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<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>$A_f$</td>
<td>Area of FRP reinforcement.</td>
</tr>
<tr>
<td>$A_{cj}$</td>
<td>Area of the concrete segment $j$.</td>
</tr>
<tr>
<td>$A_{fc}$</td>
<td>Top reinforcement area.</td>
</tr>
<tr>
<td>$b$ and $d$</td>
<td>Width and effective depth of the GFRP reinforced concrete beam.</td>
</tr>
<tr>
<td>$c_b$</td>
<td>Neutral axis depth for balanced failure.</td>
</tr>
<tr>
<td>$d$</td>
<td>Effective depth of the top reinforcement.</td>
</tr>
<tr>
<td>$d_b$</td>
<td>Steel bar diameter.</td>
</tr>
<tr>
<td>$E_f$</td>
<td>Modulus of elasticity of FRP bars.</td>
</tr>
<tr>
<td>$E_{fc}$</td>
<td>Modulus of the elasticity of the top FRP bars.</td>
</tr>
<tr>
<td>$E_c$</td>
<td>Modulus of elasticity of concrete.</td>
</tr>
<tr>
<td>$f_{cr}$</td>
<td>Modulus of rupture of concrete $= 0.62 \sqrt{f_c'}$ (N/mm²).</td>
</tr>
<tr>
<td>$f_t$</td>
<td>FRP tensile stress.</td>
</tr>
<tr>
<td>$f_f$</td>
<td>FRP stress at which the concrete crushing failure mode occurs.</td>
</tr>
<tr>
<td>$f_c$</td>
<td>Cylinder compressive strength of concrete.</td>
</tr>
<tr>
<td>$f_{fu}$</td>
<td>Ultimate tensile strength of FRP bars.</td>
</tr>
<tr>
<td>$f_{cu}$</td>
<td>Ultimate cube strength of concrete under compression.</td>
</tr>
<tr>
<td>$f_{cj}$</td>
<td>Stress in segment $j$.</td>
</tr>
<tr>
<td>$f'_{c}$</td>
<td>Stress in top reinforcement.</td>
</tr>
</tbody>
</table>
Abbreviations

\(f_r\) Ultimate strength of concrete under tension.

\(F_{ct}\) The force in top reinforcements.

\(h\) Overall height of the concrete beam.

\(I_{e}\) Effective moment of inertia of the beam section.

\(I_g\) Gross moment of inertia = \(bh^3/12\)

\(I_{cr}\) Moment of inertia of transformed cracked section.

\(I_m\) Refers to \(I_e\) at the mid-span section.

\(I_{cont.end}\) Refers to \(I_e\) at the middle support section.

\(K\) Ratio of the neutral axis depth to reinforcement depth.

\(L\) Span length.

\(l_{ci}\) Lever arm for concrete compressive force \(F_{ci}\) in segment number \(i\).

\(m\) Moment value at \(x\) location.

\(M\) Moment capacity calculated using the ACI 440.1R-06.

\(M_{cr}\) Cracking moment = \(2f_{cr} I_g / h\)

\(M_a\) Applied moment.

\(n\) Arbitrary segment numbers.

\(P\) Mid-span applied load at which the deflection is computed.

\(R\) Input value for the subdivision segments of the effective depth.

\(T_c\) Concrete tension force.

\(x\) The segment location at the stress block.

\(\beta_d\) Reduction coefficient = \([0.2\rho_\gamma/\rho_b \leq 1]\).
Abbreviations

\( \beta_1 \) Strength reduction factor for the simply supported FRP reinforced concrete beam.

\( \gamma_s \) Ratio between the area of tensile steel bars and the area of concrete in tension.

\( \lambda_t \) Parameter controlling the rate of decay of concrete tensile stress.

\( \delta \) Deflection along the beam length.

\( \varepsilon_r \) Concrete tensile strain corresponding to the tensile stress.

\( \varepsilon_f \) Strain in top FRP reinforcement.

\( \varepsilon_e \) Strain of concrete.

\( \varepsilon_{cu} \) Ultimate strain in concrete.

\( \rho_f \) \((- A_f/bd)\) is the FRP reinforcement ratio.

\( \phi \) Curvature along the beam length.

\( \theta \) Slope along the beam length of the beam.
CHAPTER ONE

Introduction

1.1 General

Over the last few years several investigations have been conducted, in order to find solutions for the corrosion problem of concrete members reinforced with steel bars. As a result, methods such as galvanisation, use of stainless steel bars, cathodic protection and epoxy coatings have been tried. Such remedies are effective in some situations and the measures developed so far to mitigate the problem have met with varying degrees of success [3].

The outstanding properties of the fibre-reinforced polymers (FRP) suggest that this material may provide the solution for the problems of corrosion, high strength-to-weight ratio and the relatively low fatigue resistance of steel [4, 5]. Most FRP structural applications in civil engineering fall into two areas. The first application concerns the strengthening of structurally deficient reinforced concrete beams with FRP sheets or plates. The second involves using FRP bars, such as carbon fibre-
reinforced polymer (CFRP), glass fibre-reinforced (GFRP) or aramid fibre-reinforced polymer (AFRP) instead of steel reinforcement. The use of FRP reinforcement bars as an alternative to steel reinforcement appears very promising, especially if concrete structures are exposed to corrosive environments. However, some properties of FRP reinforcement, such as its brittleness, could have adverse effects on its performance as reinforcement for concrete members. Therefore, a better understanding of the mechanical behaviour of FRP reinforcement is needed in order that it can be used practically [5].

The current research is an attempt towards an increased understanding of the use of FRP bars as reinforcement for simply as well as continuously supported concrete beams.

### 1.2 Research aim and Objectives

The aim of this research is to investigate the significance and shortcomings of using FRP bars as longitudinal reinforcement for concrete beams. This aim will be established through the achievement of the following objectives:

1. The exploration of the behaviour of FRP reinforced concrete beams in their continuously supported state in relation to their simply supported state.

2. Performance comparison of FRP reinforcement, in relationship to the conventional steel reinforcement, using two different reinforcement types, commonly used in the field of FRP bars, carbon and glass FRP bars. This comparison will also consider the application of these two types of FRP materials for both cases of the above mentioned supporting conditions.
3. An enhanced computer programme, for the analysis of the behaviour of simply and continuously supported FRP reinforced concrete beams, in relation to load capacity, failure mode and deflection. This enhanced computer programme will be validated by the current and other research experimental results.

1.3 Research Strategy

To achieve the above aim and objectives the following research strategy approach has been employed:

- A test programme has been carried out in the laboratory on ten different FRP reinforced concrete beams in addition to a continuously supported Steel reinforced concrete beam. Five of the FRP reinforced concrete beams have been reinforced with CFRP reinforcing bars and the rest have been reinforced by GFRP bars. Each of the two sets of the FRP Reinforced beams contained three continuously and two simply supported concrete beams respectively. Each set of the FRP reinforced concrete beams had different reinforcement arrangements in terms of over and under reinforcement ratio as will be detailed in chapter three.

- The ACI 440.1R-06 design code have been evaluated against the experimental results.

- A computational programme based on sectional and longitudinal analyses has been developed.
• The experimental results obtained from this investigation as well as from other studies have been compared to the prediction of the ACI 440.1R-06 in addition to the prediction of developed computer programme.

1.4 Research Significance

Several researchers introduced FRP material for the construction industry as external strengthening for existing concrete beams. The overall success of this introduction paved the way for others to investigate the possibility of using the FRP bars as an internal reinforcement for concrete beams as mentioned earlier.

In comparison to the investigations that have been carried out in the field of using the FRP bars as internal reinforcement for concrete beams, this research has attributed the following significances:

• The limited availability of previous researches in the field of using FRP as an internal concrete reinforcement, investigated only the behaviour of simple supported concrete beams, with certain attempts to establish an analytical models that is able to predict the behaviour of the application of such reinforcement as will be reviewed in the next chapter. The present research project goes further than the already carried out work, which only looked at simply supported beams, by considering continuously supported concrete beams. Simply supported FRP reinforced concrete beams have been considered as well, in order to investigate the differences in behaviour and
the implications of the application of FRP reinforcements when used in continuously supported beams.

- Two types of FRP reinforcements have been investigated. The most widely used FRP materials, carbon fibre reinforcement polymer bars and glass fibre reinforced polymer bars have been tested, to compare their different responses as reinforcement materials, for both continuous and simply supported beams, on the one hand, and to the conventionally continuously supported steel reinforced concrete beams on the other hand.

- Employing the principle of testing the continuously supported FRP reinforced concrete beams endorsed the way to investigate the different possibilities of allocating the reinforcing bars, based on its ratio, either at the top layer or at the bottom one or at both. Combination of these different reinforcement ratios would in return conclude different flexure behaviour of failure modes and deflections. These different behaviours will be considered in this research in order to gain an in-depth understanding of how a continuously FRP reinforced concrete beam would behave and fail as a result of such type and combination of reinforcements. This issue was not considered by previous researchers, in testing the simply supported beams, since there is only one possibility of using either the over or under reinforced sections. Thus the present research, distinctly, will test the three previous possibilities, for each type of FRP reinforcement to accomplish a
comprehensive investigation of the consequence of all possible ways of using the FRP materials as reinforcement for concrete elements.

- Projects that have been involved in the investigation of the behaviour of FRP reinforced concrete elements usually involve, intentionally or unintentionally, other different factors that could contribute, directly or indirectly, to the considered concrete member behaviour of their investigation. Such factors could include either or all of the following changes or differences like geometric dimension of the tested samples, variable supporting and loading system, using different concrete batches or casting over different time intervals, and the possible use of different curing regimes. Therefore, such investigations that could be affected by some or all of the above stated factors might results in misleading conclusions. The present research has attempted to eliminate most of, if not all, the factors that might affect the performance of the tested beams, which could be related to any other reason rather than the influence of using the FRP materials as longitudinal reinforcements for concrete beams. Therefore, the following methodologies have been followed in testing the concrete beams in this research:

1. All beams have been cast on the same day, to avoid the effect that might be caused by any changes in the concrete strength of different patches.
2. All beams, either from the continuously or simply supported groups, have the same geometric features, since it has been found that depth to span ratio, for example, has certain effects on the flexure behaviour of FRP reinforced concrete beams [5].

3. Top and bottom FRP reinforcements have been designed in order to endure similar strength capacity for all identical sections, in terms of over or under-reinforcement. Therefore it is anticipated for instance that each over GFRP or CFRP reinforced section will fail at the same ultimate applied load.

4. All beams have been reinforced with the same type of steel stirrups at the same spacing distance.

- An enhanced analytical model, has been established and validated, to anticipate different behaviours; such as flexure capacity, failure mode and deflection of such FRP reinforced concrete beams. The validation has been based on the data concluded from testing the continuously supported FRP reinforced concrete beams, as a significance factor of this research as it has been detailed previously. This validation has been also based on test results obtained from the simply supported tested beams in this research and other researches.

- The above mentioned analytical model has been also validated against four different methods of predicting the flexure behaviour of concrete beams
reinforced with FRP. These methods are: the bilinear method, mean moment of inertia, Benmokrane’s method [6], and the ACI 440.1R-06

1.5 Thesis Arrangement

This present thesis consists of seven chapters. Following the current chapter, chapter two reviews the use of FRP material in the field of construction, the existing design codes and research in the field of FRP reinforced concrete beams. Chapter three details the properties of the materials used and the methodology of the test programme carried out in this research. The results of the tested beams have been illustrated also in chapter three. These results detailed the crack propagation, failure modes, load capacity, redistribution of support reactions, mid-span deflection for the GFRP and the CFRP reinforced concrete beams tested respectively. Chapter four describes the ACI 440.1R design code and its evaluation against the results obtained from this research. The evaluation contained the prediction of loads and modes of failure as well as deflection prediction for the GFRP and the CFRP reinforced concrete beams tested. Chapter five describes the first part of an analytical modelling program developed through the research to investigate the flexural behaviour of simple and continuous beams reinforced with FRP reinforcement. This chapter is devoted to producing the moment-curvature relationship of concrete sections reinforced with FRP bars. Chapter six demonstrates the longitudinal analysis part of the program which was outlined in chapter five. In particular, the flexural capacity and load-deflection relationship along the length of FRP simple and continuously supported reinforced concrete
beams. In the final chapter, seven, the principal findings and conclusions of the research described in this thesis are presented and recommendations for future works are given.
CHAPTER TWO

Literature Review

2.1 Introduction

Fibre reinforced polymer bars are considered as a potential replacement for traditional steel reinforcement in many concrete applications, especially those in severe environments [7]. FRP composites are the latest version of the very old idea of making better composite material by combining two materials, which can be traced back to the use of straw as reinforcement in bricks used by ancient civilizations (e.g. Egyptians in 800) [8].

FRP composites have been used on a limited basis in structural engineering for almost 50 years for both new construction and repair and rehabilitation of existing structures. This material has been developed into an economically and structurally viable construction material for buildings and bridges over the last 25 years [9].

The 1980s market demanded non-metallic reinforcement for specific advanced technology; the largest demand for electrically nonconductive reinforcement was in facilities for Magnetic Resonance Imager (MRI) medical equipment. FRP reinforcement became the standard in this type of construction. Other uses developed as the advantages of FRP reinforcement became better known and
desired, specifically in seawall construction, substation reactor bases, airport
runways, and electronics laboratories [10].

In 1983, the first project funded by the U.S. Department of Transportation
(USDOT) was on “Transfer of Composite Technology to Design and
Construction of Bridges” [11]. Parallel research was also being conducted on
FRPs in Europe and Japan. In Europe construction of the pre-stressed FRP
Bridge in Germany in 1986 was the beginning of use of FRP [12]. The
European BRITE/EURAM Project, “Fibre Composite Elements and Techniques
as Non-metallic Reinforcement,” conducted extensive testing and analysis of
the FRP materials from 1991 to 1996 [13]. More recently, EUROCRETE has
headed the European effort with research and demonstration projects.

In Japan more than 100 commercial projects involving FRP reinforcement were
undertaken up to the mid-1990s. These projects were instigated in the early
1980s due to the stimulation of “Japanese Concrete Crisis”, which is mainly
caused by the problems with steel reinforcement corrosion in concrete
structures. This crisis led to the FRP composite development to replace of
conventional steel bars and tendons for concrete structures in that part of the
world. Thus, many major Japanese general contractors along with material
manufacturers began to develop various types of FRP reinforcement including
rod/tendons, cables, ground anchors and grids to replace conventional
reinforcement in case of corrosion, which are the primary products available in
today's market [14].

The annual amount of FRP reinforcement used in construction reached a peak
Carbon fiber strands comprised the largest consumption among all kinds of FRP, which was followed by Aramid rods and braids. The amounts of Carbon fiber rod and Aramid fiber plate used were relatively few [16].

With respect to its applications, the number of FRP applications increased steadily at the beginning until 1996. Typically, many FRP reinforcement were applied in coastal and water channel structures, ground anchors, underground structures and non-magnetic structures, where they were often exposed to aggressive environments. Applications in coastal and water channel structures were very common by 2001 and more recently applications in non-magnetic and ground anchor structures have been developed. Although applications began to decrease after 1998, bridges still occupy the largest consumption followed by ground anchors [17].
2.2 Production Process

Current FRP reinforcing bars are commercially produced using thermosetting polymer resins (commonly, polyester and vinylester) and glass, carbon, or aramid reinforcing fibers. The most common bars produced today are glass-fiber-reinforced vinylester bars. These are recommended for use in reinforcing applications for load-bearing concrete structures. The bars are primarily longitudinally reinforced with volume fractions of fibers in the range of 50 to 60%. FRP reinforcing bars are usually produced by a process similar to pultrusion [18] and have a surface deformation or texture to develop the bond on concrete.

However, the Fiberglass Grating Manufacturers Council [19] summarised more technically the compositions and the manufacturing process of such bars as follow:

2.2.1 Composition of FRP Composites

- Resins; the primary functions of the resin are to transfer stress between the reinforcing fibers, act as a glue to hold the fibers together, and protect the fibers from mechanical and environmental damage. The most common resins used in the production of FRP grating are polyesters (including orthophthalic-“ortho” and isophthalic-“iso”), vinyl esters and phenolics.

- Reinforcements; the primary function of fibers or reinforcements is to carry load along the length of the fiber to provide strength and stiffness in
one direction. Reinforcements can be oriented to provide tailored properties in the direction of the loads imparted on the end product.

- Fillers; fillers are used to improve performance and reduce the cost of a composite by lowering compound cost of the significantly more expensive resin and imparting benefits such as shrinkage control, surface smoothness, and crack resistance.

- Additives; additives and modifier ingredients expand the usefulness of polymers, enhance their process-ability or extend product durability

Each of these constituent materials or ingredients play an important role in the processing and final performance of the end product.

### 2.2.2 Manufacturing

In this section, those manufacturing processes typically used to make products found in the grating market are covered. Unique to the composites industry is the ability to create a product from many different manufacturing processes. There are a wide variety of processes available to the composites manufacturer to produce cost efficient products. Each of the fabrication processes has characteristics that define the type of products to be produced. This is advantageous because this expertise allows the manufacturer to provide the best solution for the customer [20].
2.2.2.1 Pultrusion Process

Pultrusion is a continuous process for manufacturing composites that have a cross-sectional shape. The process consists of pulling a fiber-reinforcing material through a resin impregnation bath and through a shaping die. The dimensions and shape of the die will define the finished part being fabricated. Inside the metal die, heat is transferred initiated by precise temperature control to the reinforcements and liquid resin. The heat energy activates the curing or polymerization of the thermoset resin changing it from a liquid to a solid. The solid laminate emerges from the Pultrusion die to the exact shape of the die cavity. The laminate solidifies when cooled and it is continuously pulled through the Pultrusion machine and cut to the desired length. The process is driven by a system of caterpillar or tandem pullers located between the die exit and the cut-off mechanism. This process allows for optimized fiber architectures with uniform color eliminating the need for many painting requirements. However, the initial capital investment for pultrusion is generally higher than the mold processes that will be explained in the following section.

2.2.2.2 Molding Process

Liquid resin and continuous fiberglass roving are systematically laid in the mould, layer after layer manually, to produce the desired thickness and panel dimensions. The finished molds are set aside for a predetermined time to allow the panel to cure. The panel is then ejected from the mould. The moulds are cleaned and prepared for the process to
begin again. The reinforcements may include a variety of fiber types, in various forms such as continuous fibers, mat or woven type construction as well as a hybrid of more than one fiber type. Vacuum is sometimes used to enhance the resin flow and reduce void formation. The part is typically cured with heat. In some applications, the exothermic reaction of the resin may be sufficient for proper cure. The mold surface can produce a high quality finish. However, size of produced parts is limited by the mold.

2.3 **FRP Applications in Civil Engineering**

The use of FRPs within civil engineering systems has been discussed in several articles [21]. These investigations have highlighted two fields of application of FRPs: the repair of existing structures and the construction of new structures incorporating FRPs as part of the primary structural system.

2.3.1 **FRP Use in Rehabilitation and Strengthening**

In the field of rehabilitation and strengthening of existing structures, including repair to damage caused by seismic vibrations, FRPs have an immense potential [22, 23]. FRPs can be bonded to the surfaces of structural members, bridging cracks in concrete to provide extra load-carrying material and thereby restoring or increasing the structural members’ capacity to withstand loadings. Columns can be wrapped with FRP to provide total constraint of concrete, thereby increasing the capacity of the column to withstand static and dynamic loadings. However, this added material, whilst increasing the capacity of structures, adds little extra dead-loading or volume to the existing structure.
Minimisation of extra dead-loading is of vital importance to avoid the necessity of strengthening other parts of a structure to compensate for the extra dead-weight added [24]. On the other hand, various parameters such as the internal steel reinforcement ratio, external plate dimension and the adhesive layer properties are found to significantly affect the behavior of a beam strengthened with external reinforcement [25].

A notable application of such use of FRP materials, as it has been stated earlier, is in the strengthening of bridges. The application of externally-bonded plates to an existing bridge can increase the allowable vehicle weights and traffic volume [26]. The repair and maintenance of existing infrastructure is of particular relevance, worldwide, since there is a significantly large number of structures that are reaching the end of their original design life and are in need of replacement or repair [27]. However, Rosenboom [28] has stated recently that the presence of a fiber reinforced polymer (FRP) strengthening material bonded to the tension face of a reinforced concrete beam will restrict but not prevent the opening of intermediate flexural cracks due to applied loading. Test results indicate that displacements at the toe of flexural cracks create stress concentrations at the interface of the FRP laminate and the beam, leading to the development of localized interface cracks which typically propagate, under the effect of the load, to join the original flexural cracks and cause de-lamination of the FRP system. This type of FRP de-lamination is commonly termed intermediate crack (IC) de-bonding. In attempt to control such phenomena Rosenboom proposed an analytical model which characterises the interface shear stress based on two distinct sources: the change in the applied moments
along the length of the member, and stress concentrations at the intermediate cracks.

2.3.2 FRP Use as the Primary Structural Reinforcement

The second field of application of FRP is in innovative structural systems, mainly in bridge design and construction. There are many advantages in this form of bridge construction because of the reduced weight, which results in savings on materials and costs. However, the following section will explore the properties of such material that entitled it to be used as reinforcement and an alternative for the traditional steel reinforcement for concrete structures.

2.3.2.1 Properties of FRP Reinforcement Bars

FRP bars produced from the previously detailed manufacturing process demonstrate mechanical properties typically quite different from those of steel bars and depend mainly on both matrix and fibers type, as well as on their volume fraction. Moreover; FRP bars have lower weight, lower Young’s modulus but higher strength than steel.

The most commonly available fiber types are the carbon, the glass and the aramid fibers. The non-structural importance of the resins, as well as their high cost, a minimum resin volume ratio is always desirable. However, the maximum fibre ratio that can be practically achieved is normally below 70% [29]. However, the American Composites Manufacturers Association [20] detailed the properties of the three types as follow:
2.3.2.1.1 Glass Fibers

Based on an alumina-lime-borosilicate composition, “E” glass produced fibers are considered the predominant reinforcement for polymer matrix composites due to their high electrical insulating properties, low susceptibility to moisture and high mechanical properties. Other commercial compositions include “S” glass, with higher strength, heat resistance and modulus, as well as some specialized glass reinforcements with improved chemical resistance, such as AR glass (alkali resistant).

Glass fibers used for reinforcing composites generally range in diameter from 0.009 to 0.023 mm (9 to 23 microns). Fibers are drawn at high speeds, approaching 200 miles per hour, through small holes in electrically heated bushings. These bushings form the individual filaments. The filaments are gathered into groups or bundles called “strands.” The filaments are attenuated from the bushing, water and air cooled, and then coated with a proprietary chemical binder or sizing to protect the filaments and enhance the composite laminate properties. The sizing also determines the processing characteristics of the glass fiber and the conditions at the fiber-matrix interface in the composite. Glass is generally a good impact resistant fiber but weighs more than carbon or aramid. Glass fibers have excellent characteristics, equal to or better than steel in certain forms. The lower modulus requires special design treatment where stiffness is critical. Composites made from this material exhibit very good electrical and thermal insulation properties. Glass fibers are also
transparent to radio frequency radiation and are used in radar antenna applications.

As an illustrative example of the characteristics of the GFRP bars, the following table presented data of current production of GFRP bars and is believed to be reliable and to represent the best available characterisation of the product as of may 2007.

<table>
<thead>
<tr>
<th>Bar Size</th>
<th>Diameter (mm)</th>
<th>Area (mm²)</th>
<th>Guaranteed Tensile Strength (MPa)</th>
<th>Ultimate Tensile Load (kN)</th>
<th>Tensile Modulus of Elasticity (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>6</td>
<td>31.67</td>
<td>825</td>
<td>26.2</td>
<td>40.8</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>71.26</td>
<td>760</td>
<td>54.0</td>
<td>40.8</td>
</tr>
<tr>
<td>4</td>
<td>13</td>
<td>126.7</td>
<td>690</td>
<td>87.3</td>
<td>40.8</td>
</tr>
<tr>
<td>5</td>
<td>16</td>
<td>197.9</td>
<td>655</td>
<td>130</td>
<td>40.8</td>
</tr>
<tr>
<td>6</td>
<td>19</td>
<td>285.0</td>
<td>620</td>
<td>177</td>
<td>40.8</td>
</tr>
<tr>
<td>7</td>
<td>22</td>
<td>387.9</td>
<td>586</td>
<td>227</td>
<td>40.8</td>
</tr>
<tr>
<td>8</td>
<td>25</td>
<td>506.7</td>
<td>550</td>
<td>279</td>
<td>40.8</td>
</tr>
<tr>
<td>9</td>
<td>29</td>
<td>641.3</td>
<td>517</td>
<td>332</td>
<td>40.8</td>
</tr>
<tr>
<td>10</td>
<td>32</td>
<td>791.7</td>
<td>480</td>
<td>382</td>
<td>40.8</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>31.67</td>
<td>825</td>
<td>26.2</td>
<td>40.8</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>71.26</td>
<td>760</td>
<td>54.0</td>
<td>40.8</td>
</tr>
</tbody>
</table>

Table 2-1 Physical and Mechanical properties of GFRP Reinforcement Manufactured by Hughes Brothers USA.

