REAL INVESTMENT AND DIVIDEND POLICY IN A DYNAMIC STOCHASTIC GENERAL EQUILIBRIUM (DSGE) MODEL

Corporate finance at an aggregate level through DSGE models

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submitted for the degree of Doctor of Philosophy

School of Management
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2010
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Abstract

In this thesis, I take a theoretical dynamic stochastic general equilibrium (DSGE) approach to investigate optimal aggregate dividend policy. I make the following contribution:

1. I extend the standard DSGE model to incorporate a residual dividend policy, external financing and default and find that simulated optimal aggregate payouts are much more volatile than the observed data when other variables are close to the values observed in the data.

2. I examine the sensitivity of optimal aggregate dividend policy to the level of the representative agent’s habit motive. My results show that, when the habit motive gets stronger, the volatility of optimal aggregate payouts increases while the volatility of aggregate consumption decreases. This is consistent with the hypothesis that investors use cash payouts from well diversified portfolios to help smooth consumption.

3. I demonstrate that the variability of optimal aggregate payouts is sensitive to capital adjustment costs. My simulated results show that costly frictions from changing the capital base of the firm cause optimal aggregate dividends and real investments to be smooth and share prices to be volatile. This finding is consistent with prior empirical observations.

4. I run simulations that support the hypothesis that optimal aggregate dividend policy is similar when the representative firm is risk averse to when it has capital adjustment costs. In both cases, optimal aggregate dividends volatility is very low.

5. In all calibrated DSGE models, apart from case 4, optimal aggregate payouts are found to be countercyclical. This supports the hypothesis that corporations prefer to hold more free cash flows for potential investment opportunities instead of paying dividends when the economy is booming, but is inconsistent with observed data.

Keywords: Dynamic Stochastic General Equilibrium (DSGE), real business cycle, utility function, habits, dividends
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Acknowledgements

My eternal gratitude goes to my supervisor, Professor Mark Freeman, whose encouragement, guidance and support from the initial to the final level enabled me to develop an understanding of the subject. Mark always patiently answered all my questions and provided professional advices and the limitless help on every possible level. This thesis would never have taken shape without his precious help. I am heartily thankful to him.

I would also like to give a special thank you to Bradford University School of Management who provided a full-time studentship to support my study. With this grateful support, I could fully concentrate on my research at all: cannot thank enough.

Many thanks also go to faculty and staff of Bradford University School of Management for providing kind support during the period of my doctoral research, particularly Professor Turalay Kenc and Professor Frank McDonald for being helpful and providing kindly support. I also appreciate the help of the “Computing and Audio Visual” and the “Library” staff in School of Management who have helped me in numerous ways.

I would like to take the opportunity to also thank those people who spent their time and shared their knowledge for helping me to complete my thesis with the best possible result, particularly Hilary Gunura, Ali Ferda Arikan, Florian Meier, Lynne Barrow, Yousef Majdalawi and Carmel de Nahlik.

Big thanks to my partner, Florian Meier, who always encouraged me in many ways and helped me overcome all the difficult moments.

Last but not least, I want to say a big thank you to my lovely family for endless support and patience during the entire PhD process.
Nomenclature

$1 - \alpha$ labour ratio
$\alpha$ capital ratio
$\beta$ the subjective discount factor of the investor’s utility function
$\beta_F$ the subjective discount factor of the firm’s utility function
$\delta$ a specific shock in the bankruptcy process
$\eta$ the capital depreciation rate
$\gamma$ the coefficient of risk aversion
$\gamma_l$ the coefficient of relative risk aversion on leisure
$\gamma_F$ the coefficient of the firm’s risk aversion
$\lambda$ Lagrangian multiplier
$x$ the deterministic trend in labour augmenting technical change
$\phi$ a discount factor to the firm’s optimisation problem
$\pi$ the operating profits
$\psi$ parameter for technology persistence
$\rho$ a value for determining the time allocated to market activities, e.g. consumption
$\sigma_{\epsilon}$ the standard deviation for the normal distribution of the technology shock
$\xi$ the parameter determining the strength of habit motive
$\zeta$ parameter for capital adjustment costs
$A$ a parameter for the dis-utilising effect caused by the labour input
$B$ debt
$B_i^{lh}$ holdings of risk-free securities
$C$ consumption
$D$ the firm’s net cash flows; dividends; shareholder’s value
EBIT: the earnings before interest and tax

\( f(\cdot) \): a production function

\( g \): the log technology growth rate

\( g(\frac{1}{K}) \): a capital adjustment cost function

\( I \): Investment

\( K \): capital

\( L \): labour (in general)

\( l \): lower bound for the probability density function of the bankruptcy point

\( L^d \): the quantity of labour from the labour demand market

\( L^s \): the quantity of labour from the labour supply market

\( M \): the stochastic discount factor; the intertemporal marginal rate of substitution

\( MPK \): marginal product of capital

\( MPL \): marginal product of labour

\( N \): the quantity of shares

\( N_t \): the quantity of shares

\( Q \): the price of equity

\( r \): the log real return on capital

\( R^e \): the gross return on equity

\( R_{t,t-1} \): the gross risk-free rate for the period \([t-1,t]\)

\( R_{t-1} \): the gross rate of return on capital for the period \([t-1, t]\). In the thesis, there are two cases for \( R \). \( R \) is the gross rate of return on capital if the external financing decision is not employed in the model. Models in this case are Replication I and II (chapter 3) and models in chapter 5. The second case is that the model has incorporated the external financing issue, then \( R \) is defined as the corporate interest rate on debt. Replication III in chapter 3 and models in chapter 4 are in this case.

\( u \): upper bound for the probability density function of the bankruptcy point

\( U(\bullet) \): the household’s utility function

\( U_F(D_{t+h}) \): the firm’s utility function of its future cash flows

\( W \): the wage rate

\( X \): the ratio of total debt payment divided by the firm value; a bankruptcy threshold value; the benchmark point of bankruptcy

\( Y \): output

\( Z \): technology shock
This table summarises model names presented in each chapter and whether the dividend policy and an external financing decision are considered. The fifth, sixth and seventh columns report the production function, the investor’s utility function, the firm’s utility function and the capital accumulation function used in each model.

Chapter 3 employs the standard Cobb-Douglas production function, chapter 4 introduces an effective labour in the production function \((Z_t L_t)^{1-\alpha}\) to capture the effect of technology shock on the labour market and directly influence output, and chapter 5 remains the technology shock hitting both capital and labour inputs, and introduces an additional factor \((x\) to capture that labour is augmenting by technical change (labour augmenting technical change). The first model in chapter 5 is a value-maximising (VM) model with fixed labour supply \((L = 1)\). Utility functions of models in chapter 4 and 5 are constructed using Constantinides (1990)'s internal habit formation. The firm’s utility function in the utility-maximising (UM) model is based on Cárceles Poveda (2003). Capital adjustment costs are considered in chapter 5. Capital accumulation functions of the basic, the advanced VM and the UM models are constructed with Gershun (2010’s) capital adjustment cost function. The control model uses Collard and Dellas (2006)’capital adjustment function.

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<td>(K_t = (1 - \eta) K_{t-1} + I_t)</td>
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<td>-</td>
<td>(Y_t = Z_t K_{t-1}^{\alpha} L_t^{1-\alpha})</td>
<td>(\log(C_t) + A(1 - L_t))</td>
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Nomenclature: \(Y\) - output; \(K\) - capital; \(L\) - labour (\(L^d\) the amount of labour from the labour demand market); \(Z\) - the technology shock; \(x\) - the labour augmenting technical change; \(C\) - consumption; \(D\) - dividends; \(I\) - investment; \(\alpha\) - a parameter for the dis-utility effect caused by the labour input; \(\rho\) - a parameter value determining the time allocated to market and nonmarket activities; \(\gamma\) - the coefficient of the investor’s risk aversion; \(\gamma_F\) - the coefficient of the firm’s risk aversion; \(\gamma_C\) - the coefficient of the investor’s risk aversion on consumption; \(\gamma_L\) - the coefficient of the investor’s risk aversion on leisure; \(\xi\) - a parameter for habit motive; \(\eta\) - the depreciation rate; \(\zeta\) - a parameter for capital adjustment costs; \(U_F(\cdot)\) - the firm’s utility function.
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CHAPTER 1

INTRODUCTION
1.1 The objective of this thesis

This thesis overlaps a number of areas in corporate finance, utility theory and macroeconomics. I take a theoretical approach to examine optimal dividend policy from an aggregate, rather than individual company, perspective. In particular, this thesis contributes to the literature on optimal corporate dividend behaviour by providing two primary findings. First, I show that market-wide dividend policy plays an important role in helping investors to smooth consumption. Second, I demonstrate that, as capital adjustment costs within the economy increase, the variability of optimal aggregate dividends decreases while share prices become more volatile. This finding is consistent with prior empirical results showing that managers aim to keep dividend policy smooth in the long run [Lintner (1956) and Brav et al. (2005)].

Standard theories of optimal dividend policy are generally based on a microeconomic foundation. Such theories include taxation theory [Brennan (1970), Miller and Scholes (1978), Poterba and Summers (1985) and Harris et al. (2001)], dividend clientele effect [Pettit (1977)], agency theory [Jensen and Meckling (1976), Easterbrook (1984), Jensen (1986) and La Porta et al. (2000)], asymmetric information and signalling theory [Bhattacharya (1979), Miller and Rock (1985), DeAngelo et al. (1996), Benartzi et al. (1997), Fuller and Goldstein (2005) and Dong et al. (2005)]. Frankfurter and Wood (2002) examine whether these theoretical models of dividend policy are consistent with observed payout behaviour, and point out that they all lack empirical support.

Recently, there has been increased focus on theoretically examining optimal payout behaviour across firms. Such an approach was originally suggested more than two decades ago by Marsh and Merton (1987, pages 4-5):

"For example, in a purely demand-driven model for dividends, the demand for dividends is not firm-specific because investors only care about the dividend-capital gain mix at the portfolio level... Thus equilibrium aggregate dividends may be determinate, but which firms service this demand and the quantity that each chooses to supply may not."
This macroeconomic approach to corporate finance has been taken by several papers in recent years including Bernanke et al. (1999), Alessandrini (2003), Covas and Wouter (2006), Levy and Hennessy (2007), Jermann and Quadrini (2009), Santoro and Wei (2009) and Amdur (2010). This thesis lies within this stream of literature. The objective of this thesis is to take a macroeconomic approach and to solve the optimisation problems of a representative corporation and a representative investor simultaneously. This approach makes it possible to analyse optimal dynamic dividend policy as an interaction between corporate and investor's activities, especially in the presence of frictions in the markets.

In particular, in this thesis, I examine how optimal aggregate dividend policy changes when (a) investors have habit formation utility functions and (b) there are frictions from changing the capital base of the firm. The reasons for examining these two issues are (a) it is well-known [Constantinides (1990)] that the stronger the habit formation, the greater the desire of investors to smooth consumption and therefore the importance of dividends as a mechanism for helping to achieve this is hypothesised to increase, and (b) capital adjustment costs limit the ability of the firm to rapidly adjust their real investment policies which will then have implications for dividends. There issues have not been examined in detail by the existing literature.

My main findings are: (a) increased habit formation leads to greater volatility in optimal aggregate dividends as these provide an important mechanism to help investors smooth consumption, (b) increasing capital adjustment costs leads to lower variability in optimal aggregate dividend payouts and real investment policy but greater volatility in share prices and (c) in contrast to empirical observations but consistent with many previous theoretical studies, dividends are strongly countercyclical in all models considered.

In the next subsection, I outline the development of the thesis and articulate my contributions.
1.2 Thesis structure, contributions and findings

In chapter 2, I provide a brief literature review in two areas. First, I look at microeconomic theory and evidence concerning dividend policy. While this is not the central focus of my thesis, it is important to contrast the macroeconomic approach that I take against the more common discussions concerning payout policy that take place in the corporate finance literature. I then review business cycle theory that underlies dynamic stochastic general equilibrium models (DSGE) which are the methodological foundations for my theoretical work. More detailed and specific literature reviews follow in subsequent chapters research focusing on variants of real-business-cycle (RBC) based DSGE models (chapter 3), habit formation utility functions in DSGE models (chapter 4) and the nature of capital adjustment costs in DSGE models (chapter 5).

In chapter 3, I initially replicate the seminal basic model of Hansen (1985), where employed working hours are indivisible and without dividends. In a second replication, I then extend this to take dividend policy into account by explicitly incorporating an optimisation function for the firm. This idea is inspired by Alessandrini (2003) who introduces optimal dynamic capital structure in a production economy and discusses its implications for aggregate investment, debt and dividends. My third investigation in this chapter involves calibrating the key model of Alessandrini (2003) which includes issues of dividend policy, default process and external financing. In this calibration, I use a different methodology for solving the default process which is an analytical solution that captures firms’ bankruptcy in closed form. Using this method, I find dividends and debt are very volatile. This is because firms can seek money from outside and that increases the flexibility of investment cash flows and consequently dividend payments move more strongly with the underlying investment.

In this chapter, and throughout the rest of the thesis, I calibrate these DSGE models by using a software package mixture of MATLAB and DYNARE. Results of the first and second replication are very similar to the results of the original papers. However, by using DYNARE in addition to MATLAB, the computation for stochastic simulations are known to be faster and more robust [Juillard (1996), Collard and Juillard (2001b), Col-
lard and Juillard (2001a) and Schmitt-Grohé and Uribe (2004)] because simulations are computed from a Taylor approximation, up to third order, of the expectation functions. My choice of using different numerical methods leads to some differences between the results that I report and those in the original paper of Alessandrini (2003) for the third calibration. I find optimal aggregate dividends and investment are significantly more volatile than Alessandrini (2003) reports.

In chapter 4, I examine aggregate payouts behaviour when households have habit formation utility functions. Habit formation has been considered in DSGE models by Jermann (1998), Boldrin et al. (2001), Lettau and Uhlig (2000), Uhlig (2007), Schmitt-Grohé and Uribe (2008) and Gershun (2010). The focus of these papers is on the business cycle. By contrast, the DSGE models that focus on optimal corporate debt and equity issues that are reviewed in chapter 3, together with DSGE models with dividend taxation [Santoro and Wei (2009) and Gourio and Miao (2010)], do not include habit formation. My contribution is to combine these two streams of literature. This is an interesting area for investigation because habit formation is closely related to the strength of investors’ consumption smoothing motive. I conjecture that the stronger the habit, and hence the stronger the desire to smooth consumption, the more important dividends become as a source of countercyclical cash flow to offset the procyclicality of labour income.

In this chapter, I model utility functions with habit formation in three different ways: multiplicative utility function with additive habit [Schmitt-Grohé and Uribe (2008)], separable utility with additive habit [Christiano and Fisher (1995), Boldrin et al. (2001) and Kano and Nason (2009) and Nutahara (2010)] and separable utility with non-additive habit [Lettau and Uhlig (2000)]. My first contribution in this chapter is to compare the sensitivity of optimal dividend policy to the precise specification of the habit motive and also to a benchmark without habit. My results are largely insensitive to the precise specification of the utility function and the slight difference is due to the various marginal substitution between consumption and leisure among utility functions. Optimal aggregate dividend payouts are countercyclical in each model. In with-habit models, aggregate dividends are less strong correlated to consumption. Consumption
smooths over periods while aggregate dividends are comparative volatile.

My second contribution is to concentrate on the multiplicative utility function with additive habit formation and then vary the strength of the habit motive. In all cases, as hypothesised, I find that optimal aggregate dividends become more variable and so does aggregate investment when the habit motive gets stronger. This implies that the role that dividends play in helping investors to smooth consumption gets stronger as the habit motive grows.

The third contribution in chapter 4 is that I calibrate the sensitivity of optimal aggregate dividends to various levels of risk aversion for different levels of habit formation. Lettau and Uhlig (2000) examine two levels of risk aversion (one is extremely high and another is a standard value) for the study of business cycle facts, but their focus is not related to (a) impact of various levels of habit motive and (b) the study of the optimal aggregate payouts. I find that, for all levels of habit formation, as risk aversion increases so the volatility of aggregate dividends decreases. Even in the presence of habit formation, the counterfactual observation that dividends are highly countercyclical continues to hold.

In chapter 5, I extend the analysis of chapter 4 to include capital adjustment costs in addition to habit formation, although in this chapter there is no debt financing for reasons of analytical tractability. Capital adjustment costs have been considered in DSGE models following Jermann (1998). Several popular papers [Boldrin et al. (2001), Danthine and Donaldson (2002), Cárcceles-Poveda (2003), Lettau (2003), Bouakez et al. (2005), Cárcceles-Poveda (2009), Santoro and Wei (2010) and Gershun (2010)] are primarily interested in the stylised economic facts of business cycle fluctuations and its implications for asset pricing but they do not report results for dividend policy. These topics, however, are not the focus of attention here. In addition, of these papers, only Jermann (1998), Boldrin et al. (2001) and Gershun (2010) include habit formation as well as adjustment costs. Their studies focus on asset returns in a production economy. My focus, however, is to investigate optimal aggregate dividend volatility and the price of equity when companies face frictions in adjusting their capital base. Again, this topic has not been addressed within the DSGE corporate finance literature
reviewed in chapter 3. This is an interesting topic for investigation because the level of capital adjustment costs affects the flexibility of the firm to adjust capital investment. Therefore my hypothesis is that greater capital adjustment costs will lead to lower volatility in both investment and dividends.

My first and second contribution in this chapter is to examine the variability of optimal aggregate dividends within a wide range of capital adjustment costs in both the case of fixed and variable labour supply. Jermann (1998) assumes that the labour supply market is fixed and the technology shock is high. This (a) restricts investors labour income to be exogenous in equilibrium and (b) the production function is hit by extremely high exogenous stochastic shocks. In this chapter I present an alternative and less restricted model where (a) labour supply is variable and (b) the presence of a standard technology shock.

My capital adjustment cost function is based on Jermann (1998) and Gershun (2010). The main findings are consistent with the hypothesis that the variability of optimal aggregate dividends is very small for firms with high capital adjustment costs. Simultaneously, equity prices become more volatile as capital adjustment costs increase. I also find that the volatility of optimal aggregate payouts in economies with low capital adjustment costs only differ in a limited way to that in high capital adjustment cost economies when I take Jermann (1998)’s assumption that labour is fixed. This phenomenon, however, becomes different when labour is assumed to be an endogenous variable. In this advanced model, I find optimal aggregate dividend volatility is sensitive to the level of capital adjustment costs. When capital adjustment costs are extremely high, optimal aggregate dividend payments are very smooth. Results show that, compared to models in chapter 4, observed dividends are not excessively smooth once high capital adjustment costs are included in DSGE models. The opposite effect occurs in share prices which are very volatile in this model. In addition, the countercyclicality of optimal aggregate dividends is also found here.

The third contribution of this chapter is that, since capital adjustment cost functions are modelled differently in previous papers, I build another DSGE model (called the control model) with a different capital adjustment cost function introduced in Collard
and Dellas (2006). I find that results in the control model are close to my previous model with extremely low capital adjustment costs. This finding implies that aggregate dividends are smooth in a DSGE model only when there are costly capital frictions existing in the economy.

The fourth contribution in chapter 5 is that I contrast the variability of optimal aggregate dividends in situations where firms are either value-maximising or utility-maximising. This work is inspired by the finding of Cárceles Poveda (2003) that the economy of firms with capital adjustment costs is analogous to another economy in which firms are risk averse but do not have capital adjustment costs. It is interesting to see that both cases show that optimal aggregate payouts are smooth due to firms’ inelastic net cash flows. In my results, I find even if both economies have the same equilibrium states, optimal aggregate dividends are much less volatile in the utility-maximising basis of the firm.

Chapter 6 concludes and presents areas for further research. Work on corporate finance problems and DSGE models is developing rapidly and new ideas in this field are burgeoning. For example, in recent working papers Jermann and Quadrini (2009) and Amdur (2010) find that financial shocks make the results of DSGE models not only match the empirical results but also have procyclical optimal aggregate dividends. In addition, the effect of US progressive dividend taxation on corporate investment decisions has recently been studied by Santoro and Wei (2010). It would be useful to incorporate these features, in addition to habit formation and capital adjustment costs into more comprehensive study of optimal dividend policy in a DSGE model.
CHAPTER 2

FUNDAMENTAL LITERATURE REVIEW
2.1 Introduction

In this chapter, I briefly review existing research on dividend policy and business cycle theory. A number of existing theoretical and empirical studies in the corporate finance literature have considered dividend policy from a microeconomic perspective. My central contributions in this thesis are based on a macroeconomic approach but it is still important to understand corporate finance problems viewed by individual firms and to contrast them with those being discussed from an aggregated market perspective. For my central analytical methodology, the General Equilibrium approach, I shall review the literature of well-known macroeconomic studies on business cycles and economic growth theory. More specific literature reviews are contained in subsequent chapters.

The chapter includes two subsequent sections and a summarised table is displayed:

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Section 2.2 includes three parts.

Part I introduces the nature of dividend policy and the debates surrounding whether dividend policy matters or not. In standard discounted cash flow theory, the value of the firm appears to be affected by the total dividend payments. Miller and Modigliani (1961), propose a dividend irrelevance theorem and emphasise that the dividend decision does not determine the value of the
firm in a perfect world. Firm values are actually influenced by earnings of the firm’s investment policy not by the output of the earnings distribution. The existence of homemade dividends presents that investors can manufacture cash incomes by selling their shares so the value of the firm is determined by its free cash flow. Despite this paper’s case for dividend payments irrelevancy, previous empirical investigations indicate that dividend increases strongly influence earnings growth and return on assets [e.g. Benartzi et al. (1997) and Grullon et al. (2002)], posing a conundrum. In addition, managers clearly pay close attention to their dividend policy, with Lintner (1956), Marsh and Merton (1986) and Kumar (1988) finding that dividends are smoothed over time. Black (1976) terms this “The Dividend Puzzle” because the “pieces” of the explanations of corporate behaviour regarding paying dividends do not fit together from one theory to another.

Part II reviews major works seeking to resolve the dividend puzzle from various viewpoints; including dividend taxation, transaction costs, asymmetric information, incomplete contracts and institutional constraints.

Part III studies the relationship between corporate net cash flows and dividend payouts, clarifying why previous papers have defined dividend payments in different ways. In general, net cash flows are the sum of aggregate dividends and stock repurchases minus equity issues. In my thesis, aggregate dividends represent net cash flows, which mean the net flow of dividends, stock repurchases and equity issues combined.

The second section of this chapter reviews the foundation of macroeconomics: business cycles and economic growth theory which supports my analytical technique. I take a Dynamic Stochastic General Equilibrium (DSGE) approach in this thesis to examine the movement of optimal aggregate dividends. This method was initially used for capturing business cycle fluctuations and has been adopted by researchers studying asset pricing issues in recent decades [Jermann (1998), Boldrin et al. (2001) and Uhlig (2007), amongst others]. The section opens with a basic introduction to the business
cycle literature and stylised facts in the economy. Business cycle theory focuses on economic stabilisation, especially over the short-term. Next, economic growth theory, which emphasises that there is a growth pattern in the long term economy when there are positive technology shocks is reviewed. An important restriction on the type of utility function for looking at the steady-state path of a DSGE model is also pointed out in this section.

In subsequent chapters, more specific literature in different areas is reviewed: a detailed study on the evolution of DSGE models (chapter 3); DSGE models with habit formation (chapter 4) and DSGE models with frictions of the capital accumulation process (chapter 5).

2.2 Dividend policy theory

2.2.1 Dividend puzzle

Dividend policy (also called payout policy) is the decision by a corporation regarding the amount, the form (e.g. cash, stock, or property dividends) and the timing (e.g. once, twice or quarterly a year) of corporate dividend payments. It is influenced by investment and financing decisions such as debt issuance, investment in positive NPV projects, the previous dividend level and forms of repurchases.

Several issues have to be considered carefully by the firm when making a dividend decision, including:

(a) Whether shareholders prefer receiving dividends or not?
(b) What is the impact of paying dividends on the stock price?
(c) What proportion of cash flows should be paid out?

Dividend policy is discussed in the research context of capital structure theories, capital budgeting, asset pricing and mergers and acquisitions discuss dividend policy [Allen and Michaely (2002)].

Five decades ago, a widely held economic belief was that the value of a firm is positively correlated with its dividend payments. Miller and Modigliani (1961) proved,
though, that dividends do not influence the value of the firm in a fully informed and efficient world (i.e. one in which there are rational investors, no taxes and no transaction costs). They showed that the current value of the firm is independent of the current dividend decision by re theorising the discounted free cash flow calculation. Their principle of dividend irrelevance is that the total amount of retained earnings and dividend payments does not affect the value of the firm as long as the investment decision does not change. The reason for that is because dividend policy does not affect current share prices and stock returns. The crucial factor is however investment decisions, neither retained earnings nor dividend payouts. Higgins (1972) supports dividend irrelevance by addressing the theory of homemade dividends that investors can still cash in their shares if they prefer cash receipts. Therefore, the firm does not need to worry about the importance of the dividend payout policy to shareholders since shareholders are able to manipulate their flows of income by selling or investing in assets. This cash-in-and-cash-out effect generated by investors eventually does not affect the total firm value.

