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Link to original published version: [http://dx.doi.org/10.1061/\(ASCE\)HY.1943-7900.0001085](http://dx.doi.org/10.1061/(ASCE)HY.1943-7900.0001085)

Citation: Guo Y (2015) Flow in Open Channel with Complex Solid Boundary. Journal of Hydraulic Engineering. Online before print. DOI [http://dx.doi.org/10.1061/\(ASCE\)HY.1943-7900.0001085](http://dx.doi.org/10.1061/(ASCE)HY.1943-7900.0001085)

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Flow in open channel with complex solid boundary

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Abstract: A two-dimensional steady potential flow theory is applied to calculate the flow in open channel with complex solid boundaries. The boundary integral equations for the problem under investigation are firstly derived in an auxiliary plane by taking the Cauchy integral principal values. To overcome the difficulties of nonlinear curvilinear solid boundary character and free water surface being not known *a priori*; the boundary integral equations are transformed to the physical plane by substituting the integral variables. As such, the proposed approach has the advantages of (1) the angle of the curvilinear solid boundary as well as the location of free water surface (initially assumed) is a known function of coordinates in physical plane; and (2) the meshes can be flexibly assigned on the solid and free water surface boundaries along which the integration is performed. This avoids the difficulty of the traditional potential flow theory which seeks a function to conformally map the geometry in physical plane onto an auxiliary plane. Furthermore, rough bed friction induced energy loss is estimated using the Darcy-Weisbach equation and is solved together with the boundary integral equations using the proposed iterative method. The method has no stringent requirement for initial free water surface position, while traditional potential flow methods usually have strict requirement for the initial free surface profiles to ensure that the numerical computation is stable and convergent. Several typical open channel flows have been calculated with high accuracy and limited computational time, indicating that the proposed method has general suitability for open channel flows with complex geometry.

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23 Key words: Potential flow; boundary value problem; iterative computation; free water surface
24 flow; open channel

25

26 **Introduction**

27 Open channel flow, a gravity driven free surface flow, is a frequently encountered flow pattern in
28 civil engineering. Such flow can be considered as inviscid and irrotational potential flow
29 provided that the flow velocity is relatively high and there is no serious separation between flow
30 and solid boundaries (Batchelor 2000; Hager 1985; White 1986; Montes 1998). Due to its
31 practical engineering relevance, extensive studies have been conducted by researchers who have
32 developed various potential flow theories and numerical methods to solve such flows in past
33 decades. Among these methods, analytical/complex function theory was the first approach used
34 to treat the problem (Guo *et al.* 1996; Guo 2005). Traditional analytical/complex function
35 approach applied conformal mapping and obtained the solution of the problem as integral
36 equations in the complex potential plane or an auxiliary plane (Birkhoff and Zarantonello 1957;
37 Gurevich 1965). However such approach can only treat the flow with simple geometry and
38 without the presence of gravity (Dias *et al.* 1987). For flow moving in a complex geometry (e.g.
39 curvilinear solid boundary occurring in most open channel flows) and/or gravity being present, it
40 is impossible to obtain the analytical solution using the complex function theory (Cheng *et al.*
41 1981; Yeung 1982; Guo *et al.* 1998). The difficulty in solving the gravity driven free water
42 surface flows, as indicated by von Kármán (1940), is due to (1) the nonlinear character of
43 curvilinear solid boundary conditions and (2) the boundary itself (free water surface) being
44 unknown *a priori*. In traditional complex variable function theory method, the solution of the
45 problem is expressed either in the complex plane or in an auxiliary plane. In those planes,

46 however, not only the free water surface needs to be solved, but also the initially known solid
47 boundaries become unknown and need to be determined as part of the solution (Wen and Wu
48 1987). This makes the problem more difficult to solve. As a result, the application of complex
49 function theory to treat open channel flows with curvilinear solid boundaries is greatly limited
50 (Diersch, *et al.*, 1977). As such, various numerical methods have been developed to treat this
51 flow pattern. The advantages and application of the numerical methods have greatly dwarfed the
52 application of the complex variable function theory in potential flow.

53

54 Lauck (1925) could be considered as one of the pioneers dealing with the potential flow using
55 complex variable function theory and numerical computation. He calculated the flow over an
56 infinite high sharp crested weir using a successive numerical approximation (Lauck 1925). He
57 found that the computation was only convergent for a certain flow discharge per unit width.
58 Since his pioneering work, Southwell and Vaisey (1946) applied finite difference method to
59 simulate the free overfall. Thom and Apelt (1961) demonstrated the advantage of computation in
60 complex plane. The same approach was applied by Markland (1965) to treat the free overfall and
61 by Cassidy (1965) to calculate the flow over spillway in which the convergent free water surface
62 was obtained only for a certain flow discharge. Flow over sharp crested weir of finite height was
63 simulated by Strelkoff (1964) and Strelkoff and Moayeri (1970) who obtained integral equations
64 using potential and stream functions as variables. Similar to Cassidy, they only obtained the
65 convergent solution for a certain flow discharge and Strelkoff and Moayeri (1970) only
66 simulated the problem with horizontal and vertical walls. Free overfall was also simulated by
67 Clarke (1965) and Montes (1992) who applied potential flow solutions and by Hager (1983) and
68 Marchi (1993) who applied analytical approach. More recently, Castro-Orgaz and Hager (2013)

69 applied potential flow theory to treat the open channel flow. The semi-inverse mapping of the
70 Laplace equation was applied by Castro-Orgaz (2013a, b) to simulate open channel flow and free
71 overfall.

72

73 In this study, the problem under investigation is expressed as a boundary value (the Riemann-
74 Hilbert) problem and the general solution is obtained in an auxiliary plane. The boundary
75 integral equations are then derived by taking the Cauchy integral principal values. Using arc
76 (streamline) length to substitute the integral variables, the boundary integral equations in the
77 physical plane are obtained with the integration performed only along solid and free water
78 surface boundaries. For rough channel bed, the wall friction induced energy loss along channel
79 bed can be estimated using the Darcy-Weisbach equation. A numerical iterative method is
80 proposed to solve the boundary integral equations and the Darcy-Weisbach equation. As the
81 convergence and stability of the iterative method is ensured (Wen and Wu 1987), this approach
82 has no stringent requirement for initial free water surface profiles while the aforementioned
83 methods usually have strict requirement for the initial free surface profiles to ensure the stability
84 and convergence of the numerical algorithms (Montes 1992, Castro-Orgaz 2013a). As such, the
85 difficulties of nonlinear solid boundary character and unknown free water surface (von Karman
86 1940) are overcome. The method has been applied to calculate several common open channel
87 flows. Good agreement is obtained between calculation and measurements, demonstrating that
88 the method has general practical engineering applications to a broad range of open channel flows
89 in complex geometry.

90

91 **Potential flow formulation**

92 Considering flow over a bump in open channel where a Cartesian coordinate system (x, y) is
 93 established with x being horizontal and y being vertical (see Fig. 1a where points A and G refer
 94 to far upstream and far downstream respectively). Assume that the flow velocity is sufficiently
 95 high and there is no serious separation between flow and solid boundary, the flow under
 96 investigation can then be considered as two-dimensional irrotational and inviscid potential flow.
 97 Let ϕ be the potential function and ψ be the stream function, both are real values and satisfy the
 98 Laplace's equation. Define a holomorphic (or analytic) function f as following (von Kármán
 99 1940; Batchelor 2000):

$$100 \quad f(x + iy) = f(z) = \phi(z) + i\psi(z) \quad (1)$$

101 where x and $y =$ real values, $z=x+iy$, $i = \sqrt{-1}$. This analytical function will conformally map the
 102 flow field in the physical domain $((x, y)$ in Fig. 1a) to an infinite rectangular strip in the
 103 transformed domain $((\phi, \psi)$, i.e. complex potential plane, see Fig. 1b). The lower and upper
 104 boundaries of the strip in the complex potential plane correspond to the solid boundary and free
 105 water surface in the physical plane, respectively. Solid boundary BCDE and free water surface
 106 KJ are streamlines. Without loss of generality, let the stream function ψ be zero along streamline
 107 BCDE and be q ($q =$ flow discharge per unit width) along streamline KJ. Differentiating f with
 108 respect to z yields the complex conjugate of the velocity:

$$109 \quad \frac{df(z)}{dz} = u_x - iu_y \quad (2)$$

110 where $u_x, u_y =$ velocity components in the x, y directions respectively. Rewriting Eq.(2) as
 111 following:

$$112 \quad \frac{df(z)}{dz} = u(z)e^{-i\beta(z)} \quad (3)$$

113 where u = flow velocity magnitude ($u = \sqrt{u_x^2 + u_y^2}$) and β = direction of flow velocity. Assume
 114 the total water head at point K of upstream free water surface be h_0 . Taking x-axis as datum and
 115 setting the constant atmospheric pressure at the free water surface be zero, one obtains:

$$116 \quad h_0 = y_0 + \frac{u_0^2}{2g} \quad (4)$$

117 where y_0 =vertical coordinate of point K ; u_0 = flow velocity magnitude at point K ; g = the
 118 acceleration due to gravity (see Fig.1a). Applying the Bernoulli equation along the free water
 119 surface (a streamline) yields:

$$120 \quad u(z) = \sqrt{2g(h_0 - y)} \quad (5)$$

121 where y = ordinate of free water surface KJ.