### 2.3.2.1.2 Carbon Fibers

Carbon fiber is created using polyacrylonitrile (PAN), pitch or rayon fiber precursors. PAN based fibers offer good strength and modulus values up to 6 $x10^5$ MPa. They also offer excellent compression strength for structural
applications up to 7000 MPa.[20]. Pitch fibers are made from petroleum or coal tar pitch. Pitch fibers extremely high modulus values (up to $1 \times 10^5$ MPa.) and favorable coefficient of thermal expansion make them the material used in space/satellite applications. Carbon fibers are more expensive than glass fibers, however carbon fibers offer an excellent combination of strength, low weight and high modulus. The tensile strength of carbon fiber is equal to glass while its modulus is about three to four times higher than glass [20].

Carbon fibers are supplied in a number of different forms, from continuous filament tows to chopped fibers and mats. The highest strength and modulus are obtained by using unidirectional continuous reinforcement. Twist-free tows of continuous filament carbon contain 1,000 to 75,000 individual filaments, which can be woven or knitted into woven roving and hybrid fabrics with glass fibers and aramid fibers.

Carbon fiber composites are more brittle (less strain at break) than glass or aramid. Carbon fibers can cause galvanic corrosion when used next to metals. A barrier material such as glass and resin is used to prevent this occurrence.

2.3.2.1.3 Aramid Fibers (Polyaramids)

Aramid fiber is an aromatic polyimid that is a man-made organic fiber for composite reinforcement. Aramid fibers offer good mechanical properties at a low density with the added advantage of toughness or damage/impact resistance. They are characterized as having reasonably high tensile strength, a medium modulus, and a very low density as compared to glass and carbon. The
tensile strength of aramid fibers are higher than glass fibers and the modulus is about fifty percent higher than glass. These fibers increase the impact resistance of composites and provide products with higher tensile strengths. Aramid fibers are insulators of both electricity and heat. They are resistant to organic solvents, fuels and lubricants. Aramid composites are not as good in compressive strength as glass or carbon composites. Dry aramid fibers are tough and have been used as cables or ropes, and frequently used in ballistic applications.

2.3.2.2 FRP and Steel Reinforcement Bars

A Fiber Reinforced Polymer composite is defined as a polymer (plastic) matrix, either thermoset or thermoplastic, that is reinforced (combined) with a fiber or other reinforcing material with a sufficient aspect ratio (length to thickness) to provide a discernable reinforcing function in one or more directions. FRP composites are different from traditional construction materials such as steel or aluminum. FRP composites are anisotropic (properties apparent in the direction of the applied load) whereas steel or aluminum is isotropic (uniform properties in all directions, independent of applied load). Therefore, FRP composite properties are directional, meaning that the best mechanical properties are in the direction of the fiber placement. Composites are similar to reinforced concrete where the rebar is embedded in an isotropic matrix called concrete.

Financially; costs for FRP bars are typically 3 times that of conventional reinforcement or 50% that of stainless steel bars, which results in an additional...
cost of about 4-8% for a typical highway bridge project. However, there are several other issues that need to be considered in a cost/benefit analysis, these include: the amount of bar required for reinforcing structures in view of the different mechanical properties; the effect of redesign and reduction in concrete usage on whole component cost; the reduction in weight of the reinforcing bar and time saved during construction of cages; the reduction in weight of section and subsequent reduced craneage and transportation costs. It has been estimated that typically a 15% weight saving in pre-cast sections may be anticipated due to cover reduction, although some case study analyses have indicated significantly higher savings are achievable (soil nail spreader plates potentially from 200-47kg) [30].

Environmentally; there have been conflicting studies relating to the comparison between producing steel and FRP products. Overall it is felt that these two materials have similar environmental effects with the difference that steel is more readily recycled at the end of life. However, FRP products are designed to last longer and this advantage may therefore be challenged [30].

When considering the application of FRP reinforcing bars in concrete however, several advantages are evident; the potential reduction in concrete cover required results in lower utilisation of cement. Cement produces 20 times more CO₂ emissions during processing than steel or FRP reinforcing bars. In addition, concrete structures containing steel reinforcement are difficult to recycle since the steel needs to be extracted prior to crushing. This is not the
case with FRP reinforcing bars. Lighter weight products also provide the opportunity to reduce emissions during transportation.

Furthermore, the visual environment is enhanced since there is no rust staining or concrete spalling associated with the use of FRP reinforcing bars which in addition have no toxic or other fume emissions during use as well [30].

However; the corrosion resistance of the E-glass type of Glass fibre composites (GFRP) has not inhibited its acceptance. Nevertheless, there is concern from a series of problems, mainly from the chemical industry, of failures in service caused by corrosion and internal laminate stresses [31].

It is well recognised that one of the functions of the matrix resin is to protect the reinforcement from environmental attack. It was originally believed that the resistance of the fibre was of little or no importance, and this view still generally applies today. But at the end of the 70's, it was discovered that GFRP laminates could suffer from stress-corrosion because of internal stresses, which principally affects glass fibres in acid environments’. Stress-corrosion is the combined action of stress and environmental corrosive attack. It was recognised that fibres required additional protection, such as a pure resin surface layer, or gel coat, a resin layer with a superficial barrier film, or a resin-rich coaling incorporating a surface veil. The same approach was adopted in the case of alkali exposure, where specially formulated gel coats and resins have been developed with outstanding chemical resistance.

The performance of GFRP in aggressive alkali has been demonstrated by several studies to be largely unaffected by glass corrosion, provided that the
fibres are well protected. However, there is always a risk that the protective layer may not be intact, particularly at cut ends, or from damage during handling or installation. Such possibilities have raised concerns over the durability of these materials in structures designed to last for 25 years, or more, and this may contribute to a reluctance to specify composites in some cases.

There is still an important need for further knowledge on the corrosion properties of laminates, resins and glass fibres, as witnessed by the incidence of cracking and damage in GFRP elements due to chemical attack [31].

Access to the fibres by the agent can originate from diffusion into the resin, or through the thermoplastic coating, or via micro-cracks in the surface and or other laminate defects. If corrosive agents reach the fibres, chemical attack can weaken them. However; using a glass fibre with better resistance to stress-corrosion could minimise the risk of such diffusion [31].

However; table 2-2 lists some of the advantages and disadvantages of FRP reinforcement for concrete structures when compared with conventional steel reinforcement, as reported by ACI 440.1R-06 [32] and Guadagnini [29].

The determination of both the geometrical and mechanical properties of FRP bars requires the use of specific procedures [33]. FRP bars have density ranging from one fifth to one forth than that of steel; the reduced weight eases the handling of FRP bars on the project site [14]. The tensile properties of FRP are what make them an attractive alternative to steel reinforcement. When loaded in tension, FRP bars do not exhibit any plastic behavior (yielding) before rupture. Therefore FRP reinforcement is not recommended for moment frames.
or zones where moment redistribution is required [29]. Table 2-3 gives the most common tensile properties of FRP reinforcing bars, in compliance with the values reported by ACI 440.1R-06 [32].

<table>
<thead>
<tr>
<th>Disadvantages of FRP reinforcement</th>
<th>Advantages of FRP reinforcement</th>
</tr>
</thead>
<tbody>
<tr>
<td>GFRP reinforcement suffers from stress Corrosion.</td>
<td>Higher ratio of strength to self weight (10 to 15 times greater than steel).</td>
</tr>
<tr>
<td>Low modulus of elasticity (varies with type of reinforcing fiber)</td>
<td>Carbon and aramid fibre reinforcement have excellent fatigue characteristics</td>
</tr>
<tr>
<td>Lack of ductility (No yielding before brittle rupture)</td>
<td>Excellent corrosion resistance and electromagnetic neutrality</td>
</tr>
<tr>
<td>Higher raw material cost</td>
<td>Low thermal and electric conductivity (for glass and aramid fibers)</td>
</tr>
<tr>
<td>Low transverse strength (varies with sign and direction of loading relative to fibers)</td>
<td>Lightweight (about 1/5 to 1/4 the density of steel)</td>
</tr>
<tr>
<td>Susceptibility of damage to polymeric resins and fibers under ultraviolet radiation exposure</td>
<td>High fatigue endurance (varies with type of reinforcing fiber)</td>
</tr>
<tr>
<td>Low durability of glass fibers in a moist Environment</td>
<td>Nonmagnetic</td>
</tr>
<tr>
<td>Low durability of some glass and aramid fibers in an alkaline environment</td>
<td></td>
</tr>
<tr>
<td>High coefficient of thermal expansion perpendicular to the fibers, relative to concrete</td>
<td></td>
</tr>
</tbody>
</table>

Table 2-2 Advantage and disadvantage of FRP Reinforcement [27,28].
<table>
<thead>
<tr>
<th></th>
<th>AFRP</th>
<th>CFRP</th>
<th>GFRP</th>
<th>Steel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal yield stress, MPa</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>276 to 517</td>
</tr>
<tr>
<td>Tensile strength, MPa</td>
<td>1720 to 2450</td>
<td>600 to 3690</td>
<td>483 to 1600</td>
<td>483 to 690</td>
</tr>
<tr>
<td>Elastic Modulus, GPa</td>
<td>41 to 125</td>
<td>120 to 580</td>
<td>35 to 51</td>
<td>200</td>
</tr>
<tr>
<td>Yield strain %</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>0.14 to 0.25</td>
</tr>
<tr>
<td>Rupture strain %</td>
<td>1.9 to 4.4</td>
<td>0.5 to 1.7</td>
<td>1.2 to 3.1</td>
<td>6.0 to 12.0</td>
</tr>
</tbody>
</table>

Table 2-3 Typical Tensile Properties of Reinforcing Steel & FRP Bars[32].

One of the principle advantages of FRP reinforcement is the ability to configure the reinforcement to meet specific performance and design objectives. For example, FRP reinforcement may be configured in rods, bars, plates, and strands. Within these categories, the surface texture of the FRP reinforcement may be modified to increase or decrease the bond with the surrounding concrete. Unlike conventional steel reinforcement, there are no standardized shapes, surface configurations, fiber orientation, constituent materials and proportions for the final products. A sample of different cross sectional shapes and deformation systems of FRP reinforcing bars is shown in Figure 2-2.

Solid GFRP wrapped and sand coated bars in addition to CFRP fabric texture coated bars were used in the present investigated research. Figure 2-3 depicts the typical stress-strain behavior of FRP bars compared to that of steel bars [34]. The Italian guidelines CNR-DT 203-2006, instead, prescribes that all types of FRP bars can be used as reinforcement bars provided that the characteristic
strength is not lower than 400 MPa, and the average value of the Young’s modulus of elasticity in the longitudinal direction is not lower than 100 GPa for CFRP bars, 35 GPa for GFRP bars, and 65 GPa for AFRP bars. The compressive modulus of elasticity of FRP reinforcing bars appears to be smaller than its tensile modulus of elasticity, in fact most of FRP reinforced concrete Design Guidelines suggest not to rely upon strength and stiffness contributions provided by the compressed FRP bars [35], however, further research is needed in this area [1].
Figure 2-2 Sample FRP Reinforcement Configurations[1]
The longitudinal coefficient of thermal expansion is dominated by fiber properties, while the transverse coefficient is dominated by the resin; typical values of the coefficient of thermal expansion in the longitudinal and transversal directions, $\alpha_l$ and $\alpha_t$, respectively, of composite bars with a fibers volume fraction ranging between 50% and 70%, are reported in Table 2-4 [36]; higher values of the transversal coefficients of thermal expansion, combined with the Poisson's effect in the case of compressed reinforcements, can be responsible for circumferential tensile stresses that allow the formation of cracks in the radial direction that may endanger the concrete-FRP bond [37].

<table>
<thead>
<tr>
<th>Bar</th>
<th>$\alpha_l$ [$10^{-6} \text{C}^{-1}$]</th>
<th>$\alpha_t$ [$10^{-6} \text{C}^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>AFRD</td>
<td>-6.0/-2.0</td>
<td>60.0/80.0</td>
</tr>
<tr>
<td>CFRP</td>
<td>-2.0/0.0</td>
<td>23.0/32.0</td>
</tr>
<tr>
<td>GFRP</td>
<td>6.0/10</td>
<td>21.0/23.0</td>
</tr>
</tbody>
</table>

Table 2-4 Coefficients of Thermal Expansion

![Figure 2-3 Stress-Strain Curves for FRP & Steel Bars](image)
FRP reinforcing bars are susceptible to static fatigue phenomenon (creep rupture), which is a progressive reduction of strength under long term loads. In general, carbon fibers are the least susceptible to creep rupture, whereas aramid fibers are moderately susceptible, and the glass fibers are the most susceptible [14]; such phenomenon is also highly influenced by environmental factors, such as temperature and moisture.

The bond between the FRP bar and the surrounding concrete is ensured by propagation of stresses whose values depend on bar geometry, chemical composition and physical characteristics of its surface as well as concrete compressive strength. The latter parameter is less important for FRP bars than for steel bars [29].

The bond splitting behaviour of reinforcing bars in concrete is a critical issue in the design of reinforced concrete structures [38]. The bond between concrete and reinforcing bars is one of the main traits of reinforced concrete structures. This is the case for any type of reinforcement, including FRP composite materials. Further details of the influence of such phenomenon on the flexural behaviour of FRP reinforced concrete members will be addressed at later stage of this chapter.

2.4 Review of Existing FRP Reinforced Concrete Design Guidelines

In addition to the ACI Design Guidelines (ACI 440.1R-01, ACI 440.1R-03, ACI 440.1R-06), a number of other design guides have been published for FRP-reinforced concrete. These include Japanese (BRI, 1995; JSCE, 1997),
Canadian (ISIS, 2001; CSA, 2002) and European guides. Table 5 gives a summary of the historical development of the existing documents ruling the design of internal FRP reinforced concrete structures [39].

<table>
<thead>
<tr>
<th>Year</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>1970's</td>
<td>The use of Fibre reinforcement in Concrete</td>
</tr>
<tr>
<td>1987</td>
<td>JSCE &amp; Canadian Society of Civil Engineers established a committee on fibre reinforced materials</td>
</tr>
<tr>
<td>1991</td>
<td>ACI founded ocommittee 440 on FRP external and internal reinforcement</td>
</tr>
<tr>
<td>1996</td>
<td>JSCE &amp; EUROCRETE published a set of design recommendations for FRP RC</td>
</tr>
<tr>
<td>1998</td>
<td>CSCE published a set of design recommendations for FRP RC in bridges</td>
</tr>
<tr>
<td>2001</td>
<td>ACI Committee 440 published the first version of their design recommendation for internal FRP reinforcement</td>
</tr>
<tr>
<td>2003</td>
<td>ACI Committee 440 published the second version of their design recommendation for internal FRP reinforcement</td>
</tr>
<tr>
<td>2004</td>
<td>The National Research Council (CNR) published the Italian design recommendation for externally bonded FRP reinforcement</td>
</tr>
<tr>
<td>2006</td>
<td>(CNR) &amp; ACI Committee 440 published the Italian design recommendation and the third version for internal FRP reinforcement respectively</td>
</tr>
</tbody>
</table>

Table 2-5 Chronological Development of Documents for Internal FRP Reinforcement [1, 40]

The recommendations ruling the design of FRP reinforced concrete structures currently available are mainly given in the form of modifications to existing steel reinforced concrete Codes of Practice, which predominantly use the limit state design approach. Such modifications consist of basic principles, strongly influenced by the mechanical properties of FRP reinforcement, and empirical equations based on experimental investigations on FRP reinforced concrete elements.
With respect to steel, when dealing with FRP reinforcement the amount of reinforcement to be used has to be determined by a different approach, due to the lower stiffness and the high strength of composite materials. In fact, for FRP reinforcement, the strength to stiffness ratio is an order of magnitude greater than that of steel, and this affects the distribution of stresses along the section. Hence, when considering a balanced section, a condition desired for steel reinforced concrete design, the neutral axis depth for FRP reinforced concrete sections would be very close to the compressive end [41]. This implies that for such a section, a larger amount of the cross section is subjected to tensile stresses and the compressive zone is subjected to a greater strain gradient. Hence, for similar cross sections to that of steel, much larger deflections and less shear strength are expected [42].

The following statement reported in the ACI 440.1R-06 can be considered as a principle that is universally accepted by the referenced guidelines: “These design recommendations are based on limit state design principles in that an FRP reinforced concrete member is designed based on its required strength and then checked for fatigue endurance, creep rupture endurance, and serviceability criteria. In many instances, serviceability criteria or fatigue and creep rupture endurance limits may control the design of concrete members reinforced for flexure with FRP bars especially AFRP and GFRP that exhibit low stiffness”. Nevertheless, also significant differences occur among the available FRP reinforced concrete documents; for example, when considering the limit state philosophy, two main design approaches may be distinguished; if one takes into account the inequality:
where }$R$ is the resistance of member and }$S$ is the load effect, the two different
design approaches are:

- The American-like design approach, where Eq. (2.1) becomes:

\[ \varphi R_n \geq S_u, \] (2.2)

where }$R_n$ being the nominal strength of member (depending on the characteristic
strength of materials); }$\varphi$ is a strength reduction factor and }$S_u$ is the
corresponding design load effect, obtained by amplifying the applied loads by
appropriate coefficients.

- The Eurocode-like design approach, where Eq. (2.1) turns into:

\[ R_u \geq S_d, \] (2.3)

where }$R_u$ is the ultimate resistance of member, computed as a function of the
design strength of material, derived by multiplying the characteristic materials
strength by material safety factors; and }$S_d$ is the design load effect,
corresponding to }$S_u$ [1].

In conclusion the reduction applied on the resistance by the American
Standards through the }$\varphi$ factor in the Euro code-like Standards corresponds to
the reduction applied on the materials resistance; in other words the nominal
value of resistance computed in the American Standard is a function of the Euro
code-like characteristic (namely guaranteed in ACI codes) values of material
strengths. [1]

In particular for the flexural design, all available guidelines on FRP reinforced
concrete structures distinguish between two types of flexural failure. Depending
on the reinforcement ratio of balanced failure, $\rho_{fb}$, to be checked in the design procedure; if the actual reinforcement ratio, $\rho_f$, is less than $\rho_{fb}$, it is assumed that flexural failure occurs due to rupture of FRP reinforcement, whereas if $\rho_{fb}$ is greater than $\rho_f$, then it is assumed that the element will fail due to concrete crushing. In the ideal situation where $\rho_f$ is equal to $\rho_{fb}$, the concrete element is balanced and hence, flexural failure would occur due to simultaneous concrete crushing and rupture of FRP reinforcement. It should be noted that, for FRP reinforced concrete structures, the concept of balanced failure is not the same as in steel reinforced concrete construction, since FRP reinforcement does not yield and, hence, a balanced FRP reinforced concrete element will still fail in a sudden, brittle manner; accordingly, a concrete crushing failure can be considered as the ductile mode of failure of an FRP reinforced concrete section [29]. However, further detail on types of failure modes will be illustrated at later stage of this chapter.

2.4.1 Japanese Design Guideline

The Japan Society of Civil Engineers (JSCE) Design Guidelines (JSCE,1997) [43] are based on modification of the Japanese steel reinforced concrete code of practice, and can be applied for the design of concrete reinforced or pre-stressed with FRP reinforcement; the analytical and experimental phases for FRP construction are sufficiently complete [44]. Effects to prescribe specifications for the design and construction of concrete structures with FRP reinforcements started in Japan in the 1980s [45].
Examples of specifications for internal reinforcements completed by the middle of the 1990s are as follows:

1. Recommendation for design and construction of concrete structures using continuous fiber reinforcing materials

2. Guideline for Structural design of FRP reinforced concrete buildings in Japan


The Design and construction recommendation in item 1 above are based on the JSCE Standard specification for design and construction of concrete structures, which is for concrete structures in general (JSCE 1986a,b) [46, 47]. The recommendations for construction in Item1 deal with issues such as FRP constituent materials, FRP storage and handling, Assembly and placement of FRP reinforcements, precautions in concrete placing and tendon jacking, and quality control.

Items 2 and 3 listed above are intended for building structures. These specifications were developed on 1993 as the final output of the research and development project ‘Effective Utilisation of advanced composite on construction’ sponsored by the ministry of construction of the Japanese government. Item 2 adopts a limit state-based design method with specific provisions somewhat different from those of item 1 [48].

The guideline provides a set of partial safety factors for the ultimate, serviceability and fatigue limit states as indicated on Table 2-6.
<table>
<thead>
<tr>
<th>Limit state</th>
<th>AFRP</th>
<th>CFRP</th>
<th>GFRP</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ultimate</strong></td>
<td>1.15</td>
<td>1.15</td>
<td>1.3</td>
</tr>
<tr>
<td><strong>Serviceability</strong></td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td><strong>Fatigue</strong></td>
<td>1.15</td>
<td>1.15</td>
<td>1.3</td>
</tr>
</tbody>
</table>

Table 2-6 Partial safety factors, proposed for FRP reinforcement by JSCE (1997a)

It is noted that the partial safety factors adopted for the ultimate and fatigue limit states are slightly higher than the ones used for steel reinforcement. The design model adopted for the ultimate limit state of bending covers both types of flexural failure; however, there is no information about the predominant mode of flexural failure that would result from the application of the proposed partial safety factors. The guideline may also be utilised as a reference document, since it includes general information about different types of FRP reinforcement, quality specifications, and characterisation tests for FRP materials [44].

**2.4.2 European Design Guideline**

Practical guidance on the structural design of polymer composites was provided for the first time in Europe by Clarke [49] in his book ‘Structural Design of Polymer Composites’ [50]. This document was based on the available technical information up to 1994 in the field of composites and has been written and compiled by members of the European Structural Polymeric Composites Group (EUROCOMP). The EUROCOMP Design Code and Handbook published in the above stated book, provides design recommendations for polymer composites in general.
In 2002, the European Committee for Standardization (CEN) released the EN 13706 standard, a normative document that merely defines two classes of pultruded profiles, associated with minimum values of material properties, not providing any design guidance [51]. In 2007, the Italian National Research Council published the first national Design Guidelines for structures. However, these specifications are mandatory only in Italy [52]. It is also worth mentioning that most textbooks on the mechanics of composite materials and composite structures refer to aerospace and mechanical engineering applications with the exception of a recent publication by Bank [53] which provides a comprehensive set of design rules for FRP structures, written in a civil engineering format. It has been reported by the institute of structural engineers [54] that two additional Eurocodes could be produced to cover the increasing use of structural glass and fibre-reinforced polymer (FRP) composites in the construction industry. Following extensive industry consultation, the European Commission’s Joint Research Centre (JRC) in Italy recently submitted formal justification for new structural design standards for structural glass and FRP composites to the Directorate General Enterprise and Industry (DG ENTR). If approved, the new codes could sit alongside the six material-specific sets of Eurocodes that currently cover concrete, steel, composite steel and concrete, timber, masonry and aluminium. JRC’s proposal for a new European design code for glass structures, parts or kits resulted from consultation with manufacturers of glass products, the construction industry, design and consulting engineers, standardisation and certification bodies and academia. A similar proposal for FRP composites was
prepared by JRC in consultation with the European Composites Industry Association, the UK Highways Agency and an ad-hoc JRC working group [54]. However, The available European Design Guidelines[3] are based on modifications to British (BS8110, 1997) [55] and European reinforced concrete codes of practice (ENV 1992-1-1) [56]. The guidelines include a set of partial safety factors for the material strength and stiffness that take into consideration both the short and long term structural behaviour of FRP reinforcement; and hence, the adopted values are relatively high when compared with the values adopted by other guidelines. The guidelines do not make any distinction between the two types of flexural failure and in addition, they do not provide clear indications about the predominant failure mode, which would result from the application of these partial safety factors [42].

Research into the design and safety philosophy of the British (BS8110) and European (Eurocode 2) concrete Codes of Practice [57] demonstrated that design based on balanced reinforced concrete sections achieves its objectives and leads to safe structures. However, it was found that this principle may occasionally result in reinforced concrete elements that exhibit a brittle flexural behaviour, especially when using the new partial safety factor (1.05) adopted for steel reinforcement by BS 8110 in 1997 [58]. Examination of the design and safety philosophy of the European Design Guideline [59, 60] led to the general conclusion that the design of FRP reinforced concrete elements can not be based on the philosophy developed for steel reinforced concrete elements.

The main findings can be summarised as follows:
• For normal-strength concrete, the tensile strength of longitudinal FRP reinforcement is not fully utilised and hence, concrete crushing is the most probable type of flexural failure.

• The use of partial safety factors for longitudinal reinforcement may not be essential for the design of FRP reinforced concrete beams, if the type of flexural failure intended at design is due to concrete crushing.

• The assumption that the use of a specific value of partial safety factors for longitudinal reinforcement would always lead to the desired type of flexural failure is not valid for all design configurations, especially for the large values of partial safety factors for longitudinal reinforcement.

• The ratio of permanent to variable load (PVL ratio) has the greatest effect on the probability of failure and thus, different partial safety factors could be used for different types of structures [58].

However, it has been recommended recently by a group of five European researchers [51] that further research work is needed to obtain in-depth understanding of the structural behaviour, for the FRP reinforced members, and to provide additional validation for the design methods that have been proposed.

