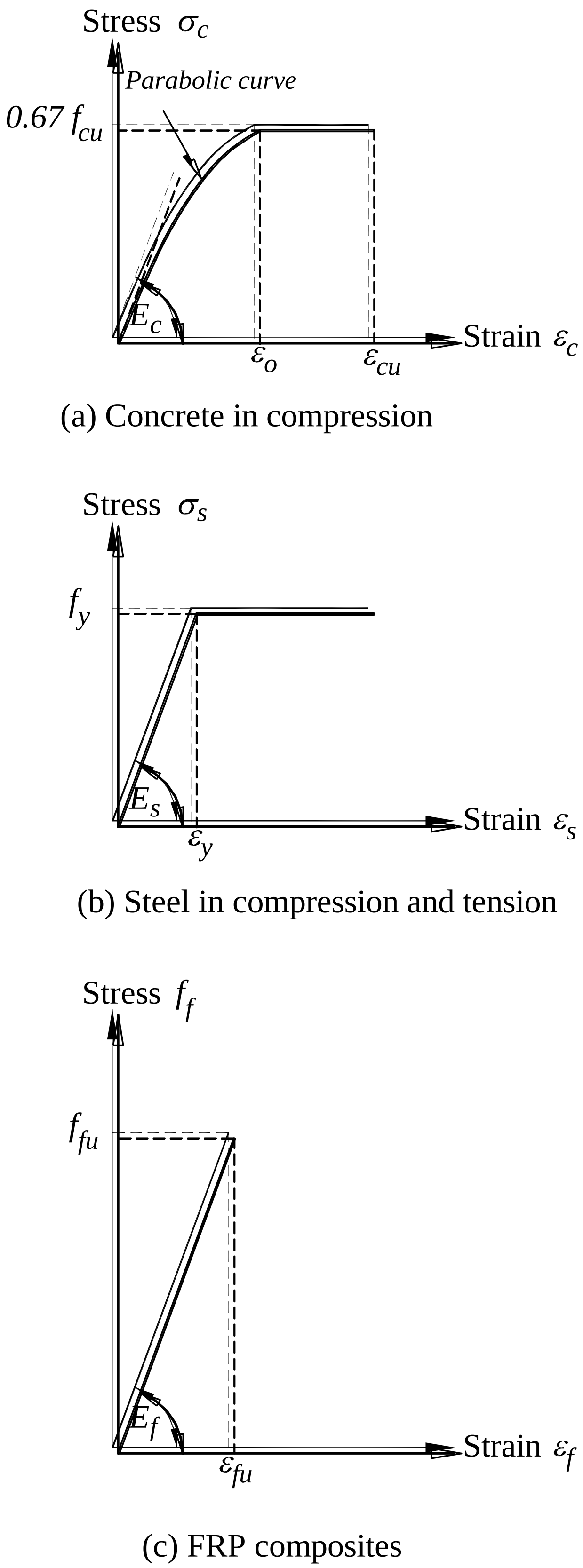
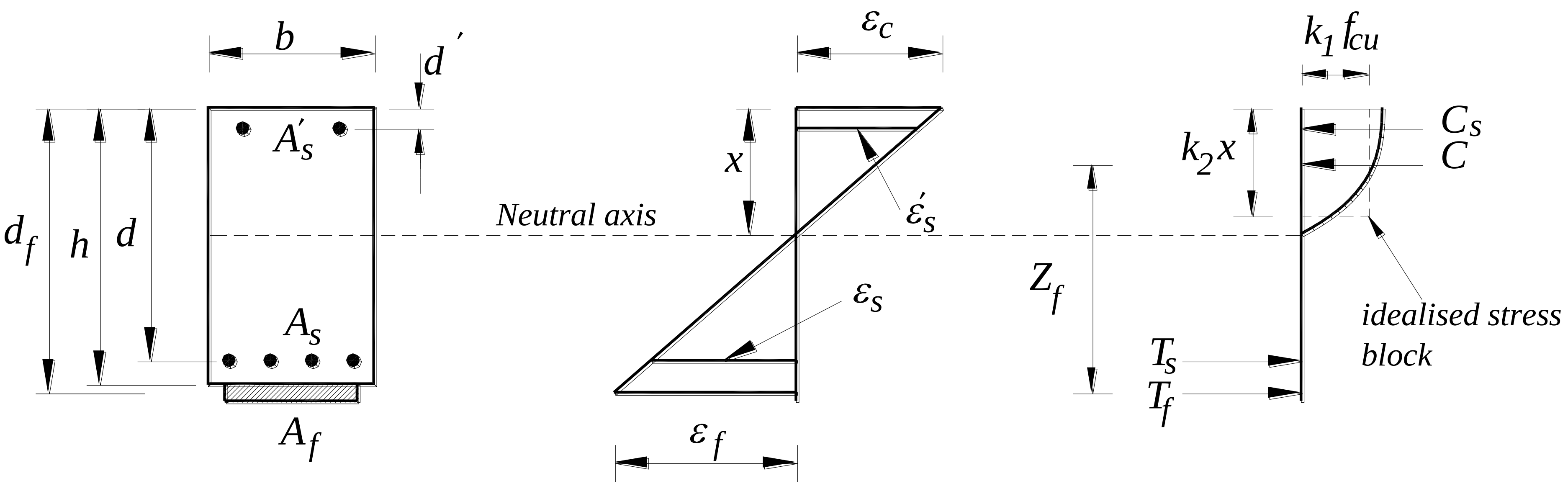