122

123 Using analytical function f to define a transformation function t :

$$124 \quad t = \eta + i\zeta = -e^{\frac{-\pi f}{q}} \quad (6)$$

125 where η and ζ =real values; π =3.14159. This transformation function t conformally maps the
 126 infinite rectangular strip in complex potential plane (f -plane) onto the upper half-plane of an
 127 auxiliary plane - t -plane (see Fig. 1c). The real axis of t -plane corresponds to the solid and water
 128 free surface boundaries of the flow domain in the physical plane (see Fig 1a) in which we
 129 assume $\phi =0$ at point B.

130

131 Using the flow velocity magnitude and its direction, an analytic function (the dimensionless
 132 logarithmic velocity) can be defined in t -plane:

$$133 \quad Fun(t) = \ln \frac{u(t)}{u_0} - i\beta(t) = \ln \frac{\sqrt{2g[h_0 - y(t)]}}{u_0} - i\beta(t) \quad (7)$$

134 The boundary conditions of the above analytical function $Fun(t)$ on the real axis of t -plane can then
 135 be determined as:

136

137 $-\infty < \eta < 0, ImFun(\eta) = -\alpha(\eta)$

138 $0 < \eta < \infty, Re Fun(\eta) = \ln \frac{\sqrt{2g[h_0 - y(\eta)]}}{u_0}$ (8)

139 where α = the bed slope of the channel (in radian), Im and Re = the imaginary and real parts of
 140 Fun , respectively. Equations (7)(8) define a boundary value (the Riemann-Hilbert) problem
 141 whose general solution can be written as (Muskhelishvili, 1965)

142 $Fun(t) = \frac{\sqrt{t_0 - t}}{\pi} \left\{ \int_{-\infty}^{t_0} \frac{-\alpha(\eta)d\eta}{\sqrt{t_0 - \eta}(\eta - t)} + \int_{t_0}^{\infty} \frac{\sqrt{2g[h_0 - y(\eta)]}d\eta}{u_0\sqrt{\eta - t_0}(\eta - t)} \right\}$ (9)

143 where $t_0=0$. Let cross section KB and JE be far away from the bottom obstacle CD. Assume that
 144 no waves occur at KB; then the flow at the cross section KB can be considered as uniform
 145 (Vanden-Broeck 1997). As we are mostly interested in the flow field bounded by BCDE and KJ,
 146 the infinite integral intervals of the two integration terms on the right hand side of Eq. (9) can be
 147 shortened and replaced by finite integral intervals:

148 $Fun(t) = \frac{\sqrt{-t}}{\pi} \left\{ \int_{t_B}^{t_K} \frac{-\alpha(\eta)d\eta}{\sqrt{-\eta}(\eta - t)} + \int_{t_J}^{t_K} \frac{\sqrt{2g[h_0 - y(\eta)]}d\eta}{u_0\sqrt{\eta}(\eta - t)} \right\}$ (10)

149 Eq. (10) is the general formula to calculate the flow field. In Eq.(10), if the solid boundary is
 150 comprised of straight segments (e.g. $\alpha = constants$) and the gravity is ignored, then Eq. (10) can
 151 be analytically integrated. For open channel flows with curvilinear solid boundaries investigated
 152 here, the mathematical difficulty in obtaining the analytical solution is so far insurmountable
 153 (Cheng et al. 1981; Yeung 1982). This is because the analytical conformal mapping function for
 154 open channel free water surface flows from flow domain in physical plane to an infinite strip in

155 f -plane (complex potential plane) or to the upper half plane of t -plane is unknown. Therefore, if η
156 or φ is chosen as independent variable; $\alpha(\eta)$ or $\alpha(\varphi)$ (known in the physical plane) will be
157 unknown functions. This will make the solution of (10) in t -plane or f -plane more difficult (von
158 Kármán 1940). On the other hand, α is a known function of coordinates x and y , or of arc length s
159 (see Fig. 2), in the physical plane. If we assume the position of free surface in advance, then u
160 (Eq.(5)) is also a known function of coordinates x and y , or of arc length s , in physical plane.
161 Therefore, we seek the solution of Eq. (10) in physical plane. This can be achieved via changing
162 the integral variables in Eq. (10).

163

164 As shown in Fig.2, points B and K are taken as the origins of streamline (arc) coordinate along
165 solid boundary BCDE and free water surface boundary KJ, respectively. Denote s as arc length
166 of the streamline measured from B and K respectively along boundaries, the potential function at
167 any point of solid and free water surface boundaries is then the function of s . From Eq.(7), we
168 have

$$169 \quad t = \eta = -e^{-\pi\varphi/q} \quad \text{on BCDE} \quad (11a)$$

$$170 \quad t = \eta = -e^{-\pi(\varphi+iq)/q} = e^{-\pi\varphi/q} \quad \text{on KJ} \quad (11b)$$

171 Differentiating η with respect to the arc length s along solid and free boundaries yields:

$$172 \quad \frac{d\eta}{ds} = \frac{d\eta}{d\varphi} \frac{d\varphi}{ds} = -\frac{\pi\eta(s)u(s)}{q} \quad (12)$$

173 where $u(s)$ = the velocity magnitude along boundaries.

174

175 For most engineering practice, the pressure distribution along the solid boundary and the position
176 of free water surface are the major concern in terms of cavitation damage studies and flooding
177 forecasting. As such, we take the Cauchy principle value of Eq. (10) on real axis of t -plane.

178 Separating the real and imaginary parts of the principle value yields the equations for calculating
 179 the inclinations of the free water surface and the velocity magnitude along the solid boundary. In
 180 light of Eq.(12), the boundary integral solutions of Eq.(10) in physical plane can be obtained:

181
 182 The velocity magnitude at a distance s from point B on the solid boundary BCDE is

$$\begin{aligned}
 \ln \frac{u_w(s)}{u_0} = & \frac{\sqrt{-\eta_w(s)}}{q} \left\{ - \int_{\eta_B}^{\eta_E} \left[\frac{\alpha(l)}{\sqrt{-\eta_w(l)}} - \frac{\alpha(s)}{\sqrt{-\eta_w(s)}} \right] \frac{\eta_w(l)u_w(l)}{\eta_w(l) - \eta_w(s)} dl \right. \\
 & + \int_{\eta_K}^{\eta_J} \left\{ \frac{\sqrt{2g[h_0 - y_s(l)]}}{u_0} \right\} \frac{\eta_s(l)u_s(l)}{\sqrt{\eta_s(l)[\eta_s(l) - \eta_w(s)]}} dl \left. \right\} - \frac{\alpha(s)}{\pi} \ln \frac{\eta_E - \eta_w(s)}{\eta_w(s) - \eta_B}
 \end{aligned} \tag{13}$$

184 The inclination of free surface KJ at a distance s from point K is

$$\begin{aligned}
 \beta(s) = & \frac{\sqrt{\eta_s(s)}}{q} \left\{ - \int_{\eta_B}^{\eta_E} \frac{\alpha(l)\eta_w(l)u_w(l)}{\sqrt{-\eta_w(l)[\eta_w(l) - \eta_s(s)]}} dl + \int_{\eta_K}^{\eta_J} \left\{ \frac{\ln\{\sqrt{2g[h_0 - y_s(l)]}\}}{\ln(u_0)\sqrt{\eta_s(l)}} - \frac{\ln\{\sqrt{2g[h_0 - y_s(s)]}\}}{\ln(u_0)\sqrt{\eta_s(s)}} \right\} \times \right. \\
 & \left. \frac{\eta_s(l)u_s(l)}{[\eta_s(l) - \eta_s(s)]} dl \right\} + \frac{\ln\{\sqrt{2g[h_0 - y_s(s)]}\}}{\ln(u_0) \times \pi} \ln \frac{\eta_J - \eta_s(s)}{\eta_s(s) - \eta_K}
 \end{aligned} \tag{14}$$

185
 186 where $\eta_B, \eta_E, \eta_J,$ and $\eta_K =$ values of η at points B, E, J and K, respectively, subscripts w and s
 187 denote solid/wall and free water surface boundaries, respectively. The integrations are performed
 188 along the solid boundary and free water surface in the direction of flow.