2.4.3 The Canadian Standards Association

The Canadian Standards Association (CSA) published two documents related to the use of FRP, namely, “Canadian Highway Bridge Design Code” [61] and “Design and Construction of Building Components with Fiber-Reinforced Polymers” [62]. The earlier stated Code applies to the design, evaluation, and
structural rehabilitation design of fixed and movable highway bridges in Canada. The latter stated standard, CSA S806-02, provides requirements for the design and evaluation of building components of fibre-reinforced polymers in buildings and of building components reinforced with FRP materials. This design guideline is the most recently issued Canadian guidelines on the design and construction of building components with FRP. In addition to the design of concrete elements reinforced or pre-stressed with FRP, the guidelines also include information about characterization tests for FRP internal reinforcement. The guideline was approved, in 2004, as a national standard of Canada, and is intended to be used in conjunction with the national building code of Canada [63].

The document prescribes that “the factored resistance of a member, its cross-sections, and its connections shall be taken as the resistance calculated in accordance with the requirements and assumptions of this Standard, multiplied by the appropriate material resistance factors”. As for the predominant mode of failure, the CSA S806-02 remarks that “all FRP reinforced concrete sections shall be designed in such a way that failure of the section is initiated by crushing of the concrete in the compression zone”. It is based on limit states design principles, but this standard does not apply to the design of fibre-reinforced concrete (FRC), except for FRC/FRP cladding as defined in Clause 7.3 and Clause 13 of this standard [64].

The Canadian network of centres of excellence on intelligent sensing for innovative structures has also published a design manual that contains design
provisions for FRP reinforced concrete structures (ISIS, 2001) [65]. The guidelines also provide information about the mechanical characteristics of commercially available FRP reinforcement. This guideline is also based on modifications to existing steel reinforced concrete Codes of Practice, assuming that the predominant mode of failure is flexural, which would be sustained due to either concrete crushing (compressive failure) or rupture of the most outer layer of FRP reinforcement (tensile failure) [65].

### 2.4.4 American Design Guideline

The American Concrete Institute (ACI) Design Guidelines for structural concrete reinforced with FRP Bars (ACI 440 1R-06) are primarily based on modifications of the ACI-318 steel code of practice (ACI 318-02). The document only addresses non-prestressed FRP reinforcement (concrete structures prestressed with FRP tendons are covered in ACI 440.4R). The basis for this document is the knowledge gained from worldwide experimental research, analytical research work, and field applications of FRP reinforcement. Such globalisation in adopting the various sources of researches promoted the ACI design code to be the most popular and widely accepted design code in the field of FRP reinforcement.

The ACI 440.1R design philosophy is based on the concept that the brittle behaviour of both FRP reinforcement and concrete allows consideration to be given to either FRP rupture or concrete crushing as the mechanisms that control failure. Both failure modes (FRP rupture and concrete crushing) are acceptable in governing the design of flexural members reinforced with FRP.
bars provided that strength and serviceability criteria are satisfied. To compensate for the lack of ductility, the member should possess a higher reserve of strength. The margin of safety suggested by this guide against failure is therefore higher than that used in traditional steel-reinforced concrete design. Nevertheless, based on the findings of Nanni [66] the concrete crushing failure mode is marginally more desirable for flexural members reinforced with FRP bars, since by experiencing concrete crushing a flexural member exhibits some plastic behaviour before failure [67]. The ACI440.1R guideline uses different values of strength reduction factors for each type of flexural failure, while for the shear design it adopted the value of $\varphi$ used by ACI318 for steel reinforcement. In addition, environmental reduction factors are applied on the FRP tensile strength to account for the long-term behaviour of FRP reinforced concrete members [68]. Chapter four of this thesis details further the implementation of this Design Guideline.

2.4.5 Design Guidelines and durability

The environmental conditions must be taken into account from the start of the design process, so that their influences with respect to the durability are considered and if needed protective measures can be taken. The durability-related aspects of FRP used as an internal concrete reinforcement and how they have been treated in existing Design Guidelines have been summarized [44] in the following points:

- The widely used ACI 440 Design Guideline divides between only two environmental conditions: wet and dry environment and have an
exclusive factor to account for environmental deterioration. Additionally, there exists a big difference between loads at the ultimate limit state and loads after creep rupture limit check.

- The JSCE Design Guideline uses a single factor that incorporates several uncertainty aspects including environmental durability. Stress limits for sustained stress are used.
- The UK Design Guideline deals with environmental degradation of FRP by using one factor that takes into account the influence of environment, sustained stress and a few other uncertainties.
- The Canadian Design Guideline uses a slightly different approach than the others. Liberal stress limit/design strengths are adopted complemented by restrictions in the use of certain FRP types in some applications; for example the use of GFRP is inadmissible for primary reinforcement in deck slabs. There are also restrictions on the selection of resin types for FRP reinforcement [40]. However, some of these restrictions in the use of certain FRP type are widely withdrawn in the 2006 version, but now three different classes of quality (class 1-3) are defined for each material group aramid, carbon and glass reinforcement [44].

It is clear that these differences in design approach to FRP durability make it difficult for the international construction community to have confidence in predictions of FRP service life in aggressive environments. Consequently, a more rigorous approach to durability specification needs to be adopted[69].
It has been acknowledged that the biggest problem is the perception that GFRP is sensitive to alkali attack and that the concrete environment is therefore intrinsically highly aggressive. Research has shown that the concrete environment is not as aggressive as the alkaline solutions that most researchers use and that alkali resistance can be significantly improved by the selection of appropriately treated glass fibres, suitable resins and better production techniques [70, 71]. The main points to note are that these Guidelines have single factors for “environmental effects”. The environmental effects identified in the literature are moisture, alkali, temperature and time and therefore with a good understanding of the exposure conditions that FRP will be subjected to in service, it should be possible to refine these factors to develop a less or more, where appropriate, conservative approach to durability specification [72].

2.5 Flexural Behaviour of FRP Reinforced Concrete Beams

Since little was known about the behaviour of FRP composite bars, extensive research investigations have been undertaken in order to use these new reinforcements for structural applications [73]. Such application necessitates the need for either developing a new design code or adopting and modifying the current one to account for the engineering characteristics of FRP materials [74]. Therefore, in the last few years, a number of tests on several types of FRP bars have been conducted in order to evaluate the interaction phenomena between FRP bars and the concrete matrix and to evidence behavioural differences with respect to deformed steel rods. The following section will introduce the
fundamental aspect of the main factors that affect the flexural behaviour of the FRP reinforced concrete beams.

2.5.1 Impacts of FRP Britteness

The application of the FRP materials on a large scale has been delayed due to the high cost of the FRP reinforcement in comparison to steel, to the lack of design codes and to the brittle behaviour of FRP resulting in poor structural ductility [75]. By contrast, steel does not suffer from all of these disadvantages, particularly the brittleness. Thus traditionally, concrete beams and slabs with steel reinforcement are usually designed for tension failure, to take advantage of the ductile behaviour of steel.

Unlike steel, FRP reinforcement has a linear stress-strain behaviour to failure. Hence while designing FRP reinforced concrete members for a required ultimate bending moment, one has to consider the nature of failure [76]. Nani and Faza [77] confirmed this fact by acknowledging that the non-ductile behaviour of FRP reinforcement necessitates reconsideration of failure modes.

2.5.1.1 Nature of Flexural Failure Modes

There are three potential flexural failure modes for FRP reinforced concrete sections; balanced failure, compression failure and tension failure [15].

The nature of failure of the concrete member that is reinforced with FRP bars is much dependent on the FRP reinforcement ratio in comparison to the certain reinforcement ratio where concrete crushing and FRP rupture occur simultaneously. This controlling ratio is commonly expressed as the balanced

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reinforcement ratio, which classifies the first nature of failure modes as the balanced failure condition.

### 2.5.1.1.1 Balanced failure condition and value

The balanced type of failure is very difficult to achieve in reality. However, the condition for such failure to take place represents a limiting value in demarcating the second and the third types of failure modes, which will be detailed later. The value of the balanced reinforcement ratio, $\rho_{fb}$, at which this condition occurs, has been approached by the ACI 440 committee by substituting the material properties of the FRP instead of steel, in section 8.4.3 of ACI 318-99 [14], to be presented as follow:

$$\rho_{fb} = (0.85 \beta_1 f_c / f_{tu}) \times (0.003 E_f / [ f_{tu} + 0.003 E_f ] ) \quad (2-4)$$

Where $\beta_1$ = factor taken as 0.85 for concrete strength up to and including 4000 psi. For strength above 4000 psi, this factor is reduced continuously at a rate of 0.05 per each 1000 psi of strength in excess of 4000 psi, but is not taken less than 0.65. $f_c$ = specified compressive strength of concrete, $f_{tu}$ = design tensile strength of FRP, considering reductions for service environment, $E_f$ = guaranteed modulus of elasticity of FRP defined as the mean modulus of a sample of test specimens minus three times the standard deviation.

AL Sayed et al [78] have approached the value of the balanced reinforcement ratio, $\rho_{fb}$, similarly as the ACI committee did, by using the following balanced steel reinforcement ratio equation given by the ACI 318 committee [79]:

$$\rho_{fb} = (0.85 \beta_1 f_c / f_y ) \times (0.003 E_s / [ f_y + 0.003 E_s ] ) \quad (2-5)$$
where \( f_y \) = the steel yield strength, \( E_s \) = the modulus of elasticity for the steel. Yet, AL Sayed [74] substituted the value of \( f_y \), the yield strength of steel reinforcement by the usable stress of the FRP or the so-called pseudo yield tensile strength, \( f_y \). He assumed that the \( f_y \) to be equal to 0.67 of the \( f_{tu} \), the ultimate tensile strength of FRP bar, where he labelled the value of \( 0.67 f_{tu} \) as the usable stress of FRP bar such that the factor of safety against its rupture is (1.5).

The balanced reinforcement ratio expression, based on Alsayed proposal, becomes:

\[
\rho_{fb} = (0.85 \beta_1 \frac{f'_c}{0.67 f_{tu}}) \times \left( \frac{0.003 E_f}{0.67 f_{tu} + 0.003 E_f} \right)
\]  

(2-6)

In comparison to the previously mentioned ACI recommended equation (2-4), this equation produces a higher balanced reinforcement ratio, which could be desired by some designers to secure a concrete crushing failure mode, avoiding any risk of failure by bar rupture failure mode.

Nevertheless, it could be noticed that the above listed formulas are based on hypothetical certain factors that needed to be verified and validated to be standardised for application in the field of FRP reinforcement concrete members.

2.5.1.1.2 **Concrete Crushing and FRP Bar Rupture Failure Modes**

Based on any of the previous detailed balanced reinforcement ratio, two other failure modes could be identified. The first takes place when the reinforcement ratio is below the balanced ratio, which will lead to the rupture of FRP bars before the exhaustion of all the concrete compression strength. The second
failure mode arises when the reinforcement ratio exceeds the balanced reinforcement ratio, which will conclude in a concrete crushing failure mode at the compression zone of the member, before reaching the ultimate strength of FRP bars due to its extensive presence, more than required for the balanced condition.

Since the stress-strain curve of FRP bars does not have a yield plateau, the possibility of having a failure due to rupture of the FRP bars, which is more brittle than the failure due to crushing of concrete, should be avoided [80]. Thus researchers recommended some reduction factors to be applied to the FRP ultimate strength, $f_{fu}$. Nanni [66] suggested that the strength reduction factor should be taken as 0.70 and the minimum FRP ratio, be greater than 1.33 the balanced ratio. Faza & Ganga Rao [81] recommended that the maximum permissible strength to be 0.80 of the ultimate strength. The range between these two values of reduction, 0.70 and 0.8, have been recommended as well by other researchers [82].

These studies clearly show that the behaviour of concrete beams reinforced with FRP is different from that of beams reinforced with steel. This difference in behaviour led several researchers to urge for the need for developing altogether new design code provisions or revising the current ones to account for the properties of FRP materials [74].

### 2.5.2 Failure Modes and Flexural Capacity

For safety reasons, it is always required that the reinforced concrete member possess a certain flexural capacity that exceed the flexural demand by the
users of this element. The previously detailed failure modes have a great effect on the flexural capacity of concrete members reinforced with FRP bars, the ACI 440-01 committee report [14] assured that “The flexural capacity of an FRP reinforced flexural member is dependent on whether the failure is governed by concrete crushing or FRP rupture”. The report concluded two different equations to anticipate theoretically the nominal flexural capacity of each failure mode, as well as three different conservative strength reduction factors that should be adopted to provide a higher reserve of strength in the member. Chapter four details these equations and their application in anticipation of the flexural capacity of FRP reinforced concrete beams.

### 2.5.3 Flexural Behaviour Exploration Attempts

Experimentally, several researchers explored the behaviour of concrete beams that reinforced with FRP bars via different approaches, as it could be exemplified in the following attempts.

Theriault and Benmokrane [83] investigated the effects of the reinforcement ratio and the concrete strength on the flexure behaviour of simply supported concrete beams reinforced with FRP. They concluded that as concrete strength and reinforcement ratio increases, the ultimate moment capacity of concrete beams increases, but this increase is limited by the concrete compressive failure strain of over-reinforced concrete beams.

Saadatmanesh and Ehsani [82] have studied the prospect of using GFRP as a primary reinforcement for concrete. The result of their research concluded that
the failure of the tested beams may be due to the failure in tension of the reinforcing bars, followed by crushing of concrete.

Some other researchers investigated the use of FRP bars as stirrups. Zia [84] attempted this issue, as well as Faza [85] when they investigated the flexure performance of four-point bending of concrete beams using stirrups made from FRP bars. They observed failure mode as rupture of the longitudinal CFRP bars. In a similar trend, Grace et al [86] introduced an analytical model to determine the ductility of FRP reinforced concrete beams. They concluded that the use of GFRP stirrups led to significant shear deformations that increased beam deflections and reduced ductility. They added in their conclusion that the flexural-shear failure modes were caused by GFRP stirrups used with FRP reinforcing bars or steel and the failure mode was governed by the type of FRP reinforcement bars and stirrups. They recommended that a new specifications are needed for ductility and failure modes.

Recently El-Mogy et-al [87] conducted a research to evaluate the flexure response of the GFRP-reinforced continuous beam. This research presented the experimental results of two reinforced concrete beams continuous over two spans with rectangular cross-section. One of these beams was reinforced with GFRP bars and the other was reinforced with steel bars. It is concluded that the GFRP-reinforced concrete beam was able to redistribute the connecting moment over the intermediate support.

At a later stage the above mentioned research group [88] presented the experimental results of four reinforced concrete beams with rectangular cross section of 200×300 mm continuous over two spans of 2,800 mm each. Two
beams were reinforced with GFRP bars in to different configurations while one beam was reinforced with carbon FRP bars. A steel-reinforced continuous concrete beam was also tested to compare the results. It was concluded that increasing the GFRP reinforcement at the mid-span section compared to middle support section had positive effects on reducing mid-span deflections and improving load capacity. The test results were compared to the available design models and FRP codes. It was concluded that the Canadian Standards Association Code (CSA/S806-02) could reasonably predict the failure load of the tested beams; however, it fails to predict the failure location.

In another approach, Benmokrane et al [89] considered the effect of the surface texture of FRP bars as well as the beam span-to-height ratio on the behaviour of GFRP reinforced concrete beams in comparison to that of conventional steel reinforced concrete beams. They concluded that the use of GFRP bars in concrete structures is possible and the optimal design is achievable if, not only an appropriate reinforcement ratio is used, but also the appropriate height-to-span ratio is computed, and the two types of bars used in their study behaved in a similar manner despite the fact that they were manufactured differently and have different surface deformations. They also briefly criticized the ACI code formulas because of their overestimation of the ultimate moment. The above mentioned researchers recommended that these formulae should be reviewed to be used safely for GFRP bars.

It has been reported that the earlier mentioned ACI 440.1R-01 equations are a very important first step toward the implementation of FRP composites in civil engineering field; however the guidelines could be revised when more data
become available [90]. Consequently verification of those equations is necessary whenever new data becomes available to elaborate the potential users on the competent of these equations. This aim has been tackled by few recent researchers. In a comprehensive study Vijay [76] presented a simple mathematical model quoting the ACI 318-99 and ACI-440.IR-01 equations to identify failure modes and to compute moment capacities for 77 GFRP reinforced simply supported concrete beams, extracted from 14 different experimental investigations. The mathematical model presented in Vijay’s study provided excellent correlation with respect to ultimate moment capacities of the GFRP beam test results. On the other hand Toutanji [91] confirmed on the ability of the ACI-440.IR-01 equations in predicting the deflections of the GFRP simply supported concrete beams. Generally the work presented in the literature recognised the potential of the ACI-440.IR-01 equations to predict the moment capacity in addition to the deflections of simply supported FRP reinforced concrete beams, particularly GFRP reinforced beams due to the availability of the data on such beams. Grace et al. [86] presented the results of the deflection and load capacity of continuously supported T-section concrete beams reinforced with different combinations of FRP and steel bars and stirrups. The research concluded that the failure mode is governed by the type of FRP reinforcement bars and stirrups, and the crack pattern is dependent mainly on the type of stirrups.

Several other experimental studies has been performed since the early eighties [92] till now, probing the previously mentioned areas of interest, in an attempt to finalise certain standards and conclusions that might help in benefiting from the
distinguished characteristics of FRP materials to be used in construction in safe and hazardless manner.

2.5.4 FRP Characteristics Effects on Serviceability

A structure must be safe, as it has been stated earlier, as well as serviceable. Serviceable means that the member must be satisfactory at working, or service, loads. Normally this means satisfactory as regards:

(a) deflection; this must be limited so as not to cause damage to partitions and finishes nor undesirable appearance nor inefficiency, and
(b) cracking; this should not adversely affect the appearance or durability of a member [93].

FRP bars are light and strong and, since they are less prone to corrosion can lead to more durable structures [94]. However, FRP reinforcement has got a lower elastic modulus than conventional steel bars and that will inevitably lead to large deflections in normally reinforced concrete sections [95].

2.5.4.1 FRP Bond characteristics

Bond constitutes a critical aspect of structural behaviour of all types of reinforcing materials including the steel and FRP reinforcing bars. The strength and performance of these structures depend on the bar strength as well as on the development of adequate bond between the bars and concrete. Thus, a good understanding of bond behaviour of FRP bars with concrete is essential to harness its full potential. However, it has been argued that before cracking occurs, strains in concrete are small and fairly uniform over the length of the beam, so the degree of bond has relatively little effect on the behaviour of the
beam. However, once cracking occurs in concrete, the amount of bond can have a significant effect [96, 97].

Several theoretical and experimental studies conducted to evaluate the bond characteristics and strength of FRP bars have indicated their dependence on many variables. These variables include size, shape, surface configuration (e.g., ribs) and surface roughness of bars, position of bars in the concrete element and the confinement pressure, compressive strength of concrete, embedment length, environmental conditions, service/operating temperatures, temperature change, moisture absorption, mechanical interlock of bars against the concrete, chemical adhesion, and hydrostatic pressure against the FRP bars due to shrinkage of hardened concrete. It has been amply recognized that the bond behaviour of FRP bars cannot be modelled after the bond behaviour of steel reinforcing bars because of the fundamental differences between the physical and mechanical properties of two materials and their interaction with concrete [98].

A state-of-the-art report on the bond of FRP bars to concrete was launched by Cosenza [99] for research that have been carried out in this field. The report investigated the effect of the surface and material properties, confinement pressure, bar diameter, bar position in the cast, top-bar, embedment length, temperature change, and environmental conditions on the bond behaviour between concrete and FRP bars. The report briefly concluded, through the discussion of more than sixty FRP reinforced concrete tested beams, that the bond strength is seen to be dependent on fibre and resin properties, whereas it is not affected by concrete strength.
At a later stage of the above mentioned study Tighiouart [100], carried out a bond test on 64 specimens of GFRP reinforced concrete beams. He also conducted 18 pullout tests to investigate the top bar effect of GFRP bars. The test results discussed the effect of the bar's type, diameter, embedded length in addition to the top bar effect on the bond behaviour of FRP bars in concrete. This research concluded, generally, similar conclusions as that of Cosenza [99], in relation to the lower bond strength values that FRP bars show than that of steel bars, and to the reduction ratio of the bond strength of top bars to that of the bottom bars. The report concluded also that for GFRP bars it is the adhesion and the friction that control the bond strength.

Svecova [101] investigated the effect of using the carbon reinforced polymer pre-stressed concrete prisms as a flexural reinforcement in concrete beams for the purpose of limiting deflection and crack width and improving bond strength under service load. It is experimentally demonstrated that the use of CFRP pre-stressed concrete prisms, instead of CFRP bars, can solve the serviceability problem made by the low modulus of elasticity of CFRP. Nevertheless, despite the effort to improve bond strength, one sample failed due to de-bonding.

Lees [102] investigated the nature of the bond between an FRP tendon and concrete. The study concluded that both the surface profile and the material properties of an FRP tendon can have a significant effect on the bond behaviour of a tendon.

Malvar [103], analyzed experimentally the bond characteristics of four different types of FRP bars with different surface deformations embedded in lightweight
concrete. He found that small surface indentations were sufficient to yield bond strengths comparable to that of steel bars.

Bond behaviour was studied with 27 pullout specimens by Belarbi [104]. Bar surface and embedment length's effect on bond characteristics were investigated. The following conclusions were made:

1. The addition of polypropylene fibers did not increase the ultimate bond strength, while providing much more ductile bond behaviour.

2. Totally different bond mechanisms were observed for CFRP and GFRP due to their different surface treatments. Bond strength decreased with increasing of embedment length for GFRP bars; while opposite results were observed for CFRP.

It must be stated here that none of the above listed concluded remarks, as regards the bond characteristics of the FRP reinforcement, were based on testing continuously supported tested beams, which puts such conclusions to the test. However, recently Gravinaa et al [105] conducted a theoretical study of the flexural behaviour of concrete beams reinforced with FRP bars using a deformation model developed by the authors. This study was conducted by applying the model to continuous beams tested in this thesis in order to predict the bending moment distribution, as well as the progressive formation of flexural cracks, associated crack spacings and crack widths. The results of this study were found to be particularly dependent on the bond characteristics between the FRP bar and surrounding concrete, as it has been stated by the research group.
2.5.4.2 ACI Attempts for Assessment of Deflection and Crack Width

The ACI 440-01 committee report [14], validated most of the above mentioned conclusions by asserting that “the serviceability provisions given in ACI318 need to be modified for FRP reinforced members due to differences in properties of steel and FRP, such as lower stiffness, bond strength, and corrosion resistance”. The report quoted the investigations of Gao et al [106] and Tighiouart et al [100] to conclude that the substitution of FRP for steel on an equal area basis, for example, would typically result in larger deflections and wider crack widths.

Based on their research, which involved the statistical analysis of a large amount of experimental data, Gergely and Lutz [107] proposed the following equation for predicting the maximum width of crack at the tension face of a beam:

$$w = 0.076 \beta f_s \frac{3}{\sqrt{d_c A}}$$ \hspace{1cm} (2-7)

in which $w$ is the maximum width of crack, in thousandth inches, and $f_s$ is the steel stress at the load for which the crack width is to be determined, measured in ksi.

where $d_c =$ thickness of concrete cover measured from tension face to centre of bar closest to that face and $\beta =$ ratio of distances from tension face and from steel centroid to neutral axis. $A =$ concrete area surrounding one bar, equal to total effective tension area of concrete surrounding reinforcement and having same centroid, divided by number of bars, in$^2$.

Based on some modification of the above stated equation, the ACI 440 committee recommended an equation to calculate the crack width for FRP
Chapter Two: Literature Review

reinforced members. However they also recommended further investigation to verify the effect of surface characteristics of FRP bars on the bond behaviour and on crack widths. As for the deflection the same report recommended to use the usual structural analysis techniques to calculate the short-term deflections, but with inserting what is called the effective moment of inertia, $I_e$. The effective moment of inertia is a modification of Branson’s equation. Branson’s method was originally developed for steel reinforced concrete, and uses an effective moment of inertia $I_e$ to compute deflection in conjunction with elastic deflection [108]. This relationship was empirically derived and represents a gradual transition from the gross un-cracked moment of inertia $I_g$ to the cracked transformed moment of inertia $I_{cr}$ as shown in the following:

$$I_e = \left[\frac{M_{cr}}{M_a}\right]^3 I_g + \left[1 - \left[\frac{M_{cr}}{M_a}\right]^3\right] I_{cr} \leq I_g$$ (2-7)

where $M_{cr}$ represents the cracking moment of the beam and $M_a$ applied service load moment at the critical section.

However, Al Sunah et al.[109] examined the ACI 440 provisions for deflection in further detail and reported that the form of the equation for $I_e$ is not fundamentally sound and cannot be used to predict deflections of FRP RC in all cases. Nevertheless, details of the above mentioned equations and its effectiveness in flexure behaviour prediction are explored in Chapter four.

2.5.4.3 Experimental and Theoretical investigations on Deflection and Crack Width Assessments

At the early stage of investigations that dealt with the behaviour of fiber glass-reinforced beams (1971), Nawy et al [110] reported that fiber glass-reinforced
beams were able to take a higher load than the corresponding steel-reinforced control beams. Their cracking and deflection behaviour were also favourably comparable to the steel-reinforced beams at working stress levels. At later stage of the above mentioned investigation the same group of researchers[111] developed analytical expressions for evaluation of crack width and deflection. One of their findings is that when FRP bars of low elastic modulus are used as concrete reinforcement, the beams, once the concrete cracked, are likely to deflect at a faster rate for a unit increase in load.