However empirical observations show that the market often responds in extreme ways to dividend decisions. For instance, the results in Ambarish et al. (1987) support the notion of a significant impact of dividends on stock prices. Jais et al. (2009) provide empirical evidence that investors react positively (negatively) to dividend increase (decrease) announcements. So the empirical findings are not consistent with Miller and Modigliani’s dividend irrelevance theorem. Jensen and Meckling (1976)’s free cash flow theory emphasises that managers’ interests are different to investors’. Managers focus on profit maximisation and in order to accomplish this aim, they may even overinvest cash flows in negative NPV projects.

Black (1976) introduces the term of “Dividend Puzzle” to describe anomalies in observed payout policies. Why do some corporations pay dividends but others do not? Why do some investors appear to care about dividends and others do not? Responding

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1Miller and Modigliani (1961) propose that the effect of a firm’s dividend policy on its current market valuation is irrelevant. They make three basic assumptions for an ideal economy: (a) The capital markets are perfect. It is costless to access markets and obtain information of transaction rules on prices, (b) investors are rational in the markets, and (c) investors certainly assure their situation as to the future investment program and their invested firms’ future profits.
to Miller and Modigliani (1961)’s dividend irrelevance theorem, Black (1976) offers several explanatory factors to support his Puzzle Theory, including taxation, asymmetric information, transaction costs and incomplete contracts. Having discussed dividend policy theory in general, the next section briefly reviews these, and other, explanations. The taxonomy follows that of Frankfurter and Wood (2002).

### 2.2.2 Models of full or symmetric information - the effect of taxation

The traditional implication of the tax effect on dividends is that taxes can deter firms from paying dividends because of the heavy double taxation at both the corporate and personal levels [Miller and Scholes (1978) and Poterba and Summers (1985)]. Elton and Gruber (1970) and Pettit (1977) term the situation that investors invest in shares depending on their desire for dividends the “clientele effect”, which assumes investors have different concerns to buy shares, such as to tax inefficiency of dividends, company payout policy, investment plans and so on.

Are dividend-paying stocks less attractive and less valuable than non-dividend-paying stocks? Brennan (1970) and Litzenberger and Ramaswamy (1979) use the after-tax Capital Asset Pricing Model to define a relation between expected returns and expected dividend yields. In theory, the tax effect can be mitigated if higher expected returns in the form of higher dividend yields compensates for the differences between dividend taxes and capital gains taxes. Elton and Gruber (1970) develop a different model to capture the differences between ex-dividend and after-dividend stock prices and their statistic results support the clientele effect. Allen et al. (2000) examine tax driven clientele effects from a theoretical and empirical perspective and conclude that firms prefer to pay dividends because taxes for institutional investors are smaller than dividend taxes for individual investors, so for the former investment group, dividends become more desirable than stock repurchases.

Empirical studies also provide a mix of results regarding the dividend tax issues. Harris et al. (2001) examine the influence of dividend taxes on firm value and argue
that taxation does not deter companies from paying dividends because they find that dividend taxes are capitalised in share prices. This dividend tax capitalisation can be regarded as a signal. Paying dividends becomes less costly for strong companies than for weak companies because strong companies anticipate greater expected future earnings, leading to greater projected internal funds for investment projects. Therefore, strong companies do not need to issue new equities and can use their expected earnings to pay dividends.

Another argument relates dividend yields to marginal tax rates. Investors appear to calculate the benefits of selling stocks around the ex-dividend date. Chetty and Saez (2004) use a survey to provide evidence that there is an increase of dividend initiations following a tax cut in 2003 in the US. Consistent with their finding, Brav et al. (2005) find that the dividend tax issue is not the first-order dividend policy concern to firms. However, using surveys and interviews targeting US firms, they show that nearly 42% of companies consider stock repurchases because of the tax efficiency. Respondents emphasise that taxation on dividend payments is not the major driver of their dividend decisions but favour signalling, agency and clientele effects instead.

Dividend taxation as a determinant of dividend policy remains a contested area. One view favours a negative effect of the dividend taxation on investment and dividend decisions [Poterba and Summers (1985)]. Another view argues that dividend taxation should not influence a firm’s cost of capital as it does not alter the distributions of cash flows to dividend policy and investment projects [Auerbach (1979)].

Chetty and Saez (2007) analyse the effects of dividend taxation by using an agency approach and provide evidence supporting both of the views expressed above. They claim that conflicts of interest between managers and shareholders are the major factors affecting the impact of dividend taxation on shareholders. If managers are also the major shareholders of the firm or are monitored by a large group of shareholders, dividend payments increase after a tax cut, suggesting a link to institutional ownership [Michaely et al. (1995), Brav and Heaton III (1998), and Binay (2001) and Perez-Gonzalez (2002)]. As to a firm owned by a number of different shareholders, the tax efficiency costs on dividends are relatively high because monitoring costs are high due
to a complex ownership structure.

Consistent with Chetty and Saez (2007), Blouin et al. (2007, 2010) also find that the effect of the tax cut is an input to the decision about dividend policy. Their empirical results, responding to the US 2003 reductions in shareholder taxes, show a positive correlation between dividend increases and the percentage of the firm owned by individual investors.

2.2.3 Models of incomplete or asymmetric information

2.2.3.1 Signalling models

Research by Bhattacharya (1979), Miller and Rock (1985) and John and Williams (1985) investigate the “signals” spread from dividend announcements. If the market is inefficient, investors cannot obtain complete up-to-date information about those companies in which they are interested, forcing decisions to be based upon limited information, such as an announcement of increasing dividends. Aharony and Swary (1980) find that dividend announcements deliver a signal of a firm’s alternative to its future prospects. Several empirical studies examine whether dividend announcements convey vital signals which determine whether stock prices would go up or down in the future. Three influential signalling models of the above papers postulate that changing dividend decisions would send signals to investors about the companies’ future prospects.

Bhattacharya (1979) constructs a two-period model in which taxed cash dividends are signals of anticipated cash flows. In his Signalling Equilibrium Model, it is assumed that investors have imperfect information about the firm’s profitability. He shows that paying dividends is considered to be a signal of the firm’s prospects. He develops a signalling cost structure showing that signalling costs are a significant additional expense. If a firm pays dividends but investors then do not receive the expected share price rise, this dividends signalling strategy fails and then the firm may be forced to seek external capital in order to provide the necessary cash flows for future investments. These signalling costs are actually expensive and these situations often happen to bad
firms.

The Miller and Rock (1985) model assumes that the investor does not have information about the profitability of the firm’s investment projects and current or the future investment decisions. In their two-period model, the firm pays dividends to investors by using the earnings obtained in the second period from the project. They claim that the firm benefits because investors cannot observe the firm’s earnings and investment decisions. Weak firms use their cash flows to pay more dividends, instead of investing in more projects, and thus signal or appear to have a high level of earnings all the time. This strategy of signalling high earnings by cutting the investments and paying high dividends can induce investors, as outsiders, possessing asymmetric information to invest in firms with counterfeit earnings because of the inefficiency of the market.

Both the Bhattacharya and Miller and Rock models use dividends as a positive signal concerning the firm’s prospects. However, dividend taxation can also be viewed as undesirable, especially when dividend payments are taxed at a higher rate compared to capital. So is it attractive to use dividends as a form of signal? Compared to dividends, stock repurchases could save investors more taxes. The weakness of the Bhattacharya (1979) model is that a perfect substitution between dividends and stock repurchases is assumed, but this is false when tax is considered. Unlike Bhattacharya (1979), John and Williams (1985) build a model in which taxes are the dissipative costs and find that it is less costly to use stock repurchases as the form of signal instead of dividends.

Contrary to previous papers, DeAngelo et al. (1996), who examine a sample of 145 firms’ annual earnings growth, point out that there is no significant indication of dividend signalling for future earnings performance. Similar work by Benartzi et al. (1997) finds no evidence that dividend changes convey information about future earnings growth. More empirical research contributes to this area but with arguments for the effect of the signalling theory. Fuller and Goldstein (2005) suggest that the signalling theory better explains the dividend puzzle, than the prospect theory. They use monthly

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2Dissipative costs are costs that occur to firms rearranging its future financing decisions and investment decisions when they signal dividends.

3Prospect theory was developed by Kahneman and Tversky (1979). This theory focuses on de-
returns data and distinguish between dividend-paying firms and non-dividend-paying firms\(^4\). Their results show that dividend-paying stocks outperform non-dividend-paying stocks when the market is in a downside. A recent study of Nam et al. (2010) also provides results that dividend-paying stocks perform better than non-dividend-paying stocks.

Regarding investors’ willingness to receive cash and stocks dividends, Dong et al. (2005) investigate a sample of Dutch individual investors and find that investors prefer dividends due to the consideration of transaction costs of selling stocks. They emphasise that signalling theory is supported by their survey when they study the preferences of receiving dividends or not by investors.

Besides studying the information content of dividends per se, several researchers believe that dividend policy changes do convey information to investors. For additional papers focusing on the signal of dividend changes, see Bajaj and Vih (1990), Akhigbe et al. (1993), Brav et al. (2005), Li and Lie (2006) and Leary and Michaely (2010) amongst others.

2.2.3.2 Agency models - asymmetric information

When shareholders face incomplete contracts or regularities, they prefer getting dividends because it is a way to remind managers to maximise the share value. Additionally, if firms are under insolvency, shareholders can only get residual profits after debtholders. To shareholders, receiving dividends is an imminent profit they can have. However, it is difficult and costly to make a contract complete which involves the consent from the board of directors, the management team and investors. The separation of managers, bondholders and shareholders is the major factor that causes agency problems, which is compounded by the fact that there is asymmetric information among these parties. Shleifer and Vishny (1986) emphasise that it is better to control and manage a firm if the ownership of the firm’s shares belongs to a large group rather than many individuals.

\(^4\) Fuller and Goldstein (2005) assume that a firm having quarterly-dividend-paying stocks is considered as a dividend-paying firm, otherwise, as a non-dividend-paying firm.
It also helps to minimise the costs resulting from the separation of corporate ownership and control. Villalonga and Amit (2006), studying the agency costs associated with Founder-CEO\(^5\) and nonfamily firms, find that the owner-manager conflict is less costly in the Founder-CEO firms than the nonfamily firms.

Jensen and Meckling (1976) consider the agency costs incurred by three interacting parties: managers, debtholders and shareholders. Managers are hired by shareholders to assist them running the business. Intuitively, managers’ incomes do not come solely from the value of the firm. Therefore, managers might make less effort or take advantage of the company’s properties to benefit themselves privately. They carefully consider how much risk they may have to take and appear to avoid any letting other people to take risk (i.e. moral hazard theory [Pauly (1968), Arrow (1968), Zeckhauser (1970) and Mirrlees (1999)]). Moreover, managers may abandon some valuable investment plans due to high risks which affect the security of their jobs if firms go bankrupt (i.e. adverse selection [Greenwald (1986) and Balakrishnan and Koza (1993)]). Jensen and Meckling term these as agency problems. Shareholders may wish to minimise agency problems, but this enhanced control may be costly, impacting on profitability.

Cadenillas et al. (2004) introduce a model of firms with risk-averse managers. Mauer and Sarkar (2005) incorporate agency conflicts and optimal capital structure into a real option model to examine the timing difference of exercise between value-maximising and equity-maximising firms. DeMarzo and Fishman (2007) examine the incentive problems with external financing by presenting a dynamic investment structure model. Their findings show that these incentive problems are highly related to the firm’s investment decisions, rate of dividend payments etc, rather than to any kind of specific moral hazard problems.

Berkovitch et al. (2000) consider managers’ incentives in their model. By contrast, Berk et al. (2010) focus on analysing the human capital risk rather than managerial incentives. They build a model to present the implications of compensation contracts. In their model, they set the managerial entrenchment as the decisive element for the optimal leverage ratio.

\(^5\)A firm is founded by its CEO is called the Founder-CEO firm.
2.2.3.3 Agency costs - free cash flow hypothesis

While investors wish to maximise the share price of the firm, managers may instead prefer to maximise turnover. This is because turnover is highly positively correlated with CEO compensation [Coughlan and Schmidt (1985), Crawford et al. (1995), Hubbard and Palia (1995) and Murphy (1999)]. Therefore, the more cash that is left in the firm, the higher the probability that free cash flows will be over-used. To avoid this kind of agency problem, Easterbrook (1984) and Jensen and Meckling (1976) suggest that dividend payments are adjusted to make it more difficult for managers to overinvest. It is, however, still a worry that managers would use other ways to overinvest since it is impossible to absolutely monitor managers as monitoring costs are very high. The following studies find that the most important trigger is the manager’s self-interest on making profitable investment projects and providing shareholders stable growth dividend payouts. This not only solves the agency problem, but also gives positive dividend payments to shareholders [see Zwiebel (1996), Fluck (1999), Myers (2000) and Allen et al. (2000)].

The next substantial agency problem is the underinvestment problem. It is risky for growth firms to have high debt levels. Managers, facing a high leverage ratio, appear to limit flows of investment to minimise any uncertain risk. Hence it is possible that managers would skip potentially positive NPV investment projects. A lot of potential profits therefore are missed. Some papers then argue that an optimal payout decision helps to balance the firm’s cash flows. For papers discussing under- and over-investment see Jensen (1986), Stulz (1990), Berkovitch and Kim (1990), Morellec and Smith Jr (2005) and Hirth and Uhrig-Homburg (2007).

2.2.3.4 Agency costs - life-cycle theory

DeAngelo et al. (2006) assert that dividend payments vary over the firm’s life cycle. Firms have to keep cash flows for profitable investment opportunities in the early years and thus there are lower cash payments. When the status of a firm becomes stable and mature, firms receive profits and will increase dividends. The aim of the firm is to
maximise its optimal dividend payments by adjusting its free cash flows for investment opportunities in the long-run. By maximising a firm’s optimal dividend payment, there are no any cash flows wasted and hence agency problems in terms of under- and over-investment are solved. Denis and Osobov (2008) also find empirical evidence that important determinants of the propensity to pay dividends are strongly related to firm size, profitability, growth opportunities, the ratio of retained earnings to total equity.

2.2.4 Transaction costs & institutional constraints

The reason that investors prefer paying-dividend stocks may be because investors seek steady incomes rather than dynamic capital gains. If investors plan to sell stocks, they have to consider transaction costs. On the contrary, they do not need to worry about this issue if they choose to receive dividends [Allen and Michaely (2002)]. Constantinides (1990)’s habit formation theory indicate that investors want consumption go be very smooth over time. In order to achieve this aim, investors would use their incomes (wages and dividends) as a help to smooth consumption. Therefore, a stable income apart from salaries is important to them.

Some circumstances, however, do not support this explanation for why investors prefer dividends. For instance, in an environment with low transaction costs, dividends are then not attractive to investors anymore. From this point, investors may wish to have share repurchases, which makes the firm to take investors’ preferences into consideration when they make dividend decisions. Dividend payments are hence substituted by share repurchases, which may be more tax efficient.

Even if the transaction costs theory has been questioned, Dong et al. (2005) provide supportive evidence showing that investors prefer to receive dividends. Their survey investigates Dutch individual investors and shows that investors still think cashing in dividends is less costly than selling stocks.

From the corporation’s point of view, a firm may choose to use dividends rather than share repurchases because of institutional constraints. In some countries, it is forbidden for firms to purchase their shares back. This kind of institutional constraint limits the company’s potential to eliminate a heavy tax burden. Based on these legal restrictions,
managers need to find an optimal way for paying dividends to their investors. For this reason, dividend policy is relevant to the firm’s earnings.

### 2.2.5 Theoretical behavioural models

In the life-cycle theory of Shefrin and Statman (1984), it is believed that investors have more self-control when they view their incomes from a long-term perspective. As mentioned before, investors plan to have a stable consumption pattern over time and cash dividends may be better than stock dividends to help them to achieve this aim. Shefrin and Statman (1984) find that this is more clearly true for retired people. This is because they do not have labour income and thus prefer to cash in dividends. Fuller and Goldstein (2005) also find that investors tend to prefer receiving dividends in bear markets. This can be explained by the prospect theory of Kahneman and Tversky (1979). Investors respond heavily to losses rather than to gains. In a recession, the probability of getting losses is higher than having gains. Shareholders worry about their income because of uncertainty. From this point of view, investors prefer dividends than capital gains. However, it can be counter-argued that it is not better to have cash dividends in a bad economy as the firm may not be able to pay out promised dividends. If the firm has to pay dividends and then goes bankrupt, the insolvency process does not provide any better solution. Therefore, prospect theory cannot fully explain why investors prefer receiving cash dividends.

### 2.2.6 Managerial surveys

Lintner (1956) asserts that corporate dividends actually move in a smooth manner over time. He surveyed corporate CEOs and CFOs for the study of corporate dividend policy and found out that corporations prefer smoothing dividend payments to minimise negative information delivered to investors. He emphasises that major factors influencing managers’ payout policy decision are the level of current retained earnings and expected future earnings of a firm. For similar survey studies see Baker et al. (1985), Farrelly et al. (1986), Pruitt and Gitman (1991), Baker and Powell (2000), and Baker et al.
A number of empirical investigations provide evidence to support Lintner’s (1956) findings [Fama and Babiak (1968), DeAngelo et al. (1992), Benartzi et al. (1997), Baker et al. (2001)]. Farrelly et al. (1986) emphasise that an optimal dividend policy exists and the value of the firm is influenced by its payout policy [Lease (2000) also shows that dividends have impacts on the value of the firm]. Kumar (1988) point out that managers notice that dividend changes can be a signal of the firm’s prospect, and therefore believe that a steady dividend policy is the best choice. Baker et al. (2001) carry out a survey of NASDAQ-listed firms to study the managerial point of view on dividend policy. Their results support Lintner’s (1956) findings and show that dividend policy is influenced by the industry type.

2.2.7 Residual dividend policy

In signalling theory, scholars assume that dividends lead earnings. By contrast, some researchers claim that earnings lead dividends from the viewpoint of a residual dividend policy. Lang Robert and Larry (1989) show that the market reacts greater to dividend changes because of a signal of a low Tobin’s $q$ ratio$^6$. Dividends will decrease if the firm has unstable earnings. This implies earnings lead dividends, which is different to the claim in signalling theory. Alli et al. (1993) support the residual dividend theory that firms pay dividends after making their investment decisions. Baker and Smith (2006) examine the characteristics of firms taking residual dividend policy and provide results that firms with residual dividend policy (a) have low levels of free cash flow, (b) have a low leverage ratio, (c) are in a large firm scale, and (d) are more profitable.

$^6 q = \text{market value of installed capital}/\text{Replacement cost of capital}$
2.2.8 The relationship between net cash flows and aggregate dividends

In this thesis, aggregate dividends are defined as net cash flows to equity investors, which can be defined as

\[
\text{Net cash flows} = (\text{Gross}) \text{ Dividends} + \text{Stock repurchases} - \text{Equity issues}
\]

Some theoretical models assume there are no stock repurchases and equity issues in the economy. In this case, net cash flows are equal to aggregate dividends. In some other cases, there are no equity issues but stock repurchases. Therefore the net cash flows are the sum of dividends and stock repurchases [Grullon and Michaely (2002)].

In order to analyse the cyclicity of net cash flows and gross dividends individually, data for gross dividends, stock repurchases and equity issues are collected, so is data for Gross National Product (GNP). Annual data for the nominal total dollar amount of dividends declared on the common stock and nominal equity repurchases are borrowed from Grullon and Michaely (2002). Nominal equity issues are taken from Baker and Wurgler (2000). Total nominal net cash flows (NCF) are the sum of nominal gross dividends (GDiv), nominal equity repurchases and nominal equity issues. Data for GNP is the seasonally adjusted annual rate and obtained from the US Department of Commerce: Bureau of Economic Analysis. All the nominal data is detrended into real data by the customer price index (CPI) used in Shiller (1992).7

NCF take into account not only GDiv but also stock repurchases and issues. One can also consider NCF as total payouts. Table 2.2.1 contains results of (a) the correlations between NCF and GNP and (b) the correlations between GDiv and GNP for several sample periods. For the whole period 1973 - 2000, NCF and GDiv are procyclical but NCF is less strongly correlated with the movement of GNP, i.e. \( \text{Corr}(NCF, GNP) = 0.26 \) and \( \text{Corr}(GDiv, GNP) = 0.40 \). The results from breaking down the whole period into two sub-periods show that the correlation between NCF and GNP is stronger.

---

during the second non-overlapping period 1987 - 2000 \((Corr(NCF, GNP) = 0.49)\), compared to the first period 1973 - 1986 \((Corr(NCF, GNP) = 0.25)\). The third set of sub-periods covers three non-overlapping periods (1973 - 1982, 1983 - 1992, 1993 - 2000). The results show that total net cash flows and dividends have a positive relationship with GNP during the three sub-periods, despite a weak correlation between NCF and GNP in early years is obtained. GDiv is however less strongly connected to GNP during 1983 - 1993. In brief, the empirical data displays that both NCF (i.e. the total payouts) and GDiv (gross dividends) are procyclical during 1973 - 2000 to some extent. Moreover, NCF has a weak positive relationship with GNP in the early years, e.g. the years between 1973 - 1982.

The co-movement between GNP and either NCF or GDiv is plotted and displayed in Figure 2.2.1. This figure presents time-series data of log on real NCF, real GDiv and real GNP from 1973 to 2000. The secondary axis is provided for the time-series data of GNP. NCF are much more volatile and do not always have a similar movement trend with GNP, which means that sometimes it occurs that NCF decreases during booms and vice versa. Compared to NCF, the movement trend of GDiv is much more consistent with that of GNP.

Overall, it is found that the empirical data shows (a) both total payouts and dividends are procyclical, and (b) total payouts are occasionally less strongly correlated to GNP even though they are procyclical. The difference between these two sets of results \((Corr(NCF, GNP) \text{ and } Corr(GDiv, GNP))\) is because NCF concern not only gross dividends but also equity repurchases and equity issues.

In the situation in which a residual dividend policy is considered in the model, cash flows after the investment and financing decisions are left for payout decisions. If there are no events of buying back equity and issuing more equity, one can therefore consider NCF to be GDiv. The empirical data shows that both NCF and GDiv are positively correlated to GNP. In this thesis, ignoring stock repurchases and issues, net cash flows are considered as total dividends and paid out as cash dividends to investors. However, in theoretical studies, it is not always the case that theoretical models simulate procyclical dividends. Some prior studies obtain procyclical dividends and several theoretical stud-
2.2. DIVIDEND POLICY THEORY

Table 2.2.1: The relationships between real total net cash flows to real Gross National Product and between real gross dividends and real Gross National Product

This table presents two sets of correlations during three different periods. The first set includes correlations between real net cash flows (NCF) and real Gross National Product (GNP). The second set includes correlations between real gross dividends (GDiv) and GNP. Figures are calculated based on three periods: (i) the whole period from 1973 to 2000, (ii) two non-overlapping sub-periods between 1973 and 2000, (iii) three non-overlapping sub-periods between 1973 and 2000. Net cash flows are the sum of gross dividends, equity repurchases and equity issues which are obtained from Grullon and Michaely (2002) and Baker and Wurgler (2000). Data for Gross National Product is collected from the US Department of Commerce: Bureau of Economic Analysis. All the data is detrended into real data by Shiller (1992)'s customer price index (CPI).

<table>
<thead>
<tr>
<th>Sub-periods (not overlapping)</th>
<th>Corr(NCF, GNP)</th>
<th>Corr(GDiv, GNP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>The whole period</td>
<td>1973-2000</td>
<td>0.26</td>
</tr>
<tr>
<td>Two sub-periods</td>
<td>1973-1986</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>1987-2000</td>
<td>0.49</td>
</tr>
<tr>
<td>Three sub-periods</td>
<td>1973-1982</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>1983-1992</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td>1993-2000</td>
<td>0.60</td>
</tr>
</tbody>
</table>

Figure 2.2.1: Log on real total net cash flows, real aggregate dividends and real Gross National Product between 1973 and 2000
ies simulate countercyclical dividends. For example, a recent paper by Amdur (2010) presents that equity payouts are positively correlated with output. Alessandrini (2003)’s model, however, simulates countercyclical theoretical aggregate dividends in dynamic stochastic general equilibrium.