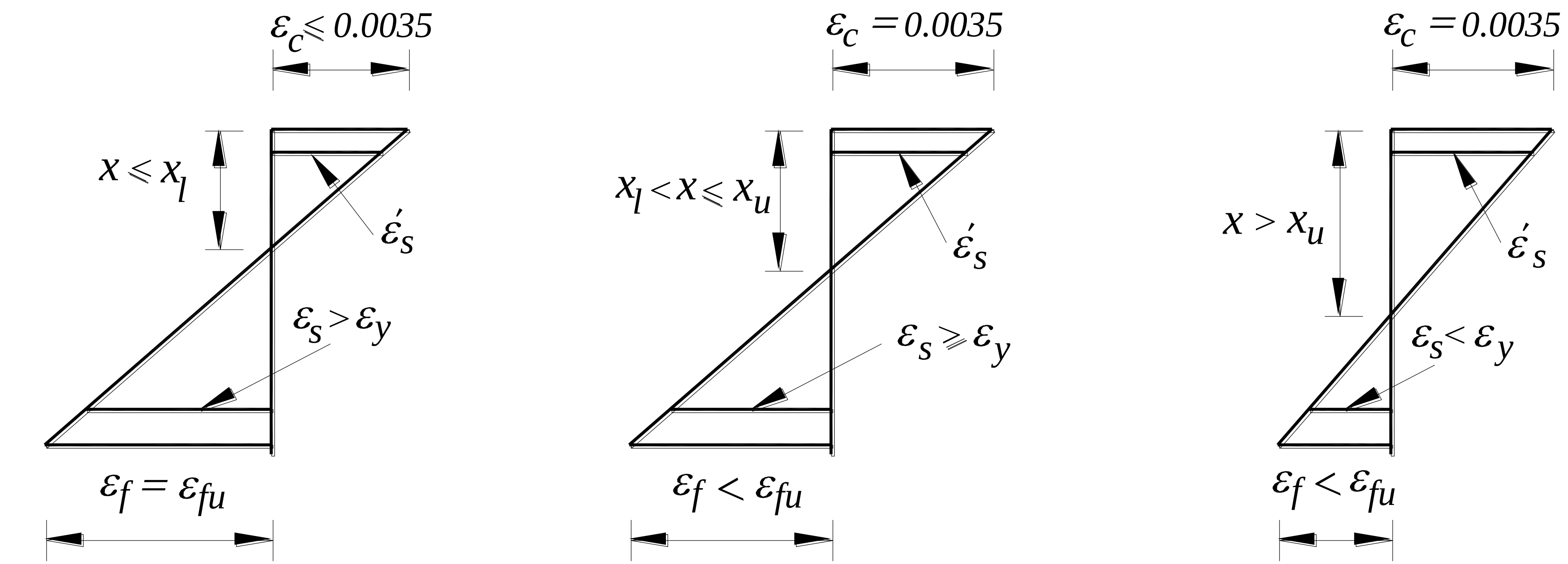