At the early 90’s Nakano et al [112] carried out experimental investigation to study and evaluate the flexural performance of concrete beams reinforced with fiber bars. One of the main conclusions of this experimental investigation was that the flexural performance of concrete beams reinforced with continuous fiber bars can be evaluated by conventional methods used in concrete beams reinforced with steel bars. It has been stated also in this experimental study that in general, one could expect the flexural rigidity of beams with FRP bars to increase, after initial cracking, with the increase of both the elastic modulus of the bars and the reinforcement ratio.

Within the same frame of time of the above mentioned study, some other investigations [82],[111],[113], concluded similar findings to that of Nakano’s. These investigations indicated that deflection and cracking of FRP reinforced concrete beams can be predicted with the same degree of accuracy as the behaviour of regular steel reinforced concrete beams and that theoretical correlation is possible.
Cosenza et al [99] discussed the effectiveness of the linear cracked model in the control of the stress in concrete and in the evaluation of the deflections for beams. They concluded that this model, very useful for the analysis of beams reinforced with steel bars under serviceability conditions, becomes not reliable also at low levels of bending moment. Therefore in analysis of beams reinforced with FRP bars it is necessary to introduce a non-linear approach in order to evaluate the maximum stress in concrete, the deformability of the section and the deflection of the elements in a reliable way.

In more productive effort, Faza and Ganga Roa [114],[115] studied the load-deflection behaviour of FRP reinforced concrete beams by extending the methods used for steel reinforced beams to compute post-cracking deflections in FRP reinforced beams. They proposed a modified effective moment of inertia, \( I_m \), based on their experimental results, expressed as a function of the effective moment of inertia, \( I_e \), and the moment of inertia of the cracked transformed section, \( I_{cr} \). Thus, \( I_m \) replaces \( I_e \) in the calculation of deflection by ACI-318 [116].

Al Sayed [117], benefited from Faza’s research [114] in presenting a study comparing the predicted and the measured load-deflection relationships for 9 simply supported GFRP and similar 3 steel reinforced concrete beams. The Numerical part of the study was carried out using (a) the computer model that was suggested by Faza & Ganga Roa [114], (b) The ACI 318-95 load deflection model [116], (c) a modified load-deflection model, developed by the author, for beams reinforced by FRP bars. The researcher authenticated the effectiveness of Faza & Ganga Roa modification through his investigation’s results. Al Sayed
found out that the application of this modification resulted in reduction of the error in predicting the service load deflection by the ACI model, which underestimates the actual deflection of the GFRP reinforced concrete beams, from 70% to less than 15%. He also praised his own proposed computer model because it was capable to narrow the errors between predicted and measured cracking and service load deflections of GFRP reinforced concrete beams by less than 20 and 10% respectively. Al Sayed [117] mentioned also that his developed computer model can be extended to consider properties of other types of FRP materials and then used to modify the currently practised design formula that were generated assuming that reinforcement is provided by steel. Finally he recommended that “more experimental results are needed to further check the proposed modifications under different load cases, reinforcement configurations, and variations in properties of the FRP materials.” In coherent with this early staged conclusion Al Sunah et al [109] stated more recently that more research work is required as well to investigate the accuracy of their proposed new form of the $I_e$ equation in addition to the earlier mentioned conclusion of the unreliability of the ACI 440 equation in predicting deflection. However, Toutanji et al [91] disputed this statement in their investigation for verification of the ACI 440 methods for predicting deflections and crack widths for GFRP reinforced concrete beams. Toutanji concluded that deflections in concrete beams reinforced with GFRP bars can be accurately predicted using ACI 440 equations and the experimental results obtained in their study compared well with those predicted by ACI. Nevertheless, Victor et al [118] reported that Large reinforcement ratios are typically used in FRP reinforced
members for deflection control. This report did not match well the recent conclusion by Issa et al [119] in their evaluation of the flexural behaviour of GFRP reinforced concrete cantilever beams. This evaluation concluded that the area of the GFRP reinforcement has a small effect on the load-deflection relation up to close to the ultimate load. However, it has been reported recently by Ascione et al [120], in their investigation on deflections at mid-span and crack widths, that more investigation is required in order to better predict the deflection and actual failure mode.

2.6 Concluding Remarks

From the literature survey carried out in this section, it is evident that considerable research has been carried out attempting to explore the flexural characteristics of FRP reinforced concrete beams. The increased awareness of the distinguishing characteristics of FRP reinforcements, compared to the conventional steel, necessitated the authentication of the available design guidelines for a wider spectrum of application for the use of FRP as an internal reinforcement for concrete members.

The main conclusions drawn from the literature survey given in this chapter are:

- Studies show that the behaviour of concrete beams reinforced with FRP is different from that of beams reinforced with steel. This necessitated the need for developing altogether new design code provisions and revising the current ones to account for the properties of FRP materials.
• One of the principal advantages of FRP reinforcement is the ability to configure the reinforcement into varieties of cross-section and texture features to meet specific performance and design objectives.

• In addition to its corrosion resistance, the tensile properties of FRP bars are what make them an attractive alternative to steel reinforcement. When loaded in tension, FRP bars do not exhibit any plastic behavior (yielding) before rupture. Therefore FRP reinforcement is not recommended for moment frames or zones where moment redistribution is required.

• The nature of failure of the concrete members that reinforced with FRP bars are greatly dependent on the FRP reinforcement ratio in comparison to the balanced reinforcement ratio, where concrete crushing and FRP rupture occur simultaneously.

• The flexural capacity of an FRP reinforced flexural member is dependent on whether the failure is governed by concrete crushing or FRP rupture.

• Since the stress-strain curve of FRP bars does not have a yield plateau, the possibility of having a failure due to rupture of the FRP bars, which is more brittle than the failure due to crushing of concrete, should be avoided.

• FRP bars show lower bond strength values than that of the steel bars. FRP bars exhibits also less ratio of the bond strength of top bars to that of the bottom bars.
• Several reports concluded that for GFRP bars it is the adhesion and the friction that control the bond strength. Different bond mechanisms were observed for CFRP and GFRP due to their different surface treatments.

• The recommendations ruling the design of FRP reinforced concrete structures currently available are mainly given in the form of modifications to existing steel reinforced concrete Codes of Practice, which predominantly use the limit state design approach. Such modifications consist of basic principles, strongly influenced by the mechanical properties of FRP reinforcement, and empirical equations based on experimental investigations on simply supported FRP reinforced concrete elements.

• The ACI 440.1R equations are a very important first step toward the implementation of FRP composites in civil engineering field; however the guidelines could be revised when more data become available. Consequently verification of those equations is necessary whenever new data becomes available to elaborate the potential users on the competent of these equations.

• It has been recommended that “more experimental results are needed to further check the above stated conclusions under different load cases, reinforcement configurations, and variations in properties of the FRP materials
2.7 Topics for Further Research

It is evident from the literature review that there are many aspects of the flexural behaviour of FRP reinforced concrete beam that are in need of future investigation. The main aspects of flexural behaviour that could still need further investigation are summarised as follows:

a) Till the time of the start of this research all the available investigations in the above stated field of flexural behaviour were based on results obtained from testing simply supported FRP reinforced concrete beams, apart from the previously indicated limited research conducted by Grace et al [86] on T-section concrete beams. These results formed the ground of the established Design Codes as has been discussed earlier. Therefore, there is a call for more realistic, practical and applicable investigations into the behaviour of FRP reinforced concrete beams to be carried out accordingly. Subsequently, investigating the continuously supported FRP reinforced concrete beams could be a valid option towards achieving such required quality of investigations.

b) The improvement of beams flexural behaviour, in the form of enhanced moment capacity and serviceability, are the common target for most of the Design Guidelines and Codes. Different parameters have been tested against their effects on this behaviour. As it has been addressed before, all these parameters were tested on simply supported beams, and the results of these tests led to series of published recommendations. By testing such parameters on continuously supported beams, the concluded results would provide the chance to
revise and authenticate the earlier achieved recommendations. Therefore, there is a need to carry out systematic and detailed investigations on the influence of different parameters on the flexural behaviour of continuous beams as it has been done before on simply supported ones. The concluded results should be compared against the previously established recommendations.

c) Despite the adaptation of the modifications to the existing steel reinforced concrete Codes of Practice, there is no evidence of investigations to compare the difference in behaviour between continuously supported steel and FRP reinforced beams using this modification.

d) There are few computational studies which have been developed for the analysis of simply supported FRP reinforced concrete beams and no attempt was evident in any other case of support.

In this thesis, the research carried out is devoted to investigate the above mentioned topics.
CHAPTER THREE

Experimental Investigation

3.1 Introduction

The results of testing two simply and three continuously supported concrete beams reinforced with glass fibre reinforced polymer (GFRP) bars are presented in this chapter. Test results of similar number of simply and continuously supported concrete beams but reinforced with (CFRP) bars are also illustrated. Furthermore, the experimental test results of concrete continuous beam reinforced with steel bars are demonstrated in the present chapter for comparison purposes.

3.2 Tested Parameters and Illustrated Results

The type of reinforcement, GFRP, CFRP or steel, is one of the main parameters investigated in the present experimental tests. Type of support, either simple or continuous, for the tested reinforced concrete beams is an added core parameter. In addition, the amount of reinforcement is also explored as one of the key parameters. Over and under FRP reinforcement ratios were applied for the simply supported concrete beams. Three different FRP reinforcement
combinations of over and under reinforcement ratios were used for the top and bottom layers of the continuous concrete beams tested.

The present chapter illustrates the failure modes, crack propagation, reinforcement strains, load capacity, reaction redistribution and the deflection characteristics, as experimental results, of each tested beams.

### 3.2.1 Test Specimens and Materials

All beams were 200 mm in width and 300 mm in depth. The continuously supported beams had two spans, each of 2750 mm, as shown in Figures 3.1 and 3.3 whereas the simply supported beams had a span of 2750 mm, as shown in Figures 3.2.

The simply supported beams were designed to achieve two different modes of failure, namely bar rupture and concrete crushing. Thus a reinforcement ratio less than the balanced reinforcement ratio $\rho_{fb}$ as defined in the ACI 440.1R-06 guidelines, was assigned to achieve the bar rupture mode. Reinforcement ratio greater than $\rho_{fb}$ was designated to achieve the concrete crushing mode.

The GFRP reinforced concrete continuous beams were reinforced with three different reinforcement combinations at the top and bottom layers. Beam GcOU was reinforced with six GFRP bars of 15.9mm diameter (over reinforcement) at the top layer and three GFRP bars of 12.7mm diameter (under reinforcement) at the bottom layer, whereas beam GcUO was reinforced with an opposite arrangement of GFRP longitudinal bars as given in Table 3.1 and Figure 3.1.

The top GFRP reinforcement of beam GcOO was the same as the bottom reinforcement, each consisting of six 15.9mm diameter GFRP bars (over
reinforcement). Table 3.1 details the properties of the entire bar reinforcements used in the beams tested.

Nevertheless, the experimental tensile tests of the CFRP reinforcing bar specimens revealed that the bar’s manufacturer has overestimated the tensile strength characteristic of the supplied bars. Consequently, the concrete crushing failure mode for the CFRP reinforced beams did not experimentally occur. Table 3.2 demonstrates the actual characteristics of the CFRP reinforcement bars used. Based on these characteristics, Table 3.1 discloses the actual reinforcement ratio and their allocation in the tested beams.

The CFRP reinforced concrete continuous beams were reinforced in a way to accomplish three possible reinforcement combinations at the top and bottom layers. These combinations were: $2\phi 12$mm at the top layer and $2\phi 7.5$mm at the bottom layer in beam C-C-3, $2\phi 7.5$mm at the top layer and $2\phi 12$mm at the bottom layer in beam C-C-4 and $2\phi 12$mm at the top as well as the bottom layer in beam C-C-5 as shown in Fig. 3.3 and Table 3.1.

The steel reinforcement (4 bars of 12mm diameter) of the continuous beam S-C-6 was selected to achieve tensile strength of 240 kN, equivalent to that of CFRP reinforcement (2 bars of 12mm diameter) used at the bottom layer of beams C-C-4 and C-C-5 and top layer of beams C-C-3 and C-C-5. Similarly, the steel reinforcement was also selected to have the same tensile strength as the three GFRP bars of 12.7mm diameter used in beams GcUO, GcOU and GsU. Vertical steel stirrups of 8 mm bar diameter, spaced at 140 mm centres were provided throughout each beam length in accordance with ACI 318-05[121]
Figure 3-1 Experimental Loading System and Cross-Section Details of GFRP Reinforced Continuous Concrete Beams

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Figure 3-2 Experimental Loading System and Cross-Section Details of GFRP Simply Supported Concrete Beams
Chapter Three: Experimental Test & Results

Figure 3-3 Test Set-up and Cross-Section Details of GFRP Reinforced Continuous Concrete Beams

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3.2.2 Material Specifications

All materials used, such as steel bars and stirrups as well as the FRP reinforcing bars, in addition to the concrete have been presented by samples, assembled according to certain standards and tested to reflect the properties of these materials. Each material test will be detailed below, illustrating the results that have been concluded from each test.

3.2.2.1 Concrete

Sand, gravel coarse aggregate (10mm maximum size) and ordinary Portland cement were used to produce concrete with a target compressive strength of 40 N/mm² at 28 days. In total, thirty three 100mm cubes and eighteen 150mm diameter x 300mm high cylinders were made for all beams tested. All test
specimens were de-moulded after 24hrs, wet cured and covered with polyethylene sheets until the date of testing. Beams testing started 60 days later after casting. Three cubes and three cylinders were tested immediately after testing of each beam to provide values for cube compressive strength, $f_{cu}$, and cylinder compressive strength, $f'_c$, as presented in Tables 3.1 and 3.4.

### 3.2.2.2 Steel Bar and Stirrup properties

Uni-axial tensile tests were carried out on three samples of steel bars, 12mm. diameter, according to BS EN 10002-1 (1990) to find out the mechanical properties for the steel reinforcement used in beam (S-C-6). The same testing method has been employed to assess the mechanical properties for the steel stirrups, 8mm. diameter, which have been applied to prevent the shear failure mode for all beams. Table 3-2 presents the properties of the steel bars and stirrups used for beam (S-C-6).

### 3.2.2.3 FRP Bars Properties

As for the FRP bars the same test method mentioned above were used but with different settings to the bar ends to cope with the brittle nature of these bars that will not be able to endure the gripping of the jaws of the uni-axial tensile test machine. Each specimen was cut into 850-mm long section and anchored with potted anchor system at each end as shown in Figure 3-5. This method bonds the rod to the steel cylinder utilizing a mortar (resin-based grout) [122]. The potted anchors used in this project consisted of 250-mm long steel cylinders for the GFRP specimens. Due to the soft surface texture of CFRP bars in
comparison to the GFRP bars, the length of the steel cylinders were increased by another 50-mm to become 300-mm for the CFRP specimens, to allow more contact area with the bars in order to enhance the bond between the bars and the cylinder. The potted anchors filled with a high performance resin grout. Tensile tests were carried out using a hydraulic loading machine, ‘Universal Testing Machine’, having a capacity of 500 kN. The specimen was inserted into the test frame and gripped by the two thick plates at the anchored ends. The internal load cell was used to monitor the applied loads. A clip-on 50-mm extensometer was attached to the specimen at the onset of the test and used to measure the longitudinal deformations of the specimen by means of built-in attachment springs figure. The extensometer usually removed at 70 % of the anticipated failure load to avoid any damage that might be caused to it. The tension force was applied at a constant specification loading rate of 250 MPa / min [123]. All tensile tests were conducted until rupture of the FRP rods. Figure 3-6 shows the typical tensile failure phenomenon of the GFRP bars. Table 3-2 presents the properties of the FRP bars used for the tested beams.
Figure 3-6 FRP Tensile Specimens

Figure 3-7 Overview of the GFRP Rod in Tensile Tests and Failure Phenomenon
3.2.3 Results Collecting and Recording

The total Load exerted by the hydraulic ram on the simply supported beams, which represents in the case of the continuous support beams the sum of the two points loads applied to the two spans, was applied in increments of 6.2 kN. After each load increment, any cracks were marked on the beam surface with ineradicably marker to trace the crack propagation. A photo of the crack marked at the right mid span of the continuous supported beam is usually taken against each load increment for crack width analysis purpose. For the same aim a scaled magnifying lens of the power of (1/50) mm. has been used to measure manually the crack width at the same side of the photos taken in the continuous beams as well as over the middle support.

A manual dial gages were inserted opposite the LVDTs attached to the data logger to check the validity of the recorded deflection readings as shown in Figure 3.1. Beams failure was judged to occur when the beam under testing could not uphold any additional applied load. Instantaneously after beam failure, the applied load was released and no further data were registered by the data logging equipment. Any load increment by the calibrated dial gage, that controls the hydraulic ram, was recorded manually, to be compared against the load recorded by the data logger for validation purpose.

3.2.3.1 Applied and Recorded Load Difference

In attempt to validate the records of the load applied to the beams during the tests, the differences between the manual gage readings and the corresponding
loads, that have been recorded using the data logger, have been tabulated as it could be seen in Figure 3-7, as an illustrative example. The comparisons show a minimum difference of (0) ton, and a maximum difference of (0.6) ton. However the maximum difference, in percentage, recorded 9.6%. The highest percentage in differences always recorded at the beginning of each test, which reflects some inaccurate calibration in the dial gages at the early stage of the test.

Figure 3-8 Applied and Recorded Load Difference
### Table 3-1 Details of Test Specimens

<table>
<thead>
<tr>
<th>Beam Notation</th>
<th>Length (mm.)</th>
<th>Top Reinforcement Details</th>
<th>Bottom Reinforcement Details</th>
<th>Concrete Compressive Strength (N/mm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>No.</td>
<td>Bar Diameter (mm)</td>
<td>Reinforcement</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\rho$%</td>
</tr>
<tr>
<td>GcOU</td>
<td>5500</td>
<td>6</td>
<td>GFRP</td>
<td>15.9</td>
</tr>
<tr>
<td>GcUO</td>
<td>5500</td>
<td>3</td>
<td>GFRP</td>
<td>12.7</td>
</tr>
<tr>
<td>GcOO</td>
<td>5500</td>
<td>6</td>
<td>GFRP</td>
<td>15.9</td>
</tr>
<tr>
<td>GsO</td>
<td>2750</td>
<td>2</td>
<td>GFRP</td>
<td>15.9</td>
</tr>
<tr>
<td>GsU</td>
<td>2750</td>
<td>2</td>
<td>GFRP</td>
<td>12.7</td>
</tr>
<tr>
<td>S-C-6</td>
<td>5500</td>
<td>4</td>
<td>GFRP</td>
<td>12.0</td>
</tr>
<tr>
<td>C-S-1</td>
<td>2750</td>
<td>2</td>
<td>CFRP</td>
<td>12.0</td>
</tr>
<tr>
<td>C-S-2</td>
<td>2750</td>
<td>2</td>
<td>CFRP</td>
<td>7.50</td>
</tr>
<tr>
<td>C-C-3</td>
<td>5500</td>
<td>2</td>
<td>CFRP</td>
<td>12.0</td>
</tr>
<tr>
<td>C-C-4</td>
<td>5500</td>
<td>2</td>
<td>CFRP</td>
<td>7.50</td>
</tr>
<tr>
<td>C-C-5</td>
<td>5500</td>
<td>2</td>
<td>CFRP</td>
<td>12.0</td>
</tr>
</tbody>
</table>

All beams tested had identical cross section of 200mm width and 300mm height.

---

Chapter Three: Experimental Test & Results
### Properties of Reinforcement Used in the Tested Beams

<table>
<thead>
<tr>
<th>Type of Bars</th>
<th>Bar Diameter (mm.)</th>
<th>Young’s Modulus, $E$ (kN/mm$^2$)</th>
<th>Ultimate Strength $f_u$ (N/mm$^2$)</th>
<th>Yield Strength $f_y$ (N/mm$^2$)</th>
<th>Ultimate Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel (Stirrups)</td>
<td>8.0</td>
<td>206.8</td>
<td>611.6</td>
<td>525.5</td>
<td>N/A</td>
</tr>
<tr>
<td>Steel (longitudinal bars)</td>
<td>12.0</td>
<td>200.0</td>
<td>594.4</td>
<td>510.8</td>
<td>N/A</td>
</tr>
<tr>
<td>GFRP</td>
<td>12.7</td>
<td>44.2</td>
<td>605.0</td>
<td>N/A</td>
<td>0.015</td>
</tr>
<tr>
<td>GFRP</td>
<td>15.9</td>
<td>38.7</td>
<td>703.0</td>
<td>N/A</td>
<td>0.018</td>
</tr>
<tr>
<td>CFRP</td>
<td>7.5</td>
<td>200</td>
<td>2000</td>
<td>N/A</td>
<td>0.01</td>
</tr>
<tr>
<td>CFRP</td>
<td>12</td>
<td>200</td>
<td>1061</td>
<td>N/A</td>
<td>0.005</td>
</tr>
</tbody>
</table>

Table 3-2 Properties of Reinforcement Used in the Tested Beams
3.2.4 Beam Notations

The notation of each beam is based on the type of reinforcement and support system. As for the GFRP reinforced beams, the first letter in the notation represents the type of reinforcement, ‘G’ for GFRP. The second letter corresponds to the supporting system, either ‘c’ for continuously supported beams or ‘s’ for simply supported beams. The third letter, which could be ‘U’ for under reinforcement ratio or ‘O’ for over reinforcement ratio, illustrates the bottom reinforcement ratio for the simply supported beams, meanwhile represents the top reinforcement ratio for the continuously supported beams. The fourth letter, which is used only for the continuously supported beams, demonstrates its bottom reinforcement ratio, ‘U’ or ‘O’. As an illustrative example, the beam notation GcOU indicates a GFRP reinforced continuously supported beam, with an over and under reinforcement ratios of GFRP bars located at the top and bottom layers of the beam, respectively.

As for the CFRP reinforced beams, the first letter in the notation represents the type of reinforcement, ‘C’ for CFRP. The second letter corresponds to the supporting system, either ‘C’ for continuously supported beams or ‘S’ for simply supported beams. The third number, which ranges between 1 to 5, gives the beam order. As an illustrative example, the beam notation C-C-3 indicates CFRP reinforced continuously supported beam, which is listed at the third place among the CFRP reinforced tested beams. The only steel reinforced beam has been notated in similar way to the CFRP reinforced beams and numbered as
six, since it will be always the sixth beam in the list of the CFRP and GFRP reinforced beams groups for the comparison purposes.

### 3.2.5 Test Set-Up

Each continuous test beam comprised of two equal spans supported on two roller supports, one at the end and the other at the middle, in addition to a hinge support at the other end of the beam, as shown in Figures 3.1 and 3.3. Each span of continuous beams was loaded at its midpoint via a hydraulic ram and an independent steel reaction frame bolted to the laboratory floor. The simply supported beams were similarly loaded at its mid-span and supported on hinge support at one end and a roller support at the other end as depicted in Figures 3.2 and 3.4. Three load cells were used to measure the reactions at one end support, the middle support in case of continuous beams and at the main applied load from the hydraulic ram as shown in Figures 3.1 and 3.3. The mid-span deflections were measured by positioning linear variable differential transformers (LVDTs) at the two mid-spans of the continuous beams and the mid-span of the simply supported beams. For quality control purposes, dial gages were also placed adjacent to each LVDT to measure the mid-span beam deflection manually. Additional dial gages were located at the three supports of continuous beams to assess any settlement that might take place during the loading process, which would affect the mid-span deflection readings and the reaction distribution. Load cells and LVDT readings were registered automatically at each load increment at a rate of 6.2kN, using data logging equipment. Failure of the tested beams was judged to occur when the beam
under testing could not uphold any additional applied load. At such stage the applied load was released and no further data were registered by the data logging equipment.

3.3 Test Results and Discussions

The following section discusses the results obtained from experimental tests carried out in this research. The obtained results address the following aspects: crack propagation, failure modes, load capacity, redistribution of support reactions and mid-span deflection for the tested beams. Each of these aspects will be discussed separately for GFRP and CFRP reinforced tested beams in comparison to the tested steel reinforced beam.

3.3.1 Crack Propagation and Failure Modes for GFRP Beams

Four different failure modes were observed throughout the experimental tests as given in Table 3.5 below.