In subsequent chapters, I show both situations in which some theoretical models simulate countercyclical dividends and some obtain procyclical dividends. Relevant details are discussed later.

2.3 Business cycles and economic growth theory

2.3.1 Business cycles and stylised facts

The business cycle\(^8\) refers to fluctuations in production over an extended period of time. These fluctuations are derived from economic activities but do not go along with any automatic predictable prototype. A cycle consists of expansion, recession, contraction and revival then converges to the next cycle. Any section of this cycle is formed by a number of economic activities which occur nearly at the same time. The major financial fluctuations are random, seasonal, trend and business cycle. A widespread method, the Hodrick-Prescott filter\(^9\), can assist us in distinguishing which general trend of growth follows after the cyclical movement and how smoothly it will develop. It will also help us to classify non-steady fluctuations.

Empirical research based on market data has found three primary regularities over the business cycle (called “stylised facts”):

1. Persistence

Fluctuations in the economy are volatile over time and are predictable in the short run. It is challenging, however, to try to foresee movements in the long

\(^8\) Burns and Mitchell (1946) is the earliest paper, to the best of my knowledge, to define business cycle.

\(^9\) The Hodrick-Prescott filter was suggested in the field of the economics by Conrad Emanuel Victor Leser (an econometrician) and become popular at the point of time that Hodrick, R.J. and Prescott, E.C launched it into their model. It is used broadly for non-linear models in time-series for analysing long-term fluctuations. It applies a multiplier for the adjustment of the sensitivity of the trend to short-term fluctuations between the extent. This filter then has been used frequently in real business cycle theory.
run. Lucas (1975) provides evidence that variable capital stock has a vital impact on persistence.

2. Cyclic variability

An estimated standard deviation of each real variable involved in the economy helps to study the relative scale of fluctuation of each variable in business cycles. Some variables have a much more unstable variation over time than others.

3. Co-movement of output with other macroeconomic variables

It is interesting to observe that different cyclical types of variables have diverse correlations with output. Normally, procyclical variables (e.g. labour, consumption and investment) have positive correlations with the economic fluctuations and vice versa. That is, the quantities of procyclical variables will increase when the economy is booming, such as Gross Domestic Product. Unemployment is, however, countercyclical. The lower quantity of unemployment can boost the business growth during booms. If the measure of the correlation between output and another variable is nearly zero, then there is no strong co-movement relationship with the business cycle. In this case, it is called "acyclical". Empirical data shows that capital stock seems to be acyclical.

2.3.2 The long-term path - the economic growth

Economic growth theory centres on the long-run dynamic movements of the economy while business cycle theory focuses on short-term economic stabilisation. Technological progress plays a vital and necessary role in determining growth.

Rebelo (2005) has pointed out that Kydland and Prescott (1982) indicate that general equilibrium models can interpret business cycles. Kydland and Prescott (1982) demonstrate that it is possible to merge studies of business cycle theory and growth theory into one model if the business cycle model is calibrated to what empirical research has found about long-term growth. The major finding is that real variables have grown steadily over cycles. The empirical data of the United States from 1954 - 1985 shows
that per capita values of the main indices (e.g. real GDP, capital and consumption) are growing steadily over time (see Plosser (1989)).

A paper by Solow (1957) points out that technological progress is a major factor for determining economic growth. Economic growth theory, in contrast, works on the trend of business activities in the long-run, while business cycles theory studies fluctuations of variables in the short-run. Hence, technological progress plays a vital and necessary role in determining growth.

An influential paper by King et al. (1988) shows the necessary restrictions on additively and multiplicative separable momentary utility forms on production and preferences if the model is intended to obtain balanced growth. They discuss restrictions on preferences under with three key points:

1. The model requires holding the elasticity of intertemporal substitution (EIS) in consumption constant in order to have a constant steady state of the marginal product of capital (MPK, i.e. the firm’s capital stock divided by its total output). The economy, therefore, is in a state of balanced-growth equilibrium. Based on this limitation, the kind of utility function for consumption in an additively separable RBC model can be power and log utility, but not exponential utility. The objection function of exponential utility for consumption and any type utility for leisure could not reach a balanced growth.

2. Balanced growth theory emphasises that the condition of a constant steady-state labour supply needs to hold in income and substitution effects of consumer theory. The income effect is related to the changes of consumption resulting from changes in real wage income, while the substitution effect is the effect of one variable resulting from changes in price of another relative variable. Based on this condition, in order to get a constant steady-state labour supply, it is required that utility for consumption has to be a log form if it is an additively separable utility function is assumed in the model.

3. Leisure per capita appear to be stable between centuries while real wage have grown steadily. This implies that the elasticity of substitution between consump-
tion and leisure is near to unity. It implies that both intertemporal and intratemporal elasticity of substitution of parameters in the model ought to be constant. This is significant for defining utility over consumption and leisure across periods. If the elasticity of substitution between consumption and leisure is equal to one, the representative household’s utility function must be in an additive form consisting of a log utility for consumption. Otherwise, when the value of the elasticity of substitution is not equal to one, the investor’s utility function for consumption and leisure is in a nonadditive form. That is, if the elasticity of substitution is equal to one, the utility function is a log-log or log-“linear derived” formula for consumption and leisure respectively. This momentary utility form is the so-called additively separable model.

2.3.3 Dynamic macroeconomic models

The basic neoclassical model captures two types of dynamic macroeconomic models, the growth models and business-cycle models (or called Keynesian macroeconomic models). The growth models include three components of economic activity: capital formation, population and productivity growth while Keynesian macroeconomic models study the mutual influence of consumption and investment. Solow (1956) initially modifies a growth model over the real business cycle (RBC hereafter) which is called the neoclassical dynamic growth model. Contemporary researchers also contribute to this dynamic model [see Cass (1965) and Koopmans (1965)]. Afterwards, Brock and Mirman (1972) introduce shocks to technology into the standard neoclassical growth model. This model shows that the economy not only dynamically evolves over time but also stochastically moves with the aggregate technology shocks.

In RBC, macroeconomic models are constructed by introducing real shocks rather than nominal shocks (e.g., a shock to nominal interest rates). These models are used to examine business cycle fluctuations in a large long-term growth trend rather than short-term movements. Dynamic stochastic general equilibrium models (DSGE hereafter) are transitional models derived from the basic neoclassical model. DSGE models investigate the development of the economy over time through random shocks (e.g.
2.3. BUSINESS CYCLES AND ECONOMIC GROWTH THEORY

macroeconomic variables, such as technological changes) based on the general equilibrium theory. DSGE modelling aims to explore aggregate economic fluctuations over the business cycle in a long term prospect. The literature of DSGE models will be discussed comprehensively in Section 3.2.

Kydland and Prescott (1982) propose the application of DSGE modelling and took the results into account for the implications of stylised facts. That is, they developed an equilibrium model with growth theory and business cycle theory and used the model to explain the autocovariances of output and the covariances of output with other variables. The substantial work of Kydland and Prescott (1982) evaluates a preference-technology-information structure of the model with non-time-separable utility. Their model shows that the variability of investment is high and that of consumption and capital stock are relatively low. In addition, consumption, investment, labour hours and productivity have high correlations with output. Moreover, they test the same model with generated parameters and empirical U.S. economic data in the post-war period for comparing the similarities and deviations. A part of their findings, labour hours for instance, shows that the variability of labour is larger than that of productivity in the model. This is of greater magnitude than is observed in the empirical data.

The technology shock that Kydland and Prescott (1982) employ in their model is the sum of a permanent component and a transitory component with the permanent shock and transitory shock individually subject to stochastic processes. Unlike Kydland and Prescott (1982), Long and Plosser (1983) assume that the technology shock is independent and identically distributed in a time-homogeneous Markov process. That is, there is no technological change and hence the cyclical regularities cannot be explained by the model directly. Long and Plosser (1983) attempt to understand the

10 General equilibrium theory discusses an equilibrium across the whole market (the supply, demand and prices)
11 Lucas (1975) finds that a non-time-separable utility function is essential to account for the fluctuations in business cycle in employment and consumption. Based on the theory of household production theory and cross-sectional evidence, non-time-separable utility provides greater intertemporal substitution of leisure which helps to account for the aggregate movements in the perspective of employment in equilibrium models.
12 Independent and identically distributed (i.i.d.) expresses that each variable has an equal probability distribution and is mutually independent.
13 The time-homogeneous Markov process gives the uncertainty of production with respect to that the conditional distribution of the future technology shock relies on the time interval and the value of the present technology shock.
magnitude of consumption-production plans in competitive general equilibrium models. Although these two papers have different assumptions of concerning technology shocks, both of them highlighted that technology shocks are essentially significant for explaining the fluctuations of business cycles.

RBC models have also been studied by Rebelo (2005) and McGrattan (2006). Rebelo (2005) puts the emphasis on four points: (a) the performance of stock prices; (b) what factors drove the Great Depression; (c) what are the sources of fluctuation over business cycle; and (d) the importance of role of technology shocks. McGrattan (2006) then concentrates on introducing the Kydland-Prescott modelling process and offers a brief review on extensions of RBC-based models. King and Rebelo (2000) and Rebelo (2005) have constructed extensions of neoclassical growth RBC models.
2.4 Summary

This chapter has briefly reviewed a literature relating to dividend policy, business cycle and economic growth theory. It has examined how Modigliani and Miller (1958)'s Dividend Irrelevance Theorem and Black (1976) Dividend Puzzle Theory have inspired both theoretical and empirical approaches to study of the role of dividend policy to corporation and investors, the determinant of dividend payout decisions and consequences related to the stock market. In addition, it has identified the relationship between corporate net cash flows and net equity payouts. It has also explored the general macroeconomic environment and the evolution of the dynamic stochastic general equilibrium modelling. Compared to the standard literature corporate finance approach to study dividend policy from a micro-based perspective, this thesis focuses on the sensitivity of aggregate dividend policy to corporate and investor's activities, particularly investment decisions, production processes and consumer behaviour.

In subsequent chapters, I model the economy in which corporate dividend policy is endogenous in an aggregated market, together with various frictions, and examine the simultaneous interaction between aggregate payouts and market activities.
CHAPTER 3

THE IMPLICATION OF CAPITAL STRUCTURE FOR DSGE MODELS
3.1 Introduction

In this chapter, I calibrate three general equilibrium models. The first model is a stochastic growth model in business cycle theory: the Hansen (1985) model. I then extend it by incorporating dividend policy, which was not considered by Hansen (1985). For my third model, I extend the second model to take external financing schemes into account and to examine optimal aggregate dividends in dynamic stochastic general equilibrium (DSGE) when an optimal dynamic capital structure is obtained. The results I obtain for the first and second calibrations are similar to those reported by Hansen (1985) and Alessandrini (2003), but, for all three models, I use a mix of software techniques (MATLAB and DYNARE) which was not employed by Alessandrini (2003). This leads to some differences in the results reported for the third calibration.

Initially I replicate the Hansen model and examine business cycle statistics in a stochastic state because of random technology shocks. The Hansen model incorporates the feature that employees are hired to work full time or they are unemployed, thus, restricting the ability of employees to choose working hours in each period. In order to capture this feature, a probability of working is introduced in the labour variable and a representative household’s optimisation problem is solved for labour and consumption.

This model, however, is based only on the representative household’s problem and thus it lacks information on optimal corporate behaviour. Following literature number of previous studies [Jermann (1998), Lettau and Uhlig (2000), Alessandrini (2003), Santoro and Wei (2009), Gourio and Miao (2010), etc], my second study in this chapter calibrates a DSGE model with corporate dividend policy in a competitive equilibrium environment. My results, very similar to Alessandrini (2003)’s results, show that optimal aggregate dividends are (a) very volatile, which is higher than the observed data and (b) highly correlated to output but countercyclical. In addition, other real variable such as investment, labour and output are slightly variable in the with-dividend-policy DSGE model compared to the basic stochastic model.

My third calibration in this chapter is to extend the second model to additional issues including external financing, bankruptcy and taxation. The motivation comes from that
the effect of a tax advantage on debt (also called the tax shield effect) encourages each individual firm to seek for an optimal capital structure. While individual firms maximise their tax savings from issuing debt, they also need to take care of their leverage levels in case they go bankrupt. As a consequence, a dynamic optimal capital structure is obtained in equilibrium. In between this situation, investment cash flows are influenced by the financing decisions of firms and thus dividends as a residual cash flow are more volatile.

In this chapter, a detailed table is provided including information of steady state, variability, correlation and cyclicality for aggregate dividend payouts and other major economic statistics (consumption, investment, output, labour, etc). My results are similar to those reported in previous studies but are more robust because the solution method for stochastic simulations uses a mixture of software; MATLAB and DYNARE. Previous papers Juillard (1996), Collard and Juillard (2001b), Collard and Juillard (2001a) and Schmitt-Grohé and Uribe (2004) use DYNARE in addition to MATLAB for computing stochastic simulations and obtain more robust simulated results through a faster route. This is because this software is designed to use a Taylor approximation which is equal to third order expansion of the expectation functions, as opposed to the second order used more commonly.

In conclusion, the third investigation of this chapter, in contrast to the first and second calibrations, is more delicate and paves the way for further studies reported in subsequent chapters of this thesis. These seek for the optimal level of aggregate payouts in the presence of market frictions.

This chapter is organised as follows. The next section provides an extensive literature review on real business cycle (RBC) based DSGE models. Section 3.3 describes the three models that are being investigated in this chapter. Section 3.4 shows the model implementation. Section 3.5 calibrates each model. Solving methods are provided in section 3.6. Section 3.7 presents results and discussion. Section 3.8 concludes. Appendices for this chapter are in section 3.9.
3.2 Literature review: variants of real-business-cycle (RBC) based dynamic stochastic general equilibrium (DSGE) models

The purpose of this section is to briefly recap the similarities and differences among four major specifications of DSGE models. A review of several papers that touch upon the implication of investment and financing decisions for DSGE models is also done.

Table 3.2.1 sums up a set of comparisons across four types of DSGE model. The major development of the specifications of DSGE models is:

1. **Kydland and Prescott (1982) vs. Hansen (1985)** (cell A2 in Table 3.2.1)

   The standard RBC model was originally derived by Kydland and Prescott (1982). This paper holds a central and fundamental position in stochastic growth modelling over the business cycle, not only in the stream of descriptive research on studying the implications of business fluctuations but also in the stream of methodological model building. Their model assumes an environment in which individuals are always working; i.e. the event of unemployment was not considered in their model setting.

   Their model fails to account for the phenomena of the enormous volatility in working hours and comparatively small movements in productivity. One of their findings showed that the elasticity of labour supply to the changes in wages is high, which is not compatible with the results of the empirical data. These conflicting findings triggered a main strand of extensions of RBC models which then tried to identify the cause of the problem and to improve it by studying the labour (supply) market [Hansen (1985), Rosen (1986), Cho and Rogerson (1988), Rogerson and Wright (1988) and Cho and Cooley (1994)].

   Hansen (1985) starts to bring in the issue of an indivisible labour supply market into the model calibration. The key assumption is that he argues that the number of continuously employed hours is the major cause for these phenomena. He
Table 3.2.1: Comparison between RBC stochastic models

This table firstly shows that general equilibrium models have developed from the neoclassical growth to an advanced dynamic stochastic general equilibrium (DSGE) environment. The classic neoclassical growth model is Kydland and Prescott (1982), then Hansen (1985) employed an indivisible labour market into the model. More extended DSGE models are calibrated by incorporating endogenised investment and financing decisions, or with credit market imperfections. This table compares the major specifications and lists the similarities and differences between one and another.

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<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Kydland and Prescott (1982) plays a vital role in stochastic growth modelling. That is, RBC modelling researchers derived calibrations from it.</td>
<td>same as 2A</td>
<td>same as 3A</td>
<td>same as 4A</td>
</tr>
</tbody>
</table>
| 2 | The Stochastic Growth Model in an Indivisible Labour Market [Hansen (1985)] | Differences:  
- Kydland and Prescott (1982): Individuals are always employed.
- Hansen (1985) model endogenises the labour supply.  
Modelling concepts:  
The concept of introducing a labour market mainly focuses on that individuals are either working full time or not working at all. The labour is determined endogenously. | same as 3B | same as 4B |
|   | Similarities:  
- No tax.  
- No default occurs.  
Differences:  
- Labour supply is endogenised.  
- Endogenised optimal capital structure with external financing fund (i.e., debt issuance) | Similarities:  
- The originality of methodology of RBC modelling.  
- Labour supply is endogenised.  
- No default occurs and no taxes.  
Differences:  
- Specific DSGE model: Introduces simple labour supply which is divisible labour; endogenises financing decisions. | Modelling concepts:  
The model is built by incorporating investment and optimal financing decisions which are determined simultaneously.  
- No default occurs.  
- Dividend policy is considered. | same as 4C |
| 3 | DSGE Models of Endogenised Investment and Optimal Financing Decisions | Differences:  
- Endogenised labour supply.  
- Endogenised optimal capital structure with external financing fund (i.e. debt issuance).  
- Tax payments.  
- Default occurs. | Similarities:  
- Endogenised indivisible labour.  
Differences:  
- Endogenised optimal capital structure with external financing fund (i.e. debt issuance).  
- Tax payments.  
- Default occurs. | Similarities:  
- Endogenised indivisible labour.  
Differences:  
- Simultaneously determined investment and optimal financing decisions.  
- Dividend policy is considered.  
Differences:  
- Taxes  
- Default occurs. | Modelling concepts:  
- Consider an exogenous variable of probability of no-default into model.  
- Dividend policy is considered. |
| 4 | Extensions of DSGE Models with Credit Market Imperfections | Differences:  
- Endogenised labour supply.  
- Endogenised optimal capital structure with external financing fund (i.e. debt issuance).  
- Tax payments.  
- Default occurs. | Similarities:  
- Endogenised indivisible labour.  
Differences:  
- Endogenised optimal capital structure with external financing fund (i.e. debt issuance).  
- Tax payments.  
- Default occurs. | Modelling concepts:  
The model is built by incorporating investment and optimal financing decisions which are determined simultaneously.  
- No default occurs.  
- Dividend policy is considered. | same as 4C |
observes that the number of hours worked should be determined along with the probability of unemployment, i.e. individuals may not work at all. This modified variable of aggregate employment hours dominates and makes the individual’s preference of utility function non-convex. The individual’s marginal productivity from working is monotonic increasing but concave due to the existence of the elasticity of substitution between leisure and working.

2. Hansen (1985) vs. DSGE Model with Endogenised Investment and Optimal Capital Structure (OCS) decisions (cell B3 in Table 3.2.1)

Most of the revised DSGE models follow the calibrating process of Hansen (1985). This is because Hansen (1985) makes a sensible assumption that the labour market should be determined over time. The major difference between his paper and the followers’ models is that the latter also consider endogenising financing decisions. Their dynamic equilibrium stochastic models allow external financing. The firm has options to loan money from the bank or issue equity. Due to the conflicts between debtholders and equityholders, the firm needs to solve an OCS to meet the needs of both sides.

Some papers study the interaction between capital structure and the industry dynamics by incorporating the theory of stationary equilibrium [Hopenhayn (1992), Hopenhayn and Rogerson (1993), Cooley and Quadrini (2001), Hackbarth et al. (2006), Titman and Tsylakov (2007)]. Hopenhayn (1992)’s model examines financial-market frictions and account for the size dependence of firm dynamics. In Cooley and Quadrini (2001), they factor in the persistent shocks into a model of industry dynamics. They generate dimensions of heterogeneity for both size and age of the firm and discover that by using the combination of persistent shocks and financial frictions, the model is able to justify the firm dynamics simultaneously on size of equity and age of the firm which is not done by Hopenhayn (1992). Hopenhayn (1992) and Miao (2005) both incorporate competitive equilibrium models under idiosyncratic shocks. The difference between their works is that Miao (2005) use contingent claims analysis for providing advanced infor-
3.2. LITERATURE REVIEW: VARIANTS OF REAL-BUSINESS-CYCLE (RBC) BASED DYNAMIC STOCHASTIC GENERAL EQUILIBRIUM (DSGE) MODELS

Information about the relationship between the firm's leverage (the capital structure) and firm turnover. Hackbarth et al. (2006) construct a dynamic capital structure model with macroeconomic conditions. They argue that the benefit and cost of debt from the trade-off theorem should depend on macroeconomic conditions. Moreover, expected bankruptcy costs should depend on the current state of the economy condition as well. Hence, the optimal leverage should be affected by macroeconomic conditions. Their model predicts that the market leverage should be countercyclical. Titman and Tsyplakov (2007) calibrate a dynamic model and suggest that firms should adjust their debt ratios for the optimal target without delay if the costs stemming from financial distress are greater than those from conflicts between the interests of shareholders and debtholders. In their model, the market value of the firm is endogenously resolved by its earnings. Meanwhile, the investment choices, the capital structure, the earnings and the target debt ratio are endogenously settled on the firm's product price.

3. DSGE Model with Endogenised Investment and OCS decisions vs. Its Variant (cell C4 in Table 3.2.1)

Both of these two types of models endogenise the investment and financing decisions simultaneously. The latter type of model incorporates real issues (e.g. taxation and default possibilities) or market frictions (e.g. additional technology shocks, capital adjustment costs), or investor's utility (the most cases are to consider habit formation in consumption).

Alessandrini (2003) and Amdur (2008) have a similar theoretical foundation for their model calibrations, which is, their objective is to study the implication of capital structure in the production economy through DSGE models. Both of them use the same computational technique proposed by Uhlig (1995) to solve their models.

Amdur (2008) introduces financial frictions into the general equilibrium model by incorporating two elements: the monitoring costs and the adjustment costs. Monitoring costs occur when the firm tries to delay the time of going into bankruptcy.
Adjustment costs are the costs when the firm plans to change the payout to its shareholders. Normally firms would not often execute this option. Therefore, this causes the dividend payments to change smoothly over time. This indicator, the adjustment cost, is not only used by Amdur (2008) but also by Jermann and Quadrini (2006). The difference between these two papers is that Amdur (2008) calculates the deviation of today’s equity payout from yesterday’s as the adjustment factor instead of that of the deviation of today’s equity payout from its long-term target taken by Jermann and Quadrini (2006). Amdur (2008)’s results show that firms appear to have higher leverage ratios during booms to finance their investment plans and also to pay high level of dividends to their shareholders.

Levy and Hennessy (2007) construct a computable general equilibrium model of optimal financing and investment over the business cycle by incorporating the condition that firms can seek the source of external finance, issuing debt or equity. The financing constraints in the model are endogenously determined. There is a constraint on the ratio of external equity to managerial equity. Their model incorporated managerial agency problems. The first agency problem is related to shareholders, that managers may appropriate corporate earnings into their own pockets by producing distorted annual reports. Another agency problem relates to bondholders, that managers may appropriate corporate properties for private use. In their calibration, the model shows that there is a countercyclical variation in leverage ratios for firms with less financing constraints.

I further review papers that consider habit formation in their DSGE models in the literature review section in chapter 4. A detailed review on papers considering the friction of capital adjustment costs in DSGE models can be found in chapter 5.
3.3 The models - Replications I, II and III

In this section, a standard Hansen (1985) model is replicated first. Then I extend the replication by incorporating the issue of dividend policy. Third, the issue of external financing decision is considered. The objectives of the three replications are:

**Replication I:** to show that the methodology of solving a social planner problem. The replication work includes the theory of indivisible labour from Hansen (1985).

**Replication II:** to show that the methodology of solving problems in a competitive equilibrium. It adds up a residual dividend policy to the typical Hansen (1985) RBC model. The objective of this replication is to solve problems of a representative agent and a representative firm in a competitive equilibrium. Three market conditions (capital, labour and goods markets) are defined and equilibrium conditions are specified. The dividend policy is determined by the firm’s investment decision. Problems of two representatives (agent and firm) are solved simultaneously.

**Replication III:** to extend the Replication II by allowing the firm to have an external financing decision.

The three replicating models are based on the RBC-based economic environment and solved in a dynamic stochastic equilibrium economy. The traditional Hansen (1985) model does not include the issue of paying dividends. The second model integrates a residual dividend policy into the basic Hansen (1985) model. The third model is more complex than the previous two as four issues are taken into account: (a) a residual dividend policy; (b) taxation: company’s earnings are now taxed by a certain rate from the government; (c) external financing decision: it allows the company to borrow money from the bank for positive NPV investment projects although issuing corporate debts is not discussed in the model; and (d) a probability of default is made possible through a bankruptcy process. Table 3.3.1 summarises the specification of each replication. Next, the required environment for Replication I, II and III is specified (information about the
Table 3.3.1: The specification of Replication I, II and III

Replication I aims to solve the social planner’s problem while Replication II and III work on solving both the representative firm’s and the household’s optimisation problems. The model assumption on the market specification is the main difference between these three replications. Replication I calibrates the standard Hansen (1985) model in which employees either work full time or are unemployed in each period. To extend Replication I, capital structure is considered in Replication II. Endogenised optimal investment and financing decisions are decided simultaneously. Replication III is built based on Replication II and additionally employs tax and default issues. All assumptions for the main evolved DSGE models refer to Table 3.2.1.