189
 190
 191 As aforementioned, cross section BK is uniform. Assume flow is two-dimensional, this cross
 192 section is then isopotential. Without loss of generality, let the potential function at BK be zero.
 193 The potential functions at a distance s from the cross section BK along solid boundary (BCDE)
 194 and free water surface (KJ) can then be calculated as:

$$\varphi_w(s) = \int_0^s u_w(l) dl \tag{15a}$$

196
$$\varphi_s(s) = \int_0^s u_s(l) dl \quad (15b)$$

197 With the inclination of free water surface having been determined, the location (coordinates) of
 198 free water surface KJ can be calculated by

199
$$x(s) = x_K + \int_0^s \cos \beta(l) dl \quad (16a)$$

200
$$y(s) = y_K + \int_0^s \sin \beta(l) dl \quad (16b)$$

201 where (x_K, y_K) = coordinates of point K.

202

203 When flow field is calculated, the pressure distribution along the channel bed can be determined
 204 by applying the Bernoulli's equation between far upstream and downstream cross section. For
 205 rough bed, the solid wall friction induced energy loss should be taken into account in order to
 206 have accurate calculation of pressure. This energy loss, which is usually ignored in potential flow
 207 methods, can be evaluated using the following iterative method. To this end, the Darcy-
 208 Weisbach equation is applied to estimate the continuous friction loss along solid boundary.
 209 Applying the modified energy equation yields the pressure distribution along the solid boundary
 210 (taking x-axis as basic datum):

211
$$\frac{P(s)}{\rho g} = h_0 - \frac{u^2(s)}{2g} - y(s) - h_f(s) = h_0 - \frac{u^2(s)}{2g} - y(s) - \frac{1}{8g} \int_0^s \frac{\lambda}{R_h} u^2(l) dl \quad (17)$$

212 where P = the pressure at the channel bed, h_0 = total energy head at cross section BK, ρ = water
 213 density, $h_f(s)$ = continuous energy loss generated by wall friction along the channel bed, R_h =
 214 hydraulic radius (taken as water depth); $\lambda = 8g n^2 / R_h^{1/3}$ = the Darcy-Weisbach coefficient (in

215 metric units); n = the Manning coefficient, which is taken as 0.014 for concrete (Sivakumaran et
216 al. 1983).

217
218 The boundary integral equations (13) and (14) together with Eqs. (16) and (17) form the basic
219 equations used to calculate the flow field, pressure distribution on the bed and the position of
220 free water surface in open channel flow. The boundary integral equations and the pressure
221 equation will be solved using iterative method proposed below.

222

223 **Boundary conditions**

224 Boundary conditions are summarized as following: at lower solid boundary, the stream function
225 ψ is set as zero and the geometries are prescribed, namely α is known. At upper free water
226 surface boundary, the stream function ψ is set as q (flow discharge per unit width) and the
227 constant atmospheric pressure is assumed to be zero. At far upstream section BK and
228 downstream section EJ where the effect of bottom topography is assumed to be negligible, the
229 flow is assumed to be parallel to the channel bottom (Montes 1994; Castro-Orgaz 2013a).

230

231 **Computational procedure**

232 A numerical iterative method is proposed to solve the boundary integral equations and pressure
233 distribution equation. The computational procedure is as following:

- 234 1. Specify the inlet boundary conditions (e.g. water depth and flow velocity) according to
235 the experiments. Assume the energy loss due to friction be zero.
- 236 2. Assume free surface KJ. Assign non-uniform meshes along boundaries with finer meshes
237 in the regions of rapid change of flow (e.g. near the bottom obstacle). Meshes are

238 assigned between points *B* and *E* along solid boundary and between points *K* and *J* along
239 free water surface. Figure 3 is an illustrative sketch to demonstrate the strategy of
240 assigning non-uniform meshes along boundaries.

241 3. Assume velocity along solid boundary BCDE and calculate velocity on free water surface
242 KJ using the Bernoulli equation.

243 4. Calculate potential functions using (15a, b) and η using (11a, b) on boundaries KJ and
244 BCDE.

245 5. Bring these values into Eqs. (13)(14) to calculate new velocity along solid boundary
246 BCDE and new inclination of free water surface KJ.

247 6. Calculate the new position of free water surface KJ using (16a, b), then calculate new
248 velocity on free surface KJ using the Bernoulli equation (5).

249 7. Calculate new potential functions and η on boundaries KJ and BCDE. Estimate new
250 friction induced energy loss.

251 8. Repeat steps 5 to 7 until

$$252 \quad \frac{\varphi^n - \varphi^{n-1}}{\varphi^n} < \varepsilon_1; \quad \frac{h_f^n - h_f^{n-1}}{h_f^n} < \varepsilon_2 \quad (18a, b)$$

253 where superscript n denotes the iterative number; $\varepsilon_1, \varepsilon_2 =$ prescribed computational accuracy for
254 potential function and energy loss, respectively; the values of potential function and energy loss
255 are taken at point E.

256 9. Calculate the pressure distribution using Eq. (17).

257

258 Numerical examples

259 Several frequently encountered open channel flows with various boundary conditions are
260 simulated using the proposed method. Simulated results are well compared with measurements,

261 demonstrating that the approach has general suitability to broad open channel flows with
262 satisfactory accuracy.

263

264 *Open channel flow over irregularities*

265 Cavitations often occur when high speed water flows around objects, resulting in the damage of
266 the object surface and reducing mechanical efficiency. Typical example is the cavitation caused
267 by high velocity flow over the surface irregularities (Ball 1976). Cavitation damage takes place
268 when the absolute pressure is equal to or lower than the vapor pressure of water at the given
269 temperature (Douglas et al. 2001). Therefore, the most potential place that cavitation may appear
270 is the place where the minimum pressure occurs and this can be represented using the pressure
271 coefficient (Guo et al. 2007).

272

273 Therefore, to investigate the likelihood of cavitations around the irregularity, applying modified
274 energy equation along the irregularity surface (which is a streamline) yields the pressure
275 coefficient on the irregularity (taking the x-axis as datum):

$$276 \quad C_p(s) = \frac{P(s) - P_0}{\rho u_0^2 / 2} = 1 - \frac{u(s)^2}{u_0^2} + \frac{2g\{h_f(s) - y(s)\}}{u_0^2} \quad (19)$$

277 where C_p =pressure coefficient on the irregularity surface, P_0 = the pressure at far undisturbed
278 upstream, $y(s)$ =the height of irregularity, other symbols have the same meanings as those in
279 Eq.(17). In general, the last item on the right hand side of Eq. (19) is much smaller than other
280 items for relatively small irregularity and can be ignored. Equation (19) shows that the minimum
281 pressure, thus the minimum pressure coefficient, appears at the point that the maximum velocity
282 takes place. This is the position that the most likelihood of cavitations damage takes place.

283

284 Two types of irregularity are simulated in this study: one is arc irregularity and another is semi-
285 arc step. Both the arc irregularity and semi-arc step were placed at the centre of the channel. The
286 front and end edges of arc irregularity were at the same level of the channel bed (see the sketch
287 inside Fig. 4). For semi-arc step, the channel bed at the end of semi-arc step was raised to the
288 same level of the end edge of the step (see the sketch inside Fig. 5). Therefore, there was no
289 abrupt drop at the back of the semi-arc step.