Mode 1: Bar rupture—This mode was illustrated by beams GcOU and GsU. These beams were reinforced with an under-reinforcement ratio of GFRP bars at the bottom layer. Thus, it was expected that the strain in the GFRP reinforcement would reach its ultimate limit, at the mid-span failed section, before the full exhaustion of the ultimate concrete strain, which usually leads to such failure mode, as shown in Figure 3. 8.
Table 3-3 First Visible Cracking Loads, Failure Modes and Loads of the Tested Beams

<table>
<thead>
<tr>
<th>Beam Notation</th>
<th>First visible cracking load, $P$ (kN)</th>
<th>Experimental Total failure load, $2P$ (kN)</th>
<th>Experimental modes of failure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mid-span</td>
<td>Middle support</td>
<td></td>
</tr>
<tr>
<td>GcOU</td>
<td>17.7</td>
<td>14.6</td>
<td>290.2</td>
</tr>
<tr>
<td>GcUO</td>
<td>8.3</td>
<td>17.7</td>
<td>328.7</td>
</tr>
<tr>
<td>GcOO</td>
<td>8.0</td>
<td>24.0</td>
<td>333.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GsO</td>
<td>16.6</td>
<td>N/A</td>
<td>163.0</td>
</tr>
<tr>
<td>GsU</td>
<td>9.2</td>
<td>N/A</td>
<td>118.8</td>
</tr>
<tr>
<td>S-C-6</td>
<td>46.9</td>
<td>46.9</td>
<td>332.3</td>
</tr>
</tbody>
</table>

Mode 2: Concrete crushing—This failure mode was experienced by beams GcUO and GsO, which were reinforced with an over-reinforcement ratio of GFRP bars at the bottom layer. Such reinforcement was the reason behind the concrete crushing at the mid-span section before reaching the ultimate strain value of the GFRP reinforcing bars as revealed in Figure 3.9. In case of the continuous beam GcUO, wide cracks appeared over the middle support section before concrete crushing at the mid-span section. However, the strain recorded at the top GFRP reinforcement was much less than the GFRP rupture strain, indicating that de-bonding of top GFRP reinforcement may have occurred as explained later.
Mode 3: Concrete crushing combined with shear failure—Beam GcOO exhibited this mode of failure as shown in Figure 3.10. The diagonal shear cracks, which emerged at a late stage of loading, propagated simultaneously with the flexural concrete crushing mode of failure up to the sudden collapse and the disintegration of the beam. In comparison to beam GcUO, beam GcOO
developed a higher compression resistance at the top layer of the failed section due to the presence of a higher amount of GFRP reinforcement. Such enhancement in compression allowed the shear force to participate in the failure process; whereas the top layer of beam GcUO did not resist the compression force to a limit that would allow enough time for the shear force to participate fully in the failure process.

Mode 4: Conventional ductile flexural failure mode—This mode was demonstrated by the steel reinforced concrete beam S-C-6. It occurred due to yielding of tensile steel reinforcement followed by concrete crushing at both middle support and mid-span sections as shown in Figure 3.11.

![Figure 3-11 Flexure-Shear Failure of Beam GcOO](image-url)
The first cracking load of each beam tested is presented in Table 3.3. The table indicated that the steel reinforced concrete beam cracked at a later stage, in comparison to its similar continuously supported GFRP reinforced concrete beam. This could be attributed to the higher modulus of elasticity of steel bars than that of GFRP bars. The table also revealed that the two over-reinforced GFRP beams at the bottom layer, beams GcOO and GcUO, showed the first mid-span crack at a slightly higher load than that of the mid-span under-reinforced GFRP beam GcOU. It could be also noticed that, for the same two beams GcOO and GcUO, the first crack over the middle support started earlier than that at the mid-span. In contrast, beam GcOU developed an earlier crack at its mid-span than middle support.

In general, cracking in the flexural span of all beams tested consisted predominantly of vertical flexural cracks. The steel beam S-C-6 demonstrated similar vertical flexural crack pattern as the under-reinforced GFRP continuous beam GcOU. As the load was increased, shear stresses became influential and induced inclined cracks in beams GcOO and GcUO. These cracks diagonally
propagated toward the vicinity of load points on the compressive side of these two beams.

Figures 3.13 and 3.14 present the crack width at the middle support and mid-span of the beams tested, respectively. Beam S-C-6 demonstrated the least crack width among all beams tested. It could be also noticed that the GFRP beams reinforced with over reinforcement ratio at their bottom layer, beams GcUO, GcOO and GsO, had considerably less crack width at mid-span sections than the under reinforced GFRP beams, GcOU and GsU, as shown in Figure 3.13. The simply supported GFRP beams accomplished a similar crack propagation trend as its corresponding continuously supported beams that reinforced with identical bottom reinforcement, as seen in Figure 3.13.
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Figure 3.14: Mid-Span Crack Width of the Tested Beams

Figure 3.15 presents the total applied load against tensile strains in the top reinforcement over the central support and bottom reinforcement at mid-span for the continuously supported beams tested. Tensile strains in GFRP reinforcing bars increased significantly after concrete cracking. At a given load, strains in GFRP bars were higher than those in steel bars before yielding. Strains in top GFRP bars over the central support were larger than those in bottom bars at midspan for the three continuous beams GcOU, GcUO and GcOO. Strains in GFRP bars of beam GcOU are larger than those of the other two continuous beams GcUO and GcOO. Although wide cracks over the central support of beams GcUO and GcOO were observed before failure, recorded
strains in top GFRP bars of these two beams were much less than the rupture strains as shown in Figure 3.15, indicating local de-bonding between top GFRP bars and concrete.

Figure 3-15 Load-Bar Strain Relation for Continuously Supported Beams Tested

3.3.2 Crack Propagation and Failure Modes for CFRP Beams

As for the CFRP reinforced beams, Figures 3-16 and 3-17 illustrated the crack width at the middle support of the continuously supported beams, and at the mid-span of each tested beam respectively. The first visible cracking load of each beam tested is presented in Table 3.4. The steel reinforced concrete beam cracked at a later stage in comparison with its similar continuously supported CFRP reinforced concrete beam. For the CFRP continuous beams, the earliest crack initiation, at the mid-span sections, was observed in beam C-
C-3 reinforced with the smallest CFRP reinforcement ratio at its bottom layer. Beam C-C-4 reinforced with 2 $\phi$ 7.5mm CFRP bars at the top layer, was the first beam to be cracked over the middle support section among the three CFRP reinforced continuous beams. Beam C-C-5 demonstrated the largest cracking load, at either mid-span or middle support section among the three continuous CFRP reinforced concrete beams.

It was observed that crack patterns of CFRP reinforced concrete beams at early stage of loading were similar to those of the steel-reinforced concrete beam, but as loads increased, crack spacing decreased and crack width increased compared with the steel beam. This is mainly attributed to the reported lower bond strength values that FRP bars demonstrate in comparison with steel bars since it has been found by several researchers [7, 100, 124-126] since an adequate bond between reinforcing bars and concrete arrests flexural cracking. It was also observed that the continuous CFRP reinforced concrete beams developed a remarkable wide crack over the intermediate support as shown in Fig. 3.18, indicating bond slip between CFRP bars and concrete. Belarbi and Wang[124] reported that the bond strength of GFRP bars was about twice as much as that of CFRP bars, which consequently would cause an un-tolerated width of cracks in CFRP reinforced concrete beams at large loads.

All CFRP beams tested exhibited bar rupture failure mode. These beams were reinforced with an under-reinforcement ratio of CFRP bars. Thus, it was expected that the strain in the CFRP reinforcement would reach its ultimate limit, at the failed section, before the full exhaustion of ultimate strain of
concrete, which usually results in such failure mode, as shown in Fig. 3-18 for beam C-C-5. In continuous CFRP reinforced concrete beams, the bottom CFRP bar ruptured at the mid-span section, while over support section experienced wide cracks indicating that bond slip would have been occurred as explained above and shown in Fig. 3.18.

<table>
<thead>
<tr>
<th>Beam</th>
<th>Visual cracking load</th>
<th>Failure Load (kN)</th>
<th>Mode of failure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>At mid-span</td>
<td>Over middle support</td>
<td></td>
</tr>
<tr>
<td>C-S-1</td>
<td>17.3</td>
<td>N/A</td>
<td>93.2</td>
</tr>
<tr>
<td>C-S-2</td>
<td>15.9</td>
<td>N/A</td>
<td>64.4</td>
</tr>
<tr>
<td>C-C-3</td>
<td>18.3</td>
<td>18.3</td>
<td>148.6</td>
</tr>
<tr>
<td>C-C-4</td>
<td>21.2</td>
<td>12.2</td>
<td>187.9</td>
</tr>
<tr>
<td>C-C-5</td>
<td>22.1</td>
<td>24.9</td>
<td>180.4</td>
</tr>
<tr>
<td>S-C-6</td>
<td>47.1</td>
<td>47.1</td>
<td>332.5</td>
</tr>
</tbody>
</table>

Table 3-4 Experimental Failure Loads
Figure 3-16 Crack Width at the Middle Support for the Tested Continuously Supported Beams

Figure 3-17 Crack Width at the Mid-Span for the Tested Beams
Figure 3-18: Wide Cracks at the Middle Support of Continuous CFRP Reinforced Concrete Beams test
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Figure 3-19 Bar Rupture Failure Mode of Beam C-C-5

3.3.3 Load Capacity for GFRP Reinforced Beams

Failure loads of the beams tested are plotted in Figure 3.20 and presented in Table 3.3 Over-reinforced simply supported beam GsO failed at nearly 50% of the total failure load of beams GcUO and GcOO. Similarly beam GsU failed at nearly 40% of the total failure load of beam GcOU. Such harmony in comparison between the load capacity of the simply and continuously supported GFRP reinforced concrete beams is attributed to the identical reinforcement ratio at the bottom layer of each compared set of beams. Beams GcUO and GcOO have tolerated more loads than beam GcOU as beam GcOU is reinforced with under-reinforcement ratio of GFRP bars at the bottom layer.
Even though the area of the top GFRP reinforcement used in beam GcOO was three times higher than that used in beam GcUO, beam GcOO resisted a slightly higher failure load (1.3 %). This indicates that GFRP top reinforcement was ineffective in enhancing the beam load carrying capacity.

In spite of the under-reinforcement ratio used for the top and bottom layers of steel reinforced concrete continuous beam S-C-6, this beam accomplished similar load capacity as beam GcOO, which in contrast to beam S-C-6, reinforced with over-reinforced ratio of GFRP bars at the top and bottom layers as indicated in Table 3.3

![Experimental Failure Loads](image)

**Figure 3-20 Experimental Failure Loads**

### 3.3.4 Load Capacity for CFRP Reinforced Beams

Failure loads of the beams tested are presented in Table 3.4 and Fig. 3.22. The simply supported reinforced concrete beam C-S-1 failed at nearly 50% of
the total failure load of beams C-C-5 and C-C-4. Similarly beam C-S-2 failed at nearly 43% of the total failure load of beam C-C-3. Such harmony in comparison between the load capacities of the simply and continuously supported CFRP reinforced concrete beams is attributed to the identical reinforcement ratio at the bottom layer of each compared set of beams. Beams C-C-5 and C-C-4 have tolerated more loads than beam C-C-3 as beam C-C-3 is reinforced with less reinforcement ratio of CFRP bars at the bottom layer. Similar trend has been also illustrated by the higher failure load of beam C-S-1 in comparison with that of beam C-S-2.

Even though the reinforcement ratio of the top CFRP bars used in beam C-C-4 was less than half that used in beam C-C-5, beam C-C-4 resisted similar failure load to that of beam C-C-5. This would indicate that the CFRP top reinforcement was ineffective in enhancing the beam load carrying capacity. But owing to the previously stated de-bonding issue of the top CFRP reinforcement, this conclusion could be challenged.

Although beam S-C-6 was reinforced with steel bars having similar strength to that used in beam C-C-3, it accomplished the highest load capacity among all the continuous beams tested. The failure load achieved by the continuous CFRP reinforced concrete beams tested could have been much higher and significantly comparable to that of the steel reinforced concrete beam, if it was not arrested by the de-bonding occurred for the top layer reinforcement over the middle support of these beams. That has been illustrated by the strain values of the top bars of the continuously supported CFRP beams, which were less than the ultimate strain values that have been tested experimentally, as shown in Figure 3.21.
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Figure 3-21 Load-Bar Strain Relation for Continuously Supported Beams Tested

Figure 3-22 Experimental Failure Loads of CFRP Beams Tested
3.3.5 Redistribution of Support Reactions for GFRP Reinforced Beams

Reactions recorded at middle and end supports for each continuous beam are presented in Figure 3.23. To assess the load redistribution of each beam the elastic reactions at the middle and end supports, considering a uniform flexure stiffness ‘$EI$’ throughout the entire beam, are also plotted in the same figure. Due to the ductile behaviour of the steel bars, it was expected that beam S-C-6 would demonstrate distinctive load redistribution in comparison to the GFRP reinforced concrete beams. Such anticipation has not been shown in Figure 3.23 due to the following reasons:

- The loading system illustrated in Figure 3.1 produced a small difference between the moment values at mid-span and the middle support.
- The identical reinforcement (4 bars of 12mm. diameter) used at the top and bottom layers of the steel reinforced concrete beam tested caused similarity in strains of the top and bottom bars as illustrated in Figure 3.15, where the yielding point for the top and bottom steel reinforcement was near enough to be compatible.

Beam GcOO demonstrated similar unremarkable moment redistribution behaviour to beam S-C-6. This would be mainly accredited to the brittle nature of the GFRP bars.

Beams GcUO and GcOU behaved slightly dissimilar to beam GcOO, due to the variation of the flexure stiffness ‘$EI$’ throughout the entire length of these two beams, which is attributed to the difference in the amount of reinforcement at the top and bottom layers. Further to that, the reverse reinforcement
arrangement used for beam GcOU in comparison to beam GcUO demonstrated the opposite way of reaction response for these two beams.

![Graph of Load-Reaction Relation for Continuously Supported Beams Tested](image)

**Figure 3-23 Load-Reaction Relation for Continuously Supported Beams Tested**

### 3.3.6 Redistribution of Support Reactions for CFRP Reinforced Beams

The reactions recorded at the middle and end supports of each continuous beam tested are presented in Fig. 3.24. To assess the load redistribution of each beam, the elastic reactions at the middle and end supports, considering a uniform flexure stiffness ‘EI’ throughout the entire beam, are also plotted in the same figure. Owing to the ductile behaviour of the steel bars, it was expected that beam S-C-6 would demonstrate distinctive load redistribution in comparison
with the CFRP reinforced concrete beams tested. Such anticipation has not been exhibited by beam S-C-6 tested as shown in Fig. 3.24 due to the reasons mentioned earlier.

![Figure 3- 24 Support Reactions of Construction Beams Tested](image)

All continuous CFRP reinforced concrete beams tested exhibited very similar trend of end and middle support reactions, however, they failed at different loads. They also demonstrated similar unremarkable moment redistribution behaviour to beam S-C-6 till certain load. Further to that load, the support reaction distribution is suddenly shifted as more loads transferred to the end supports leaving the middle support with less load. This sudden shift could be mainly accredited to the previously mentioned wide cracks over the middle support that eventually changed the reaction system of continuously supported
beams to that of simply supported beams. However, there is no remarkable moment redistribution after the above mentioned loading shift owing to the brittle nature of the CFRP bars as depicted in Fig. 3.24.

### 3.3.7 Mid-Span Deflection for GFRP Reinforced Beams

The experimental load against mid-span deflection curves of the all beams tested are presented in Figures 3.25.a and b. Each curve represents the average of two readings of deflection obtained from LVDTs and dial gauges at the mid-span of each beam tested. For continuous beams, recorded mid-span deflections at one side were similar to those at the other side, therefore one side mid-span deflections are presented in Figure 3.25.

Initially, all tested beams were un-cracked where they exhibited linear load-deflection behaviour. This is accredited to the linear elastic characteristics of concrete, FRP bars, as well as steel bars before reaching the yielding point. With the increment of additional loading, cracking occurred at the mid-span of each beam, causing a reduction in stiffness.

As the GFRP reinforced concrete beams demonstrated wider crack openings than the steel reinforced concrete beam, they exhibited higher mid-span deflections, as it could be seen from Figure 3.25 a.

Beam GcOU exhibited the highest deflection among the GFRP continuous beams, due to the low stiffness of its bottom reinforcement ($E_f A_f = 16808$ kN) in comparison to that of the other two continuous GFRP reinforced concrete beams GcOO and GcUO ($E_f A_f = 46057$ kN). The over-reinforcement ratio used at the top layer of beam GcOO, which was equivalent to more than 3.3 times of the reinforcement ratio used for the same layer in beam GcUO, had a marginal
effect on the reduction of the deflection of this beam in comparison to that of beam GcUO as shown in Figure 3.25.a. Beam GsO deflected less than beam GsU as the bottom GFRP reinforcement used in beam GsO had higher stiffness, $E_f A_f$, than that of the bottom GFRP bars in beam GsU.

![Figure 3-25 Experimental Deflection for GFRP Beams Tested](image-url)
As for the CFRP beams, with the increase of loading, cracking occurred at mid-span of each beam, causing a reduction in stiffness. As the CFRP reinforced concrete beams demonstrated wider crack openings than the steel reinforced concrete beam, they exhibited lower stiffness and consequently higher mid-span deflections, as it could be seen from Fig. 3.26.b.

Beam C-C-3 exhibited the highest deflection among all continuous CFRP reinforced concrete beams tested, due to the low stiffness, $E_f A_f$, of its bottom reinforcement in comparison with that of the other two continuous CFRP reinforced concrete beams C-C-4 and C-C-5. The large amount of top layer CFRP reinforcement of beam C-C-5, which was equivalent to more than 2.5
times of that used in beam C-C-4, had a small effect on the reduction of the
deflection of this beam in comparison with that of beam C-C-4. Beam C-S-1
deflected less than beam C-S-2 as the bottom CFRP reinforcement used in
beam C-S-1 had higher stiffness, $E_i A_i$, than that of the bottom CFRP bars in
beam C-S-2.

As expected, the steel reinforced concrete continuous beam S-C-6 exhibited
the highest stiffness among all continuous beams tested owing to the higher
modulus of elasticity of steel than FRP reinforcement. In addition, only beam S-
C-6 demonstrated ductile behaviour before failure due to yielding characteristics
of steel bars.

3.4 Conclusions

Based on the experimental work described in this chapter, the following
conclusions are drawn:

- Due to the lower elastic modulus of GFRP bars, continuously supported
  GFRP reinforced concrete beams can develop earlier and wider cracks
  than similar steel reinforced concrete beams.
- The proposition of the ACI 318-05 [121], regarding the spacing of steel
  stirrups, has to be reconsidered to avoid shear failure when FRP bars
  are used as longitudinal reinforcement for continuously supported
  concrete beams with steel stirrups.
- Continuously supported GFRP and CFRP reinforced concrete beams do
  not demonstrate any remarkable load redistribution.
- Over reinforcing the bottom layer of simply and continuously supported concrete beams by GFRP bars could be a key factor in enhancing the load capacity, controlling the deflection, in addition to the delay of crack propagation at the mid-span section of such beams.

- Increasing the top layer reinforcement of continuously supported GFRP reinforced concrete beams does not contribute significantly in improving the load capacity and deflection reduction.

- Increasing the CFRP reinforcement of the bottom layer of simply and continuously supported concrete beams is a key factor in enhancing the load capacity and controlling deflection.

- Increasing the top layer CFRP reinforcement of continuous beams slightly reduced deflections, but, has not shown such improvement for the beam load capacity.

- In the continuous CFRP reinforced concrete beams tested, de-bonding of CFRP bars from concrete appears to be an important issue that needs to be further investigated.

- The above concluded points indicated, in general, that increasing the reinforcement ratio of the CFRP bars has a favourable effect on the concrete beams behaviour reinforced with such reinforcement. Thus it is recommended to investigate more on the applying of the over-reinforcement ratio of such bars, since the present research has not had the chance to implement such application. The investigation should test the affect of such application on the de-bonding, crack propagation and deflection of CFRP reinforced concrete beams.
CHAPTER FOUR

ACI 440.1R-06 Evaluation against Experimental Results

4.1 Introduction:
This chapter evaluates the ACI 440.1R-06 [32] equations for moment capacity and deflection against the experimental results of continuously and simply supported GFRP and CFRP reinforced concrete beams tested.

4.2 Prediction of Loads and Modes of Failure
The ACI 440.1R-06 report, based on the balanced FRP reinforcement ratio $\rho_{fb}$ obtained from Eq. 4-1 below, predicted the moment capacity $M$ of beams reinforced with FRP bars using Eqs. 4-2 and 4-3 when the reinforcement ratio $\rho_f$ is greater than $\rho_{fb}$, and by applying Eqs. 4-4 and C 4-5 when the reinforcement ratio $\rho_f$ is less than $\rho_{fb}$.

$$\rho_{fb} = 0.85 \beta_1 \frac{f_c'}{f_{fu}} \frac{E_f \varepsilon_{cu}}{E_f \varepsilon_{cu} + f_{fu}}$$

(4-1)

$$M = \rho_f f_f (1 - 0.59 \frac{\rho_f f_f}{f_c'}) bd^2$$

(4-2)

$$f_f = \sqrt{\frac{(E_f \varepsilon_{cu})^2}{4} + \frac{0.85 \beta_1 f_c'}{\rho_f} E_f \varepsilon_{cu} - 0.5 E_f \varepsilon_{cu} \leq f_{fu}}$$

(4-3)
\[ M = A_f f_u (d - \frac{\beta c_b}{2}) \]  
\[ c_b = \left( \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{fu}} \right) d \]

where \( \rho_f \) (= \( A_f / bd \)) is the FRP reinforcement ratio, \( A_f \) is the area of FRP reinforcement, \( b \) and \( d \) are the width and effective depth of the GFRP reinforced concrete beam, \( f_c' \) is the cylinder compressive strength of concrete, \( f_u \) is the ultimate tensile strength of FRP bars, \( \varepsilon_{cu} \) is the ultimate strain in concrete, \( E_f \) is the modulus of elasticity of FRP bars, \( f_t \) is the FRP stress at which the concrete crushing failure mode occurs, \( c_b \) is the neutral axis depth for balanced failure as defined in Eq. (4-5) and \( \beta_t \) is a strength reduction factor taken as 0.85 for concrete strength up to and including 27.6 MPa. For strength above 27.6 MPa, this factor is reduced continuously at a rate of 0.05 per each 6.9 MPa of strength in excess of 27.6 MPa, but is not taken less than 0.65.

For the simply supported FRP reinforced concrete beam, the beam load capacity \( P' \) is estimated by satisfying the equilibrium condition at the mid-span critical section \((M=P' l/4, \text{ where } M \text{ is the moment capacity calculated using the ACI 440.1R-06 equations presented above and } l \text{ is the beam span})\).

The load capacity of continuously supported FRP reinforced concrete beams is chosen to be the lower load that causes the accomplishment of the moment capacity of either mid-span or middle support section. This is mainly due to the brittle nature of concrete crushing or FRP rupture mode of failure. The elastic moments at the designated critical sections of a continuously supported beam...
are \(0.156P_l\) and \(0.188P_l\) for the mid-span and middle-support sections, respectively, where \(P\) and \(l\) are the mid span applied load and beam span, respectively. The self weight of beams tested is negligible compared with the failure load; therefore it is not included in the above calculation.

### 4.2.1 Prediction of Loads and Modes of Failure for the GFRP Reinforced Concrete Beams

The predicted and experimental failure loads for each beam tested is presented in Table 4-1 and plotted in Figure 4-1. The ACI 440.1R-06 equation reasonably predicted the load capacity of the simply supported beam GsU and significantly underestimated the load capacity of beam GsO.

<table>
<thead>
<tr>
<th>Beam Notation</th>
<th>Total failure load, (2P) (kN)</th>
<th>Experimental</th>
<th>ACI prediction*</th>
<th>Experimental modes of failure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Experimental</td>
<td>Mid-span</td>
<td>Middle support</td>
</tr>
<tr>
<td>GcOU</td>
<td>290.2</td>
<td>287.7</td>
<td>321.4</td>
<td>Mid-span FRP rupture</td>
</tr>
<tr>
<td>GcUO</td>
<td>328.7</td>
<td>357.7</td>
<td>229.0</td>
<td>Mid-span concrete crushing</td>
</tr>
<tr>
<td>GcOO</td>
<td>333.0</td>
<td>336.2</td>
<td>279.0</td>
<td>Mid-span concrete crushing combined with shear failure</td>
</tr>
<tr>
<td>GsO</td>
<td>163.0</td>
<td>112.5</td>
<td>N/A</td>
<td>Concrete crushing</td>
</tr>
<tr>
<td>GsU</td>
<td>118.8</td>
<td>92.3</td>
<td>N/A</td>
<td>GFRP rupture</td>
</tr>
<tr>
<td>S-C-6</td>
<td>332.3</td>
<td>350.9</td>
<td></td>
<td>Flexure/Tension</td>
</tr>
</tbody>
</table>

* ACI 440-06 and ACI 318-05 were used for GFRP and steel beams, respectively.

| Table 4-1 ACI and Experimental Failure Loads of GFRP Beams Tested |
This may be attributed to the difficulty in predicting the concrete crushing strain, $\varepsilon_{cu}$, which, in turn, has a major impact on stresses $f_t$ in GFRP bars obtained from Eq. (4-3) when failure occurs due to concrete crushing. In addition, the ACI 440 equation ignores the reinforcement at the compression zone which would have some additional influence on the underestimation of the predicted moment capacity. The tensile rupture and concrete crushing modes of failure are correctly predicted by the ACI 440.1R-06 method for beams GsU and GsO, respectively.

![Figure 4-1 ACI and Experimental Failure Loads](image)

As for the continuously supported GFRP reinforced concrete beams, the load and location of failure of beam GcOU were reasonably predicted by the ACI
Chapter Four: ACI.1R-06 Evaluation against Experimental Results

440.1R-06 equations. A continuous beam reinforced similarly to beam GcOU in this study was tested by Grace et al. [86]. The ACI 440.1R-06 equations also reasonably predicted the load capacity for the Grace's beam. Such successful predictions strengthen the belief in the credibility of the ACI 440.1R-06 equations in estimating the failure of continuous beams under-reinforced with GFRP bars in the bottom layer.