<table>
<thead>
<tr>
<th>Objective of study</th>
<th>Replication I</th>
<th>Replication II</th>
<th>Replication III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Definition</td>
<td>Social planner’s problem</td>
<td>Competitive equilibrium</td>
<td>Multi-markets in the economy. Equilibrium is satisfied when the objective function for each sector (firms, investors) is solved.</td>
</tr>
<tr>
<td>Equilibrium condition(s)</td>
<td>First-order conditions (FOC) from solving the household’s optimisation problem.</td>
<td>FOCs from solving the firm’s and the household’s optimisation problem. Market clearing conditions for an equilibrium of multi-markets.</td>
<td></td>
</tr>
<tr>
<td>Assumptions (refer to Table 3.2.1)</td>
<td>B2</td>
<td>C3</td>
<td>D4</td>
</tr>
</tbody>
</table>

essential environment that a standard DSGE model should have is provided in Appendix 3.9.1).

3.3.1 Replication I: the Hansen (1985) model

The first replication model is a one-sector stochastic growth model with indivisible labour\(^1\) (refer to Hansen (1985)). The social planner aims to solve the representative agent’s problem by maximising the expected discounted utility over the agent’s lifetime. The optimisation process aims to solve the household’s problem.

The representative household’s preferences are captured by a time-separable utility function that includes consumption and leisure:

\[
\sum_{h=0}^{\infty} \beta^h U(C_{t+h}, 1 - L_{t+h})
\]  

\(^1\)Hansen (1985) claims that employees shall be employed either full or not be unemployed at all at each period. It is not possible for employees to choose and change their working hours all the time once they are hired (i.e. indivisible labour). He then introduces an endogenous probability of working hours into the stochastic growth model to capture that there are a proportion of people employed at each period in the aggregated labour demand market. The representative household solves his/her optimisation problem by maximising the utility with respect to consumption and labour following a given condition of indivisible labour in the agent’s utility function.
where $\beta$ is the subjective discount factor, $U(\cdot)$ is the period utility function for consumption and leisure, $C_{t+h}$ is the level of consumption at time $t+h$, $L^*$ is the number of hours worked in the labour supply market and the total number of hours available is normalised to one. This results in $1 - L^*_t$ being available for leisure at time $t+h$.

The household looks to maximise its expected lifetime utility. At time $t$ it can choose the current level of consumption and the amount of labour. The representative agent’s optimisation problem at time $t$ therefore becomes:

$$
\max_{C_t, L^*_t, K_t} E_t \left[ \sum_{h=0}^{\infty} \beta^h U(C_{t+h}, 1 - L^*_t) \right]
$$

subject to

$$
C_t + I_t = Y_t
$$

$$
Y_t = Z_t K_{t-1}^\alpha L_t^{1-\alpha}
$$

where the output, $Y_t$ of the representative firm at time $t$ is given by a constant returns to scale Cobb-Douglas production function. $K_t$, $L_t$ and $Z_t$ denote the capital employed by the firm, the labour supplied to the firm and a stochastic technology shock respectively, while $\alpha$ is the output elasticity of capital. The technology shock is assumed to follow a first order autoregressive process in logs:

$$
\log Z_t = (1 - \psi) \log \bar{Z} + \psi \log Z_{t-1} + \epsilon_t
$$

where $\psi$ is the parameter of technology shock persistence and $\epsilon_t$ is an independently and identically normally distributed random variable; $\epsilon_t \sim N(0, \sigma^2_{\epsilon})$. The capital base of the firm evolves according to:

$$
K_t = I_t + (1 - \eta) K_{t-1}, \quad 0 < \eta < 1
$$

where $I_t$ represents net investment by the firm at time $t$ and $\eta$ denotes the fixed capital
3.3. THE MODELS - REPLICATIONS I, II AND III

depreciation rate. Equation 3.3.5 represents the capital accumulation process. Capital stocks evolve over time, capital in the next period is the current depreciated capital plus investment.

3.3.2 Replication II: the Hansen (1985) model with dividend policy

The second replicating study considers the issue of an optimal dividend policy [Alessandri (2003)]. The major purpose of replicating this modified model is to show that the economy is in a competitive equilibrium where dividend payouts are decided after the firm’s investment decision, which is sensitive to the investor’s need to maximise his/her utility. The model economy is populated by a continuum of identical firms and households. Two optimisation problems from firms and households are solved simultaneously.

1. The household’s problem

The representative household’s specification in the competitive equilibrium is borrowed from Uhlig (1995). I also consider an optimal endogenous labour in equilibrium. The optimisation problem of the representative agent is given as:

$$\max_{C_t, L_t, K_t} E_t \left[ \sum_{h=0}^{\infty} \beta^h U(C_{t+h}, 1 - L_{t+h}) \right]$$

subject to the budget constraint

$$C_t + K_t = W_t L_t + R_{t-1} K_{t-1}$$

where $W_t$ is the wage rate for labour, and $R_{t-1}$ represents the gross rate of return on capital supplied for the period $[t-1, t]$ and is determined endogenously in this model.

2. The firm’s problem
\[
\max_{L_t,K_t} D_t + E_t \left[ \sum_{h=1}^{\infty} M_{t,h} D_{t+h} \right]
\]  

(3.3.6)

where

\[
D_t = Y_t - W_t L_t - I_t
\]  

(3.3.7)

\[
\pi_t = Y_t - W_t L_t
\]  

(3.3.8)

\[
I_t = K_t - (1 - \eta)K_{t-1}
\]  

(3.3.9)

\(M_{t,h}\) is the stochastic discount factor and is determined by the representative household’s optimisation process. The operating profits (\(\pi\)) of each firm is determined. The firm’s profits are used for investment and then the remaining part (i.e. the retained earnings) is distributed to investors as the payment of dividends.

### 3.3.3 Replication III: a DSGE model with capital structure theory

This part shows how to develop an extended DSGE model (called Replication III) of Kydland and Prescott (1982) and Hansen (1985) when there is an indivisible labour market. In this framework, the assumption is made that there is one representative firm, one representative household and one representative bank that all potentially survive forever in the absence of bankruptcy. Equilibrium is reached when each is simultaneously able to maximise its individual objective function subject to budget and market clearing constraints.

Within this proposed model, there are a continuum of identical firms and households in the model economy. A representative firm has a constant return to scale Cobb-Douglas production function incorporating stochastic technical shocks to capital and the demand on labour. Optimal levels of consumption of the single consumption good,
labour and capital will be determined simultaneously by the firm and the household.

1. Specification of the household’s problem

The representative household’s preferences are captured by a time-separable utility function that includes consumption and leisure:

$$\sum_{h=0}^{\infty} \beta^h U(C_{t+h}, 1 - L_{t+h}^s)$$

There are capital markets that allow the household to adjust its consumption profile across time. In particular, it can purchase $N_t$ shares in the representative firm and will hold $B_t^H$ in risk-free securities over the interval $[t, t+1]$. The representative agent’s optimisation problem at time $t$ therefore becomes:

$$\max_{C_t, L_t^s, N_t, B_t^H} E_t \left[ \sum_{h=0}^{\infty} \beta^h U(C_{t+h}, 1 - L_{t+h}^s) \right]$$

subject to the budget constraint

$$C_t + Q_t N_{t+1} + B_t^H \leq W_t L_t^s + (Q_t + D_t)N_t + R_{f,t-1}B_{t-1}^H$$

where $Q_t$ is the share price of the representative firm at time $t$, $N$ denotes the quantity of shares, $D_t$ denotes the dividends paid by the firm and $R_{f,t-1}$ represents the gross risk-free rate for the period $[t - 1, t]$ and is determined endogenously (details in the Appendix 3.9.2).

2. The firm’s problem

The output is defined the same as in Replication I and II. However, as it is in a competitive equilibrium, the labour in the production function is denoted specifically to show that it is from the labour demand market:

$$Y_t = Z_t K_t^\alpha (L_t^d)^{1-\alpha}$$

where $L_t^d$ denote the labour demanded by the firm. In addition, Replication III
assumes that the firm can augment its capital base by borrowing an amount $B_{t-1}$ for the interval $[t-1, t]$ at a gross corporate interest rate $R_{t-1} \geq R_{f,t-1}$. It is assumed that the firm pays corporation tax at a fixed rate $\tau$ and that the interests payment at time $t$, $(R_{t-1} - 1)B_{t-1}$, is tax-deductible. Therefore, the free cash flow of the firm before accounting for investments is given by:

$$\pi_t = (1 - \tau)(Y_t - W_t L_t^d - (R_{t-1} - 1)B_{t-1})$$

(3.3.10)

Net investment in the firm, $I_t$, is generated from this "profit", $\pi_t$, the net cash flow from paying off last period’s debt and re-borrowing this period, $B_t - B_{t-1}$, minus any dividend paid, $D_t$. In this framework, the dividend level is a residual variable whose optimal value emerges naturally from the chosen investment and capital structure choices. This defines $I_t$ as

$$I_t = (1 - \tau)(Y_t - W_t L_t^d - (R_{t-1} - 1)B_{t-1}) + B_t - B_{t-1} - D_t$$

$$\Rightarrow D_t = (1 - \tau)(Y_t - W_t L_t^d - R_{t-1}B_{t-1}) - \tau B_{t-1} + B_t - I_t$$

$$= (1 - \tau) \left[ Z_t K_{t-1}^\alpha (L_t^d)^{1-\alpha} - W_t L_t^d - R_{t-1}B_{t-1} \right]$$

$$- \tau B_{t-1} + B_t - K_t + (1 - \eta)K_{t-1}$$

(3.3.11)

The objective of the firm’s managers is to maximise the current share price, or equivalently, the present value of future dividend payments. They have three control variables to enable them to do this; labour, capital employed and how much borrowing is raised; $L_t^d, K_t, B_t$:

$$\max_{L_t^d, K_t, B_t} D_t + E_t \left[ \sum_{h=1}^{\infty} M_{t,h} D_{t+h} \right]$$

subject to the production function, intertemporal capital stock function, technology shock process (refer to equations 3.3.5, 3.3.3 and 3.3.4) and the definition of dividends given in equation 3.3.11.

3. The banking sector
In our economy, the representative household has access to a risk-free asset for saving at an interest rate of $R_{f,t}$ and the firm can help fund its investment opportunities by borrowing at a corporate rate $R_t$. It is therefore assumed that there exists a representative bank that is risk-neutral and, in expectation, non-profit making; it is prepared to lend to the firm at a rate that gives an expected rate of return equal to $R_{f,t}$. However, because of the risk of default, in general, the quoted interest rate to the firm is greater than the risk-free rate. The relationship between these two variables is described in this section. Before introducing the construction of these two variables, the theory and the fundamental environment for employing a banking sector in a RBC-based model presented in a key paper is summarised below.

**Inspiration**

Introducing the banking sector is motivated by Alessandrini (2003) which employs a bankruptcy process in the economy. Alessandrini (2003) argues that issues related to financial markets have rarely been discussed by prior standard RBC-based literature. His paper focuses on variations of debt over the business cycle when it comes to how to balance the tax shield from debt and the cost of potential financial distress. This thesis is in line with the paper to study corporate finance issues but puts the emphasis on the impact of either the firm’s or the household’s specification on fluctuations of dividend policy.

**Environment**

Alessandrini (2003) investigates determinants of capital structure in a production economy. He assumes the firm seeks funds for investment from both internal (retained earnings) and external (raise a loan from the bank) sources. By linking the relationship between financing and investment decisions, an optimal capital structure (OCS) is obtained in a dynamic general equilibrium model. Specifically, the representative firm maximises its present value of future cash flows by solving the variables labour, capital and debt while the representative household
maximises his/her discounted utility on consuming goods and leisure by solving consumption, labour, the quantity of equities and the amount of risk-free bonds. The equilibrium economy can be obtained once (a) a relationship between the risky interest rate on the firm’s debt and the risk-free rate on the household’s bonds is defined, and (b) each first order condition of the firm and the household and market clearing conditions are satisfied simultaneously.

A firm considers several issues at the same time, including (a) a leverage level, (b) a financial distress point in terms of default risk, and (c) the tax benefit from the committed debt, and then the model provides an OCS implying a best benchmark that balances the maximum level of the tax shield on debt and the cost of financial distress. Finally, one can analyse the impact of the level of debt and its relevant tax issues on fluctuations of debt and investment over the business cycle when the external financing decision is constructed in the model. The interest rate on debt is vital to the firm when it estimates its OCS as this factor is decided by the bank, not by the firm.

The effect of external financing

Standard RBC-based models have not yet considered external financing for firm’s investment sources. By incorporating external financing, Alessandri (2003) shows that the variance of debt is relatively higher than that of dividends. Dividends are smooth over the cycle. From his model, debt varies together with the level of investment once the firm considers external funds. These results point to the implication that the firm takes an action of increasing the amount of debt to finance its investment plans instead of cutting flows for dividend payouts. Dividends, as a result, fluctuate less in the with-external-financing models than in the standard RBC model. Moreover, he tests a relatively higher leverage ratio in the same model. The calibrated results show that the variance of debt is less than in the model with a relatively lower leverage ratio. This is because the cost of potential financial distress goes up when the leverage ratio becomes greater. The short-term fluctuation of investment after the technology shock is therefore
also smaller in the high-leverage-ratio model than that in the low-leverage-ratio model. The firm with a high leverage ratio limits its investment plans. This is consistent with empirical studies reporting a negative relationship between leverage and investment [Fama (2002)]. This extended RBC model clearly shows changes in variations of debt, investment and dividends when the firm’s optimal capital structure is considered.

Specification

The banking sector in Alessandrini’s theoretical model plays an important role for determining the level of two interest rates according to the debtholder’s (i.e. the firm in this case) probability of default. The two interest rates are the rates the bank is willing to offer to the lender (the household) and the borrower (the firm). It is, therefore, providing a simultaneous environment to determine the involved variables: a risk-free interest rate \( R_f \) and a risky interest rate \( R \) when the default point and the recovery rate are considered. The results of Alessandrini’s model show that debt is significantly related to output and investment. When the firm’s leverage is higher, the variance of debt, investment and dividends is smaller. This implies that the firm has relatively sticky investment and payout decisions if an OCS exists. The firm is more sensitive to high default stress in the case of high leverage, even though the tax benefit increases by more debt.

The assumption of the specification of the bankruptcy process is used commonly in the area of monetary policy (Carlstrom and Fuerst (1997) and Bernanke et al. (1999)). The bankruptcy process helps to define an estimated break point, named the threshold point, which also leads to the issue of the probability of default. In Alessandrini (2003), it is understood that the probability of default is endogenous. The method to solve the default condition and the relationship between risk-free rate and the quoted interest rate is not specified. I provide an alternative method to estimate the threshold value and the probability of default. The default process is still based on his assumption that there is a specific shock affecting the gross return on capital, that is, the specific shock is uniformly distributed on an interval.
determined by a given measure. A detailed layout capturing the impact of the banking sector and related issues on the firm and the investor’s activities is given in the following.

The relationship of $R_f$ and $R$

Define the earnings before interest and tax as $EBIT_t = Y_t - W_tL_t^d$. At time $t$, this then gives the firm a total capital base of $EBIT_{t+1} + (1 - \eta)K_t$, which is subject to a multiplicative specific shock, $\delta_{t+1}$, which is i.i.d.. This specific shock hits the firm and the firm may default, a condition captures this scenario:

$$B_tR_t > \delta_{t+1}[EBIT_{t+1} + (1 - \eta)K_t]$$

Another variable $X_{t+1}$ is defined as the ratio of total debt payment divided by the firm value:

$$X_{t+1} = \frac{B_tR_t}{EBIT_{t+1} + (1 - \eta)K_t} \quad (3.3.12)$$

When $\delta_{t+1} = X_{t+1}$, the firm is on the verge of bankruptcy as, in this case, $\delta_{t+1}[EBIT_{t+1} + (1 - \eta)K_t] = B_tR_t$; there is just enough capital in the firm to repay the debt with interest. The relationship between $R_t$ and $R_{f,t}$ is then given by:

$$B_tR_t \int_{X_{t+1}}^{\infty} f(\delta_{t+1}) + \int_{-\infty}^{X_{t+1}} \theta\delta_{t+1}(EBIT_{t+1} + (1 - \eta)K_t)f(\delta_{t+1}) = B_tR_{f,t} \quad (3.3.13)$$

where $\theta$ is defined as the recovery ratio of the firm’s value when bankruptcy occurs and $f(\cdot)$ is a probability density function. The first term of the left-hand side of equation 3.3.13 captures the fact that the bank fully receives the principle and interest at time $t+1$ if there is no default. The second term on the left-hand side captures the fact that the bank receives all the remaining capital from the firm in the event of bankruptcy, but this is less than the promised amount. By integrating over $f(\delta_{t+1})$, the left hand side is the expected payoff to the bank. As the bank is risk-neutral and non profit making in expectation, this expected
payoff should be equal to \( B_t R_{f,t} \), as given on the right-hand side of equation 3.3.13.

To operationalise this set-up, I assume that \( \delta_{t+1} \) is drawn from a rectangular distribution:

\[
f(\delta_{t+1}) = \begin{cases} 
0 & \text{if } \delta_{t+1} < l \text{ and } \delta_{t+1} > u \\
\frac{1}{u-l} & \text{if } \delta_{t+1} \in [l, u]
\end{cases}
\]

for some upper and lower bound \( u, l \). By substituting this probability distribution function into equation 3.3.13, the bank can set the corporate interest rate, conditional on knowing the level of debt and the risk-free interest rate, through the relationship:

\[
R_t = \left\{ B_t R_{f,t} - \frac{\theta}{2(u - l)} [EBIT_{t+1} + (1 - \eta)K_t][X_{t+1}^2 - l^2] \right\} \frac{u - l}{B_t(u - X_{t+1})} 
\]

Appendix 3.9.3 displays a detailed process for deriving the above equation.

### 3.4 Models implementation

#### 3.4.1 General procedures for solving nonlinear DSGE models

This section summarises a typical solving process for nonlinear DSGE models. Major papers discussing the methodology and toolkits are Campbell (1994) and Uhlig (1995). Both have provided explicit information for solving stochastic growth models, especially the method of undetermined coefficients. Campbell (1994) introduces an explicit explanation of the stochastic growth models and the analytical solution. The paper provides the approach of log-linearising the equations. In order to demonstrate the log-linearisation, Uhlig (1995) has simplified Campbell (1994)’s approach while also showing how to solve the neoclassical growth model by hand. It is not efficient, however, to log-linearise first order conditions (FOCs) by hand when the model is sophisticated. In that case, DYNARE\(^2\) software can help to deal with complicated tasks. A general

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\(^2\)DYNARE is a pre-processor and a collection of MATLAB for solving nonlinear models. It is the research work done at the Centre Pour La Recherche Economique Et Ses Applications (CEPREMAP;
3.4. MODELS IMPLEMENTATION

The process of solving nonlinear DSGE models is summarised below.

1. **First order conditions (FOCs)**
   
The first step is to find the necessary equations by solving the Lagrangian maximisation formula of representatives with constraints. The maximised variables are generally consumption, capital and labour.

2. **Steady state**
   
   To observe the steady state of each variable is to understand its property under a condition of behaviour that lasts (or continues) into the future until all variables reach to a state in which each variable is in an optimal level under those given first order conditions. To solve the steady state, what we need to do is to rewrite the first-order conditions by dropping the time indices.

3. **Log-linearise the first-order conditions and constraints**
   
   Log-linearisation is a vital technique for nonlinear dynamic stochastic models. It ensures that the first-order equations and the log-deviations from the steady state have an approximately linear relationship.

   Campbell (1994), Uhlig (1995) and King et al. (2002) have proposed the solution of log-linearisation. The practical approach by hand is to form a logarithmic deviation of each variable from its steady state value, and use the Taylor approximation to estimate the logarithmic deviation. The result will be a roughly linear equation for the variable, its steady state and its log-deviation.

   A technology shock process is a nonlinear equation. To characterise its equilibrium in the system, a method called log-linearisation aims to obtain an equation approximately linear in the log-deviations from the steady state.

   The procedures of log-linearising the technology shock are

---


3It follows an approximate computation by solving the logarithm deviation of the targeted variable from its steady state. Refer to Uhlig (1995) for more details.
(a) To define the difference between the logarithmic deviation of $Z_t$ and its steady state value $\bar{Z}$ as

$$Z_t = \bar{Z} e^{z_t} \approx \bar{Z} (1 + z_t)$$

where, approximately, $z_t$ is the deviation between $Z_t$ and $\bar{Z}$.

(b) To define the stochastic technology shock:

$$\log Z = (1 - \psi) \log \bar{Z} + \psi \log Z_{t-1} + \epsilon_t$$

(c) To log-linearise the above equation, we substitute equation (a) into equation (b). After rearranging, an approximately log-linearised constraint for the technology shock is obtained:

$$\log \bar{Z} e^{z_t} = (1 - \psi) \log \bar{Z} + \psi \log \bar{Z} + \psi z_{t-1} + \epsilon_t$$

$$z_t = \psi z_{t-1} + \epsilon_t$$

4. Solving for the dynamics

Uhlig (1995) provides explicit instructions for solving the dynamic stochastic neoclassical models via the method of undetermined coefficients by hand. The method he presents is to postulate a linear recursive law of motion first and then solve for the yet to be determined coefficients. This part can also be done by some software packages e.g. MATLAB.

5. Analysing the results

After running the simulation for the models, results of the steady state of each variable, theoretical moments of each variable, coefficients of autocorrelation
between one and another, relative standard deviations (RSD)\(^4\) (i.e. \(100\times\text{the standard deviation/the mean}\)) can be analysed. Additionally, it is also useful to study the movement of each variable to the technology shock via the impulse-response analysis.

### 3.4.2 First order conditions (FOCs)

The momentary utility function for the representative household is:

\[
U(C_t, 1 - L_t) = \log C_t + A(1 - L_t)
\]

where \(A\) is the parameter of disutility of labour. This monetary utility function considers indivisible labour, which is introduced in Hansen (1985). The above objective function has utility for consumption and leisure. The household has log utility for consumption and linear derived utility for leisure. The former utility is the special case of the time-separable power utility function where the coefficient of relative risk aversion is equal to one; the latter one is a specification of power utility for leisure where the elasticity of intertemporal substitution for leisure is infinite. This function shows that the household’s preferences are separable in consumption and work.

To demonstrate how the proposed Replication I, II and III models are solved, each process of solving the optimisation is individually displayed:

#### 3.4.2.1 FOCs for Replication I

The Lagrangian \((\mathcal{L})\) in Replication I shows

\[
\mathcal{L} = \max_{C_t, L_t, K_t} E_t \left[ \sum_{t=1}^{\infty} \beta^t \left[ (\log C_t - AL_t) - \lambda_t(C_t + K_t - (1 - \delta)K_{t-1} - Y_t) \right] \right]
\]

To solve the optimisation, I take the first order conditions (FOC) of consumption

\(^4\text{The values of relative standard deviations (RSD) shown in this chapter are in per cent. RSD is calculated as each variable's standard deviations divided by its mean value. It is the value of the coefficient of variation. RSD considers not only the variable's standard deviation but also its mean. As each variable has its mean value, RSD can utilise risks of real variables.}\)
(C_t), labour (L_t) and capital (K_t):

\[
\frac{\partial L}{\partial C_t} : \quad \frac{1}{C_t} - \lambda_t = 0
\]
\[
\frac{\partial L}{\partial L_t} : \quad -A + \lambda_t(1 - \alpha)Z_t K_t^{\alpha - 1} L_t^{-\alpha} = 0
\]
\[
\frac{\partial L}{\partial K_t} : \quad -\lambda_t + \lambda_{t+1}\beta(1 - \delta) + \lambda_{t+1}\beta \alpha Z_{t+1} K_t^{\alpha - 1} L_t^{1-\alpha} = 0
\]

Rearranging the results, I obtain:

\[
AC_t = (1 - \alpha)Z_t K_t^{\alpha - 1} L_t^{-\alpha} \Rightarrow AC_t = (1 - \alpha)\frac{Y_t}{L_t} \quad (3.4.1)
\]

\[
1 = \beta E_t[(\frac{C_t}{C_t + 1})(\alpha Z_{t+1} K_t^{\alpha - 1} L_t^{1-\alpha} + 1 - \delta)] \quad (3.4.2)
\]

\[
R_t = \alpha \frac{Y_{t+1}}{K_t} + 1 - \delta \quad (3.4.3)
\]

The second term on the right-hand side of the second equation is the total returns which are equal to the sum of the capital share and one minus depreciation. The third equation is the Euler equation for gross returns (R_t) for period \([t, t+1]\), the net returns (r_t) is then obtained as \(\alpha \frac{Y_t}{K_{t-1}} - \delta\).