Fig. 1 Stress-strain relationships for material



(a) RC section with FRP (b) Strain Distribution (c) Stresses and Forces

Fig. 2 Strains, stresses and forces on RC section with externally bonded FRP laminates at failure

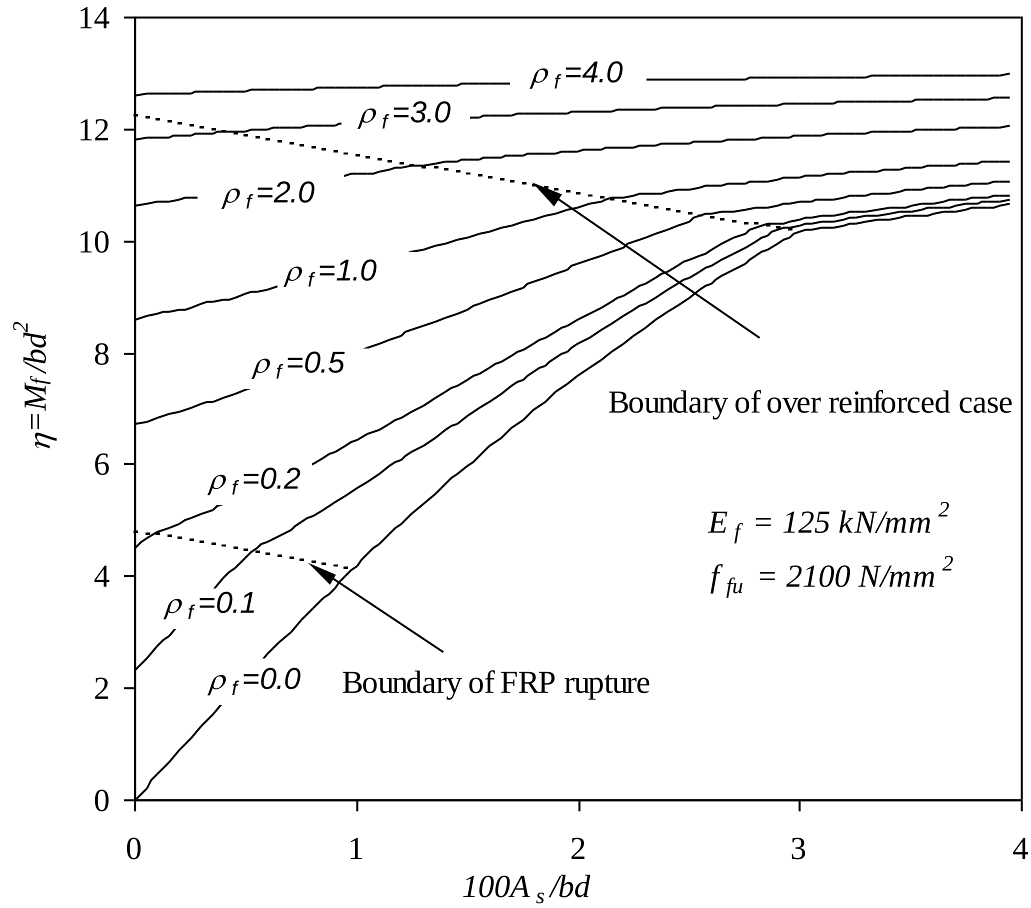


(a) Plate rupture before concrete crushing ($A_f \leqslant A_{fl}$)