However, the ACI 440.1R-06 equations predicted that beams GcUO and GcOO would fail at much lower loads than those recorded in experiments as it could be seen from Table 4-1 and Figure 4-1. The application of the previously detailed principle, of choosing the lower load that would achieve the moment capacity at either the mid-span or middle support section, predicted the failure location of beams GcOO and GcUO to take place at the middle support that has not been fulfilled in the actual experimental tests. This is mainly attributed to the wide cracks that were developed over the middle support section of both beams (see Figure 3-12) due to de-bonding of top GFRP bars which turned the continuously supported beam into two over reinforced simply supported beams. Thus the failure was eventually occurred at the mid-span sections. To validate this justification, Figure 4-1 illustrated the ACI 440-1R-06 prediction of the load capacity for one span of beams GcOO and GcUO, as a simply supported beam subjected to a mid-span point load. The comparison between the ACI 440.1R-06 failure load prediction of beams GcOO and GcUO at their simply and continuously supported status authenticated the above assumption. This behaviour explains the strain values of the top reinforcement bars of both
beams near failure, which were much less in comparison to their ultimate strain limit, as it could be demonstrated in Figure 3-14 and Table 3-2.

The capacity reduction factor recommended by ACI 440-06 was developed based on test data of simply supported beams. As the ACI 440 equations are more conservative in predicting failure loads for the simply supported beams than continuous beams tested, as shown in Figure 4-1 and Table 4-1, the use of the same capacity reduction factor recommended by ACI 440-06 when designing continuous beams would be less conservative. Such conclusion should be further investigated when more test results become available.

4.2.2 Prediction of Loads and Modes of Failure for the CFRP Reinforced Concrete Beams

The predicted and experimental failure loads for CFRP beams tested is presented in Table 4-2 and Fig. 4-2. The ACI 440.1R-06 equation reasonably predicted the load capacity of the simply supported beam C-S-2 and slightly underestimated the load capacity of beam C-S-1. The tensile rupture modes of failure are correctly predicted by the ACI 440.1R-06 method for both simply supported CFRP reinforced concrete beams, C-S-1 and C-S-2.
<table>
<thead>
<tr>
<th>Beam</th>
<th>Exp. (A)</th>
<th>ACI (B)</th>
<th>B/A</th>
<th>Mode of failure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Continuous</td>
<td></td>
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</tr>
<tr>
<td></td>
<td></td>
<td>Simple</td>
<td>Cont.</td>
<td>Simple</td>
</tr>
<tr>
<td>C-S-1</td>
<td>93.2</td>
<td>N/A</td>
<td>N/A</td>
<td>84.6</td>
</tr>
<tr>
<td>C-S-2</td>
<td>64.4</td>
<td>N/A</td>
<td>N/A</td>
<td>67.1</td>
</tr>
<tr>
<td>C-C-3</td>
<td>148.6</td>
<td>203.9</td>
<td>212.9</td>
<td>203.9</td>
</tr>
<tr>
<td>C-C-4</td>
<td>187.9</td>
<td>261.3</td>
<td>171.5</td>
<td>171.5</td>
</tr>
<tr>
<td>C-C-5</td>
<td>180.4</td>
<td>216.9</td>
<td>265.4</td>
<td>216.9</td>
</tr>
<tr>
<td>S-C-6</td>
<td>332.5</td>
<td>N/A</td>
<td>N/A</td>
<td>350.9</td>
</tr>
</tbody>
</table>

Table 4-2 ACI and Experimental Load of GFRP Beams Tested
Figure 4-2 Experimental and ACI Predicted Failure Loads of Beams Tested
As for the continuous CFRP reinforced concrete beams, the failure load values and locations of the beams tested were differently predicted by the ACI 440 equations from those experimentally measured. As shown in Table 4-2 and Fig. 4-2, the ACI 440.1R-06 equations predicted that these beams would tolerate much more load than the experimental failure load if they were to fail at the mid-span as occurred in the physical tests. Furthermore, by applying the previously detailed principle, of choosing the lower load that would achieve the moment capacity at either mid-span or middle support section, the failure locations of beams C-C-4 and C-C-5 were predicted to take place at the middle support, which has not been fulfilled in the actual experimental tests. This is mainly attributed to the wide cracks that were developed over the middle support section of the continuous CFRP reinforced concrete beams (see Fig. 3-15) due to de-bonding of the top CFRP bars which turned the continuously supported beam into two simply supported beams. Thus the failure was eventually occurred at mid-span section. To validate this justification, Table 4-2 and Fig. 4-2 illustrated the ACI 440-1R-06 prediction of the load capacity for one span of the three continuous concrete beams reinforced with CFRP reinforcement, as a simply supported beam subjected to a mid-span point load. The comparison between the ACI 440.1R-06 failure load predictions of these beams assuming simply supported status were closer to the experimental failure loads, authenticated this assumption.
4.3 ACI 440.1R Deflection Prediction

The immediate deflection of simply and continuously supported reinforced steel concrete beams loaded with a mid-span point load illustrated in Figures 3-1 and 3-2, could be calculated by Eqs. (4-6) and (4-7), respectively, as given below:

\[
\Delta = \frac{P l^3}{48 E_c I_e} \quad (4-6)
\]

\[
\Delta = \frac{7}{768} \frac{P l^3}{E_c I_e} \quad (4-7)
\]

where \( P \) is the mid-span applied load at which the deflection is computed, \( l \) is the span length, \( E_c \) is the modulus of elasticity of concrete and \( I_e \) is the effective moment of inertia of the beam section. A modified expression for the effective moment of inertia \( I_e \) to be used for predicting the deflection of FRP reinforced concrete beams is given by ACI 440 committee as follows:

\[
I_e = \left[ \frac{M_{cr}}{M_a} \right]^3 \beta_d I_g + \left[ 1 - \left( \frac{M_{cr}}{M_a} \right)^3 \right] I_{cr} \leq I_g \quad (4-8)
\]

where \( M_{cr} \) is the cracking moment = \( 2 f_{cr} l_g / h \), \( M_a \) is the applied moment, \( \beta_d \) is a reduction coefficient = \( [(0.2 \rho_f / \rho_{fb}) \leq 1] \), \( I_g \) is the gross moment of inertia = \( bh^3/12 \), \( h \) is the overall height of the concrete beam, \( I_{cr} \) is the moment of inertia of transformed cracked section = \( \left( bd^3 / 3 \right) k^3 + n_f A_f d^2 (1-k)^2 \), \( k \) is the ratio of the neutral axis depth to reinforcement depth = \( \sqrt{(\rho_f n_f)^2 + 2 \rho_f n_f - \rho_f n_f} \), \( n_f = (E_f/E_c) \) is the modular ratio between the FRP reinforcement and concrete, \( E_c = \)
$4750 \sqrt{f_c}$ (N/mm$^2$) and $f_{cr}$ is the modulus of rupture of concrete $= 0.62 \sqrt{f_c}$ (N/mm$^2$).

**4.3.1 ACI 440.1R Deflection Prediction of GFRP Reinforced Beams**

Comparisons between experimental load-deflection curves obtained in this study and those predicted by the ACI 440.1R-06[32] equations at mid-span of the simply supported GFRP reinforced concrete beams are presented in Figure 4-3. The curves show that there is a good agreement between the experimental and predicted deflection values for the simply supported GFRP reinforced concrete beams tested.

![Figure 4-3 ACI and Experimental Deflection for Simply Supported GFRP Reinforced Beams Tested](image)

Figure 4-3 ACI and Experimental Deflection for Simply Supported GFRP Reinforced Beams Tested
For continuous members, the ACI 318-05[127] stated that the use of the mid-span section properties is considered satisfactory in approximate deflection calculation primarily because the mid-span rigidity (including the effect of cracking) has the dominant effect on deflection. Furthermore, the ACI 318-05 also suggests, alternatively, using the average $I_e$ at the critical positive and negative moment sections for deflection calculation. The ACI 318-95[128] recommends that the above mentioned average of $I_e$ could be obtained, for beams with one continuous end, by the following formula:

$$I_e = 0.85I_m + 0.15I_{cont.end} \quad (4-9)$$

where $I_m$ refers to $I_e$ at the mid-span section and $I_{cont.end}$ refers to $I_e$ at the middle support section. The above mentioned two methods were implemented to predict the deflection of the GFRP continuous beams tested. Comparisons between experimental load-deflection curves obtained in this study and those predicted by the above mentioned suggestions for the continuously supported GFRP tested beams are presented in Figures 4-4 (for Beam GcOU), 4-5 (for Beam GcUO) and 4-6 (for Beam GcOO).
Chapter Four: ACI.1R-06 Evaluation against Experimental Results

Figure 4.4 ACI and Experimental Deflection for Beam GcOU

Figure 4.5 ACI and Experimental Deflection for Beam GcUO
Figure 4-4 illustrates that the experimental deflection of beam GcOU compared well with the predicted deflection, by applying Eq. 4-8 for the mid-span section only. Meanwhile, using the same method for beams GcUO and GcOO, shows a reasonable comparison to the experimental results, with a steady underestimation of the deflection up to nearly 50% of the failure load. As the load was increased, this underestimation has progressively increased till the end of loading due to the sudden increase of the over support crack width, as shown in Figure 3-12, which took place almost at the same percentage of 50 % of the failure load, as given in Figures 4-5 and 4-6.
To overcome the shortcomings of the previous methods, a reduction factor, $\gamma_G$, has been applied to the second term of the ACI 440-06 equation 4-8, which represents the post cracking phase, as below:

$$I_s = \left[ \frac{M_i}{M_a} \right]^3 \beta_s I_s + \left[ 1 - \left[ \frac{M_i}{M_a} \right]^3 \right] I_s \gamma_G$$ \hspace{1cm} (4-10)

A reduction factor, $\gamma_G$, of 60% was found to be an effective tuning parameter for the deflection prediction as it could be seen in Figures 4-4, 4-5 and 4-6. Further test results would be required to authenticate the proposed value of $\gamma_G$.

### 4.3.2 ACI 440.1R Deflection Prediction of CFRP Reinforced Beams

Comparisons between predicted by ACI 440 and experimental deflections for the two simply supported beams, C-S-1 and C-S-2, are presented in Fig. 4-7. It shows that the experimental deflection of the simply supported beams tested reasonably compared with the predicted deflection at early stages of loading. But as the load increased, a proportionally stiffer trend is predicted by the ACI 440 equations.
For continuous members, the two previously discussed methods, mid span section properties, formula 4-8, and average properties, formula 4-9, were implemented in the current analysis to predict also the deflection of the continuous CFRP reinforced concrete beams tested.

Comparisons between experimental load-deflection curves obtained in this study and those predicted by the above mentioned methods for continuous CFRP reinforced concrete beams C-C-3, C-C-4 and C-C-5 are presented in Fig. 4-8. At early stages, the ACI 440 equations closely predicted the two beam deflections. But, after the occurrence of cracks over the intermediate support,
the ACI 440 predictions using the mid span section properties or average properties of mid-span and over support sections as presented by Eq. 4-9 are far much stiffer. However, deflections predicted using mid-span section properties were slightly softer and closer to experiments. This is mainly attributed to the wide cracks over the middle support which transformed the continuous beams tested into two simply supported beams. Deflections of continuous beams tested were also predicted by assuming the beam is converted into two simply supported beams. The ACI 440 equations has shown a closer prediction to that experimentally measured when the beam is assumed to be simply supported as shown in Fig. 4-8.

Figure 4-8 (a) C-C-3
Figure 4-9 (b) C-C-4

Figure 4-10 (C) C-C-5

Figure 4-11 Comparison Between ACI and Experimental Deflection of Continuous CFRP Reinforced Concrete Beams
4.4 Conclusion

- Load capacity and deflection of GFRP and CFRP simply supported concrete beams could be reasonably predicted using ACI 440.1R-06 equations. However, their potential capabilities for predicting the load capacity of continuous GFRP and CFRP reinforced concrete beams need to be verified against further test results making an allowance for the bond strength of GFRP and CFRP bars and concrete.

- The ACI 440.1R-06 equations appear to be effective in predicting the deflection of the under-reinforced at the bottom layer GFRP continuously supported concrete beams. As for the over-reinforced at the bottom layer GFRP continuously supported beams, the prediction process has been negatively affected by the wide cracks appeared over the intermediate support, which eventually turned the continuous beams into two simply supported beams.

- The ACI 440.1R-06 equations could reasonably predict the deflection of GFRP continuous beams with a slight steady under-estimation before the occurrence of wide cracks over the middle support. A proposed reduction factor of 60% for the post cracking term for estimating the effective moment of inertia proposed by ACI 440-06, has been found to be effective in tuning the deflection prediction of continuously supported GFRP reinforced concrete beams.

- The ACI 440.1R-06 equations seem to be effective in predicting the deflection of CFRP simply and continuously supported concrete beams up to the initiation of excessive cracks. Further to that, the prediction
process, particularly for the continuously supported CFRP reinforced concrete beams, has been unconstructively affected by the loss of bond between CFRP top reinforcement and concrete.
CHAPTER FIVE

Flexural Analysis of FRP Reinforced Concrete Cross-Sections

5.1 Introduction

This chapter describes the first part of an analytical modelling program developed by the author to investigate the flexural behaviour of simple and continuous beams reinforced with FRP reinforcement bars. The first part of the program is devoted to producing the moment-curvature relationship of concrete sections reinforced with FRP bars. Hence, the influence of design parameters such as the reinforcement ratio, internal reinforcement type, GFRP or CFRP, top reinforcement ratio and concrete compressive strength could be studied. First, mechanical properties of concrete and FRP bars including stress-strain relationship of both materials will be discussed and the concluded characteristics of these materials will be taken to action in the body of this computer programme. Using the analytical modelling program, a design procedure and a sample of a design chart to calculate the moment capacity of the FRP reinforced sections are developed in this chapter.

The moment capacities calculated by the model developed will be compared and discussed with their actual corresponding experimental results presented in chapter 3 and other experimental samples recorded by others. Finally, ACI
prediction for the above stated experimental samples will be also compared against the previously mentioned moment capacities calculated by the proposed model in the current research.

5.2 Material Modelling

This section will address the mechanical properties of the material used for the tested beams, namely concrete and FRP bars. These properties will be discussed and the concluded characteristics of these materials will formulate the fundamental aspects for the numerical method used for the earlier mentioned analytical program.

5.2.1 Concrete in Compression

Concrete is categorized by considerably nonlinear deformation behaviour. The material non-linearity is assumed to occur due to cracking of concrete in tension and crushing of concrete in compression. Bangash [129] stated that experimental tests show that concrete behaves in a highly nonlinear manner in uni-axial compression. Fig.5-1 shows a typical stress-strain relationship of concrete specimen subjected to uni-axial compression. This stress-strain curve is linearly elastic up to 30% of the maximum compressive strength. Above this point the curve increases gradually up to about 70-90% of the maximum compressive strength [129]. Eventually it reaches the peak value, which is the maximum compressive strength \( f_{cu} \). Immediately after this peak point the stress decreases; this part of the curve is termed as softening. After curve descends, crushing failure occurs at an ultimate strain of \( \varepsilon_{cu} \). A numerical expression has been developed by Hognestad [130] which treats Fig 5-1 the ascending part as
parabola and the descending part as a strain descending line. This numerical expression is given as:

$$f_{c} \over f_{cu} = 2 \frac{\varepsilon}{\varepsilon_0} \left(1 - \frac{\varepsilon}{2\varepsilon_0}\right)$$ \hspace{1cm} (5-1)

$$f_{c} \over f_{cu} = 1 - 0.15 \left(\frac{\varepsilon - \varepsilon_0}{\varepsilon_{cu} - \varepsilon_0}\right)$$ \hspace{1cm} (5-2)

where $\varepsilon_0$ is the strain at the peak stress and $\varepsilon_{cu}$ is the ultimate strain.

British Standard BS-8110 [131] simplified the above mentioned curve into the curve presented in Fig. 5-2, which exhibits short-term stress-strain relationship.

The current research benefited from this simplification and adopted this curve in the computer analytical modelling program. Fig 5-2 shows a parabola up to $\varepsilon_0 = 2.4 \times 10^{-4} \sqrt{f_{cu}}$, $f_{cu}$ (N/mm$^2$), and a straight horizontal line up to ultimate failure strain $\varepsilon_c = 0.0035$. The term $\gamma_m$ is a safety factor for design purposes only and has not been used in the computational calculations for the present research.
Figure 5-1 Concrete stress-strain relationship under uni-axial compression [129]

Figure 5-2: Short-term stress-strain relationship presented by BS8110
5.2.2 Concrete in Tension

Before the onset of vertical cracking due to flexure, the stress-strain relationship of the concrete in tension is assumed to be linear elastic, such that:

\[ f_t = E_c \varepsilon_t \] (5-3)

where \( f_t, \varepsilon_t \) and \( E_c \) are the tensile stress, strain and elasticity modulus of concrete, respectively, as shown in Figure 5-3.

After cracking of concrete due to flexure, the tensile force in concrete is assumed to be zero. However, concrete between cracks can still carry tensile stress and thus may increase the stiffness of the member. This is known as tension stiffening [132]. Therefore, the stress-strain relationship shown in Figure 5-3 proposed by Stevens [133] is adopted in the current investigation; it has the following mathematical expression:

\[ \frac{f_t}{f_r} = (1 - \alpha) e^{-\lambda_t (\varepsilon - \varepsilon_r)} + \alpha \] (5-4)

\[ \lambda_t = \frac{270}{\sqrt[3]{\alpha}} \leq 1000 \] (5-5)

\[ \alpha = 75 \frac{\gamma_c}{d_p} \] (5-6)

where \( f_r \) and \( \varepsilon_r \) are the concrete tensile strength and corresponding concrete tensile strain, respectively; \( \lambda_t \) is a parameter controlling the rate of decay of concrete tensile stress as shown in Figure 5-3; \( \gamma_c \) is the ratio between the area
of tensile steel bars and the area of concrete in tension and $d_b$ is the steel bar diameter, which will be replaced by FRP bar diameter in the current research.

![Stress vs. strain relationship for concrete in tension](image)

Figure 5-3: Stress vs. strain relationship for concrete in tension [133]

### 5.2.3 FRP Composite

FRP composites are assumed to exhibit linear elastic behaviour in tension up to failure as shown in Figure 5-4. The equation governing the relationship is as follows:

$$\frac{f_t}{f_r} = (1 - \alpha)e^{-\lambda'(\varepsilon_t - \varepsilon_r) + \alpha}$$

where $\varepsilon_t$, $f_t$ and $E_f$ are the strain, stress and modulus of elasticity of FRP composite bars respectively.

The compressive strength of FRP bars are not reliable, but test results show far much lower strength for FRP bars in compression than tension, for instance it
has been reported that compression stress is 55 and 78% of the tensile strength for GFRP and CFRP bars respectively [134]. Furthermore, the modulus of elasticity of FRP bars in compression is lower than that in tension. According to reports, the compressive modulus of elasticity is approximately 80, 85, and 100% for GFRP, CFRP, and AFRP respectively of the tensile modulus of elasticity for the same product [134]. Fig. 5-4 shows a typical CFRP, AFRP, GFRP stress-strain relationship up to failure [2].

![Figure 5-4 Typical Stress-Strain Relationship for FRP Bars up to Failure][2]

### 5.3 Moment Capacity Numerical Method

The following section explains the numerical method used in this research for calculating the moment capacity of the FRP reinforced concrete sections. This method will also be used for producing the moment-curvature relationship that will be discussed later in this chapter.
The above mentioned method, which is used for the rectangular reinforced concrete cross-section shown in Figure 5-5, is used based on the following assumptions:

- Plane sections before bending remain plane after bending. Thus the strain profile shown in Figure 5-5 is plotted and the strain in each concrete element is determined by linear interpolation.
- Complete bond exists between concrete and internal FRP reinforcement.

5.3.1 Moment Capacity

Based on the previous material modelling the following steps have been established to accomplish the prediction of the moment capacity of FRP reinforced concrete sections.

Step 1: Establishing the balanced reinforcement area for the investigated section in relation to the concrete and FRP reinforcement properties.

Step 2: Establishing the neutral axis of the investigated section using the property of this section in relation to its reinforcement area, which should be compared to the previously calculated balanced reinforcement area. Thus, two scenarios will be investigated, namely over and under-reinforcement cases, to accomplish the neutral axis in each case, to counter for any type of reinforcement ratio for the investigated sections.

Step 3: Establishing the moment capacity of the investigated section, based on the above allocation of the neutral axis to calculate the internal forces that generate the ultimate moment of the proposed section.
Illustration of how to accomplish each of the above listed steps numerically could be detailed below.

### 5.3.1.1 Establishing the Balanced Reinforcement Area

Finding the balanced reinforcement area $A_{fbal}$ for any section of the FRP reinforced beams is the milestone in predicting the failure of such section. This area could be described as the area of the FRP bars that fulfils the equilibrium status between the internal tension force, $F_t$, at its ultimate strain, and the internal compression force, $F_c$, when the extreme compression layer reaches its failure strain $\varepsilon_{cu}$, which will be 0.0035. Thus the following equation should be established:

$$F_c = F_t$$  \hspace{1cm} (5-8)

The first step in estimating the balanced reinforcement area is establishing the depth, $x$, of the neutral axis at the balanced failure state, which is measured from the extreme concrete top surface. In this case of simultaneous failure the triangle similarity of the strain distribution block, shown in Fig. (5-5), could be used to estimate the distance of the neutral axis, as detailed in the following equation:

$$0.0035/X = \varepsilon_{tu} / (d - X)$$  \hspace{1cm} (5-9)

Where $d$ is the effective depth of the section and $\varepsilon_{tu} = f_{tu}/E_f$ as been detailed in equation 5-7 earlier.
Since \( F_i = (f_{fu} A_{fbal}) + T_c \), \hspace{1cm} (5-10) 

where \( T_c \) is the concrete tension force, Eq.(5-8) could be written in the following form:

\[ F_c = (f_{fu} A_{fbal}) + T_c \] \hspace{1cm} (5-11)

Therefore the balanced reinforcement area, \( A_{fbal} \), could be obtained by finding the values of \( F_c \) and \( T_c \) only, as shown in Eq.(5-9), since the value of \( f_f \) is either measured experimentally or given by the manufacturer. The earlier detailed stress-strain relationship presented by BS8110 and Stevens [133] for concrete in compression and tension, respectively, will be advantageous in assessing the appropriate concrete stress.

Details of establishing the required values of \( F_c \) and \( T_c \) are illustrated in the following two sections, 5.3.1.1.1 and 5.3.1.1.2.

### 5.3.1.1.1 Estimating Total Compression Force:

For accuracy purposes the computer model divides any of the tested sections into number of divisions ‘\( n \)’ say 3900. Each strain segment \( \varepsilon_{ci} \), in the strain
distribution diagram, has its own corresponding concrete stress value $f_{ci}$ that could be obtained from the stress-strain relationship presented by BS8110 where $i$ is the range of numbers of the section divisions $n$ for the neutral axis depth $X$. Thus $i$ is taken as the iterative increment value of 1 up to $n$.

Subject to the position of the strain segment, the value of concrete strain in each segment $\varepsilon_{ci}$ could be estimated from the triangle similarity as follow:

$$\varepsilon_{ci} = (X - \left(i \left(\frac{X}{n}\right)\right))(0.0035/X) \quad (5-12)$$

![Fig. 5-6 Strain Distribution and Triangle Similarity for Reinforcement Strain Calculation.](image)

Depending on the value of concrete strain in each segment ($\varepsilon_{ci}$), being smaller or greater than the concrete strain at the end of parabolic section of stress-strain curve ($\varepsilon_0$), the stress in the segment can be calculated using the following equations:

$$0 < \varepsilon_{ci} \leq \varepsilon_0 \quad \Rightarrow \quad f_{ci} = E_c \varepsilon_{ci} - (E_c \varepsilon_{ci}^2 / 2 \varepsilon_0) \quad (5-13)$$

$$\varepsilon_0 < \varepsilon_{ci} \leq \varepsilon_c \quad \Rightarrow \quad f_{ci} = 0.67 f_{cu} \quad (5-14)$$
where $E_c$ is concrete modulus of elasticity, $f_{cu}$ is ultimate cube strength of concrete under compression and $f_{ci}$ is the stress in segment $i$.

The concrete compression force, $F_{ci}$, for each segment $i$, could be obtained by the following equation, using the previously estimated stress value $f_{ci}$:

$$F_{ci} = f_{ci} A_{ci}$$ (5-15)

where $A_{ci}$ is the area of the concrete segment $i$. This area is equivalent to the product of the segment width $b$, which corresponds to the beam width, and the segment height, $X/n$.