Equilibrium can finally be solved when first-order conditions are satisfied with the given constrains and the log-linearised technology shock equation \(z_t = \psi z_{t-1} + \varepsilon_t\).

3.4.2.2 FOCs for Replication II

In Replication II, it is assumed that the quantity of labour supply is equal to that of labour demand, and the economy is in an equilibrium status. The same assumption is applied to the capital market. In order to obtain an equilibrium state, both the firm’s and the household’s optimisation problems are solved by taking the approach of Lagrange.
The partial derivatives of labour and capital for maximising the firm’s problem are obtained:

\[ W_t = (1 - \alpha)Z_tK_i^{\alpha}L^{-\alpha} \]

\[ 1 = E_t[M_{t,1}(\alpha Z_{t+1}K_t^{\alpha-1}L^1_{t+1} + 1 - \eta)] \]

The partial derivatives of Lagrangian multiplier, consumption, labour and capital for maximising the household’s problem are obtained:

\[ \lambda_t = \frac{1}{C_t} \]

\[ AC_t = (1 - \alpha)Z_tK_i^{\alpha}L^{-\alpha} \]

\[ \lambda_t = \beta E_t[\lambda_{t+1}R_t] \Rightarrow 1 = \beta E_t\left[\frac{C_t}{C_{t+1}}R_t\right] \]

The necessary equations are obtained and summarised as follows:

\[ AC_t = (1 - \alpha)\frac{Y_t}{L_t} \]

\[ 1 = \beta E_t\left[\frac{C_t}{C_{t+1}}R_t\right] \]

\[ M_{t,1} = \frac{C_t}{C_{t+1}} \]

\[ R_t = \alpha Z_{t+1}K_i^{\alpha-1}L^1_{t+1} + 1 - \eta \]

\[ D_t = \alpha Y_t - I_t \]  

(3.4.4)
3.4. MODELS IMPLEMENTATION

\[ Y_t = Z_t K_t^{\alpha} L_t^{1-\alpha} \]

\[ K_t = I_t + (1 - \eta) K_{t-1} \]

\[ C_t = Y_t + I_t \]

\[ z_t = \psi z_{t-1} + \varepsilon_t \]

Results of steady state values and fluctuations of the theoretical optimal real variables \((K_t, L_t, C_t, Y_t, I_t, R_t, D_t, Z_t)\) can be acquired simultaneously while these necessary conditions are satisfied by solving them with the method of simulation with the HP-filter.

Equation 3.4.4 shows that dividend payouts are determined after investment decisions. This replicating study presents how dividend payments stochastically interact with other variables and investor’s consumption in a general equilibrium model.

3.4.2.3 FOCs for Replication III

I solve the firm and the household’s optimisation problems of Replication III (details in Appendix 3.9.2) and summarise the obtained equilibrium conditions:

\[ AC_t = (1 - \alpha) \frac{Y_t}{L_t} \quad (3.4.5) \]

\[ 1 = E_t [M_{t,1} R_{f,t}] \quad (3.4.6) \]

\[ R_{f,t} = E_t [(1 - \tau)\alpha Z_{t+1} K_t^{\alpha-1} L_{t+1}^{1-\alpha} + 1 - \eta] \quad (3.4.7) \]

\[ R_t = \left( \frac{1}{1 - \tau^2} \right) [R_{f,t} - \tau] \quad (3.4.8) \]

\[ Q_t = E_t [M_{t,1} (Q_{t+1} + D_{t+1})] \quad (3.4.9) \]
where $M_{t,1} = \beta \frac{C_t}{C_{t+1}}$ is identified from the representative household’s optimisation (equation 3.9.12 in Appendix 3.9.2).

The next step is to obtain market clearing conditions. This economy begins with a capital stock $K_t > 0$ and an initial level of the technology shock $Z_t > 0$. In equilibrium, consumption, capital and labour need to be determined simultaneously. For the market to clear, a number of conditions must hold. First, aggregate borrowing must equal aggregate lending:

$$B_t^H = B_t \quad (3.4.10)$$

Second, the equity market must clear so that the total equity of the firm, which is normalised to one, is owned by the representative household.

$$N_t = 1 \quad (3.4.11)$$

Third, the labour market must clear so that the hours provided by the representative household matches the labour used by the representative firm.

$$L_t^d = L_t^s = L_t \quad (3.4.12)$$

The final market condition is derived from the goods market. Since the economy consists of identical individual firms, each individual firm is under stochastic process and hit by the credit shock, even though aggregate firms have no difference by the credit shock, the aggregate amount of output ($Y_t^a$) is determined from that each individual firm does not go bankrupt and survives in the model economy. The aggregate output is defined as follows

$$Y_t^a = Y_t - (1 - \theta) \int_0^{X_t} (Y_t - W_t L_t) f(\delta_t) \quad (3.4.13)$$

where the aggregate output is the net amount after allowing for bankruptcy. After a default, some proportion of the total output seized by the bank cannot be recovered, i.e. $\theta \int_0^{X_t} (Y_t - W_t L_t) f(\delta_t)$. The recovered portion of the seized output by the bank will be sold on the goods market and therefore it is considered in aggregate output. From the demand side, the total aggregate demand includes aggregate investment ($I_t^a$)
and consumption. As with aggregate output, aggregate investment is considered while the firm is still running its business (i.e. no default). Aggregate investment is denoted as follows

\[ I^a_t = \int_{X_t}^{\infty} I_t f(\delta_t) \]  

Substituting the equations of aggregate output and investment into the goods market clearing condition \((Y = C + I)\) gives

\[ Y^a_t = I^a_t + C_t \]

\[ \Rightarrow Y_t - \frac{(1-\theta)}{u-l}(Y_t - W_t L_t)(X_t - l) = \frac{1}{u-l}I_t(u - X_t) + C_t \]  

A detailed computation is given in Appendix 3.9.4. The value of aggregate dividends are with respect to a condition of the firm’s non-default state as the household can only get dividends when the firm runs the business and gives dividend payouts. Aggregate dividends is defined as

\[ D^a_t = \int_{X_t}^{\infty} [(1-\tau)(Y_t - W_t L^d_t - R_{t-1} B_{t-1}) - \tau B_{t-1}] f(\delta_t) \]

\[ = \frac{(u - X_t)}{u-l} [(1-\tau)(Y_t - W_t L^d_t - R_{t-1} B_{t-1}) - \tau B_{t-1}] \]

Finally, the general equilibrium can be solved once these conditions (3.4.5, 3.4.6, 3.4.7, 3.4.8, 3.4.9, 3.3.12 and 3.3.14) are satisfied with respect to the market clearing conditions (equations 3.4.10, 3.4.11, 3.4.12, 3.4.15 and 3.4.16).

### 3.5 Calibration

For Replication I, all values used for the parameters are standard and borrowed from Hansen (1985). For technology parameters, the constant capital share in a Cobb-Douglas production function, \(\alpha\), is set to 0.36. The quarterly capital depreciation rate, \(\eta\), is 0.025. The persistence of the idiosyncratic technology shock \(\psi\) is 0.95. The standard deviation of the idiosyncratic shock is assumed to be 0.00712. The parameter \(A\) in the utility function denotes the disutility of labour (or utility of leisure) and is set
Table 3.5.1: Parameter values

This table lists the parameter values used to solve and simulate the three replications. All the parameters are categorised into three groups. The first group, Technology, includes parameters in the firm’s production function. The second group, Financial Credit Default, includes parameters in the bank’s default process and the firm’s operating profits after tax. The final group of parameters is calibrated for the investor’s utility function. The parameters of Replication I are taken from Hansen (1985) and those of Replication II and III from Alessandrini (2003).

<table>
<thead>
<tr>
<th>Replications</th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic Hansen (1985) model</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Replication I</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Replication II</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Replication III</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Technology</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capital share to output</td>
<td>$\alpha$</td>
<td>0.36</td>
<td>0.35</td>
</tr>
<tr>
<td>Depreciation ratio</td>
<td>$\eta$</td>
<td>0.025</td>
<td>0.019</td>
</tr>
<tr>
<td>Persistence of the technology shock</td>
<td>$\psi$</td>
<td>0.95</td>
<td>0.95</td>
</tr>
<tr>
<td>Standard deviation of the technology shock</td>
<td>$\sigma_\epsilon$</td>
<td>0.00712</td>
<td>0.00712</td>
</tr>
<tr>
<td>Disutility of labour</td>
<td>$A$</td>
<td>2.85</td>
<td>2.85</td>
</tr>
<tr>
<td><strong>Financial Credit Default</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Recovery rate</td>
<td>$\theta$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Tax rate</td>
<td>$\tau$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Upper bound</td>
<td>$u$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Lower bound</td>
<td>$l$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>Preferences</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subjective time discount factor</td>
<td>$\beta$</td>
<td>0.99</td>
<td>0.99</td>
</tr>
</tbody>
</table>

equal to 2.85. The quarterly intertemporal subjective time discount factor $\beta$ is set to 0.99.

For Replication II and III, most values for parameters are borrowed from Alessandrini (2003) because the replicating results are to be compared with the results in his paper. Therefore, most values for the parameters in Replication II and III remain the same as used for Replication I except the capital share and the depreciation ratio, which are, 0.35 and 0.019 respectively.

Replication III considers more factors than I and II. The recovery rate, $\theta$, and tax
rate, $\tau$, are assigned values of 0.92 and 6\% respectively. The specific shock is distributed in an interval of 0.75 suggested by Alessandrini (2003). The value of $(u - l)$ determines the interval which the specific shock is distributed in. Accordingly, the range between parameters of the upper bound ($u$) and the lower bound ($l$) is assumed to be fixed given the value of 0.75, that is to say, if the value of the upper bound is given to 1.00 and the value of the lower bound is then adjusted to be 0.25. By calibrating Replication III for various cases, values of 1.125 and 0.375 (150\% of 0.75 is set to be the upper bound and 50\% of 0.75 is for the lower bound) for example, the model shows results which are close to the second calibration of Alessandrini (2003) when the values of the upper/lower bounds are 1.225 and 0.475. Therefore I use the values of 1.225 and 0.475 for $u$ and $l$ respectively for keeping the interval as a value of 0.75. Table 3.5.1 gives a summary of parameter values.

The results and discussion sections summary the US economy statistics are also provided. The US data for capital, consumption, labour and investment are taken from Hansen (1985). They are quarterly data from 1955.3-1984.1 detrended with a Hodrick-Prescott filter. The US quarterly data for dividends and debt are taken from Alessandrini (2003). Alessandrini (2003) gets the quarterly data from 1955-2001 from US National Income and Product Accounts (NIPA) statistics for dividends and from the US Flow of Funds statistics for debt. The multiplier of the HP filter aims to adjust the sensitivity of short-term fluctuations. I take the suggestion from Hodrick and Prescott and set the multiplier at 1600 for quarterly data. All data are detrended by the Hodrick and Prescott filter (HP filter) and are quoted in percent.

3.6 Solving methods

The optimisation problems of the representative firm and investor are solved simultaneously and to reach an equilibrium state. Carrying out the simulation through MATLAB and DYNARE for DSGE models requires the four first order conditions (FOCs) of the optimisation problems. DYNARE codes for the three replications are provided in Appendices 3.9.5, 3.9.6 and 3.9.7.
I summarise two solving methods for DSGE models, the first method is a standard method using technique MATLAB alone while the second method considers MATLAB with DYNARE.

1. Method one: *technique MATLAB only (refer to the toolkit of Uhlig (1995))*
   
   Step 1: Solve first-order conditions by hand.
   
   Step 2: Obtain steady state by hand.
   
   Step 3: Log-linearise constraints by hand.
   
   Step 4: Create the matrices by hand.
   
   Step 5: Solve for recursive equilibrium law of motion by hands (method of undetermined coefficients) or via MATLAB.
   
   Step 6: Get the results.

2. Method two: *multi-techniques MATLAB and DYNARE*
   
   Step 1: Solve for first-order conditions by hand
   
   Step 2: Write codes for DYNARE which has been installed in MATLAB and simulate it in MATLAB
   
   Step 3: Get the results

From the above comparison, one can see that method two can effectively solve time-consuming problems. An additional advantage is that it avoids errors in doing the mathematics for the matrix of first-order conditions. Overall, taking method two for nonlinear models is more efficient and comparatively accurate than using method one.

### 3.7 Results and discussion

The focus of this study is on the properties of aggregate dividends. By taking dividend policy into account in DSGE models, the results show that the theoretical variability of optimal dividends is much greater than in empirical findings. Additional issues and frictions can be incorporated into theoretical DSGE models, such as the investor’s habit
formation and the firm facing costly adjusting capital inputs costs. More advanced research in these areas is contained in the next two chapters.

Using multi-techniques helps not only to calibrate results precisely for the standard Hansen (1985) model, but also to successfully solve the extended model used in Alessandrini (2003). The next more complex advanced DSGE model, in which the firm has both a residual policy and an external financing decision, shows that outcomes are close to Alessandrini (2003).

Results of business statistics of real variables are reported in table 3.7.1. This table has two sections. Section I has four sets of columns consisting of (a) empirical US data; (b) the basic Hansen model taken from the original paper Hansen (1985); (c) the extended Hansen model with a dividend policy [Alessandrini (2003)]; and (d) an advanced DSGE model incorporating a dividend policy and an external financing decision [Alessandrini (2003)]. Section II contains three sets of columns which show the simulated results for my three replications, named Replication I, II and III, respectively.

### 3.7.1 Business cycle statistics

#### 3.7.1.1 Aggregate dividends

In Table 3.7.1, Replication I simulated a standard stochastic growth economy in which a dividend policy is not involved. My simulated results are very close, nearly the same, to the original paper. This model captures that real investment is the most volatile variable in the production economy; its variability is approximately three times aggregate output volatility. All the real variables in this model are procyclical, particularly consumption, labour, investment and return on capital are strongly correlated to output. I calculate that the steady-state consumption output ratio is 0.74 which is consistent with Campbell (1994)’s analytical result.\(^6\)

---

\(^5\)The reason of obtaining different but close outcomes between this research and Alessandrini (2003)’s paper lies in the methods used to solve. The solving methods used here enable me the generation of matrices for first order conditions, that has not been used by Alessandrini (2003). He takes a log-linear approximation method of undetermined coefficients.

\(^6\)Steady-state values show that each variable reaches to a stable and unchanged state and each variable’s relatively size in the economy once all equilibrium conditions are solved simultaneously at each period. In this case, the steady-state consumption output ratio of 0.74 represents that, when an equilibrium exists in the economy, the value of consumption is relatively 74 percent of the value of
Replication II and III display the moments of aggregate payouts and show that the figures for the theoretical relative standard deviation (RSD) of aggregate payouts are 7.57% and 6.93% respectively. The first number is close to 7.62% as reported in Alessandrini (2003) but significantly greater than the observed US data. The empirical US data shows that the RSD of aggregate dividends for the period 1955-2001 was 3.73%. In Alessandrini (2003)’s DSGE model with investing and financing decisions, optimal aggregate dividends are, 2.06%, still lower than the empirical observation.

Both the theoretical models fail accurately capture the variability of corporate payouts for the aggregated market. The reason is that there are many unknown factors that influence, directly and indirectly, the movement of dividend policy. That is the purpose of this thesis: to examine the variance of aggregate dividends by incorporating different factors or frictions. Chapters 4 and 5 will have further explicit studies on this issue.

From the simulated results of Replication II and III, one can see that optimal dividends are countercyclical to the business cycle. This phenomenon has also been observed in Alessandrini (2003). In the observed US economy, gross dividends are positively correlated with the output but the strength of the correlation is much weaker, the value of $corr(D,Y)$ is 0.34. In Section 2.2.8 of Chapter 2, the empirical data shows that net cash flows and gross dividends are procyclical. My simulated results however are not in line with this.

### 3.7.1.2 Business statistics for other main variables

Replication I simulates similar results to the basic Hansen (1985) model and further generates steady-state values for major real variables. Steady state values help us to study the relative optimal state-relationship between variables. For instance, in a numerical way, the goods condition states that the sum of consumption and investment is equal to output. That is, consumption of 0.83 and investment of 0.29 are summed output.

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7 Table 3.7.1, Section II, “Replication III”, Column (a) for $D$.
8 Table 3.7.1, Section I, “Hansen model with dividends”, Column (a) for $D$.
9 Table 3.7.1, Section I, “US data”, Column (a) for $D$.
10 Table 3.7.1, Section I, “DSGE model with investing and financing decisions”, Column (a) for $D$.
11 Table 3.7.1, Section I, “US data”, Column (c) for $D$. 
### Table 3.7.1: Results of Replications

This table reports summary statistics for the US data (from Alessandrini (2003)), Hansen (1985)’s model, the Hansen model with dividends (from Alessandrini (2003)), a dynamic stochastic general equilibrium (DSGE) model with investment and financing decisions (from Alessandrini (2003)) and three replications from this chapter, including dividends, $D$, consumption, $C$, output, $Y$, capital, $K$, labour, $L$, investment, $I$, debt, $B$, the gross risky interest, $R$, and the risk-free rate, $R_f$. This table has two sections. Section I includes the estimated and simulated results from Hansen (1985) and Alessandrini (2003). Section II includes results of Replication I, II and III. Column (a) is the Relative Standard Deviation (%) of the variable (RSD = 100 * the Standard Deviation / the mean). Column (b) shows the Relative Standard Deviation (%) of the variable relative to the Relative Standard Deviation (%) of output. Column (c) displays the correlation of each variable with output. Column (d) shows the steady state values. All data are detrended by the Hodrick and Prescott filter (HP filter). The multiplier of the HP filter aims to adjust the sensitivity of short-term fluctuations. A value of 1600 is given as the multiplier and is suggested from Hodrick and Prescott for quarterly data.

#### Section I: Results reported from targeting papers

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<tr>
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<tbody>
<tr>
<td></td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
<td>(a)</td>
</tr>
<tr>
<td>$D$</td>
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<td>2.14</td>
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</tr>
<tr>
<td>$B$</td>
<td>4.82</td>
<td>2.78</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
<td>$R$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_f$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Section II: Results of replications

<table>
<thead>
<tr>
<th></th>
<th>Replication I (a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>Replication II (a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>Replication III (a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>7.57</td>
<td>4.01</td>
<td>-0.98</td>
<td>0.14</td>
<td>6.93</td>
<td>3.63</td>
<td>-0.98</td>
<td>0.08</td>
</tr>
<tr>
<td>$C$</td>
<td>0.52</td>
<td>0.29</td>
<td>0.87</td>
<td>0.83</td>
<td>0.48</td>
<td>0.26</td>
<td>0.87</td>
<td>0.87</td>
<td>0.47</td>
<td>0.25</td>
<td>0.86</td>
<td>0.84</td>
</tr>
<tr>
<td>$Y$</td>
<td>1.80</td>
<td>1.00</td>
<td>1.00</td>
<td>1.12</td>
<td>1.89</td>
<td>1.00</td>
<td>1.00</td>
<td>1.13</td>
<td>1.91</td>
<td>1.00</td>
<td>1.00</td>
<td>1.10</td>
</tr>
<tr>
<td>$K$</td>
<td>0.50</td>
<td>0.28</td>
<td>0.35</td>
<td>11.47</td>
<td>0.46</td>
<td>0.24</td>
<td>0.32</td>
<td>13.57</td>
<td>0.45</td>
<td>0.24</td>
<td>0.32</td>
<td>12.42</td>
</tr>
<tr>
<td>$L$</td>
<td>1.37</td>
<td>0.76</td>
<td>0.98</td>
<td>0.30</td>
<td>1.48</td>
<td>0.79</td>
<td>0.99</td>
<td>0.30</td>
<td>1.52</td>
<td>0.80</td>
<td>0.99</td>
<td>0.30</td>
</tr>
<tr>
<td>$I$</td>
<td>5.76</td>
<td>3.20</td>
<td>0.99</td>
<td>0.29</td>
<td>6.88</td>
<td>3.65</td>
<td>0.99</td>
<td>0.26</td>
<td>6.79</td>
<td>3.56</td>
<td>0.99</td>
<td>0.24</td>
</tr>
<tr>
<td>$B$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.45</td>
<td>0.24</td>
<td>0.20</td>
<td>4.97</td>
</tr>
<tr>
<td>$R$</td>
<td>0.06</td>
<td>0.04</td>
<td>0.96</td>
<td>1.01</td>
<td>0.06</td>
<td>0.03</td>
<td>0.97</td>
<td>1.01</td>
<td>0.053</td>
<td>0.03</td>
<td>0.97</td>
<td>1.011</td>
</tr>
<tr>
<td>$R_f$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.050</td>
<td>0.03</td>
<td>0.97</td>
<td>1.010</td>
</tr>
</tbody>
</table>
to be equal to output, 1.12.\textsuperscript{12}

The RSD of consumption in the three replications of my research are around 27 percent that of output.\textsuperscript{13} This figure is less than that of the observed US economy, 73 percent.\textsuperscript{14} The three replications presented so far in this thesis estimate the correlation between consumption and output at roughly 87 percent, which is close to that of the empirical US data, 85 percent. Moreover, the simulated moments of consumption, investment, labour and return on equity are procyclical.

From the simulated results of Replication II and III, I find that, not only optimal aggregate dividends are very volatile, optimal aggregate investment is also volatile. Its variability is nearly, as high as aggregate payouts, three and half times of the output volatility.

One may notice that Replication I provides strong support for the Hansen (1985) model except the correlation between capital and output (i.e. 0.05 reported by Hansen (1985) vs. 0.35 in Replication I). In this thesis, I use $K_{t-1}$ instead of $K_t$ as the point at which the capital input is actually chosen, which is suggested by Uhlig (1995). Hansen (1985) uses $K_t$ to denote that capital input is chosen at the beginning of time $t$. Prescott (1986, 1998) shows the cyclical behaviour of (a) the Kydland and Prescott economy (i.e. the simulated production economy by Kydland and Prescott (1982)) on the current period and one period before and after the current period (see Table 3.7.2), and (b) the US economy during 1954 - 1982 (see Table 3.7.3). By comparing their simulated results with their empirical data, the theoretical correlation between capital stock in the previous period ($K_{t-1}$) and current output, $\text{corr}(K_{t-1}, \text{Output}_t)$, is -0.05 while the empirical statistic of the correlation between current nonresidential structure and output, $\text{corr}(K_t, \text{Output}_t)$, is -0.03. From this comparison, the notation difference from capital stocks matters when it comes to the correlation with output. In order to be consistent with the empirical data, it is suggested to define the chosen capital input in time, when it is actually chosen. Prior studies such as Uhlig (1995) and Alessandrini (2003) are in line with this assumption of defining the initial capital at time $t-1$.

\textsuperscript{12}Table 3.7.1, Section II, “Replication I”, Column (d) for $C$, $I$ and $Y$.
\textsuperscript{13}Table 3.7.1, Section II, “Replication I”, “Replication II” and “Replication III”, Column (b) for $C$.
\textsuperscript{14}Table 3.7.1, Section I, “US data”, Column (b) for $C$. 
3.7. RESULTS AND DISCUSSION

Table 3.7.2: Cyclical behaviour of the Kydland-Prescott economy

This table is taken from Prescott (1986) (page 9) and shows the statistics of the main real macroeconomic variables for the Kydland-Prescott economy, including standard deviation in percent and the cross correlation of Gross National Product (GNP) with each variable for the three time periods (-1 to +1, 0 is the base period).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Standard Deviation</th>
<th>Cross Correlation of GNP With</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>x(t-1)</td>
</tr>
<tr>
<td>Gross National Product</td>
<td>1.76%</td>
<td>60</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>(.13)</td>
<td>(.07)</td>
<td>(.07)</td>
</tr>
<tr>
<td>Consumption</td>
<td>.45</td>
<td>47</td>
<td>.85</td>
</tr>
<tr>
<td></td>
<td>(.05)</td>
<td>(.05)</td>
<td>(.02)</td>
</tr>
<tr>
<td>Investment</td>
<td>5.49</td>
<td>52</td>
<td>.80</td>
</tr>
<tr>
<td></td>
<td>(.41)</td>
<td>(.09)</td>
<td>(.03)</td>
</tr>
<tr>
<td>Inventory Stock</td>
<td>2.20</td>
<td>.14</td>
<td>.60</td>
</tr>
<tr>
<td></td>
<td>(.37)</td>
<td>(.14)</td>
<td>(.08)</td>
</tr>
<tr>
<td>Capital Stock</td>
<td>.47</td>
<td>-.05</td>
<td>.02</td>
</tr>
<tr>
<td></td>
<td>(.07)</td>
<td>(.07)</td>
<td>(.06)</td>
</tr>
<tr>
<td>Hours</td>
<td>1.23</td>
<td>.52</td>
<td>.95</td>
</tr>
<tr>
<td></td>
<td>(.09)</td>
<td>(.09)</td>
<td>(.01)</td>
</tr>
<tr>
<td>Productivity (GNP/Hours)</td>
<td>.71</td>
<td>.62</td>
<td>.86</td>
</tr>
<tr>
<td></td>
<td>(.06)</td>
<td>(.05)</td>
<td>(.02)</td>
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<tr>
<td>Real Interest Rate (Annual)</td>
<td>.22</td>
<td>.65</td>
<td>.60</td>
</tr>
<tr>
<td></td>
<td>(.03)</td>
<td>(.07)</td>
<td>(.20)</td>
</tr>
</tbody>
</table>
Table 3.7.3: Cyclical behaviour of the US economy, 1954.1 to 1982.4

This table, taken from Prescott (1986) (page 4), reports statistics of standard deviation in percent and correlation between the variable and Gross National Product (GNP) for the US economy during 1954.1 to 1982.4. The estimated results for cross correlation of GNP with each variable are across three periods (-1 to +1, 0 is the base period).