290

291 Fig. 4 is the plot of the simulated and measured (taken from Lin and Xu 1985) pressure
292 coefficient distribution around arc irregularity for upstream incoming flow velocity of
293 $u_0=4.52\text{m/s}$, $h=0.1\text{m}$ (corresponding to the incoming flow Froude number $F=u_0/(gh)^{1/2}=4.56$).
294 The height of arc irregularity is $\delta=0.0092\text{m}$ and radius $R=0.6\text{m}$. Simulations were performed at
295 10 water depths in both upstream and downstream where the effect of arc irregularity was
296 expected to be negligible. The maximum mesh size is 0.005m at both the far upstream and
297 downstream and the minimum mesh size is 0.0015m around the arc irregularity and the free
298 surface above it. The simulation was performed at a Dell OPTIPLEX390. For the computational
299 accuracy of 10^{-4} and 10^{-3} for potential function and energy loss respectively, the total
300 computational time for all cases is less than 1 minute with iteration number being between 15
301 and 30.

302

303 Fig.4 demonstrates that an almost symmetric pressure coefficient distribution around the centre
304 of the arc irregularity exists. It is seen that the pressure coefficient decreases along the
305 irregularity and reaches the minimum value roughly at the top of irregularity. This is the location
306 that the largest effect of the obstruction of irregularity to flow takes place, thereby producing the

307 maximum flow velocity. This position is the location where the likelihood of cavitations may
308 take place. Good agreement between the simulated and measured pressure coefficients indicates
309 that the proposed method can accurately predict the flow and pressure field for flow over an
310 irregularity in open channel.

311
312 If irregularity is sufficiently high, flow regime will be similar to that of flow over a hump
313 (discussed below) or weir flow where flow regime changes from upstream subcritical to
314 downstream supercritical.

315
316 Fig. 5 is the comparison of simulated and measured pressure coefficient around a semi-arc step
317 for incoming flow velocity $u_0=2.95\text{m/s}$, water depth $h=0.1\text{m}$, irregularity height $\delta=0.0111\text{m}$ and
318 radius $R=0.6\text{m}$. The corresponding incoming flow Froude number is $F=2.98$. The mesh
319 assignment is similar to that in arc-irregularity. Both the measured (symbols, taken from Lin and
320 Xu 1985) and simulated (solid lines) results show that the pressure coefficient decreases along
321 the semi-arc step and reaches the minimum at $s/L\approx 0.82$ where the maximum velocity occurs due
322 to the contraction effect of the step. The pressure coefficient then increases downstream.

323
324 Simulations have also been run to investigate the effect of incoming flow velocity (thus the
325 Froude number) on the flow and pressure field. For the sake of clarity and for the purpose of
326 comparison, measured results of $u_0=6.40\text{m/s}$ ($F=6.47$) are plotted in Fig. 5, while the solid line
327 represents the averaged simulated results for $F=2.98$ and 6.47 . It is seen that the incoming flow
328 velocity (or the Froude number at the inlet) has insignificant effect on the pressure distribution

329 around the semi-arc step. Simulations run for a range of the Froude number reveals similar
330 results.

331
332 Fig. 5 also demonstrates that relatively large deviation between the simulation and experiments
333 exists around the top of the semi-arc step. This may be ascribed to the fact that the asymmetric
334 step irregularity has relatively larger effect on the flow near the top step region.

335
336 Comparing Fig. 4 and Fig. 5 demonstrates that for the similar size (height and radius), arc
337 irregularity has slightly greater impact on the flow and pressure field around irregularity than that
338 semi-arc step irregularity does. There may be two reasons. The first reason is that the arc
339 irregularity is almost twice long of the semi-arc step irregularity; therefore, it has larger effect on
340 the flow than semi arc step does. The second reason is that the end of the semi-arc step is at the
341 same level as that of the channel bed. Therefore, it is unlikely that the flow separation will take
342 place at the end of step, which greatly reduces the impact of semi-arc step on the flow field. For
343 cases that there exists an abrupt enlarge cross section (e.g. the end of the semi-arc irregularity is
344 higher than the channel bed), flow separation and vortices may take place, thereby resulting in
345 larger impact on pressure distribution around the end of irregularity. In this situation, the
346 proposed potential flow approach fails and advanced turbulent models are required to capture the
347 vortex flow structures at the end corner of semi-arc irregularity.

348
349 *Open channel flow over a bottom hump*

350 The second example is the open channel flow over a relatively large hump. The case simulated is
351 taken from Sivakumaran et al. (1983) who measured the free water surface profile and bottom

352 pressure distribution around the symmetrical and asymmetrical humps. For symmetrical hump
353 which is expressed as $y=0.2\exp[-0.5(x/24)^2]$ (in m), the unit width flow discharge is $q=0.112$
354 m^2/s (high flow) and $0.036 \text{ m}^2/\text{s}$ (low flow), respectively. The corresponding upstream Froude
355 number is 0.18 and 0.08, indicating that the incoming flow is subcritical for both cases. The
356 length of the open channel simulated is about 6m with the hump located approximately centrally.
357 In the regions where flow varies rapidly (e.g. around the hump and the free water surface above
358 it); meshes are locally refined to improve the computational accuracy with the minimum mesh
359 size of 0.0012m. The computational time (run on a Dell OPTIPLEX390) for all cases is less than
360 2 minutes for the computational accuracy of 10^{-4} for potential function.

361
362 Figs. 6(a) and (b) plot the measured (symbols) and simulated (solid and dashed lines) free water
363 surface and bottom pressure distribution around the hump. Fig. 6 shows that water flow
364 accelerates as it approaches the hump. The flow continues accelerating and descends down the
365 lee side of the hump. The simulation shows that the flow transition from subcritical to
366 supercritical roughly takes place at the crest of hump for both cases. In contrast to free water
367 surface, the bed pressure decreases as flow approaches the hump and reaches the minimum in
368 the half top lee side of hump. This is caused by the variation of water depth as well as the profile
369 of hump. The water depth at the half top lee side is relatively small while the hump convex
370 profile produces the negative centrifugal force, resulting in the minimum pressure. The bed
371 pressure then slightly increases downstream due to the contribution of centrifugal force
372 generated by the concave profile of the hump. The good agreement between the simulated and
373 measured free water surface and bed pressure for both cases indicates that the proposed potential
374 flow method performs well for both subcritical and supercritical flow.

375 Flow over asymmetrical hump is also simulated. In this case, the hump is a B-splined shape
376 (Sivakumaran et al. 1983). The flow discharge per unit width is $q=0.11165 \text{ m}^2/\text{s}$. The
377 corresponding upstream Froude number is 0.12, indicating that the upstream incoming flow is
378 subcritical. The mesh assignment is similar to that for symmetrical hump. The simulated and
379 measured free water surface and bed pressure distribution are plotted in Fig. 6c. The simulation
380 shows that the flow transition from subcritical to supercritical takes place roughly at the hump
381 crest, while this flow transition takes place slightly downstream of the hump crest in the
382 experiments. It is also seen that the potential flow method underestimate the bed pressure. Runs
383 using finer meshes (the minimum meshes around asymmetrical hump is 0.0008m) didn't
384 improve the computational accuracy. This discrepancy between simulation and measurement
385 may be ascribed to the fact that the asymmetrical hump shape has larger impact on flow field
386 with larger extensive flow transition region (Sivakumaran et al. 1983); which the potential flow
387 approach may fail to accurately capture.

388

389 *Flow transition in curved open channel*

390 Flow in an open channel with the bed slope changing from mild to steep (or other way round) is
391 frequently encountered in men-made flows. Free overfall usually takes place at the slope break.
392 The water depth upstream the slope break is larger than the critical water depth, while the water
393 depth downstream the slope break is smaller than the critical water depth. The determination of
394 the free water surface is of importance due to its practical engineering applications (Chow 1959;
395 Montes 1994). As flow regime changes from subcritical in the upstream mild slope section to
396 supercritical in the downstream steep slope reach, free water surface goes through a sharp drop
397 which provides a challenge for accurate simulation. In this study, the flow transition from mild

398 slope to steep slope is simulated using the proposed potential flow method. The simulation case
399 is taken from Montes (1994). Though details can be found in Montes (1994), we present a brief
400 description here for convenience. The upstream horizontal plane was connected with a steep
401 slope of either 45^0 or 60^0 via a circular fairing of radius $r=0.1\text{m}$. The flume width was 0.402m .
402 The flow discharge simulated for both cases is $0.06\text{m}^3/\text{s}$. The data of free water surface profiles
403 and bed pressure are taken from Montes (1994) for the purpose of comparison with the
404 simulation.