The computer programme will sum up all the forces that have been calculated for each segment in a counter, and the final amount of that counter will be considered as the total internal compressive force, $F_c$. Thus the final value of this force will be as follow:

$$F_c = \sum F_{ci}$$ (5-16)

When the top layer of the section is reinforced, an additional compressive force needs to be added up to the total compressive force if the depth of the neutral axis exceeds the depth, $d^\wedge$, of this reinforcement. This force can be calculated using the following steps:

If $X > d^\wedge$

$$\varepsilon_{fc} = (X - d^\wedge)(0.0035/X)$$ (5-17)

$$f_c = E_{fc} \varepsilon_{fc}$$ (5-18)

$$F_{ct} = f_c A_{fc}$$ (5-19)

where $\varepsilon_{fc}$ is the strain of top reinforcement, and $d^\wedge$ are the effective depths of the top reinforcement, $f_c$ is the stress in top reinforcement, $E_{fc}$ is the modulus of
the elasticity of the top FRP bars, $F_{ct}$ is the force in top reinforcements and $A_{tc}$ is top reinforcement area. Compressive strengths of 55 and 78% of the tensile strength have been reported for GFRP and CFRP bars respectively as it has been mentioned earlier [134]. In this case, where the depth of the neutral axis exceeds the depth of this reinforcement $d\backslash$, the final value of the internal compressive force could be estimated by altering equation (5-16) to the following equation:

$$F_c = \sum F_{ct} + 0.55 F_{ct} ; \text{ for GFRP Reinforced beams} \quad (5-20)$$

$$F_c = \sum F_{ct} + 0.78 F_{ct} ; \text{ for CFRP Reinforced beams} \quad (5-21)$$

5.3.1.1.2 Estimating Concrete Tension Force

The only factor left to be calculated to conclude the balanced reinforcement area, $A_{fbal}$, is concrete tension force, $T_c$, to fulfil the requirement of equation (5-11).

Each strain segment $\varepsilon_{cj}$, in the strain distribution diagram, has its own corresponding concrete stress value $f_{cj}$ that could be obtained from the stress-strain relationship, illustrated in Fig. (5-3), where $j$ is the range of numbers of the section divisions $n$ for the depth $d-X$. Thus $j$ is taken as the iterative increment value of 1 up to $n$.

Subject to the position of the strain segment, the value of concrete strain in each segment $\varepsilon_{cj}$ could be estimated from the triangle similarity as follow:

$$\varepsilon_{cj} = (((d - X) - (j(d - X)/n)))(\varepsilon_{fu} / (d - X))) \quad (5-22)$$
Depending on the value of concrete strain in each segment $\varepsilon_c j$, being smaller or greater than the concrete strain at the end of linear section of stress-strain curve $\varepsilon_r$, the stress in the segment can be calculated using the following equations:

\[0 < \varepsilon_c j \leq \varepsilon_r \quad f_{cj} = E_c \varepsilon_c j\]  \hspace{1cm} (5-23)

\[\varepsilon_r < \varepsilon_c j \leq \varepsilon_t \quad f_{cj} = f_r (1 - \alpha) e^{-\lambda(\varepsilon_t - \varepsilon_r) + \alpha}\]  \hspace{1cm} (5-24)

Where $E_c$ is concrete modulus of elasticity, $f_r$ is ultimate strength of concrete under tension and $f_{cj}$ is the stress in segment $j$.

The concrete tension force, $F_{cj}$, for each $j$ segment, could be obtained by the following equation, using the previously estimated stress value $f_{cj}$:

\[F_{cj} = f_{cj} A_{cj}\]  \hspace{1cm} (5-25)

where $A_{cj}$ is the area of the concrete segment $j$. This area is equivalent to the product of the segment width $b$, which corresponds to the beam width, and the segment height $d \cdot X / n$.

The computer programme will sum up and save all the forces that have been calculated for each segment in a counter, and the final amount of that counter will be considered as the total internal tension force. Thus the final value of this force will be as follow:

\[T_C = \sum F_{cj}\]  \hspace{1cm} (5-26)

By referring to Eq.(5-8) the balanced reinforced area ($A_{fbal}$) could be then estimated as follow:

\[A_{fbal} = (F_c - T_c)/f_{ju}\]  \hspace{1cm} (5-27)
5.3.1.2 Establishing the Neutral Axis

Further to the previous step, the reinforcement ratio, $\rho_f$, of the examined section should be calculated to be compared against the balanced reinforcement ratio $A_{fbal}$ established above. The comparison between the above two values will result in the following two cases:

5.3.1.2.1 Concrete Crushing Failure Mode

In such case, where balanced reinforcement ratio $A_{fbal} < \rho_f$, it is expected that the examined section will fail due to concrete crushing. Thus concrete will reach its ultimate strain, 0.0035; meanwhile the FRP bars still reserve some of their ultimate strain. For an assumed neutral axis depth that is equivalent to the value shown in equation (5-28) as a first assumption, the computer programme will start to estimate the total internal compression force following the same steps that has been explained above starting from equation (5-10) till equation (5-16).

$$X = d - (d/r) \quad \text{(5-28)}$$

where $r$ is an input value for the subdivision segments of the effective depth. It has been found, through the iterative trials of several $r$ values, that the computer model replicates the same result for any input value exceeds the value of 3900 for $r$. For that reason, the arbitrary value of $r$ was chosen to be 3900 in the current research to avoid extra replication of the calculation that might consume an unnecessary time.
Moment of each compressive force in each segment will be taken about centre of tensile force of tensile reinforcement and the value will be saved in a counter. In the case of using additional compressive reinforcement, an additional moment induced by compressive reinforcement will be added to total moment counter as detailed in the following equations:

\[
M_{total} = \Sigma F_{ci} \cdot l_{ci} + (0.55 \text{ or } 0.78) F_{ct} (d - d')
\]  
(5-29)

where \(l_{ci}\) is the lever arm for concrete compressive force \(F_{ci}\) in segment number \(i\).

At each assumed depth of the neutral axis the strain of the FRP bars in tension \(\varepsilon_f\) is calculated, using the triangle similarity, to estimate the total internal tension force \(F_i\) as detailed in the following equation:

\[
\varepsilon_f = (d - X)(0.0035/X)
\]  
(5-30)

\[
f_f = E_f \varepsilon_f
\]  
(5-31)
Here \( f_t \) corresponds to the tensile strength of FRP bars at the concrete crushing failure, not to the ultimate strength of the FRP bars, \( f_{fu} \), due to the anticipated failure mode, as illustrated in Fig.(5-7).

\[
F_t = A_f f_t
\]  

(5-32)

At this stage the computer programme will compare the difference in value between \( F_c \) and \( F_t \). The computer will repeat the above stated steps by shifting up the depth of the neutral axis by the value of \( d/r \) each time, till the achievement of the minimum difference value between the two forces. Further to this accomplishment of obtaining the optimum depth of the neutral axis, the programme will repeat the process of finding the minimum difference between the two forces but within a range of distance of \( d/r \) in each side of the previously allocated natural axis. This aims to achieve more precise results. Thus this range of distance \( 2d/r \) will be divided by \( z \) value. It has been found, through the iterative trials of several \( z \) values, that the computer model replicates the same result for any input value exceeds the value of 499 for \( z \). For that reason, the arbitrary value of \( z \) was chosen to be 499 in the current research to avoid extra replication of the calculation that might consume an unnecessary time.

A measurement of accuracy is taken at this stage to reflect a satisfactory level of finding the accurate neutral axis. This measurement is calculated as follow:

\[
\text{Accuracy measurement} = \frac{|F_c - F_t|}{F_t}
\]  

(5-33)

Using the input value of 3900 for \( r \), as explained earlier, has often accomplished an accuracy value, using equation (5-33), of less than 0.0001.
5.3.1.2.2 FRP Bars Rupture Failure Mode

In such case, where $A_{fbal} > \rho_f$, it is expected that the examined section will fail due to the rupture of FRP bars in tension. Thus these bars will reach its ultimate strain $\varepsilon_{fu}$ at the final failure, while the concrete in compression still reserve some of its strain. In this case all the previous processed steps in case one will be followed for case two apart from the following changes that refers to the changes in the triangle similarity, as shown in Figure (5-8):

Fig. 5-8 FRP Bars Rupture Failure Mode

Eq. (5-30) will be changed to the following equation:

$$\varepsilon_{ci} = (X - (iX/n)) \left( \frac{\varepsilon_f}{(d-X)} \right)$$

Eq. (5-31) will be changed to the following equation:

$$\varepsilon_{fc} = (X - d') \left( \frac{\varepsilon_f}{(d-X)} \right)$$

Equation (5-32) will be changed to the following equation:

$$F_t = A_f f_{fu}$$
5.3.1.3 Establishing the Moment Capacity

At the stage of satisfaction of the minimum possible forces difference, if not zero, the total internal moment calculated by equation (5-29) is considered as the ultimate moment for the examined section.

5.3.2 Moment-Curvature Relationship

The following section describes a computational method which leads to the moment-curvature relationship of FRP reinforced concrete beams. This relationship will be carried out by incremental increase of the concrete strain, starting from a very little proportion of concrete strain at failure. The computer programme will calculate the maximum strain of concrete, using the entered cross-section details and the predicted mode of failure, which is based on the calculated balanced reinforcement ratio. These concrete strain increments will stimulate the step-by-step load application to the beam.

In this method concrete strain will be increased, as it has been mentioned above, and for each concrete strain a relating strain in tensile reinforcement will be derived out using strain distribution and triangle similarity throughout the effective depth of the section as shown in Fig. 5-9.

Curvature of the section is the concrete strain to depth of the neutral axis ratio $\varepsilon_c / x$.

The concrete strain at failure will be given to the computer programme automatically. Nevertheless, the depth of neutral axis which corresponds to a given concrete strain and the details of the cross-section, same step by step calculations, as detailed earlier will be carried out. At the end of each cycle of
the above detailed calculations the moment of the section and the curvature will be calculated.

The increasing of the concrete strain value will continue until the section under investigation reaches its ultimate failure strain, either in concrete crushing or FRP reinforcement rupture.

The result of each step of the moment-curvature calculation for each concrete strain growth will be saved as a data pair. Finally, interpolating these data will draw the moment-curvature graph. A typical moment-curvature graph for FRP reinforced concrete section is shown in Fig. 5-10 which illustrates the moment-curvature graph for the GsU tested beam.
Flow chart (5-11) illustrates the above stated steps and their interrelation to each other in sequential order to conclude the moment capacity and the moment-curvature relationship for the FRP reinforced concrete section.
Chapter Five: Flexural Analysis of FRP RC Beams Cross-Section

Fig 5-11 Design Chart to Calculate the Moment Capacity and Moment-Curvature Relation

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5.3 Verification of the analytical modelling Program and Design Procedure

To validate the present model comparisons between the predictions from the current technique and the flexural capacity of the beams tested in this research and elsewhere are presented in Table (5-1). In addition to the four simply supported beams failed in flexure in the current experimental investigation, test results of additional 49 beams reinforced with different type of FRP bars published by other researchers have been employed for the validation purposes.

The comparison included moment capacities resulted from 11 different experimental investigations presented in past researches with details of the tested beams. The average and standard deviation of the ratio between predicted and experimental bending capacities are 98.4% and 9.3%, respectively. The predictions obtained from the current analysis are in very good agreement with the experimental results.

Further to the above validation, Table 5-1 included also the ACI 440 prediction method to assess the moment capacity for the same 53 beams that have been assessed by the present technique to validate it against the prediction of the ACI method. The average and standard deviation of the ratio between ACI predicted and experimental bending capacities are 94.6% and 16.3%, respectively. The improved prediction results achieved by the present techniques could be referred to the consideration of the concrete tension phase, compression reinforcements and the iteration in the calculation of the neutral axis that eventually will lead to the accomplishment of the most accurate
calculation of the compression force and distance needed to be considered for the moment capacity calculation.
## Chapter Five: Flexural Analysis of FRP R C Beams Cross-Section

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Chapter Five: Flexural Analysis of FRP R C Beams Cross-Section
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Chapter Five: Flexural Analysis of FRP R C Beams Cross-Section

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### Chapter Five: Flexural Analysis of FRP R C Beams Cross-Section

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<td>G2φ6</td>
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<td>150</td>
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<td>Depth</td>
<td>Stress</td>
<td>Diameter</td>
<td>Moment</td>
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<td>ACI Moment</td>
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<td>43.8</td>
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<td>73.3%</td>
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<td>695</td>
<td>400000</td>
<td>106.4%</td>
<td>81.2</td>
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</table>

* Tension Reinforcement: A = AFRP, C = CFRP and G = GFRP
** Experimental Ultimate Moment Capacity (kN.m)
*** Numerical Moment Capacity (kN.m)
**** ACI Moment Capacity (kN.m)

<table>
<thead>
<tr>
<th>Table Summary</th>
<th>Numerical</th>
<th>ACI</th>
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<tr>
<td>Average</td>
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<td>94.6</td>
</tr>
<tr>
<td>Standard Deviation</td>
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<td>16.3%</td>
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</table>

Table 5-2 Comparison between Theoretical, Experimental and ACI Flexural Capacity
5.4 Assessing the influence of Parametric Changes using the Analytical Modelling

Further to the validation of the present model the following discussion investigates on the effect of certain influential factors that affect the flexure capacity and moment–curvature relationship of FRP reinforced concrete beams. The section details of the GFRP and CFRP reinforced concrete beams tested in this research, which have been illustrated in chapter three, Figures (3-1), (3-2), (3-3) and (3-4), will be used to examine the effect of the selected factors. Theses factors, namely reinforcement ratio, concrete compressive strength and top reinforcement ratio, are taken individually to be examined as the only tested variable, keeping all other variables that might have an effect on the tested parameter at constant values. The influence of the above stated parametric changes using the current analytical model could be detailed as follow:

5.4.1 Reinforcement Ratio

The following line of investigation will examine the influence of the reinforcement ratio on the moment capacity and moment-curvature relationship of GFRP and CFRP reinforced concrete beams.

5.4.1.1 Influence of Reinforcement Ratio on Moment Capacity

This section examines the effect of the incremental increase of the reinforcement ratio over the moment capacity. The present analytical model used the
specifications of the two types of FRP reinforcements tested in this research, the CFRP and GFRP bars, to calculate the moment capacity related to each reinforcement ratio increase. Fig 5-12 demonstrates the outcome of these calculations. The figure reveals the expected general difference in moment capacity between the CFRP and GFRP reinforced sections, due to the higher strength of CFRP bars in relation to GFRP bars as indicated earlier. Also, the figure demonstrates a sudden drop in the increase rate of moment capacity at a reinforcement ratio, \( A_f / bd \), of 0.006 for the GFRP reinforced section and at the range of 0.0066 to 0.0075 for the CFRP reinforced section. That change is related to the overstepping of the balanced reinforcement ratio of each examined section, knowing that the balanced reinforcement ratio for the GFRP and CFRP sections were 0.0057 and 0.0073 respectively. Consequently, the effect of the increased ratio of the bottom reinforcement become less effective in improving the moment capacity due to the major incorporation of the concrete specification factor at that stage as discussed at early stage of this research.
5.4.1.2 Influence of Reinforcement Ratio on Moment-Curvature

Figures 5-13 and 5-14 illustrate the effect of the reinforcement ratio on the moment-curvature relationship for GFRP and CFRP reinforced concrete beams respectively. The reinforcement ratio for the investigated sections has been increased gradually by increasing the reinforcement bars number for each run keeping all other section properties unaltered.

The above mentioned figures demonstrated that at the same value of bending moment, increasing the reinforcement ratio decreased the curvature of the FRP reinforced concrete section. The figures also show that the enhanced
difference of the curvature, i.e. the reduction of the curvature at any moment value for the three reinforcement ratio, increased proportionally as the bending moment increased.

![Figure 5-13 Effect of the Reinforcement Ratio on the Moment-Curvature Relationship of GFRP Reinforced Concrete Beams](image)

Figure 5-13 Effect of the Reinforcement Ratio on the Moment-Curvature Relationship of GFRP Reinforced Concrete Beams
5.4.2 Concrete Compressive Strength

This section will address the influence of the concrete compressive strength on the moment capacity and Moment-Curvature relationship respectively for the GFRP CFRP reinforced concrete beams.

5.4.2.1 Influence of Concrete Compressive Strength on Moment Capacity

Figure 5-15 illustrates the effect of increasing the concrete compressive strength on the enhancement of the moment capacity of GFRP and CFRP reinforced concrete sections. The figure demonstrates progressive enhancement of the moment capacity of FRP reinforced concrete sections up to a certain limit. That
limit represents the concrete compressive strength that generates the balanced reinforcement ratio for which has been estimated by the analytical model to be 41.8 and 12.23 N/mm² for GFRP and CFRP tested sections respectively. The figure shows that any increase in the concrete compressive strength further than the above mentioned values will generate a neglected moment capacity improvement. This conversion could be referred to the change of the section reinforcement ratio to be under reinforced rather than over reinforced, the issue that will switch dependency of the moment capacity enhancement on properties of the FRP reinforcement rather than these of concrete.

Figure 5-15 Effect of concrete strength on Moment Capacity of FRP Reinforced Concrete Beams
5.4.2.2 Influence of Concrete Compressive Strength on Moment-Curvature relationship

Figures 5-16 and 5-17 show the moment-curvature relationship for GFRP and CFRP reinforced concrete sections with concrete compressive strength of 27, 31.1, 35 and 39 N/mm$^2$ for the GFRP reinforced sections and 27, 31.8, 35 and 39 N/mm$^2$ for the CFRP reinforced sections. These figures revealed that at the same value of the bending moment, increasing the concrete compressive strength has a very minor effect on reducing the curvatures. The figures also show that the enhancement difference of the curvature, at any moment value for the four values of concrete compressive strength, increased proportionally with a small value, as the bending moment increased. It has been found, in a similar research on steel reinforced concrete sections [141], that increasing the concrete compressive strength decreased the curvature. Pilakoutas [142] stated that the FRP strength to stiffness ratio is an order of magnitude greater than that of steel and this has a significant impact on the distribution of stresses along the section. When considering a balanced section, as usually desired in steel reinforced concrete design, the neutral axis depth, $X$, for the equivalent FRP reinforced concrete section is relatively small, as shown in Figure 5.18. For such a section this implies that a larger proportion of the cross-section is subjected to tensile stress [142] which is controlled by the FRP reinforcement properties that eventually has the influential control in determining the neutral axis depth. Thus the increase in the maximum curvature value, $X/\varepsilon_{cu}$, will be manipulated more by the FRP properties,
rather than the concrete compressive strength as indicated in figures 5-13, 5-14, 5-16 and 5-17.

Figure 5-16 Effect of Increased Concrete Strength on Moment Curvature Relation-Ship for GFRP Reinforced Concrete Beams
Figure 5-17 Effect of Increased Concrete Strength on Moment Curvature Relation-Ship for CFRP Reinforced Concrete Beams

Figure 5-18 Strain Distribution of FRP and Steel Reinforced Concrete Sections [142]
5.4.3 Top Reinforcement Ratio

This section investigates numerically the effect of the top reinforcement ratio over the moment capacity of the tested sections. The top reinforcement, in this investigation, has been counted for as a percentage of the bottom reinforcement ratio as prime ratio for beams reinforcement. The outcome of this analysis has been presented in Figure (5.19) this figure reveals that the increase of the top reinforcement ratio has a minimal affect on the moment capacity improvement. This conclusion could be disclosed from the measured increment increase of the moment capacity of 7.5 and 7.3%, for the GFRP and CFRP sections respectively, over the whole increase of the ‘top reinforcement ratio/ bottom reinforcement ratio’ from 0 to 150% as indicated in Fig. 5-19. In compliance with this results, Almusallam [143] concluded, in similar study, that the effects of the compression reinforcement on ultimate loads are small for all considered ‘tension/compression’ ratios. However, in consideration of such improvement of the moment capacity, despite of its small values, an economical consideration should be thought of if such improvement is needed due the high cost of the FRP bars.
In view of the above discussed chapter, the following points could be considered as summarised concluding remarks for the appraisal of the analytical model in predicting moment capacity and moment-curvature relationship for FRP reinforced concrete sections. As a consequence of validation of the model, the effect of the reinforcement ratio, concrete compressive strength and top reinforcement ratio on the behaviour of FRP reinforced concrete sections has been addressed in the present chapter.
1 The proposed computer module differentiates between the effect of using the GFRP and CFRP type of reinforcements on the behaviour of the concrete beams.

2 The developed analytical model proved to be a valid tool in predicting the flexure capacity of FRP reinforced sections.

3 The model could be used to investigate on several parameters that could affect the flexure capacity of the FRP reinforced sections. The three parameters been investigated in the present research revealed the following conclusions:

   • The increase of the reinforcement ratio has a considerable effect in improving the moment capacity. That increase of the reinforcement ratio becomes less effective after reaching the balanced ratio.

   • It has been found that the reinforcement ratio has a considerable effect in decreasing the curvature of the FRP reinforced concrete sections.

   • Increasing the concrete compressive strength has been proven to be effective in enhancing the moment capacity of the over reinforced FRP sections. On the other hand such increase has a disregarded effect in reducing the curvature of such sections.

   • Top reinforcement ratio has a small effect in enhancing the moment capacity. That effect is minimal if compared with the influence of the tension reinforcement ratio.
CHAPTER SIX

Longitudinal Analysis of Simple & Continuous FRP Reinforced Concrete Beams

6.1 Introduction

This chapter describes the second part of the computer program presented in the previous chapter. The investigated part in this chapter explores, in particular, the flexural capacity and load-deflection relationship along the length of FRP reinforced concrete beams.

6.2 Longitudinal Analysis Procedure

The computer program under investigation in this research examines in general the flexural behaviour of FRP reinforced continuous and simple concrete beams. The program has been employed to investigate flexure capacity, as explained earlier in the previous chapter, and deflection, as will be explained in this chapter. The analytical modelling program is based on satisfying force equilibrium and compatibility conditions. It consists of two parts, namely a sectional analysis and a longitudinal analysis. The sectional analysis part of the program has been described in chapter five. The longitudinal analysis part of the program is outlined
in this chapter. The program can be used for the analysis of simple as well as continuous concrete beams reinforced with FRP bars. Although the program was developed for the analysis of the beam arrangement used in the experimental work described in chapter three, it can be extended for the analysis of beams subjected to different load patterns and reinforced with different types of FRP bars, as well be detailed in the validation part of this chapter. The program can also be modified to accommodate different material models.

The procedure followed for the longitudinal analysis of simple and continuous beams (only one beam span is studied due to beam’s symmetry in the case of continuous beams) is summarised in the following sections.

### 6.2.1 Moment Capacity of FRP Reinforced Continuous Concrete Beams

The present analytical modelling programme under investigation has employed the previously discussed section analysis, in the last chapter, for the continuous beam to establish the ultimate moment capacity for such beams. That will be accomplished by finding the moment capacity for the mid-span \( (M_{ms}) \) and over-support \( (M_{os}) \) sections individually by following the procedure of section analysis explained in the previous chapter.

Unlike steel reinforced beams, FRP reinforced concrete beams will fail completely once any of its sections fails, either by concrete crushing or bar rupture mode. For instance, in the case of under-reinforced FRP concrete beams it is anticipated that such beams will fail completely once any of its sections fails by bar rupture mode.
due to the brittleness of the FRP bars that results in sudden rupture of the bars. This mode of failure will be achieved without going through any plastic behaviour of the bars as it would be the case in steel reinforced concrete beams. Therefore, the computer programme compares the values of the two moment capacities, at mid-span, \( M_{ms} \), and over-support, \( M_{os} \), to find the lesser value between them to be taken as the failure moment capacity of such beam.

Thus, if the moment capacity was to be represented by \( M_{ms} \), this will conclude that the failure location will be at the mid-span section, otherwise it would be at the over-support one. However, it has been found that computer model prediction has disagreed with the experimental results of the following continuously supported beams; GcUO, C-C-5 and C-C-4. That disagreement was due to immature failure of the CFRP reinforced beam, and the wide cracks appeared in the top layer of the GFRP reinforced beams as described in the earlier chapters.

### 6.2.2 Load Capacity of FRP Reinforced Concrete Beams

The ultimate load capacity could be estimated using the accomplished ultimate moment capacity from the above detailed discussion. The computer programme will establish this by equating the ultimate moment capacity at the failed section, with a formula that would conclude the relation of the load capacity with the ultimate moment at the failure section. This formula is to be embedded at the beginning of the programme inputs for each critical section. As an illustrative example the load capacity, \( P \), for the continuously supported tested beams will be obtained from either of the following equations:
The computer programme will use this value, to start the process of the longitudinal analysis by calculating the moment at each segment of the beam to estimate its curvature as it will be explained in the next step.

The same value, $P$, will be used a guide to replicate the longitudinal analysis process from the value of 0 to the value of the load capacity, $P$. This replication will explore the behaviour of the beam at different stages of loading as it will be explained in the following session of this chapter.

### 6.2.3 Curvature Distribution along FRP beams

The fundamental ground towards the establishment of the slope and deflection values at each and every point of the beam along its length will be based, in this research, on the following two principles:

a) The well established relationship between curvature, slope, and deflection

b) The discussed numerical method, in the previous chapter, for observing the moment-curvature relationship

To calculate the slope of the beam along its length, the span has been divided into a number of segments to determine the curvature at each of these segments first, using moment-curvature relationship. This will be accomplished by calculating the moment at each segment. The moment will be established by using an incremental values of the load capacity, calculated previously as explained above, multiplied by a formula that would conclude the reaction of that load multiplied by the relevant
distance of the segment. This formula is to be addressed in form of an equation for
the investigated beam's span at the beginning of the programme inputs. As a
simplified illustrative example for the moment calculation procedure the following
equations have been used for the tested simply supported beams in this research:

For \( 0 \leq x \leq L/2 \)  
\[ m = 0.5 \, P \, x \]  
(6-3)

For \( x = L/2 < x \leq L \)  
\[ m = 0.5Px - (P \, x - L/2) \]  
(6-5)

\[ n = 10000 \]  
(6-6)

\[ P = \text{Load capacity /np} \]  
(6-8)

where \( x \) is a segment location that starts with the value of 0 progressing
incrementally by \( L/n \) value till it reaches the maximum value of \( L \), \( m \) is the moment
value at \( x \) location, \( L \) is the span length, \( n \) is an arbitrary segment numbers, \( P \) is an
incremental \( np \) portions of the load capacity of the beam, calculated as explained
early, starts at a 0 value till the load capacity.