<table>
<thead>
<tr>
<th>Variable x</th>
<th>Standard Deviation</th>
<th>Cross Correlation of GNP With</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$x(-1)$</td>
</tr>
<tr>
<td>Gross National Product</td>
<td>1.8%</td>
<td>.82</td>
</tr>
<tr>
<td>Personal Consumption Expenditures</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Services</td>
<td>5.6</td>
<td>.66</td>
</tr>
<tr>
<td>Nondurable Goods</td>
<td>1.2</td>
<td>.71</td>
</tr>
<tr>
<td>Fixed Investment Expenditures</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nonresidential Investment</td>
<td>5.3</td>
<td>.76</td>
</tr>
<tr>
<td>Structures</td>
<td>4.8</td>
<td>.42</td>
</tr>
<tr>
<td>Equipment</td>
<td>6.0</td>
<td>.56</td>
</tr>
<tr>
<td>Capital Stocks</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Nonfarm Inventories</td>
<td>1.7</td>
<td>.15</td>
</tr>
<tr>
<td>Nonresidential Structures</td>
<td>4.0</td>
<td>-.20</td>
</tr>
<tr>
<td>Nonresidential Equipment</td>
<td>1.0</td>
<td>.03</td>
</tr>
<tr>
<td>Labor Input</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nonfarm Hours</td>
<td>1.7</td>
<td>.57</td>
</tr>
<tr>
<td>Average Weekly Hours in Mfg.</td>
<td></td>
<td>.76</td>
</tr>
<tr>
<td>Productivity (GNP/Hours)</td>
<td>1.0</td>
<td>.51</td>
</tr>
</tbody>
</table>
3.7. RESULTS AND DISCUSSION

In Table 3.7.1, the result of the correlation of output with debt for “DSGE model with investing and financing decisions from Alessandrini (2003)” is also dissimilar to Replication III, i.e. 0.94 and 0.20 respectively. The difference comes from the definition of the firm’s profit (see equation 3.3.10). The after-tax profit a firm has is obtained after the interest payments on debt, i.e. the net interest rate $R - 1$. Alessandrini (2003), however, estimates the after-tax profit after the interest payments and the total amount of debt. With the tax issue, the interest payments are tax deductible but not the amount of debt. In this thesis, I amend this profit equation from his paper and the model therefore shows a different result for the correlation between debt and output.

3.7.2 Impulse responses to a technology shock

Figures of impulse responses present information about how macroeconomic variables deviate from their own steady state values after an exogenous technology shock.

Figures 3.7.1, 3.7.2 and 3.7.3 plot the impulse responses of the main real variables to a technology shock for Replication I, II and III respectively.

The results show that the impulse response of dividends is greater when the economy has less friction. The graph for dividends in Replication II shows that aggregate payouts immediately decline approximately $-7 \times 10^{-3}$ from their steady state value. This is more than the value in Replication III, which is about $-4 \times 10^{-3}$.

In Replication III, corporations have external financing decisions. It means investment funding can be partly from debt issuance. Free cash flows, as a result, are less volatile and improve relatively quickly after the shock when compared to Replication II.

Other production variables including output, capital and investment are significantly influenced by the dynamic technology shock, which can be viewed from these figures. Replication III displays deviations of capital and debt from their steady state values after the shock. The capital structure (the debt over the capital ratio) is volatile immediately after the shock. It takes almost twenty years to return to steady-state levels.

The response of consumption to the shock is about 0.1% in the first ten years after the shock and then smoothly returns towards the steady state value. The differences
between the three replications are, however, not obvious to see.

Investment and labour are as volatile as dividends, in the early years after the shock. The technology shock has a direct impact on investing and labour demanding activities from the production process. The impulse responses of investment and labour are dramatically variable. It takes them nearly twenty years onward to start to improve and reach stable conditions.

Figure 3.7.1: Impulse responses to a technology shock for Replication I
3.7. RESULTS AND DISCUSSION

Figure 3.7.2: Impulse responses to a technology shock for Replication II

Figure 3.7.3: Impulse responses to a technology shock for Replication III
3.8 Summary

This chapter showed that a DSGE model is capable of capturing corporate and investor activities such as investment and financial decisions and presenting simultaneously aggregated-level movements of real variables. I calibrated three equilibrium models and found that (a) the volatility of theoretical optimal aggregate payouts is high, (b) the volatility of theoretical optimal aggregate consumption is small, (c) aggregate dividends are countercyclical, (d) productivity variables, investment, labour and return on equity, are strongly correlated with real output in the economy.

The standard stochastic growth model does not consider dividends and external financing, I then incorporated them into dynamic stochastic general equilibrium (DSGE) models. The variability of optimal aggregate dividends and real investment are approximately three to four times of the variability of optimal real output. On the contrary, the variability of theoretical optimal consumption is smaller than the empirical observation. The theoretical model captures simultaneous interaction in the economy that optimal aggregate output volatility increases while optimal aggregate investment volatility increases and optimal payouts are volatile. The countercyclicality of aggregate dividends is consistent with the hypothesis that companies appear to retain their free cash flows for future profitable NPV projects if the economy during booms.

In order to examine the high variability of aggregate dividends in terms of the sensitivity of optimal payouts to corporate production process and consumer behaviour in a deep depth, the subsequent chapters investigate the impact of specific issues and frictions on the fluctuation of optimal aggregate dividends.
3.9 Appendices

3.9.1 The essential environment for DSGE models

For the RBC-based stochastic neoclassical growth model, the economic environment is inhabited by the representative households who maximise their expected utility. A representative firm possesses a typical Cobb-Douglas production function with stochastic shocks to technology. A state of consumption, labour and capital will be determined simultaneously in order to reach the equilibrium of the economy. The economic environment is populated by three fundamental criteria (note: all the subscript notations $t$ and $t - 1$ are at the point in time when the input is actually chosen). The information is revised based on Uhlig (1995):

1. **Preference:** A representative household’s preference is determined by a utility function. A utility function can quantify the level of satisfaction of the household from consumption and working (or leisure). The household maximises the lifetime expected discounted utility according to the equation

$$\max_{C_t, L_t} E_t \left[ \sum_{t=0}^{\infty} \beta^t U(C_t, 1 - L_t) \right], \quad 0 < \beta < 1$$

where $E_t$ denotes the expectation operator conditional on time-$t$ information, $\beta$ is the subjective discount factor, $U(C, 1 - L)$ is the utility function of consumption and leisure, $C$ is consumption, $L$ is labour, and $1 - L_t$ is the level of leisure at time $t$. There are various forms of utility functions. The utility function that Kydland and Prescott (1982) use in their model is a constant-relative-risk-aversion utility function. The representative household is endowed with a constraint that the sum of the time for working and for leisure equals the total amount of time in each time period which is set to be one.

2. **Technology:** A firm has an expected process for new capital. Capital evolves over
time by this process

\[ K_t = I_t + (1 - \delta)K_{t-1}, \quad 0 < \delta < 1 \]

where \( K \) is capital, \( \delta \) is a depreciation rate, and \( I_t \) is investment. The firm’s output depends on the amounts of capital and labour along with a production function which follows

\[ Y_t = f(Z_t, K_{t-1}, L_t) \]

where \( Y \) is output and \( f(\cdot) \) denotes a production function. Production functions can be various types. Kydland and Prescott (1982) use a constant-returns-to-scale production function. Many studies have decided to use a non-constant-returns-to-scale production function instead. The most popular choice is the Cobb-Douglas production function. \( Z \) is the aggregate technology shock and exogenously evolving over time. \( Z_t \) conforms to the law of motion

\[
\log Z_t = (1 - \psi)\log \bar{Z} + \psi \log Z_{t-1} + \varepsilon_t \tag{3.9.1}
\]

where \( \psi \) is the parameter of persistence, \( \bar{Z} \) is the steady state of the technology shock. The error term, \( \varepsilon_t \), is an independent and identically distributed random variable (i.i.d.) along with the normal distribution \( \varepsilon_t \sim N(0, \sigma^2_\varepsilon) \). The aggregate technology shock assumes to follow a first order autoregressive process (AR(1)) in logs.

3. Information: In this economy, it begins with a capital stock \( K_t > 0 \) and an initial level of the technology shock \( Z_t > 0 \). The technology shock is dependent on a stochastic process. In equilibrium, three inputs “consumption”, “capital” and “labour” need to be determined simultaneously up to time \( t \) by the technology parameter (the shock).

To sum up, Kydland and Prescott (1982) holds a central and fundamental position in stochastic growth models capturing business cycles, not only in the stream of descriptive research on studying the implications of business fluctuations but also in the stream of
3.9.2 Solving the firm’s and the household’s optimisation problem of Replication III

**Step 1: Solving the firm’s optimisation problem**

The representative firm’s optimisation problem:

$$\max_{L_t^d, K_t, B_t} D_t + E_t \left[ \sum_{h=1}^{\infty} M_{t,h} D_{t+h} \right]$$

subject to

$$D_t = (1 - \tau) \left[ Z_t K_{t-1}^\alpha (L_t^d)^{1-\alpha} - W_t L_t^d - R_{t-1} B_{t-1} \right] - \tau B_{t-1} + B_t - K_t + (1 - \eta) K_{t-1}$$

Setting the partial derivative of the objective function with respect to $L = 0$, $K = 0$, $B = 0$ reveals the following three constraints for the firm:

$$W_t = (1 - \alpha) Z_t K_{t-1}^\alpha L_{t-1}^{1-\alpha} = (1 - \alpha) \frac{Y_t}{L_t}$$ (3.9.2)

$$1 = E_t \{ M_{t,1} \{ [(1 - \tau) \alpha Z_{t+1} K_t^{\alpha-1} L_t + 1 - \eta] + 1 \} \}$$ (3.9.3)

$$1 = E_t \{ M_{t,1} [(1 - \tau) R_t + \tau] \}$$ (3.9.4)

the above equations 3.9.2, 3.9.3 and 3.9.4 are the firm’s optimal conditions.

**Step 2: Solving the household’s optimisation problem**

To solve the household’s problem, I form the Lagrangian of the household’s optimisation problem (on page 47):

$$\mathcal{L} = E_t \left\{ \sum_{h=0}^{\infty} \beta^h \left[ \log C_{t+h} + A(1 - L_{t+h}) - \lambda_{t+h} (C_{t+h} + Q_{t+h} N_{t+h} + 1) \right] + B_{t+h}^H - W_{t+h} L_{t+h}^s - Q_{t+h} N_{t+h} - D_{t+h} N_{t+h} - R_{f,t+h-1} B_{t+h-1} \right\}$$
where $\lambda_{t+h}$ is a Lagrangian multiplier. I solve the optimisation problem by computing partial derivatives of the Lagrangian with respect to each of the control variables and setting each equal to zero. The following equations are yielded according to $\frac{\partial L}{\partial C_t} = 0$, $\frac{\partial L}{\partial L_t} = 0$, $\frac{\partial L}{\partial N_t} = 0$ and $\frac{\partial L}{\partial B_{t}^{H}} = 0$ respectively:

$$\lambda_t = \frac{1}{C_t}$$ (3.9.6)

$$- A + \lambda_t W_t = 0$$ (3.9.7)

$$Q_t = \beta E_t \left[ \left( \frac{\lambda_{t+1}}{\lambda_t} \right) (Q_{t+1} + D_{t+1}) \right]$$ (3.9.8)

$$\lambda_t = \beta E_t [\lambda_{t+1} R_{f,t}]$$ (3.9.9)

The above four equations can be summarised into three major closed forms for the household’s optimisation. First, by substituting $\lambda_t$ from equation 3.9.6 to equation 3.9.7:

$$W_t = AC_t$$ (3.9.10)

Second, the standard Euler equation follows directly from equation 3.9.9:

$$1 = \beta E_t \left[ \left( \frac{\lambda_{t+1}}{\lambda_t} \right) R_{f,t} \right]$$ (3.9.11)

This defines the intertemporal marginal rate of substitution (IMRS) with respect to consumption

$$M_{t,1} = \beta \frac{\lambda_{t+1}}{\lambda_t} = \beta \frac{C_t}{C_{t+1}}$$ (3.9.12)

To rewrite equation 3.9.8, I obtain:

$$Q_t = E_t [M_{t,1}(Q_{t+1} + D_{t+1})]$$ (3.9.13)

Equations 3.9.10, 3.9.11, 3.9.12 and 3.9.13 are the household’s optimal conditions.

**Step 3:** Equilibrium conditions from firm and household
3.9. APPENDICES

To merge the firm and the household’s optimal conditions for a general equilibrium, we can obtain five equilibrium conditions:

\[ AC_t = (1 - \alpha) \frac{Y_t}{L_t} \]

\[ 1 = E_t [M_{t+1} R_{f,t}] \]

\[ R_{f,t} = E_t[(1 - \tau)\alpha Z_{t+1} K_{t+1}^{\alpha-1} L_{t+1}^{1-\alpha} + 1 - \eta] \]

\[ R_t = \left( \frac{1}{1 - \tau} \right) [R_{f,t} - \tau] \]

\[ Q_t = E_t[M_{t+1}(Q_{t+1} + D_{t+1})] \]

3.9.3 The relationship of \( R_t \) and \( R_{f,t} \)

Presuming that the probability density function (pdf) of the specific shock \( f(\delta_{t+1}) \) is

\[ f(\delta_{t+1}) = \begin{cases} 0 & , \delta_{t+1} < l \text{ and } \delta_{t+1} > u \\ \frac{1}{u - l} & , \delta_{t+1} \in [l, u] \end{cases} \]

Applying this to equation 3.3.13 gives

\[ B_t R_t \int_{X_{t+1}}^{\infty} f(\delta_{t+1}) + \int_{-\infty}^{X_{t+1}} \theta \delta_{t+1}(EBIT_{t+1} + (1 - \eta)K_t) f(\delta_{t+1}) = B_t R_{f,t} \]

\[ \Rightarrow B_t R_t \int_{X_{t+1}}^{u} \frac{1}{u - \gamma} d\delta_{t+1} + \int_{l}^{X_{t+1}} \frac{\theta}{u - l} \delta_{t+1}(EBIT_{t+1} + (1 - \eta)K_t) d\delta_{t+1} = B_t R_{f,t} \]

\[ \Rightarrow \frac{B_t R_t}{u - l} (u - X_{t+1}) + \frac{\theta}{2(u - l)}[EBIT_{t+1} + (1 - \eta)K_t][X_{t+1}^2 - l^2] = B_t R_{f,t} \]

this can be re-written as

\[ R_t = \left\{ B_t R_{f,t} - \frac{\theta}{2(u - l)}[EBIT_{t+1} + (1 - \eta)K_t][X_{t+1}^2 - l^2] \right\} \frac{u - l}{B_t(u - X_{t+1})} \]
the above result shows the relationship of $R_t$ and $R_{f,t}$. The gross interest rate is determined in the credit market. It is set by the bank after having the information about debt and the risk-free interest rate.

### 3.9.4 Clearing goods market condition

In an aggregate equilibrium, the clearing condition for the goods market (equation 3.4.15) is given as

\[
Y_t = I_t + C_t
\]

\[
\implies Y_t - (1 - \theta) \int_0^{X_t} (Y_t - W_t L_t) f(\delta_t) = \int_{X_t}^{\infty} I_t f(\delta_t) + C_t
\]

\[
\implies Y_t - (1 - \theta) \int_0^{X_t} \frac{1}{(W_t - l)} (Y_t - W_t L_t) d\delta_t = \int_{X_t}^{\infty} \frac{1}{(u - l)} I_t d\delta_t + C_t
\]

\[
\implies Y_t - \frac{(1 - \theta)}{(u - l)} (Y_t - W_t L_t)(X_t - l) = \frac{1}{(u - l)} I_t (u - X_t) + C_t
\]

where $(Y_t - W_t L_t)$ can be substituted to $(\alpha Y_t)$ as $W_t L_t$ represents $(1 - \alpha Y_t)$ knowing from equation 3.9.2.

### 3.9.5 DYNARE code for Replication I

```matlab
% file name: test0904_basic_hansen
% save in ResearchWorkinSchool\Mystuff\RBCexamples\20090904_similarwith20090810ParameterAdiffer
% similar to Model0_hansen from \mystuff\20090810 but A=2.85 here
% Purpose: Hansens RBC model (1985) 7 variables
% with indivisible labor
% Points : (1) Parameters: same as in Hansen (1985)
% (2) Replicate Hansend basic RBC model with dividends in Alessandrini (2003)
% (3) no dividends, no debt , no default, no tax
% (4) 7 variables: c, i, k, l, y, z, r
%-----------------------------------------------------------------------
% 0. Housekeeping
%-----------------------------------------------------------------------
close all;
```
% 1. Defining variables
% periods 20100; var k, c, y, l, i, r, z;
% output, consumption, capital, labor, interest, investment, technology varexo e;
% parameters A, alpha, beta, delta, psi, sigma;
A = 2.85;
alpha = 0.36; % capital share
beta = 0.99;
delta = 0.025; % depreciation share
psi = 0.95; % autocorrelation of technology shock
sigma = 0.00712; % standard deviation of technology shock units
model;
A*c = (1-alpha)*exp(z)*(k(-1)^alpha)*(l^(-alpha));
1 = beta*((c/c(+1))*r(+1));
r = alpha* (k(-1)^(alpha-1))*exp(z)*(l^(1-alpha)) + 1 - delta;
y = (k(-1)^alpha)*exp(z)*(l^(1-alpha));
i = k - (1-delta)*k(-1);
i = (k(-1)^alpha)*exp(z)*(l^(1-alpha)) - c;
z = psi*z(-1) + e;
end;

% 4. Computation
initval;
k = 2;
c = 1.33;
l = 0.31;
z = 0;
e = 0;
end;
steady;
shocks; var e = sigma^2; end;
stoch_simul(hp_filter = 1600, order = 1);

% 5. Some Results
statistic1 = 100*sqrt(diag(oo_.var(1:7,1:7)))./oo_.mean(1:7);
dyntable('Relative standard deviations in %',strvcat('VARIABLE','REL. S.D.'),M_.endo_names(1:7,:),statistic1,10,7,4)
statistic2 = 100*sqrt(diag(oo_.var(1:7,1:7)));
dyntable('standard deviations in %',strvcat('VARIABLE','S.D.'),M_.endo_names(1:7,:),statistic2,7,8,4);
% file name: test0904_hansendiv
% save in ResearchWorkinSchool\Mystuff\RBCexamples\20090904_similarwith20090810_parameterAdiffer
% similar to Model1_hansendiv from \mystuff\20090810 but A=2.85 here
% Hansen with dividends only (compare to Alessandrini, same!)

% Purpose: Hansens RBC model (1985) 8 variables
% with indivisible labor

% Points: (1) Parameters: same as in Hansen (1985)
% (2) Replicate Hansend basic RBC model with dividends in Alessandrini (2003)
% (3) no debt, no default, no tax
% (4) dividends are decided after investment and financial decisions
% (5) 8 variables: var c, d, i, k, l, y, z, r, q;

var k, c, y, l, i, d, r, z, q;
% output, consumption, capital, labor, interest, investment, technology, q: the price of the equity
varexo e;
parameters A, alpha, beta, delta, psi, sigma;
A = 2.85;
alpha = 0.35; % capital share / Uhlig & Hansen: 0.36, Alessandrini: 0.35
beta = 0.99;
delta = 0.019; % depreciation share / Uhlig & Hansen: 0.025, Alessandrini: 0.019
psi = 0.95; % autocorrelation of technology shock
sigma = 0.00712; % standard deviation of technology shock units
model;
A*c = (1-alpha)*exp(z)*(k(-1)^(alpha))*(l^(1-alpha));
1 = beta*(c/c(+1))*r(+1);
r = alpha* (k(-1)^(alpha-1))*exp(z)*(l^(1-alpha)) + 1 - delta;
y = (k(-1)^(alpha))*exp(z)*(l^(1-alpha));
i = k - (1-delta)*k(-1);
i = (k(-1)^(alpha))*exp(z)*(l^(1-alpha)) - c;
d = alpha*exp(z)*(k(-1)^(alpha))*(l^(1-alpha)) - i;
q = beta*((c/c(+1))*(q(+1)+d(+1)));
z = psi*z(-1) + e;
end;
initval;
k = 2;
c = 1.33;
l = 0.31;
z = 0;
e = 0;
end;
steady;
shocks; var e = sigma^2; end;
stoch_simul(hp_filter = 1600, order = 1);
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% 5. Some Results

statistic1 = 100*sqrt(diag(oo_.var(1:9,1:9)))./oo_.mean(1:9);
dyntable('Relative standard deviations in %',strvcat('VARIABLE','REL. S.D.'),
M_.endo_names(1:9,:),statistic1,10,9,4)
statistic2 = 100*sqrt(diag(oo_.var(1:7,1:7)));
dyntable('standard deviations in %',strvcat('VARIABLE','S.D.'),
M_.endo_names(1:9,:),statistic2,9,8,4);

3.9.7 DYNARE code for Replication III

% file name: test0904_testnew6
% save in ResearchWorkinSchool\Mystuff\RBCexamples
\20090904_similarwith20090810_parameterAdiffer
% similar to testnew6 from \mystuff\20090810 but A=2.85 here
% testnew6 : similar to testnew3. now trying to put aggregate equilibrium for d and
(c=y-i)
% (not partial b and k of r, put the equation in, then can run.)
% file name: testnew5.mod
% \mystuff\20090810_startingfromDynareSummerSchool
%
% Purpose: Extended RBC model: 13 variables
% with indivisible labor, dividends, debt, risk-free rate,
% interest rate and default probability estimate.
%
% 0. Housekeeping
%--------------------------------------------------
close all;
%
% 1. Defining variables
%--------------------------------------------------
var k, c, y, l, i, z, d, b, rf, r, q, x, fau; %fau
% output, consumption, capital, labor, interest, investment, technology, q: the price
% of the equity
varexo e;
parameters A, alpha, beta, delta, psi, sigma, tau, theta, U, L, epi;
%
% 2. Calibration
%--------------------------------------------------
A = 2.85; % 2.85 self-estimated by Hansen (1985)'s parameter results
alpha = 0.35; % capital share / Uhlig & Hansen: 0.36, Alessandrini: 0.35
beta = 0.99; % subjective time discount factor
delta = 0.019; % depreciation share / Uhlig & Hansen: 0.025, Alessandrini: 0.019
psi = 0.95; % autocorrelation of technology shock
sigma = 0.00712; % standard deviation of technology shock units
tau = 0.06; % tax rate
\theta = 0.92; \ % \ recovery \ rate \\
U = 1 + 0.3*\text{epi}; \\
L = 1 - 0.7*\text{epi}; \\
\text{epi} = 0.75; \ % \ specific \ shock

% 3. Model
%---------------------------------------------------------------
model;
1 = beta* (c/c(+1))*rf; \\
rf = (1-tau)*(alpha*exp(z(+1))*(k^(alpha-1))*(l^(1-alpha)))+ 1 - delta; \\
A*c = (1-alpha)*\text{exp(z)}*(k(-1)\alpha)*\text{ln}(-alpha)); \\
y = \text{exp(z)}*(k(-1)\alpha)*\text{ln}(1-alpha)); \\
k = i + (1-delta)*k(-1); \\
c = (1 - (alpha*(1-theta)*(x-1+0.7*\text{epi})/\text{epi})/ \text{epi})*y - ((1+0.3*\text{epi}-x)/\text{epi})*i; \\
y = \text{exp(z)}*(k(-1)\alpha)*\text{ln}(1-alpha)); \\
k = i + (1-delta)*k(-1); \\
c = (1 - (alpha*(1-theta)*(x-1+0.7*\text{epi})/\text{epi})/ \text{epi})*y - ((1+0.3*\text{epi}-x)/\text{epi})*i; \\
r = (1/(1-tau))*rf-tau; \\
r = (b*rf - ((theta/(2*epi))*\text{alpha}*y(+1)+(1-delta)*k)*(x(+1)^2 - (1-0.7*\text{epi})^2))/(epi/(b*(1+0.3*\text{epi}-x(+1)))); \\
q = beta* (c/c(+1)) * (q(+1) + d(+1)); \\
x = b(-1)*r(-1) / (alpha*\text{y} + (1-delta)*k(-1)); \\
d = ((1+0.3*\text{epi}-x)/\text{epi})*(1-tau)*(alpha* y - \text{r(-1)*b(-1)} - \text{tau*b(-1)} - \text{i + b}); \\
z = psi*z(-1) + e; \text{fau} = x/(alpha*\text{y} + (1-delta)*k(-1)); \text{end};
%---------------------------------------------------------------

% 4. Computation
%---------------------------------------------------------------
initval; \\
k = 2; \\
c = 1.33; \\
l = 0.31; \\
b = 0.6; \\
x = 0.3; \\
fau = 0.03; \\
z = 0; \\
e = 0; \\
end; \\
steady; \\
shocks; var e = sigma^2; \text{end}; \\
stoch_simul(hp_filter = 1600, order = 1); 
%---------------------------------------------------------------
% 5. Some Results
%---------------------------------------------------------------
statistic1 = 100*sqrt(diag(oo_.var(1:13,1:13)))./oo_.mean(1:13); \\
dyntable('Relative standard deviations in %','strvcat(''VARIABLE'',''REL. S.D.''), \\
M_.endo_names(1:13,:),statistic1,13,10,4) \\
statistic2 = 100*sqrt(diag(oo_.var(1:13,1:13))); \\
dyntable('standard deviations in %','strvcat(''VARIABLE'',''S.D.''), \\
M_.endo_names(1:13,:),statistic2,13,8,4);
CHAPTER 4

AGGREGATE DIVIDENDS AND CONSUMPTION SMOOTHING
4.1 Introduction

In this chapter I show the simultaneous interaction between aggregate dividends and aggregate consumption in the particular case that investors have habit formation. The purpose of this investigation is to examine that investors, who prefer having smooth consumption, utilise cash dividends as an important source of countercyclical cash flows. My results are consistent with the hypothesis that aggregate dividend policy plays an important role in helping investors to smooth consumption.