405
406 For the given experimental data, the flow at upstream horizontal section is critical and transfers
407 to supercritical as it moves towards the steep slope. Simulation was performed 10 upstream
408 water depths in both upstream and downstream reaches. The maximum mesh size is 0.008m at
409 both the far upstream and downstream and the minimum mesh size is 0.004m around the circular
410 fairing section and the free water surface above it. The simulated and measured free water
411 surfaces and bed pressure distributions are plotted in Fig. 7 for a steep slope of 45^0 (Fig. 7a) and
412 60^0 (Fig. 7b) in which h is water depth at upstream mild slope section. The simulated end depth
413 ratio (depth at the end of mild slope to critical depth) is 0.703 for 45^0 and 0.692 for 60^0
414 respectively. These values are slightly smaller than the classic result of Rouse (1936) for a
415 horizontal channel and favorably compare with the experiments taken from Montes (1994). For
416 both cases, the simulation shows that water depth continues to decrease over the short circular
417 transition section and steep slope. In general, good agreement between the simulated and
418 measured free water surface profiles is obtained.

419

420 It is seen from Fig. 7 that both the simulations and experiments demonstrate that the bed pressure
421 sharply decreases at the circular section. This can be ascribed to the centrifugal force caused by
422 convex curve solid boundary as well as the decrease of water depth. The minimum bed pressure
423 appears at the lower part of the circular linkage for two steep slopes. The bed pressure then
424 increases and reaches the positive value near the beginning section of the steep slope where the
425 centrifugal force disappears. Comparison of simulation and measurements demonstrates that the
426 proposed potential flow method favorably predicts the sharp variation of the bed pressure
427 distribution around the transition from mild to steep slopes. Fig. 7 also shows that the simulation
428 slightly overestimates the minimum bed pressure.

429

430 Simulations have also been performed for various flow discharges. Similar results to Fig. 7 are
431 obtained. Simulation reveals that the absolute value of the minimum bed pressure decreases with
432 the decrease of flow discharge.

433

434 *Flow through spillway flip bucket*

435 Spillway is usually used to discharge water from a reservoir into downstream with a free jet. The
436 flow characteristics (e.g. free water surface profiles, bottom pressure, etc) in the spillway flip
437 bucket are of importance to optimize the design of the spillway. The profile of flip bucket has
438 significant effect on the free trajectory jet exiting from the bucket. In this study, a circular flip
439 bucket with different Froude numbers is simulated and compared with the measurements taken
440 from Lenau and Cassidy (1969). The central angle of bucket (or total turning angle of bucket)
441 simulated is 95° and the slope angle of upstream spillway chute is 56.16° (see the inset sketch in
442 Fig.8). Simulation was performed from 10 upstream water depths from the entrance of the flip

443 bucket. Meshes are locally refined within the bucket to ensure the computational accuracy. Two
444 upstream incoming flow Froude numbers $F=7.35$ and 10.39 are simulated, corresponding to the
445 ratios of bucket radius to the depth of flow well upstream from bucket being 3 and 6
446 respectively. Figure 8 shows the comparison of simulated and measured (taken from Lenau and
447 Cassidy 1969) bucket bottom dimensionless pressure distribution for (a) $F=7.35$ and (b) 10.39 .
448 In Figure 8, s is the arc distance from the starting point of flip bucket and L is the total arc length
449 of bucket. It is seen that bottom pressure increases sharply as flow enters into bucket. This is
450 mainly caused by the centrifugal force due to the concave curve solid boundary. The bottom
451 pressure then remains approximately constant in the bucket. The bottom pressure then sharply
452 decreases as flow approaches exit and becomes zero at the exit. Comparing Fig 8a and 8b
453 demonstrates the bottom pressure in the bucket decreases with the increase of the incoming
454 Froude number. In general, simulated bottom pressure agrees well with the measurements though
455 a slight discrepancy between simulation and measurement takes place at the entrance of bucket.

456

457 **Conclusion**

458 Open channel flow over curved boundary or irregularities/humps is frequently encountered in
459 civil engineering. Potential flow theory has been applied to calculate such flows due to its simple
460 form and easy calculations (Castro-Orgaz 2013a). However, traditional complex variable
461 function approach can only treat flow in simple geometry comprised by straight sections and
462 without the presence of gravity. For free water surface open channel flows with curvilinear solid
463 boundaries, the mathematical difficulty is so far insurmountable (Cheng et al. 1981; Yeung
464 1982) due to the fact that the boundary condition is nonlinear and the free water surface is
465 unknown *a priori* (von Kármán 1940). In this study, the boundary integral equations for the

466 problem under investigation are derived in an auxiliary plane by taking the Cauchy integral
467 principal values. Using the arc length to substitute the integral variables yields the boundary
468 integral equations in the physical plane. The advantage of expressing boundary integral
469 equations in the physical plane is that both the angle of the prescribed curved solid boundary and
470 position of free water surface (assumed in advance) are known functions of coordinates x and y ,
471 or of arc length s , in physical plane. The effect of rough boundary friction is evaluated using the
472 Darcy-Weisbach equation. An iterative computational method is proposed to solve the boundary
473 integral equations and the Darcy-Weisbach equation. As the integration is performed only along
474 the solid and free water surface boundaries, the computational meshes can be flexibly assigned
475 along boundaries to ensure simulation accuracy and save computational time. When flow
476 discharge is known, the convergence and stability of the numerical iteration has been proved
477 (Wen and Wu 1987). Therefore, the proposed method has no stringent requirement for initial
478 values and position of free water surface while other potential flow methods usually have high
479 requirement for the initial free water surface profiles in order to have a convergent numerical
480 solution (Montes 1992, Castro-Orgaz 2013a). The approach has been successfully applied to
481 calculate several common open channel flows in various boundary conditions for a range of flow
482 parameters. The position of free water surface and pressure distribution at the channel bed can be
483 accurately simulated using the proposed method. Examples carried out in this study demonstrate
484 that the proposed approach can provide quick and accurate solution to frequently encountered
485 engineering problems. Given that for most engineering practice, the location of free water
486 surface, bottom pressure distribution and flow rate are the most concerned aspects in terms of
487 flooding forecasting and prediction and cavitation damage studies; the proposed approach has the
488 broad engineering practice applications.

489 For cases that only water depth upstream is given, the iterative approach based on consistency
490 between the discharge and uniform velocity at far upstream can be used to determine flow
491 discharge (Guo et al. 1996, 1998). The iterative method for flow discharge will then be
492 incorporated into the iterative procedure for computing free water surface and bottom pressure to
493 form a synchronous iteration (Guo et al. 1996, 1998).

494

495 If serious flow separation between flow and solid boundary takes place or if the turbulent
496 properties are of importance for the problem under investigation, the proposed potential flow
497 approach cannot capture these details of flow structures. For these flow scenarios, the more
498 sophisticated and complex turbulent models should be used.

499

500 Acknowledgement: The writer is grateful for the financial support of this study by the Open
501 Fund from the State Key Laboratory of Hydraulics and Mountain River Engineering, Sichuan
502 University (SKHL1302). The comments and suggestions made by the Associate Editor and three
503 Reviewers have greatly improved the quality of the final manuscript.

504

505 Notation

506 *The following symbols are used in this paper:*

507 C_p =pressure coefficient;

508 F =the Froude number;

509 $f=\varphi+i\psi$;

510 Fun =analytical function or dimensionless logarithmic velocity;

511 g =acceleration due to gravity;

512 h =water depth at cross section BK;

513 h_0 = total water head upstream;

514 h_f = continuous energy loss due to friction;

515 $i = \sqrt{-1}$

516 L =total arc length of arc, semi-arc irregularity or flip bucket;

517 n = the Manning's roughness;

518 P = the pressure at the channel bottom

519 P_0 = the pressure at far undisturbed upstream;

520 R =radius of arc and semi-arc irregularity;

521 R_h = hydraulic radius;

522 r = radius of a circular fairing connecting mild and steep slopes;

523 s =arc length from the starting point of irregularity or flip bucket;

524 u =flow velocity magnitude;

525 u_x, u_y = velocity components in the x, y directions, respectively

526 u_0 = the velocity at far upstream;

527 α = the bed slope of the channel;

528 β = direction of flow velocity;

529 λ = the Darcy-Weisbach coefficient

530 $\pi = 3.14159$;

531 φ = potential function;

532 ψ = stream function;

533 ε_1 = prescribed computational accuracy for potential function;

534 ε_2 = prescribed computational accuracy for energy loss;