Further to the moment calculation, a simulating process for the corresponding
curvature from the calculated moment at each of the previously mentioned
segments takes place. The simulation process will be performed through an
interpolation procedure between each value of the calculated moment and the
moment-curvature relationship established earlier at the section analysis part of the
computer programme in the previous chapter.

For the continuous beams, the previous step, of calculating the corresponding
curvature for each segment, will be done twice. This is due to the reverse of the
moment diagram style, from sagging to hogging, throughout the length of the span. Such change of the moment will necessitate the rearrangement of the consideration of FRP reinforcement from being prime tension reinforcement at the sagging section of the span to be compression reinforcement at the hogging section of the same span, as shown in Figure 6.1. Consequently, each type of these reinforcement arrangements will produce a different moment-curvature relationship at each part of the moment styles of the span, the issue that required the repeating of the curvature allocation step, detailed above, once more for the over support section.

Figure 6-1 Reinforcement Arrangement for Section Analysis Purpose for FRP Reinforced Continuous Beam GcOU
6.2.4 Slope Distribution along FRP Beams

Further to the allocation of the curvature of each segment of the beam, the corresponding slope is computed as detailed in the following line of reasoning. Eq. 6-9 describes the relationship between slope and curvature:

\[ \theta = \int f(\phi)dx + \theta_0 \]  

(6-9)

where \( f(\phi) \) is the function of curvature, \( \phi \), along the beam length and \( \theta \) is slope of the beam and \( \theta_0 \) is the boundary condition for slope. This equation indicates that the slope at any point within the beam length is equal to the area below curvature graph of the beam up to that point; since the loading conditions are symmetrical throughout the beam length, all calculation for slope and deflection will be done from one end support to mid-span of the beam for simple beams, and to the middle support for continuous beams. Slope at each of these points will be the summation of areas of these segments under curvature diagram. Eq. 6-10 and Eq. 6-11 show the step by step process of calculating slope at each segment \( \theta_n \) along the beam length:

\[ A_{\phi i} = \phi_i \times (l/n) \]  

(6-10)

\[ \theta_n = \sum_{i=1}^{n} A_{\phi i} + \theta_0 \]  

(6-11)

where \( A_{\phi i} \) is the area of the segment \( i \) under curvature diagram, \( \phi_i \) is the curvature of the segment \( i \), and \((l/n)\) is the length of each segment which is equal to half of the beams effective span divided by number of the segments \( n \). \( \theta_n \) is the
slope at segment $n$, and $\theta_0=0$, which is the boundary condition for mid-span slope. Figure 6-2 below shows the procedure of calculating the slope from the curvature along simply supported beam length. The same procedure has been applied for the continuously supported beams with some alterations. These alterations are concerned with the changes in the boundary conditions; e.g. $\theta_0=0$ at the mid-support, and the change of the sign of the curvature; from being positive close to the mid-support to be negative next to the mid-span. Figure 6-13, which has been produced by the developed computer model, illustrates the symbols of the above stated alterations.

Figure 6-2 Graphical Method of Calculating Slope from Curvature
6.2.5 Deflection Distribution along FRP Beams

Having the slope calculated at each point throughout the beam length, the next step is to estimate the deflection at each of these points. Equation 6-12 presents the slope-deflection relationship:

\[
\delta = \int f(\theta)d\theta + \delta_0
\]  

(6-12)

where \( f(\theta) \) is the slope function of \( \theta \) throughout the beam, \( \delta \) is the deflection along the beam length and \( \delta_0 \) is the deflection boundary condition.

The equation above shows that to calculate the deflection at any point, the area below the slope graph needs to be considered. The deflection value of \( \delta_0 = 0 \), is the boundary condition, which is located under support points at each end of the beam span. The same number of equal segments will be used for calculating the area under slope graph. The following equations (Eq. 6-13 and Eq. 6-14) describe the details of calculation progress for deflection at each segment \( \delta_n \).

\[
A_\theta = \theta_i.(l/n)
\]  

(6-13)

\[
\delta_n = \sum_{i=1}^{n} A_\theta + \delta_0
\]  

(6-14)

where \( A_{\theta i} \) is the area of segment \( i \) under slope graph, \( \theta_i \) is the slope of the segment \( i \), \( \delta_n \) is the deflection of the beam at segment \( n \). \( \delta_0 = 0 \) is the boundary condition at the support points. Thus, the calculation of deflection, unlike the slope, starts from supports where the deflection is equal to zero as a boundary condition.

Figure 6-3 exhibits the graphical method of calculating deflection from slope as described earlier.
To satisfy the compatibility condition, the deflection at the central support should be zero or it should not exceed an accepted tolerance (say $10^{-6}$). If not, the bending moment over the central support is iteratively changed according to the sign of the deflection at that support. The whole procedure of deflection estimation starts again based on the new calculated reaction force related to the iteratively adjusted moment till the compatibility condition has been satisfied as illustrated in Figure 6-14.

Figure 6-4 illustrates a flow chart of the above stated steps and their interrelation to each other in sequential order to conclude the deflection for the FRP reinforced concrete beam.
Section analysis

Store outcome of Moment–Curvature Relationship

Calculate curvature at each segment of the span by interpolating the moment value to the Moment – Curvature relationship.

Calculate Ultimate Moment

Estimate Load Capacity

Calculate Moment at each segment of the span

Calculate Slope at each segment of the span; by the incremental area summation of the calculated curvature; Eq.(6-11)

Calculate Deflection at each segment of the span by the incremental area summation of the calculated Slope; Eq.(6-14)

Satisfy Compatibility

Plot (Load- Deflection, Moment, Curvature, Slope)

END

---

Figure 6-4 Flowchart of the Longitudinal Analysis Procedure
6.3 Validation of the Analytical Modelling Program

From the literature survey presented in chapter two, there seems to be no experimental data available for continuously supported FRP reinforced concrete beams with rectangular cross-section. Therefore, for continuous beams, the validation of the current analytical modeling program will be carried out against the obtained results from this research, which have been detailed in chapter three. As for simply supported FRP reinforced concrete beams the validation will be carried out against experimental results obtained from different researchers as well as the present research.

Due to the vast experimental results available for simply supported FRP reinforced concrete beams the validation will be carried out using the following values:

- Comparison against selected experimental results obtained from various tested beams. Part of the selected beams presents the tested simply supported beams in the present research. The other part, of the selected beams, presents beams [125, 144, 145, 146] with different loading system arrangement, from the one used in the present research, namely four-point loading bending.

- Comparison against prediction methods that have been employed to predict the deflection of the above selected beams. These methods are the bilinear method, mean moment inertia method,
Benmokrane’s method and ACI440. Bogdanovic [6] illustrated in details the first three prediction methods and their results in prediction the early mentioned selected beams. The ACI440 method and its results in deflection prediction of the selected simply supported beams from the present research were detailed in Chapter four.

To avoid repetition in results presentation for the simply supported beams, the validation of any of the deflection prediction methods, including the present computer model, will be all demonstrated as illustrative curves in one Figure against the curve that demonstrates the experimental result of each of the selected beams, as shown in Figures 6-6, 6-7, 6-8 and 6-9. By the inclusion of all of these curves in one Figure, the validity of the present computer model against the experimental results as well as against each of the selected prediction methods could be identified at the same time. As for the experimental results for the simply supported beams for the present research, the GFRP and CFRP reinforced beams will be presented in two separate Figures. Each one of these Figures will include the experimental results for the two simply supported beams reinforced with the same type of reinforcement in addition to the ACI440 deflection prediction, as well as the prediction of the present computer model prediction, as illustrated in Figures 6-11 and 6-12. However, the following sections will illustrate the validation results for simply supported beams first, followed by the second part which will exhibits the validation for continuously supported beams.
6.3.1 Simply Supported Beams Validation

The purpose of this section is to investigate the prediction of different types of deflection methods available for simply supported FRP reinforced concrete flexural beams against the present computer model under validation. These methods could be used to model long or short-term deflection, which depend on different parameters (shrinkage, creep, tensile strength, elastic modulus, reinforcement distribution, load history). There are a vast number of models available, the four that are going to be investigated are: ‘Comite Euro-International du Beton et Federation International de la Precontrainte’ (CEB-FIP) model by the bilinear method, CEB-FIP by the mean moment of inertia, the method formulated by Benmokrane’s [6] and the ACI 440.1R-06 method.

As the ACI 440-6 method has been detailed earlier, the following section will detail the equations used for the first three above stated methods

6.3.1.1 Bilinear Method (CEB-FIP)

The equation for maximum deflection of a simply supported beam under four point bending is given by:

\[
\Delta_c = \left( \frac{Pa}{24 \cdot E_c \cdot I_g} \right) (3L^2 - 4a^2)
\]

(6-15)

where, \( P \) = point load applied to the beam at a distance of “a” from the support, \( N \), \( a \) = distance between the support and the point load, \( mm \), \( E_c \) = elastic modulus of concrete, \( Mpa \), \( I_g \) = moment inertia of the gross un-cracked concrete section, \( mm^4 \) and \( L \) = span of the simply support beam, \( mm \).
The calculation of the deflection according to the mean curvature approach is relatively time consuming and will need the use of a computer [6]. Therefore the CEBFIP manual offers alternatives, such as the bilinear and mean moment of inertia method [147].

The bilinear equation is as follows:

\[ \Delta = (1 - \xi_b)\Delta_1 + \xi_b \Delta_2 \]  

(6-16)

Where

\[ \xi_b = 1 - \beta_1 \beta_2 \frac{M_{cr}}{M_a} \]  

(6-17)

\( \beta_1 = \) coefficient which depicts the bond quality of bars

\( \beta_2 = \) coefficient that distinguishes duration of loading, 0.8 for first time or short term loading; 0.5 for long term loading

\[ \Delta_1 = k_{s1} \Delta_c \]  

(6-18)

\[ \Delta_2 = k_{s2} \Delta_c \]  

(6-19)

It should be noted that tension-stiffening deflection in equations (6-18 & 6-19) are a function of the initial (instantaneous) deflections and the factors of \( k_{s1} \) and \( k_{s2} \) can be calculated by the curves given in the CEB-FIP manual [147]. The values of these factors are mainly dependent on the reinforcement ratio of the top and bottom reinforcements.

\[ M_{cr} = f_r I_g / y_t \]  

(6-20)

\( I_g = \) moment of inertia (gross section), \( mm^4 \)

\( y_t = \) distance from the neutral axis to the extreme tension fibre (un-cracked), \( mm \).
\[ M_a = \text{applied moment on the beam at the point of deflection being measured, } \ N . \ mm \]

\[ M_{cr} = \text{cracking moment, } N . \ mm \]

\[ f_r = 0.6 \sqrt{f_c'} \ 	ext{modulus of rupture of concrete, MPa}. \]

### 6.3.1.1.1 **Mean Moment Of Inertia Method**

Hall and Ghali [148] used the following equation to calculate beam deflection:

\[ \delta = \frac{l^2}{96}(\psi_a + 10\psi_m + \psi_b) \quad (6-21) \]

Where

\[ \psi_a, \psi_b = \text{curvature at the ends of the beam (for simply supported curvature both zero) and } \psi_m \text{ is the mean curvature which can be calculated by:} \]

\[ \psi_m = \frac{M_a}{E_c I_m} \quad (6-22) \]

where \( E_c \) is elastic modulus of concrete, MPa, and \( I_m \) is the mean moment of inertia which is derived by The CEB-FIP [148] as follow:

\[ I_m = I_1 I_2 \left[ I_1 + \beta_1 \beta_2 \left( \frac{M_{cr}}{M_a} \right)^2 (I_2 - I_1) \right] \quad (6-23) \]

where \( \beta_1, \beta_2, M_{cr}, M_a \) as indicated earlier, \( I_1 \) is the moment of inertia for un-cracked sections, mm\(^4\) and \( I_2 \) is the moment of inertia for fully cracked sections, mm\(^4\).

### 6.3.1.2 **Benmokrane’s Method**

The deflection equation is used for four point bending only, and is derived by linear elastic analysis:
\[ \delta = \frac{Pa}{24E_Cl_e} (3L^2 - 4a^2) \]  

(6-24)

where \( E_C \) is as indicated earlier and \( a \) is the load, \( P \), lever arm to the end support of the beam and \( l_e \) is the effective moment of inertia that is proposed by Benmokrane’s method to be calculated as follows [149]:

\[ l_e = \left( \frac{M_{cr}}{M_a} \right)^3 \beta_b l_g + \left[ 1 - \left( \frac{M_{cr}}{M_a} \right)^3 \right] l_{cr} \leq l_g \]  

(6-25)

Where \( \beta_b = 0.5 \left[ \frac{E_{frp}}{E_s} + 1 \right] \)

Figures 6-6, 6-7, 6-8 and 6-9 illustrate experimental results of the following beams respectively Beam F-2-GF by Swamy and Aburawi [125, 144], Beam V1 by Gosenza et al[125], Beam9 by Nawy and Neuwerth [145] and Beam 26 by Maruyama and Zaho [146].

Each of the above stated Figures included four prediction methods, the bilinear method, mean moment inertia method and Benmokrane’s method, in addition to the present computer model method. Before plotting the full load-deflection relationship, this model produces, first, a chart that represents the relationship between the span of the beam in one hand and moment, curvature, slope and deflection of that beam in the other hand at the ultimate load, as illustrated in Figure 6-5. Such chart reflects the capability of the computer model to satisfy the equilibrium and compatibility conditions that could be a validation tool itself for the analytical modeling program in relation to the input data that have been provided for any of the investigated beams.
Figure 6-5 Span-Moment, Slope, Curvature and Deflection Relation for Cosenza’s Beam Presented by the Computer Model under Validation

Figure 6-6 Load-Deflection Relation for Swamy & Aburawi Beam
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Figure 6-7 Load-Deflection Relation for Cosenza's Beam

Figure 6-8 Load-Deflection Relation for Nawy & Neuwerth Beam
As it could be noticed from each of the early stated four Figures for the selected beams, the present computer model demonstrated a good match and simulation for the deflection illustrated by the experimental results. In comparison to the other three prediction methods, the present module could be considered, in general, as the most acceptable predicted scenario of the deflection values in relation to the load applied, as it has been illustrated in Figures 6-6, 6-7, 6-8 and 6- 9.

These Figures also demonstrated that this computer model presents the most conservative prediction among the others as it has produced always the highest predicted values of deflection. Such conservation in prediction secures the confidence in the predicted values if they have to be used for the design procedure.
It could be noticed from Figure 6-10 that the analytical program, based on the input data, satisfied the force equilibrium and compatibility conditions for the simply supported beams tested in the present research. Furthermore, Figure 6-11 demonstrated that the computer model presented a good match to the experimental results of GFRP reinforced bars.

Considering the de-bonding issue for the tested CFRP reinforced beams, which have been discussed in the earlier chapters; the computer model presented a reasonable match to the experimental results for the simply supported CFRP reinforced beams tested in this research as it could be observed from Figure 6-12. In comparison to the ACI 440 the present investigated model demonstrated more accurate prediction than the ACI ones as it have been illustrated by the lately mentioned two Figures.
Figure 6-10 Load-Deflection, Slope, Curvature and Moment Relation for GsO Beam Presented by the Computer Module

Figure 6-11 Load-Deflection Relation for Simply Supported GFRP Reinforced Beams; GsO & GsU
6.3.2 Continuously Supported Beams Validation

Figures 6-13 and 6-14 demonstrate illustrative examples of the computer model prediction for continuously supported GFRP and CFRP reinforced concrete beams. These two Figures present the moment, curvature, slope and deflection relationships to the half-span for beams GcOU and C-C-4 respectively. The above stated Figures reveal that the analytical model, under investigation, satisfies the force equilibrium and compatibility conditions for such continuously supported beams.
Figure 6-13 End Support Half Span-Moment, Curvature, Slope and Deflection Relations for GFRP Reinforced Continuously Supported Beam, GCOU
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Figure 6-14 End Support Half Span-Moment, Curvature, Slope and Deflection Relations for CFRP Reinforced Continuously Supported Beam, C-C-4
Figures 6-15, 6-16, 6-17 and 6-18 illustrate the load deflection relation for the continuously supported FRP reinforced beams tested in the present research. Each of the above stated Figures also include two prediction methods for the deflection of the tested beams, namely the ACI and the computer model method under investigation.

The above mentioned four Figures, for the continuously supported FRP reinforced beams, demonstrate the ability of the computer model to match the actual experimental deflection behaviour in relation to the load. Such matching has been illustrated only up to certain different levels of beams loading. This limitation have been caused by the wide cracks appeared at the top of the GFRP reinforced beams and the excessive de-bonding occurred at the CFRP reinforced beams, as been discussed in previous chapters.

In comparison to the ACI 440 prediction, the present computer model produced more enhanced results in predicting the deflection of the continuously supported FRP reinforced beams.

Such enhancements have been revealed in Figures 6-15, 6-16, 16-17 and 16-18. This improvement in prediction, either for the simple or continuous beams, could be related to the method of calculation followed in this model. That could be due to the catering of this method for the full details of the section properties, eg. Top reinforcement details, top reinforcement cover, and the enhancement of deflection calculation accuracy using several replications of the calculation over a very small segment of the section; eg 3900 segment or more. In addition to that, this method has not included in its major steps of calculation any assumed factors as in the ACI
440, such as $\beta$, $\beta_d$ and $\alpha_b$ as it has been detailed in chapter four. Such factors proved to be containing insufficient accuracy. Therefore, an elimination or alteration of their values often occurs in different upgraded version; such as the elimination of the $(0.85)$ factor in calculating the moment capacity of tension-controlled sections from the 2006 version. However, more experimental results, particularly in the field of FRP reinforced continuous beams, would shed more lights on the way that could enhance the performance of the present investigated prediction method. Moreover this computer model, as it has been displayed in Figures 6-5, 6-10, 6-13, and 6-14, in contrast to the ACI 440, is capable of presenting the whole scenario of FRP reinforced beams flexure behaviour, numerically and graphically at any stage of loading. The detailed illustrated results produced by this model could explain the necessity of the inclusion of the whole features of the assessed beam into the computer program, the issue that is not fully happing when using the ACI 440 model which produces limited numerical estimated values about the beams behaviour. This difference in handling the beam data would make the ACI 440 model look easier to use if the apparent difference in the outcome data illustration would be excluded.
Figure 6-15 Load-Deflection Relation for Continuously Supported GFRP Reinforced Beam, GcOO

Figure 6-16 Load-Deflection Relation for Continuously Supported GFRP Reinforced Beam, GcUO
Figure 6-17 Load-Deflection relation for Continuously Supported CFRP Reinforced Beam, C-C-2.

Figure 6-17 Load Deflection Relation for Continuously Supported CFRP Reinforced Beam, C-C-4
6.4 **Concluding Remarks**

The analytical technique presented in this chapter and in chapter five is an iterative procedure based on satisfying force equilibrium and deformation compatibility conditions. A computer program based on such a technique was developed and employed to investigate flexural behavior, in particular the flexural strength and deflection of simple and continuously supported FRP reinforced concrete beams.

Although the program has been applied in this research for two span continuous beams with point loads concentrated at mid–span, a similar approach could be adopted for beams with different span configurations and load patterns as been illustrated earlier in this chapter.

The main conclusions drawn from the study carried out in this chapter are as follows:

The analytical modelling program is very versatile. In addition to providing flexure moment and load capacity; moment curvature relation, moment –curvature-slope and deflection relationship to the beam’s span and load-deflection output, it can also accommodate different FRP material models and load arrangements.

The analytical modelling program has been compared against different prediction methods, namely ACI 440, the bilinear method, mean moment inertia method and Benmokrane’s method. This comparison revealed the reliability of this programme in producing more enhanced results in predicting the behaviour of the FRP reinforced beams more than the above stated methods.
More results needed for the continuously supported FRP reinforced concrete beams to upgrade the validation for the analytical modelling program under investigation.
chapter seven: conclusions & recommendations

chapter seven

conclusions and recommendations

7.1 introduction

in this thesis, the influence of using FRP internal reinforcement composite on the flexural behaviour of continuous and simply supported concrete beams was studied. Never the less, a continuously supported steel reinforced beam was included in the testing programme as a control beam. Hence, crack propagation, failure modes, load capacity, redistribution of support reactions and beams deflections of such beams were experimentally investigated. However, the research was carried out through three elements: an experimental study, evaluation of the predictability of the ACI 440.1R-06 method against the collected results of this experimental study, particularly the result of the continuously supported beams for the first time, and the development of an analytical modelling program comprising sectional and longitudinal analyses.
As all FRP composites have broadly similar stress-strain relationships in tension, i.e. an elastic-brittle response, the results presented in this thesis in which CFRP and GFRP composite were used may also be valid for other types of FRP composite. It should be noted that the main conclusions drawn from each section of the work reported in this thesis have been given at the end of each chapter. This chapter summarises the principal findings of the research conducted in this thesis and identifies a number of recommendations for future work.

7.2 Conclusions from the Current Research

The principal findings drawn from the current research are summarised below:

- Due to the lower elastic modulus of GFRP bars, continuously supported GFRP reinforced concrete beams can develop earlier and wider cracks than similar steel reinforced concrete beams.

- Continuously supported GFRP and CFRP reinforced concrete beams do not demonstrate any remarkable load redistribution.

- Over reinforcing the bottom layer of the GFRP and CFRP simply and continuously supported concrete beams could be a key factor in enhancing the load capacity and controlling the deflection. It also has a significant effect as well on delaying the crack propagation at mid-span of GFRP reinforced beams.

- Increasing the top layer reinforcement of continuously supported GFRP and CFRP reinforced concrete beams does not contribute significantly in improving the load capacity and deflection reduction.
• In the tested continuous CFRP reinforced concrete beams, de-bonding of CFRP bars was the main reason behind the immature failure of such beams.

• Load capacity and deflection of GFRP and CFRP simply supported concrete beams could be reasonably predicted using ACI 440.1R-06 equations. However, this method did not illustrate a good potential capability for predicting the load capacity of continuous GFRP and CFRP reinforced beams.

• The ACI 440.1R-06 equations appear to be effective in predicting the deflection of the under-reinforced at the bottom layer GFRP continuously supported concrete beams. As for the over-reinforced at the bottom layer GFRP continuously supported beams, the prediction process has been negatively affected by the wide cracks appeared over the intermediate support, which eventually turned the continuous beams into two simply supported beams.

• ACI 440.1R-06 equations seem to be effective in predicting the deflection of CFRP simply and continuously supported concrete beams up to the initiation of excessive cracks. Further to that, the prediction process, particularly for the continuously supported CFRP reinforced concrete beams, has been unconstructively affected by the loss of bond between CFRP top reinforcement and concrete.
• A computer program based on satisfying force equilibrium and deformation compatibility conditions was developed in this research. This program has been employed to investigate flexural behaviour, in particular the flexural strength and deflection of simple and continuously supported FRP reinforced concrete beams.

• Although the program has been applied to two span continuous beams with point loads concentrated at mid-span, a similar approach could be adopted for beams with different span configurations, load patterns and FRP material models.

• The model could be used to investigate on the several parameters that could affect the flexure capacity of the FRP reinforced sections. The parameters that have been investigated were the effect of the reinforcement ratio and the concrete compressive strength on the moment capacity and curvature of the FRP reinforced beams.

• The analytical modelling program has been compared against different prediction methods, namely ACI 440, the bilinear method, mean moment inertia method and Benmokrane’s method. This comparison revealed the reliability of this programme in producing more enhanced results in predicting the behaviour of the FRP reinforced beams more than the above stated methods.
7.3 Recommendations for Future Work

Some of the important areas that still need further investigations for continuous beams reinforced with FRP composites are listed below:

- Further research is needed to explore the effect of the de-bonding phenomena of the FRP bars on the immature failure of continuously supported concrete beams.
- The earlier concluded remarks indicated, in general, that increasing the reinforcement ratio of the CFRP bars has a favourable effect on the concrete beams behaviour reinforced with such reinforcement. Thus it is recommended to investigate more on such effect and its relation to the application of different over-reinforcement ratio of CFRP reinforcing bars. This investigation should test the affect of such application on the de-bonding, crack propagation and deflection of CFRP reinforced concrete beams.
- The proposition of the ACI 318-05[121], regarding the spacing of steel stirrups, has to be reconsidered to avoid shear failure when FRP bars are used as longitudinal reinforcement for continuously supported concrete beams with steel stirrups.
- More results needed for the continuously supported FRP reinforced concrete beams, through further investigations, to upgrade the validation for the analytical modelling program under investigation as well as the ACI 440 method.
• Although all FRP composites have broadly similar stress-strain relationships in tension, it is recommended to investigate further on using the AFRP bars as an internal reinforcement for continuously supported concrete beams as that was not facilitated in this research.

• Further to the present research it would be recommended to investigate on the effect of use of the FRP stirrups on the flexure behaviour of the continuously supported FRP reinforced concrete beams.

• Enhanced design guidance particularly for continuous beams reinforced with FRP bars should be developed and validated for further practical and optimised use of such reinforcement.
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