Previous papers, which take habit formation into account in their DSGE models [Jermann (1998), Boldrin et al. (2001), Uhlig (2007), Lettau and Uhlig (2000), Schmitt-Grohe and Uribe (2008) and Gershun (2010)], have not focused on optimal aggregate dividend behaviour. Some existing papers [Santoro and Wei (2009) and Gourio and Miao (2010)] have considered dividend policy in their DSGE models but (a) their focus is on dividend taxation and (b) they do not include habit formation. To fill this gap, my contribution in this chapter is to demonstrate that the role of aggregate corporate dividend policy is related to consumption smoothing and to examine the changes in the variability of aggregate dividends over a range of habit motives.

This chapter starts with an extensive review of the literature of DSGE models with habit formation. The main feature of utility functions used in this chapter is that it includes consumption and leisure, which means the investor’s lifetime utility is determined by both market and nonmarket activities. Internal habit formation is introduced in Constantinides (1990) and defined in a way such that each individual investor’s current utility of consumption is influenced by his/her own past consumption. This is useful here because it is well known that the stronger the habit motive, the greater the desire of investors to smooth consumption. Dividends are highlighted as an important source to offset investors’ procyclical labour incomes.

There are three habit formation utility functions incorporated into previous DSGE models. The first is a multiplicative utility function with an additive habit formation [Schmitt-Grohe and Uribe (2008)]. The second utility function, introduced by Nutahara (2010), is a logarithm form for consumption and a power utility form for leisure, together
with an additive habit formation. The third function is a power utility for consumption and a power utility for leisure, incorporating a different habit formation type: a ratio of current and past consumption [Letttau and Uhlig (2000)]. My objective here is to examine the variability of optimal aggregate dividends in different type of habit formation utility functions. I compare the simulated results obtained from the three models in addition to a benchmark which involves no habit formation. My results show that the three model’s results of variability of optimal aggregate dividends and other economic variables differ slightly one from the other. In addition, optimal aggregate dividends are countercyclical cash flows in each model.

My next contribution is to investigate the relationship between optimal aggregate dividend volatility and consumption volatility for different strengths of habit motive. This work is based on a DSGE model including a multiplicative utility function with an addition habit formation. By changing the habit motive, the relationship between optimal dividend policy and consumption smoothing in equilibrium can be examined. I find that the greater the habit motive, the smoother optimal aggregate consumption while the greater optimal aggregate dividend volatility. It implies that the corporate dividend policy is an important mechanism to help investors to smooth consumption.

I also examine the sensitivity of optimal aggregate dividend payouts in a range of habit motives to a different change in the coefficient of risk aversion. Previous work has not investigated whether there are changes in optimal aggregate dividend movements when both the investor’s habit motive and risk aversion are altered. Interestingly, my results show that, for all levels of habit motive, the variability of optimal aggregate payouts increases as investors’ risk aversion decreases.

Overall, my calibrated results show that corporations are sensitive to the habit motive of investors at an aggregate level. When investors’ utility of current consumption depends more on past consumption, the volatility of the optimal aggregate payout policy increases to balance out changes in other sources of income, such as wages. There is also greater associated volatility in real investments. Aggregate corporate activities are thus influenced by investors’ consumption smoothing motives because investors need dividends to attain and maintain a balance between savings and spending across time.
This chapter proceeds as follows. The next section provides a literature review on habit formation in DSGE models. Section 4.3 introduces the specification of the model with habit formation. Section 4.4 presents the model implementation. Section 4.5 provides a brief solving method following the detailed description in the previous chapter. Section 4.6 and 4.7 show the first calibration and discusses the impact of different utility functions on aggregate dividends. Section 4.8 calibrates (a) the dynamic movements of optimal aggregate dividends and consumption when the habit motive is adjusted to different levels and (b) the sensitivity of optimal dividends to various levels of risk aversion for different strengths of habit formation. Section 4.9 presents the results and discussions for section 4.8. Section 4.10 summarises this chapter. Appendices are provided in section 4.11.

4.2 Literature review

This section starts with a brief summary of the specification of the momentary consumption-leisure utility function and habit formation. Then it moves to the specific topic of consumption-leisure momentary utility and habit persistence. Previous scholars have touched upon this specific topic in three different domains: asset pricing and business cycles; momentary and fiscal policy analysis; and estimated DSGE models for analysing aggregate variables.

4.2.1 The momentary consumption-leisure utility function

The first and standard type of momentary utility function is a multiplicative form. It is a power utility form capturing the agent’s preferences of consumption and leisure\(^1\):

\[
U(C_t, 1 - L_t) = \frac{[C_t^\rho (1 - L_t)^{1-\rho} ]^{1-\gamma} - 1}{1 - \gamma} \quad (4.2.1)
\]

where \(C_t, L_t\) and \(1 - L_t\) are the variables consumption, labour and leisure in period \(t\), respectively. The endowment of time for leisure and hours worked (labour) is normalised to be one [Hansen (1985) and Schmitt-Grohe and Uribe (2008)]. Parameter \(\gamma > 0\) is

\(^1\)Campbell (1994) and Jacobs (2007) use this specification of period utility function for their studies.
defined as the coefficient of relative risk aversion. Parameter $0 < \rho < 1$ determines the
time devoted to market activities [Campbell (1994) and Cooley and Prescott (1995)].

In addition to the above nonseparable utility function, another type of momentary
utility function is an additively separable utility function. It is the case in which the
representative household has log utility for consumption and power utility for leisure:

$$U(C_t, 1 - L_t) = \text{log}(C_t) + A(1 - L_t)^{1 - \gamma_1}$$

where a parameter $A$ stands for the dis-utilising effect caused by the labour input,
with $A > 0$. The parameter $\gamma_1$ stands for the coefficient of relative risk aversion on
leisure, with $\gamma_1 > 0$. The utility for consumption is restricted as a log utility form when
it comes to additively separable utility function in consumption and leisure [see King
et al. (1988)]. The reason for that is to obtain a constant steady-state path of the
model (more details were discussed in chapter 2.3.2 on page 28.).

Jacobs (2007) estimates nonlinear Euler equations for preferences by using panel
data for the period 1974-1987 and indicates that the risk aversion parameter, $\gamma$, is
far from the value of one. This implies the elasticity of substitution, $1/\gamma$, will not be
one. Thus, from his investigation, it can be concluded that a logarithmic preference
specification used in existing theoretical studies is far from adequate for explaining
observed phenomenon.

### 4.2.2 What is habit formation?

So far, the utility for consumption is in a *time separable* function. The standard RBC
model is built up with a time-separable momentary utility function. The representative
household’s preferences are independently determined by consumption and leisure in
each period. The representative’s utility for current consumption is independent from
past consumption behaviour. Several researchers have extended the momentary utility
function from being time independent to time dependent by incorporating habit forma-
tion. The utility function with habit formation is known as *time-inseparable* utility.\(^2\)

\(^2\)Several studies focus on the impact of habit formation on the dynamics of consumption over time. Dynan (2000) investigates consumption habit in households’ preferences by using data from the Panel
It considers that a representative household’s level of satisfaction from consumption is determined not only by current consumption but also by past consumption.

Habits are considered either internal or external. An internal habit can ensure the individual’s utility in consumption is derived by his/her own level of past consumption rather than by others’ consumption [Constantinides (1990)]. External habit formation assumes the individual’s consumption is determined by other people’s past consumption [Campbell and Cochrane (1999)]. Pennacchi (2007) presents a detailed context for time-inseparable utility and also points out the specification for internal and external habits respectively.

The focus of this chapter is to incorporate internal habit formation into the household’s utility function. A brief understanding of Constantinides (1990)’s internal habit formation is presented here.

Constantinides (1990) derives an improved utility function taking both current and past consumption into account. What the paper does is to break the condition of the time inseparability in Mehra and Prescott (1985) by considering a time-separable utility function. Constantinides (1990) relaxes the assumption of time separability and rebuilds a utility function with habit formation. He assumes that the utility function is of the constant relative risk aversion type but is extended to incorporate the agent’s past consumption as well. The modified utility function of Constantinides (1990) consists of the discounted present value of the agent’s current and past consumption, and can be defined as:

\[
U(C) = E_t \sum_{s=0}^{\infty} \beta_s \frac{(C_{t+s} - \xi C_{t+s-1})^{1-\gamma}}{1-\gamma}, \quad \xi > 0
\]  

(4.2.3)

where

- \( \beta \) is the subjective time discount factor
- \( \xi \) indicates the effect of past per capita consumption on current utility

Study on Income Dynamics (PSID). His paper finds there is no significant evidence of habit formation at annual frequency. In contrast, Gruber (2004) and Ravina (2005) indicate the importance of the evolution of consumption with habit formation for preferences.
This function is extended in this thesis by incorporating labour into the DSGE model, the details of which are provided in Section 4.4.2.

4.2.3 Habit formation in DSGE models

In a DSGE model, habit formation can be defined as either habits in consumption or habits in leisure. The majority of research studies habit formation in consumption [Jermann (1998), Boldrin et al. (2001), Uhlig (2007) and Gershun (2010)] and although a few papers discuss habits in both consumption and leisure [Lettau and Uhlig (2000) and Schmitt-Grohe and Uribe (2008)].

Habit formation has been discussed earlier in the area of consumption-based capital asset pricing models (CCAPM). Asset pricing models are in an endowment economy in which the consumption is pinned down and makes the asset’s rate of return endogenous. Habit formation not only contributes to solving the equity premium and the risk-free rate puzzles in asset pricing, but also to studies of business cycles. In a production economy, the stochastic growth model makes consumption and output endogenous, while the asset’s rate of return distribution is determined exogenously, which is an opposite assumption compared to asset pricing models. Thus, a refined RBC model with consumption habit formation will have effects on economic fluctuations endogenously.

Generally, the study of the momentary utility function with habit formation in DSGE models is about the intertemporal and intratemporal substitution with respect to consumption and between consumption and leisure. Two arguments that have been mainly discussed are related to: (a) the fluctuations caused by habit formation, especially the observed procyclical phenomenon of the aggregate labour input, and (b) the interactive behaviours between consumption and labour in multiplicative and additive-separable preferences, respectively. A very early paper that employs habit formation in a growth model is Ryder Jr and Heal (1973). The objective of their paper is to study the behaviour of the capital input in independent and dependent utility functions. They

\[3\text{Supposing the agent makes decisions with a past consumption habit, his/her utility will display extreme aversion to consumption risk. So in a market economy, the investor is willing to buy bonds rather than equities, hence the risk-free rate will decrease. That explains the risk-free rate puzzle. This method, however, still cannot resolve the equity premium unless the coefficient of risk aversion becomes unusually big [Mehra and Prescott (1985) and Weil (1989)].}\]
4.2. LITERATURE REVIEW

discuss how the intertemporal preference affects the movement of the capital input in an optimal path. Their model is a fixed labour model where labour is unity and in which the representative household’s preferences are based on consumption across periods only.

The literature review hereafter will mainly be concerned with general equilibrium models associated with (i) internal habit formation [Constantinides (1990)] and (ii) external habit formation [Campbell and Cochrane (1999)]. The review is categorised into four topics: (1) the impact of a momentary utility function with habit formation on the inter(intra)temporal marginal rate of substitution [Subsubsection 4.2.3.1]; (2) the implication for asset pricing and fluctuations over business cycles [Subsubsection 4.2.3.2]; (3) policy analysis [Subsubsection 4.2.3.3]; and (4) aggregate variables in estimated DSGE models [Subsubsection 4.2.3.4]. Table 4.2.1 summarises previous papers working on stochastic general equilibrium models with habit formation.

4.2.3.1 The impact of the momentary utility function with habit formation on the substitution effect

Researchers find that consumption and leisure move in opposite directions when a DSGE model incorporates a multiplicative momentary utility function. For studies on the procyclical movement of labour input and the covariance of consumption and leisure in a multiplicative utility function refer to Kydland and Prescott (1982), Eichenbaum et al. (1988), Lucas (1995) and Seckin (2001). Seckin (2001) uses a multiplicative specification of momentary utility form to study how habit formation alters the range of intertemporal substitution between current and future leisure and the intratemporal substitution between current consumption and leisure. His paper concludes that the intratemporal marginal rate of substitution between consumption and leisure in a multiplicative utility function with habit formation is higher than in the case without habit formation, as well as the intertemporal marginal rate of substitution between current and future leisure. These results imply that the household is less willing to substitute between consumption and leisure, as well as unwilling to change the substitution of current and future leisure. The household takes more leisure today than in the future.
Table 4.2.1: DSGE models with habit formation
This table reports a summary of previous dynamic stochastic general equilibrium (DSGE) models with specific utility functions and habit formation. There are two types of utility function, multiplicative and additively separable, and two types of habit formation, internal and external. Multiplicative form means the household’s utility on consumption and leisure is constructed in a power utility function. Additively separable form is the special case when the coefficient of relative risk aversion of the multiplicative form is equal to one. Internal habit assumes that the individual consumer’s consumption is affected by his/her past consumption while external habit assumes that the individual consumer’s consumption is determined by other people’s past consumption. The form of current and past consumption in the internal habit is found to have two types, non-separable and separable. Models with fixed labour are under an assumption that the labour variable is exogenous. Models with variable labour make the labour endogenous and are simulated simultaneously with other variables (e.g., consumption, output, etc.).

<table>
<thead>
<tr>
<th>Internal habit</th>
<th>Non-separable Preferences in Consumption</th>
<th>Separable Preferences in Consumption</th>
<th>Multiplicative Utility Function</th>
<th>Additively Separable Utility Function</th>
</tr>
</thead>
</table>
Moreover, the household prefers having more leisure and less consumption. Lettau and Uhlig (2000) employ an additively separable momentary utility function which is a power utility function for both consumption and leisure. They emphasise that using the additively separable utility function rather than multiplicative utility function is because the latter would cause a lower conditional volatility of the intertemporal marginal rate of substitution (IMRS) in consumption compared to the former\textsuperscript{4}. It is therefore not plausible to calibrate a multiplicative utility function for a study explaining the observed equity premium, as a lower conditional volatility will make the asset pricing implications counter-factual. The authors also employ habit motives to consumption and leisure individually. As before, log utility in consumption is required in an additively separable utility function [King et al. (1988)]. Lettau and Uhlig (2000), however, do not follow this restriction. They suggest two solutions to allow a power utility form for consumption in an additively separable consumption-leisure utility function. One solution is, relatively simple, to change the form of current and past consumption. They suggest the use of a ratio style instead of an additive style, i.e. $\frac{current\, consumption}{function(past\, consumption)}$ instead of $current\, consumption - function(past\, consumption)$. Thus, they could still look for a steady-state path in the model without restricting the coefficient of relative risk aversion to be one. Uhlig (2007) builds a multiplicative utility function with external habits and wage rigidities in a DSGE model to explain asset prices. Following Lettau and Uhlig (2000), which points out the lack of ability to explain asset pricing anomalies when using a multiplicative utility function, Uhlig (2007) emphasises that the multiplicative utility function works well when it is restricted to real wage stickiness or sufficient curvature on preferences. In his paper, the sufficient curvature on preferences is with habits in both consumption and leisure.

\textsuperscript{4}The intertemporal marginal rate of substitution (IMRS) for the multiplicative utility function includes not only consumption but also leisure. With the assumption that consumption and leisure are negatively correlated and the coefficient of risk aversion is greater than one, the volatility for IMRS is smaller than in the case of additively separable utility function. This is because the IMRS for the additively separable utility function only contains consumption.
4.2.3.2 Studies of asset pricing and business cycles

Habit formation helps to account for the equity premium puzzle by calibrating not only CCAPM but also DSGE models. It creates a smooth consumption process which implies low elasticity of substitution (EIS) between current and future consumption [Lettau and Uhlig (2000), Seckin (2001) and Otrok et al. (2002)]. The studies in this field can be summarised into two streams: (a) with fixed labour supply and (b) with variable labour supply.

(a) Models with fixed labour supply

Fixed labour in the model means that the labour is constrained to equal one (equivalently, leisure is zero) and the production function is determined by capital (and the technology shock) only. The objective function of the representative household studied in Jermann (1998) includes consumption but no labour input, which means it maximises the expected lifetime utility of consumption only. The author tries to explain the historical equity premium by modelling a real business cycle model with an internal habit formation. Boldrin et al. (1995) also adopt internal habit formation into a basic RBC model. But the major difference is that Boldrin et al. (1995) introduce multi-sector technologies in order to account for the behaviour of asset prices, especially the equity premium and risk-free rate. The household’s preferences are based on a form of power utility with additive intertemporal consumption and fixed labour supply. Their model shows a higher correlation of consumption and asset returns, compared to empirical statistics. Moreover, they indicate that the model works well for interpreting the equity premium puzzle and business fluctuations.

(b) Models with variable labour supply

Models with variable labour supply consider that labour is endogenous and determined by both the firm’s and the household’s objectives simultaneously. Christiano and Fisher (1995) incorporate an additively separable momentary utility function with internal habits to interpret the fact that Tobin’s q is different from one and analyse the observed asset returns and the implications for business cycles. The momentary utility function in their model is in the form of a log utility for consumption and a linear utility for leisure,
which is borrowed from Hansen (1985) (see chapter 3). Boldrin et al. (2001) employ the same momentary utility function as Christiano and Fisher (1995) but introduce a two-sector RBC model with habit persistence. They extend their previous paper (Boldrin et al. (1995)) to also take endogenous labour into account. Their results show that habit formation in a modified RBC model, with low elasticity of capital supply, can generate a large equity premium. Elasticity of capital is low due to costly reallocation of capital and labour across sectors. Jermann (1998) and Boldrin et al. (2001) examine the effect of habit formation on business cycles mainly for the study of asset pricing, while Lettau and Uhlig (2000) put the focus on business cycles. The latter is a rich paper in which the impact of habit formation on RBC models is studied in four scenarios: (i) no habit; (ii) habit in consumption; (iii) habit in leisure; and (iv) habit in both consumption and leisure. Furthermore, existing work finds that habit formation gives a smoother consumption process from actual U.S. data [Lettau and Uhlig (2000), Otrok et al. (2002) and Nason and Kano (2004)]. Otrok et al. (2002) consider habit formation in consumption asset pricing models and discuss the potential to resolve the equity premium puzzle. Nason and Kano (2004) doubt the implication of models with habit formation for business cycles for two reasons. Firstly, models with habit formation show lower frequency fluctuations in output growth compared to empirical U.S. data. Secondly, there are no clear consequences of output to permanent shocks through the impulse response functions.

4.2.3.3 Studies of monetary and fiscal policy analysis

RBC models also have been used by researchers who sought to study the impact of habit formation on optimal fiscal and monetary policy. Most importantly, the RBC-based monetary models are with nominal frictions where prices and wages are sticky. Christiano et al. (2005) present a general equilibrium model with habit formation and nominal rigidities to account for the observed behaviour of economic inflation and fluctuations. The utility function of the household’s preferences is an additive form employing a function of consumption with internal habit, a function of hours worked and a function of real cash balances. Kano and Nason (2009) study the implications of
propagation and monetary transmission in New Keynesian DSGE models with internal habit in consumption where the model is borrowed from Christiano et al. (2005). In their model, the household’s momentary utility function overall is an additively separable form including log utility for consumption with internal habit, power utility for disutility of labour and a log form for real balances that is a ratio of the household’s stock of cash divided by the aggregate price level.

4.2.3.4 Studies of estimating parameters of DSGE models

A strand of research studies the parameters that should be used in stochastic growth RBC models. Smets and Wouters (2003) employ a DSGE model with a RBC framework to study the business cycle characteristic of the actual data in E.U. economies. The household’s preferences in this estimated model are an additively separable utility function of power forms for consumption and labour supply individually. Smets and Wouters (2003) put emphasis on business fluctuations in the short-run. Smets and Wouters (2005) and Smets and Wouters (2007) present an estimated linearised DSGE model which proffers a steady-state growth path through a multiplicative (equivalently, non-separable) utility function with external formation. Smets and Wouters (2007) estimate a DSGE model for the U.S. economy by taking a Bayesian likelihood approach. The environment of this estimated structural DSGE model is with an exogenous technology shock and a multiplicative utility function of consumption and leisure with external habit formation in consumption. Their estimated model shows the capability to fit the U.S. macro data by incorporating frictions and shocks. In addition, their paper discusses the effect of a productivity shock on hours worked. They indicate that the technology shock has a weighty, but not dominant, role. With various frictions (nominal price rigidities, habit formation and capital adjustment costs) in DSGE models, labour is influenced greatly by the effect of a productivity shock and drops immediately from its steady state. This phenomenon is counterfactual as empirical studies show that there is a strong correlation between output and labour input. Schmitt-Grohe and Uribe (2008) is another paper working on estimating structural Bayesian models. They display an estimated RBC-based model with capital adjustment costs, variable
4.3. THE MODEL WITH EFFECTIVE LABOUR AND HABIT FORMATION

capacity utilisation, consumption habit formation and leisure habit formation. Moreover, they employ three shocks (permanent and stationary neutral productivity shocks, permanent investment-specific shocks, and government spending shocks) to explain predicted aggregate fluctuations over business cycles in the post-war United States. Their estimation works well, predicting over two thirds of observed aggregate fluctuations. The household’s momentary utility function in their paper is assumed to be a multiplicative function with habits in both consumption and leisure.

In conclusion, with habit formation, the household’s consumption volatility is not only determined by current consumption but also by past consumption. The resulting consumption is expected to be smooth over time. Table 4.2.1 clearly shows that the majority studying habit formation in macroeconomics take an additively separable utility function of consumption and leisure into account (at the same time, it is assumed that there is a separable form of current and past consumption). That is, the momentary utility function is assumed to have a log utility for consumption, for example, \( U(C_t, C_{t-1}, L_t) = \log(C_t - \xi C_{t-1}) + A(1 - L_t) \) or \( U(C_t, C_{t-1}, L_t) = \log(C_t - \xi C_{t-1}) + A \log(1 - L_t) \). This, however, serves the purpose of simplicity in calibration. This thesis will use a multiplicative utility function rather than an additively separable function. It provides the flexibility for studies in both log and power utility forms. Furthermore, the investor’s overall utility is determined by a mixed effect of leisure, consumption and its habit formation.