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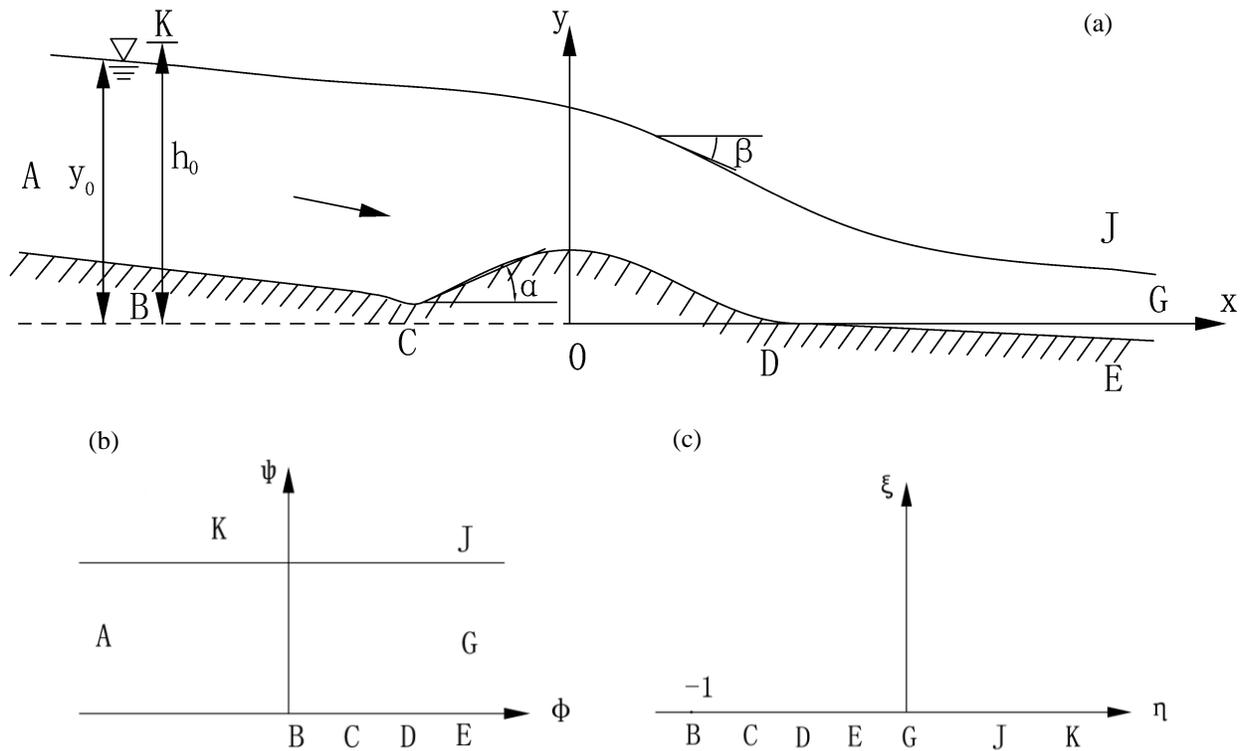


Figure 1. Schematic diagram of open channel flow over a bottom obstacle; (a) flow domain in physical plane; (b) in complex potential plane ((φ, ψ) plane) and (c) in an auxiliary plane - t -plane.

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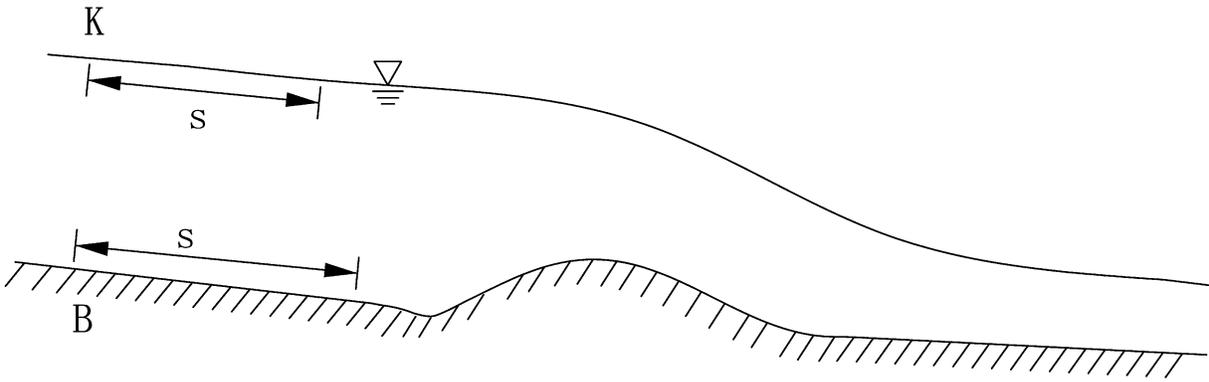


Figure 2. Sketch of streamline coordinate system.

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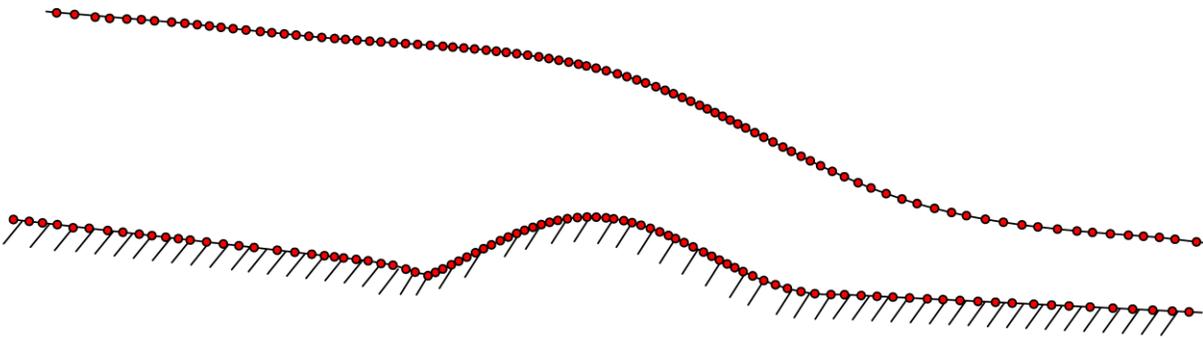


Figure 3. Sketch of meshes along solid and free water surface boundaries for demonstrating the strategy of meshes assignments.

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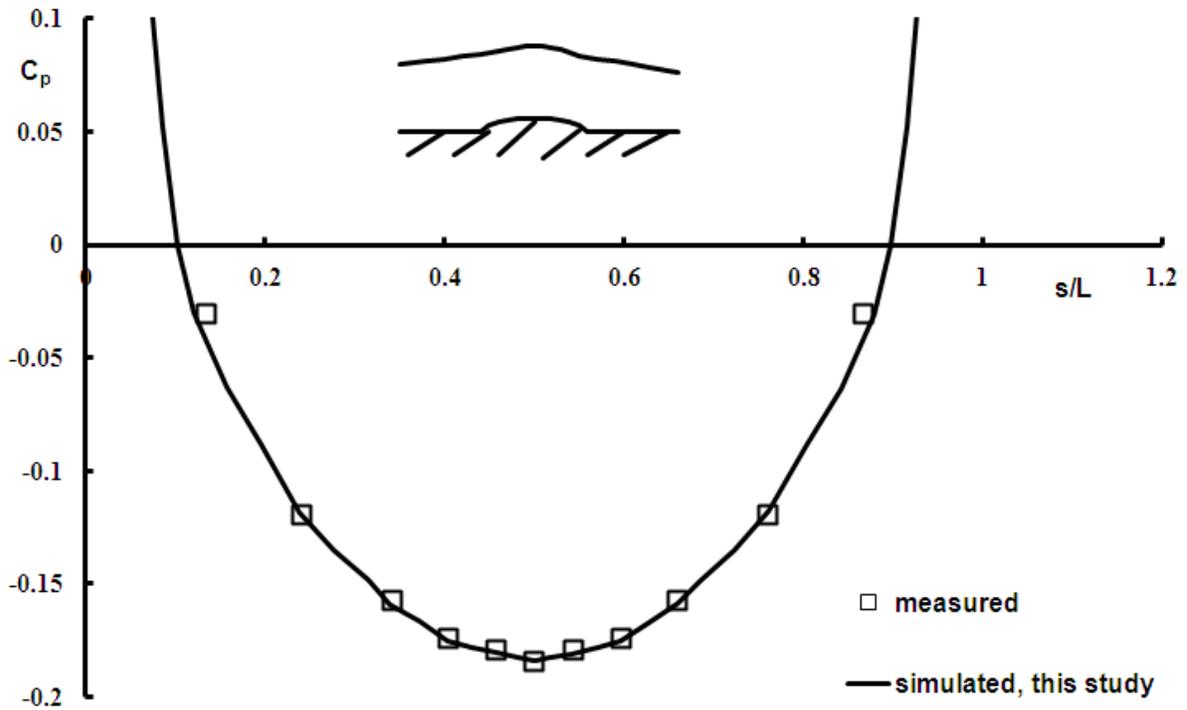


Figure 4. Comparison of the simulated (solid line) and measured (symbol) pressure coefficient distribution around the arc irregularity, s is the streamline distance from the starting point of the arc irregularity and L is the total streamline length of the arc irregularity. Inset is the sketch of experimental set-up.