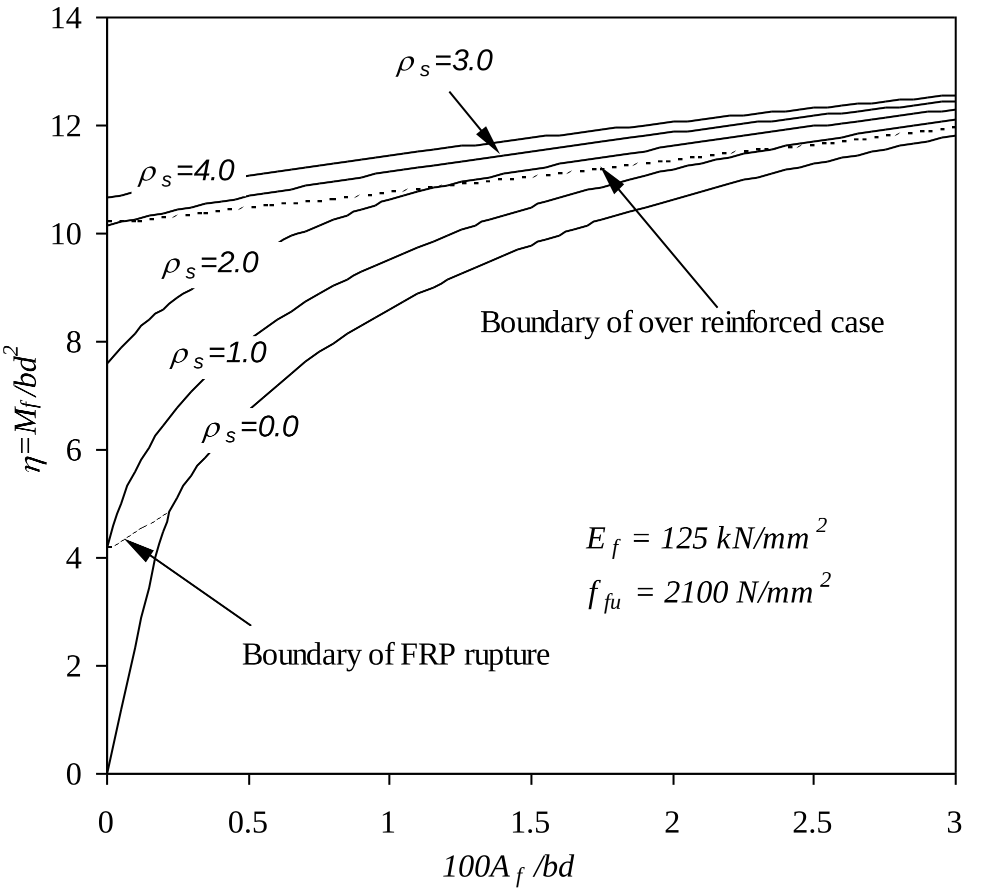
(b) Under reinforced case ($A_{fl} < A_f \leqslant A_{fu}$)

(c) Over reinforced case ($A_f > A_{fu}$)

Fig. 3 Strain distribution for different sizes of FRP laminates at failure



(a) Variation of moment capacity against the area of internal steel reinforcement



(b) Variation of moment capacity against the size of externally bonded FRP composites

Fig. 4 Effect of internal and external reinforcements on the normalised bending capacity

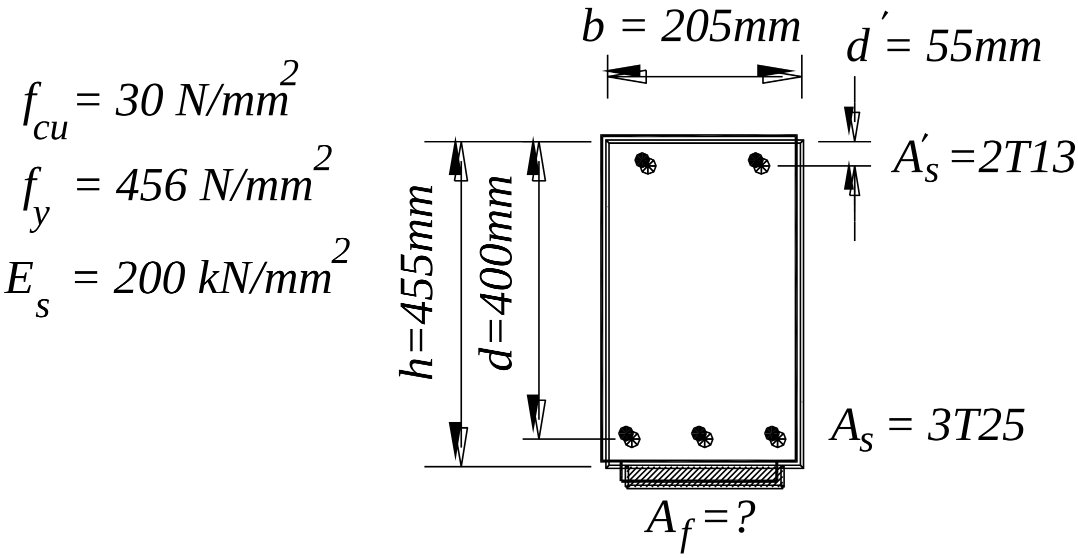


Fig. 5 Details of RC section with FRP laminates