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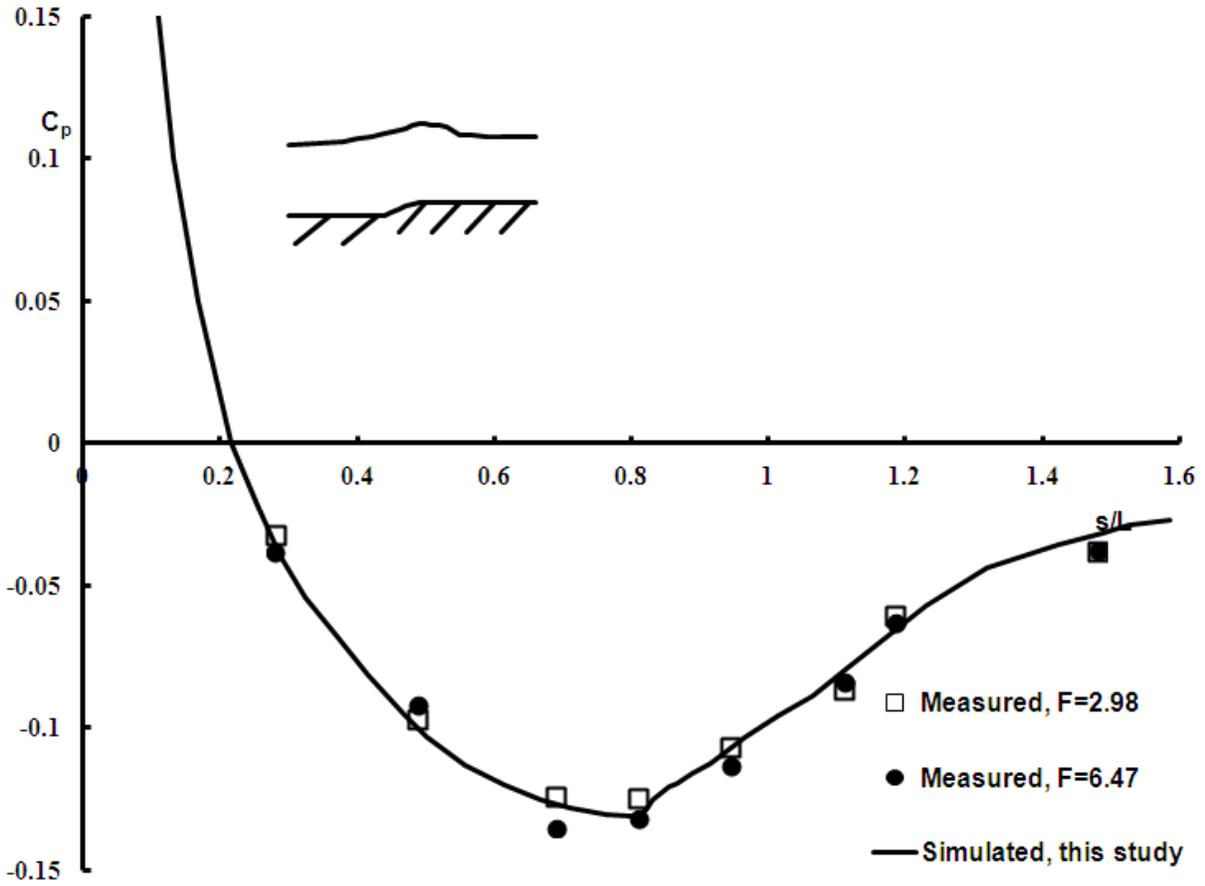


Figure 5. Comparison of the simulated (averaged, solid line) and measured (symbols) pressure coefficient distribution around the semi-arc step irregularity for $F=2.98$ and 6.47 . Inset is the sketch of experimental set-up. s and L are the same as in Fig. 4.

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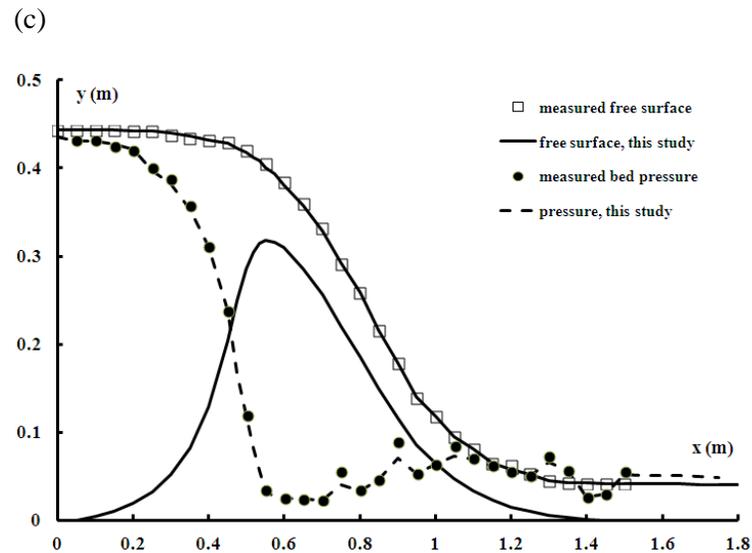
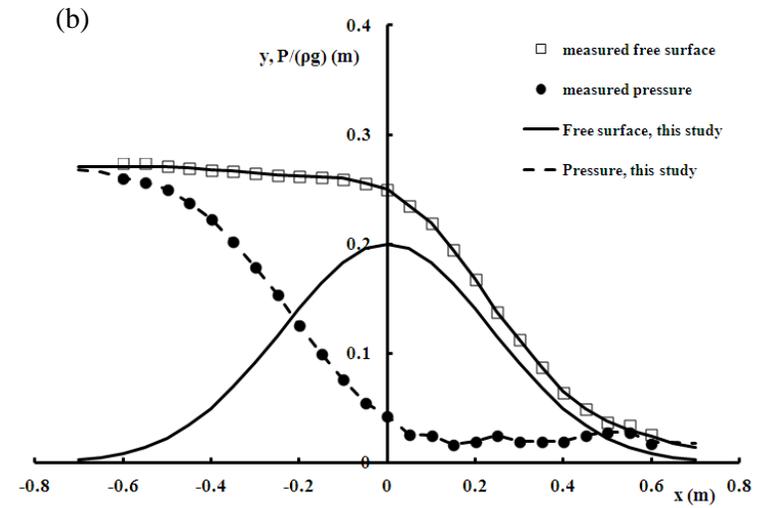
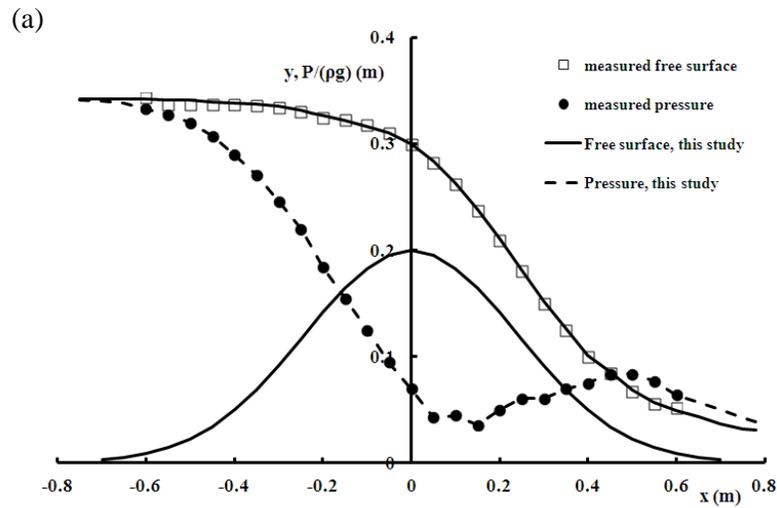
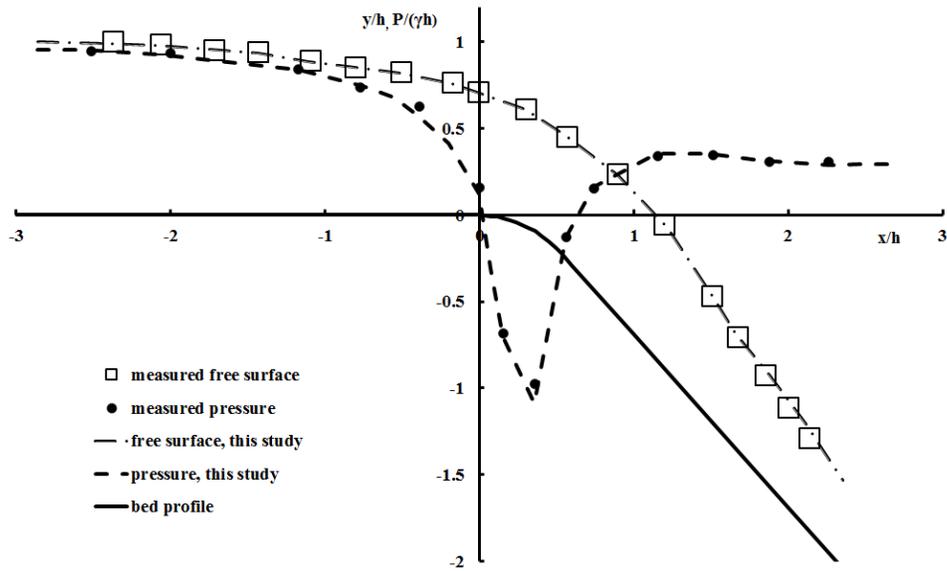
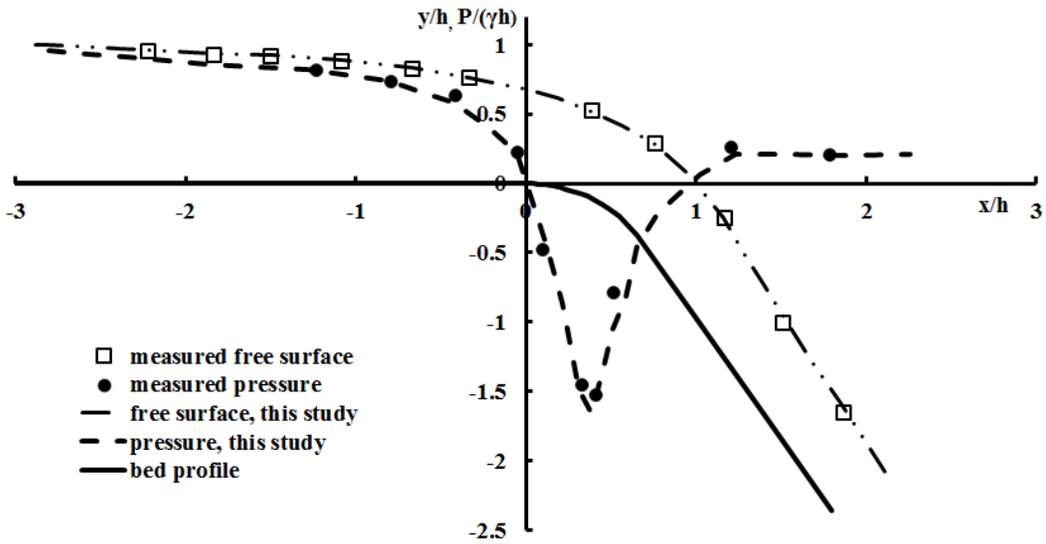


Figure 6. Plot of simulated (solid and dashed lines) and measured (symbols) free water surface and bottom pressure distribution of potential flow. (a) over a symmetrical hump for $q=0.112 \text{ m}^2/\text{s}$ and $F=0.18$; (b) over a symmetrical hump for $q=0.036 \text{ m}^2/\text{s}$ and $F=0.08$; and (c) over an asymmetrical hump for $q=0.11165 \text{ m}^2/\text{s}$ and $F=0.12$.



(a)



(b)

Figure 7 Simulated and measured free surface profile and bottom pressure distribution of the potential flow in transition from horizontal to steep slope for (a) steep slope = 45° , $F=1$ and (b) steep slope = 60° , $F=1$.

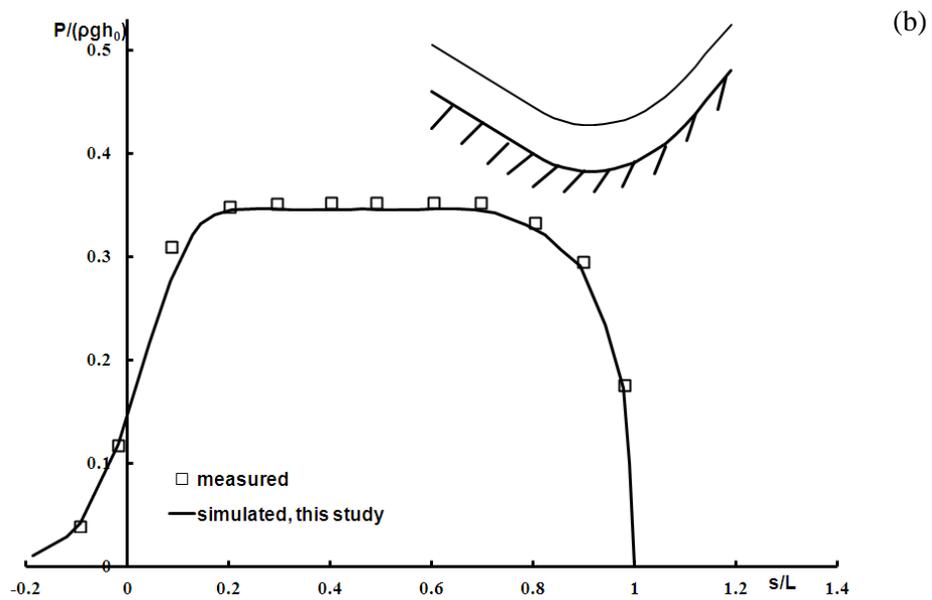
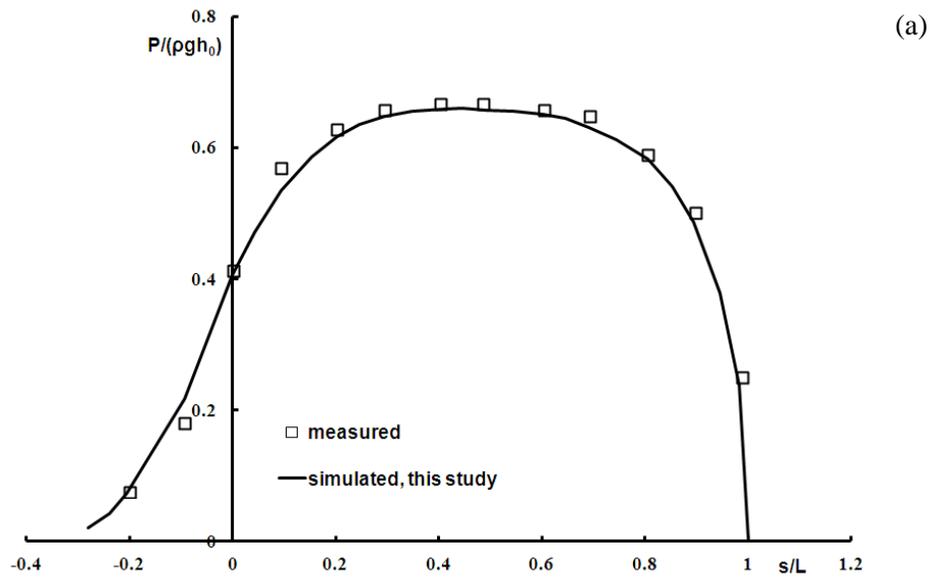


Figure 8 Comparison of simulated and measured flip bucket bottom pressure. The central angle of bucket is 95° and the slope angle of upstream spillway chute is 56.16° (inset is the sketch of experimental set-up). The incoming flow Froude number is (a) $F=7.35$; (b) $F= 10.39$.