4.3 The model with effective labour and habit formation

This section shows how I develop the extended DSGE models. The standard framework of DSGE models can be traced back to the pioneers Kydland and Prescott (1982) and Hansen (1985). In the proposed framework, it is assumed that there is one representative firm, one representative household and one representative bank that all potentially survive forever in the absence of bankruptcy. Equilibrium is reached when each is si-
multaneously able to maximise its individual objective function subject to budget and market clearing constraints.

The modified RBC-based DSGE models demonstrated here are the extension of my previous work: Replication III (page 46). The main difference between Replication III and the model here is that there are modifications on two elements:

1. Effective labour: In the real world, the labour supply market would enhance its technology. This technology growing in a positive increasing rate can influence output. The labour unit, therefore, is “effective labour”. In order to improve the efficiency of labour, a factor reflecting stochastic technical change in labour is incorporated into the production function.

The output, $Y_t$ of the representative firm at time $t$ is given by a constant returns to scale Cobb-Douglas production function:

$$Y_t = K_{t-1}^\alpha (Z_t L_d^b)^{1-\alpha}$$  \hspace{1cm} (4.3.1)

2. The momentary consumption-leisure utility function with consumption habit formation: The representative household’s preferences are captured by a time-inseparable utility function ($U(.)$) that includes consumption and leisure:

$$U_t \equiv U(C_t, C_{t-1}, 1 - L_t^s)$$  \hspace{1cm} (4.3.2)

that the utility function for time $t$ is determined by not only current consumption and leisure but also past consumption.

(a) The first model is to incorporate consumption habit formation into the multiplicative momentary utility function.

(b) The second model is to incorporate consumption habit formation into the additively separable utility function.

(c) The third model is a DSGE model with non-additive habit formation (i.e. the habit formation is in the form of a ratio).
The specifications of the representative agent's and corporate optimisation problems and the bank remain the same as in the Replication III, introduced in previous chapter (pages 46 - 53). Optimal levels of consumption of the single consumption good, labour and capital will then be determined simultaneously by the firm and the household.

4.4 Model implementation

4.4.1 First order conditions and market clearing conditions

To solve the corporate and investor’s optimisation problem, five FOCs and four market clearing conditions are obtained as:

\[
(1 - \alpha) \frac{Y_t}{L_t} = - \frac{\frac{\partial U(C_t, C_{t-1}, 1 - L_t^s)}{\partial L_t^s}}{\frac{\partial U(C_t, C_{t-1}, 1 - L_t^s)}{\partial C_t} + \beta \frac{\partial U(C_{t+1}, C_t, 1 - L_{t+1}^s)}{\partial C_t}} \quad (4.4.1)
\]

\[
1 = E_t[M_{t,1} R_{f,t}] \quad (4.4.2)
\]

\[
R_{f,t} = E_t[(1 - \tau)\alpha K_t^{\alpha - 1}(Z_{t+1} L_{t+1})^{1-\alpha} + 1 - \eta] \quad (4.4.3)
\]

\[
R_t = \left(\frac{1}{1 - \tau}\right) [R_{f,t} - \tau] \quad (4.4.4)
\]

\[
Q_t = E_t[M_{t,1}(Q_{t+1} + D_{t+1})] \quad (4.4.5)
\]

\[
B_t^H = B_t
\]

\[
N_t = 1
\]

\[
L_t^d = L_t^s = L_t
\]

\[
Y_t - \frac{(1 - \theta)}{u - l} (Y_t - W_t L_t)(X_t - l) = \frac{1}{u - l} I_t (u - X_t) + C_t
\]
where $Y_t$ is subject to equation 4.3.1. The definition of $X_t$ has been assumed as the ratio of total debt payment divided by the firm value (the detailed definition please refer to chapter 3 (page 52). This chapter will use three types of utility functions. $M_{t,1}$ is identified from the representative household’s optimisation (see Appendix 4.11.1 for an example). Aggregate dividends is subject to

$$D_t = \frac{(u - X_t)}{u - l}[(1 - \tau)(Y_t - W_tL_t^s - R_{t-1}B_{t-1}) - \tau B_{t-1}]$$

The relationship between the risk-free interest rate on bond and gross interest rate on debt is given in equation 3.3.14 in Chapter 3 (page 53).

Finally, a dynamic stochastic general equilibrium can be obtained with respect to the technology shock $Z$ which is assumed to follow a AR(1) process.

### 4.4.2 Utility function implementation (with internal habit)

To solve models with the above conditions, this section demonstrates how to calibrate the marginal rate of substitution ($\partial U_t/C_t$, $\partial U_{t+1}/C_t$, $\partial U_{t+1}/C_{t+1}$, $\partial U_t/L_t$), the Lagrangian multipliers ($\lambda_t$ and $\lambda_{t+1}$), and the form of the intertemporal marginal rate of substitution (IMRS) (which is denoted as $M_{t,h}$).

The study of Campbell (1994) demonstrates both multiplicative and additively separable utility functions but it does not consider habit persistence. So far, there are two types of utility functions (multiplicative and additively separable) and two types of internal habit formation (additive and of ratio). The next step is to extend these momentary utility functions by applying internal habit formation to three cases. The purpose of demonstrating three models is to compare the effects of habit persistence on aggregate dividends among different functions of utility.

**A nonseparable model (Model 1)**

In model 1, the representative household’s utility is determined by the multiplied effect of

\[\text{The ratio formation of habits is applied to a special case that an additively separable utility function is with power utility for consumption [Lettau and Uhlig (2000)]. In a multiplicative form, it is possible to use an additive form of habit formation. Thus, I skip the fourth combination: multiplicative utility function with the ratio of habit formation.} \]
of consumption and leisure:

\[ U(C_t, C_{t-1}, 1 - L_t) = \frac{[(C_t - \xi C_{t-1})^\rho (1 - L_t)^{1-\rho}]^{1-\gamma} - 1}{1 - \gamma} \tag{4.4.6} \]

where parameter \( \rho \) determines the time allocated to market activities by the representative agent. \( \gamma \) is the coefficient of risk aversion. The household’s utility is determined by the multiplied effect of consumption and leisure. The consumption process partly considers its past values. Schmitt-Grohe and Uribe (2008) also use this utility function with internal habit. Another similar case can be found in Uhlig (2007).

With the given momentary utility function for Model 1, I calculate \( \frac{\partial U_t}{\partial C_t}, \frac{\partial U_{t+1}}{\partial C_t}, \frac{\partial U_{t+1}}{\partial L_t}, \lambda_t, \lambda_{t+1}: \)

\[ \frac{\partial U_t}{\partial C_t} = \left( (C_t - \xi C_{t-1})^\rho (1 - L_t)^{1-\rho} \right)^{-\gamma} \left[ \rho(C_t - \xi C_{t-1})^{\rho-1}(1 - L_t)^{1-\rho} \right] \tag{4.4.7} \]

\[ \frac{\partial U_{t+1}}{\partial C_t} = \left( (C_{t+1} - \xi C_t)^\rho (1 - L_{t+1})^{1-\rho} \right)^{-\gamma} \left[ \left(-\xi C_{t+1} - \xi C_t\right)^{\rho-1}(1 - L_{t+1})^{1-\rho} \right] \tag{4.4.8} \]

\[ \frac{\partial U_t}{\partial L_t} = \left( (C_t - \xi C_{t-1})^\rho (1 - L_t)^{1-\rho} \right)^{-\gamma} \left[ (-1)(1 - \rho)(C_t - \xi C_{t-1})^\rho (1 - L_t)^{-\rho} \right] \tag{4.4.9} \]

\[ \lambda_t = \frac{\partial U_t}{\partial C_t} + \beta \frac{\partial U_{t+1}}{\partial C_t} \tag{4.4.10} \]

\[ \lambda_{t+1} = \frac{\partial U_{t+1}}{\partial C_{t+1}} + \beta \frac{\partial U_{t+2}}{\partial C_{t+1}} \tag{4.4.11} \]

Take these formulae to summarise the form of IMRS.
4.4. MODEL IMPLEMENTATION

\[ M_{t,1} = \beta \frac{\lambda_{t+1}}{\lambda_t} = \beta \left( 1 - [\xi(\frac{C_{t+2} - \xi C_{t+1}}{C_{t+1} - \xi C_t}) - \gamma + \rho - 1(\frac{1 - L_{t+2}}{1 - L_{t+1}}) - \gamma(1 - \rho) + 1 - \rho] \right) \]  

(4.4.12)

An additively separable model with additive habit formation

(Model 2)

The momentary utility function for Model 2 is:

\[ U(C_t, C_{t-1}, 1 - L_t) = \log(C_t - \xi C_{t-1}) + A \frac{(1 - L_t)^{1-\gamma_L}}{1 - \gamma_L} \]  

(4.4.13)

This type of form is an extension of Hansen (1985) by considering an internal consumption habit and giving a power form for leisure. For an additively separable utility function, Kydland and Prescott (1982) have shown that a presumption of \( \gamma_c = 1 \) is necessary for balanced growth. The utility form for consumption, therefore, is a logarithmic form. Campbell (1994) also uses this specification of utility function but without considering habit formation. I intend to extend his model with internal habit in consumption.

Regarding the utility form for leisure, it is assumed to be a power form. The specification of a power function for capturing the utility on leisure (or labour) provides various possibilities. For example, when \( \gamma_L \) is one (and \( \xi = 0 \)), the form of the utility for leisure is the case of Hansen (1985)’s divisible labour. When \( \gamma_L \) is equal to zero (and \( \xi = 0 \)), it then changes to the case of a model with indivisible labour supply.

The necessary equations for equilibrium are calculated as below:

\[ \frac{\partial U_t}{\partial C_t} = \frac{1}{(C_t - \xi C_{t-1})} \]  

(4.4.14)

\[ \frac{\partial U_{t+1}}{\partial C_t} = \frac{-\xi}{(C_{t+1} - \xi C_t)} \]  

(4.4.15)

\(^6\)Nutahara (2010) uses the same utility function with internal habit. For similar specifications see Christiano and Fisher (1995), Boldrin et al. (2001) and Kano and Nason (2009).
\[
\frac{\partial U_t}{\partial L_t} = (-1) A (1 - L_t)^{-\eta L}
\]

\[\lambda_t = \frac{\partial U_t}{\partial C_t} + \beta \frac{\partial U_{t+1}}{\partial C_{t+1}} \tag{4.4.17}\]

\[\lambda_{t+1} = \frac{\partial U_{t+1}}{\partial C_{t+1}} + \beta \frac{\partial U_{t+2}}{\partial C_{t+2}} \tag{4.4.18}\]

\[M_{t,1} = \beta \frac{\lambda_{t+1}}{\lambda_t} = \beta \left(1 - \xi \beta \left(\frac{C_{t+1} - \xi C_t}{C_{t+2} - \xi C_{t+1}}\right)\right) \tag{4.4.19}\]

**An additively separable model with a ratio of habit formation**

*(Model 3)*

The momentary utility function used so far is an additive form, which means $C_t - \xi C_{t-1}$. Some papers (see Seckin (2001) and Lettau and Uhlig (2000)) take non-additive habit formation, that is, a ratio of current and past consumption: $\frac{C_t}{\xi C_{t-1}}$. In this section, an additional DSGE model is introduced with a one-period non-additive habit formation. Bouakez et al. (2005) also use the ratio of habit formation for their study of the persistence of monetary shocks.

The model (called Model 3 hereafter) uses a separable utility function of consumption and leisure, that is, a power form for consumption and a power form for leisure. King et al. (1988) suggests that this kind of additive utility function needs to have a log form for measuring utility in consumption otherwise there is lack of necessary condition for obtaining a balanced growth path. From Lettau and Uhlig (2000)’s suggestion, it is possible to build a model under non-log utility function for consumption with habit, and the model can still reach to a balanced growth path over business cycles. The advice is to assume a ratio of a consumption process instead of an additive form. Therefore, it is assumed that there is a consumption process with internal habit given by $\frac{C_t}{\xi C_{t-1}}$ instead of the standard one-period internal habit formation $C_t - \xi C_{t-1}$. The momentary utility function is displayed as:
\begin{equation}
U(C_t, C_{t-1}, 1 - L_t) = \left( \frac{C_t}{\xi C_{t-1}} \right)^{1-\gamma_C} + A \left( \frac{1 - L_t}{1 - \gamma_L} \right)^{1-\gamma_L} \tag{4.4.20}
\end{equation}

This momentary utility function shows that consumption and leisure determining utility are in an additive form. Both variables are in a power utility function. The idea of this specification is adopted from Lettau and Uhlig (2000). They use an additive utility function in consumption and leisure. They also consider external habits for both variables. King et al. (1988) have asserted that it is necessary to use a log form for consumption in an additively separable period utility function. Lettau and Uhlig (2000) have suggested approaches avoiding the restriction given by King et al. (1988). One of their suggestions is to replace the additive form of consumption and its habit persistence with a ratio form. As a result, the momentary utility function would still be consistent with a balanced path when \( \gamma_C \) is generated across time. Their suggestion is duly taken into consideration for the purposes of this research and therefore it is assumed that there is a ratio for the consumption process in Model 3. The utility in leisure is a power form in which the \( \gamma_L \) is not restricted particularly.

The calculations of \( \partial U_t/C_t, \partial U_{t+1}/C_t, \partial U_{t+1}/C_{t+1}, \partial U_t/L_t, \lambda_t, \lambda_{t+1} \) and \( M_{t,1} \) are shown below respectively:

\begin{align*}
\frac{\partial U_t}{\partial C_t} &= \left( \frac{C_t}{\xi C_{t-1}} \right)^{-\gamma_C} \left( \frac{1}{\xi C_{t-1}} \right) \tag{4.4.21} \\
\frac{\partial U_{t+1}}{\partial C_t} &= (-1) \left( \frac{C_{t+1}}{\xi C_t} \right)^{-\gamma_C} \left( \frac{C_{t+1}}{(\xi C_t)^2} \right) \tag{4.4.22} \\
\frac{\partial U_t}{\partial L_t} &= (-1) A (1 - L_t)^{-\gamma_L} \tag{4.4.23} \\
\lambda_t &= \frac{\partial U_t}{\partial C_t} + \beta \frac{\partial U_{t+1}}{\partial C_t} \tag{4.4.24} \\
\lambda_{t+1} &= \frac{\partial U_{t+1}}{\partial C_{t+1}} + \beta \frac{\partial U_{t+2}}{\partial C_{t+1}} \tag{4.4.25}
\end{align*}
\[ M_{t,1} = \beta \frac{\lambda_{t+1}}{\lambda_t} = \beta \left( \frac{C_{t+1}/C_t}{C_t/C_{t-1}} \right)^{-\gamma_C} \left( \frac{1}{C_t/C_{t-1}} \right)^{\left(1 - \beta \left( \frac{C_{t+2}/C_{t+1}}{C_{t+1}/C_t} \right)^{1-\gamma \psi} \right)} \] (4.4.26)

...in conclusion, this section has succeeded in calibrating the household’s preferences for three types of momentary utility functions. We can afterwards substitute these formulae for the previous obtained FOCs. Followed by giving parameter values, we then can obtain results for the moments of aggregate dividends, consumption, output and the other main real variables for each model.

4.5 Solving methods

To solve the three models, I use the software packages DYNARE that is compatible with MATLAB. DYNARE codes are provided in Appendices 4.11.5, 4.11.6 and 4.11.7.

4.6 Calibration (I)

Table 4.6.1 presents a summary of parameter values. Most of the values for the parameters are standard and borrowed from Alessandrini (2003). For technology parameters, the constant capital share in a Cobb-Douglas production function, \( \alpha \), is set to 0.35. The quarterly capital depreciation rate, \( \eta \), is 0.019. The persistence of the idiosyncratic technology shock \( \psi \) is 0.95. The standard deviation of the idiosyncratic shock is assumed to be 0.00712. The recovery rate, \( \theta \), and tax rate, \( \tau \), are assigned values of 92\% and 6\% respectively. The values for \( U \) and \( L \) are given equal 1.225 and 0.475; details are given in Chapter 3.

The quarterly intertemporal subjective time discount factor \( \beta \) is set to 0.99. The coefficients of relative risk aversion of consumption in Model 1 and Model 3, denoted \( \gamma \) and \( \gamma_C \) respectively, are set to 2. The coefficient of relative risk aversion of labour, \( \gamma_L \), is set to 5 for both Model 2 and Model 3.

In the momentary utility function, the habit persistence level is set to 0.8 as used in
Constantinides (1990)\textsuperscript{7}. I take 0.36 as the parameter value of $\rho$, which is taken from Campbell (1994)\textsuperscript{8}.

A benchmark has been demonstrated in this research that is a model without habit formation. The utility function is log utility for consumption and linear utility form for leisure, as used by Hansen (1985):

$$U(C_t, 1 - L_t) = \log C_t + A(1 - L_t)$$

The reason to choose this specification for a benchmark utility function is because that log utility function of consumption is a simple form which has been used often in studies of business cycles [such as Jermann (1998) and Campbell (1994)] and that the linear-derived utility for leisure implies an infinite elasticity of substitution in leisure in different periods [Hansen (1985)\textsuperscript{9}]. Even though the utility function of the benchmark is the same as the previous chapter Replication III, simulated results with this benchmark are different than reported there. This is because that the production process in the equilibrium economy in this chapter is improved by applying an effective labour scheme. DYNARE codes for the benchmark are provided in Appendix 4.11.4.

4.7 Results and discussion (I)

The discussion in this section is much more technical than before as the objective here is to compare the impact of different specifications of momentary utility functions with habit formation on economic statistics by calibrating the three models and one

\begin{itemize}
\item \textsuperscript{7}The value for the habit persistence varies. In Jermann (1998), it is 0.82. In Cochrane and Hansen (1992), they use two values - 0.5 and 0.6.
\item \textsuperscript{8}The preference parameters $\rho$ and $1 - \rho$ are the consumption and leisure share parameter. Ghez and Becker (1975) [cited by Prescott (1998) and Kydland and Prescott (1982)] emphasise that the ratio of the household’s productive time to nonmarket and market activities is approximately two to one. Nearly two thirds of the time is devoted to nonmarket activities while one third of the time is allocated to market activities. Campbell (1994) calculates an implied value of 0.36 for parameter $\rho$ with a given labour input ($L = 0.33$). A value for the leisure share parameter, therefore, is close to two thirds. These values are taken as the principle of the allocation of time for consumption and leisure choice for the Model 1. An expected optimal endogenous steady state value of labour input, approximately 0.33, is obtained.
\item \textsuperscript{9}The idea of the linearity in leisure is derived from Hansen (1985). He emphasises that labour should be indivisible, which means employees either work fully or not at all. It is much more reasonable than divisible labour that hours worked are chosen by individuals.
\end{itemize}
**4.7. RESULTS AND DISCUSSION (I)**

Table 4.6.1: Parameter values

This table lists the parameter values used to solve and simulate the three with-habit dynamic stochastic general equilibrium (DSGE) models. All the parameters are categorised into three groups. The first group, Technology, includes parameters in the firm’s production function. The second group, Financial Credit Default, includes parameters in the bank’s default process and the firm’s operating profits after tax. The upper and lower bound are used to solve a default probability density function given that the credit shock follows a rectangular distribution, \( f(\delta_{t+1}) = \begin{cases} 0 , & \delta_{t+1} < l \text{ and } \delta_{t+1} > u \\ \frac{1}{u-l} , & \delta_{t+1} \in [l, u] \end{cases} \). The final group of parameters is calibrated for the investor’s utility function. Model 1 is a nonseparable consumption-leisure model and its utility function is \( U(C_t, C_{t-1}, 1 - L_t) = \log(C_t - \xi C_{t-1}) + A \left(1 - L_t\right)^{1-\gamma_L} \) and the utility function of model 3 is \( U(C_t, C_{t-1}, 1 - L_t) = \left(\frac{C_t}{1-C_t}\right)^{1-\gamma} + A \left(1-L_t\right)^{1-\gamma_L} \). The parameter value for the habit persistence level is taken from Constantinides (1990). For determining the time allocated to market activities (consumption), the parameter value used here is suggested by Campbell (1994).

<table>
<thead>
<tr>
<th>Technology</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital share to output ( \alpha )</td>
<td>0.35</td>
</tr>
<tr>
<td>Depreciation ratio ( \eta )</td>
<td>0.019</td>
</tr>
<tr>
<td>Persistence of the technology shock ( \psi )</td>
<td>0.95</td>
</tr>
<tr>
<td>Standard deviation of the technology shock ( \sigma_\epsilon )</td>
<td>0.00712</td>
</tr>
<tr>
<td>Disutility of labour (Model 2) ( A )</td>
<td>2.85</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Financial Credit Default</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recovery rate ( \theta )</td>
<td>0.92</td>
</tr>
<tr>
<td>Tax rate ( \tau )</td>
<td>0.06</td>
</tr>
<tr>
<td>Upper bound ( u )</td>
<td>1.225</td>
</tr>
<tr>
<td>Lower bound ( l )</td>
<td>0.475</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Preferences</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor ( \beta )</td>
<td>0.99</td>
</tr>
<tr>
<td>Coefficient of relative risk aversion (Model 1) ( \gamma )</td>
<td>2</td>
</tr>
<tr>
<td>Coefficient of relative risk aversion (Model 3) ( \gamma_C )</td>
<td>2</td>
</tr>
<tr>
<td>Coefficient of relative risk aversion for labour (Model 2 and 3) ( \gamma_L )</td>
<td>5</td>
</tr>
<tr>
<td>Habit persistence ( \xi )</td>
<td>0.80</td>
</tr>
<tr>
<td>Parameter determining the time allocated to market activities ( \rho )</td>
<td>0.36</td>
</tr>
</tbody>
</table>
The first main finding is that these modified DSGE models provide the same constant steady growth path for the economy regardless of the specification of utility function. The second contribution here is that models with separable habit formation have similar results of moments of aggregate dividends and other main real variables. All the models with habit formation have a smoother consumption growth than the model that is without habit formation. In a model with a ratio of habit formation, the variability of aggregate dividends is closer to that in a benchmark which is without habit formation. It appears that the ratio of habit formation used in this research does not provide a strong effect of habit formation on business cycles. It only affects the variability of consumption.

In addition, I find that aggregate dividend policy is countercyclical to business cycles. Although this finding is inconsistent with the empirical result that the correlation between net cash flows and Gross National Product is positive during several periods (see Section 2.2.8, Chapter 2), it implies that the structure of the firm’s production process, investment and financing decisions is complex and results in a negative relationship with output. The outcome of countercyclical payouts is related to the theorem that corporations prefer to keep free cash flows for potential positive investment plans in a bull market rather than pay dividends. It does not affect investors because, on one hand, the value of profitable investments will be reflected in share prices. On the other hand, agents with habit formation prefer to have smooth consumption. If the economy is growing, labour income increases while output profit increases. Meanwhile, the agent would like to keep consumption smooth and balance out changes in labour income and other income (i.e. dividends). My results show that the procyclicality of labour and the countercyclicality of dividends are consistent with this hypothesis.

4.7.1 Steady state values

The steady-state values for the marginal product rates of capital and labour (denoted $MPK$ and $MPL$ respectively) should be constant in a steady-state growth stochastic model as highlighted by King et al. (1988), as well as the ratios of $Y/K$, $C/Y$ and
$C/K$. These results have confirmed these conditions and are displayed in Table 4.7.1.

With internal habits, steady-state values in Model 1 are slightly greater than in the benchmark. Steady-state values in Model 2, have the opposite result and are about half of the values in the benchmark. The value of the threshold bankruptcy point is about 0.40 in the all models. This shows that the firm will have to close its business if it is unable to pay back the loan with the key threshold value being forty per cent of the firm’s earnings and its residual capital.

Model 1 and Model 2 are with additive habit formation and Model 3 is with non-additive habit formation. The steady state values of real variables in Model 3 are considerably lower than in others. This is because the consumption process in Model 3 is computed in a ratio form. Despite this, Model 3 reaches a general equilibrium economy and the steady state values of capital to output ($Y/K = 0.0885$), consumption to output ($C/Y = 0.7665$) and consumption to capital ($C/K$) are the same as Model 1 and Model 2. It is a significant finding that steady-state growth on output, capital and consumption will remain the same no matter what utility function is adopted.

The benchmark, Model 1, Model 2 and Model 3 have obtained steady-state growth paths over business cycles in which technology, output, capital and consumption all grow at a constant rate around $0.002$ ($0.8\%$ at an annual rate) (details in Appendix 4.11.2). In Campbell (1994)'s paper, the values of the log technology growth rate $g$ and the log real return on capital $r$ are $0.005$ ($2\%$ annualised) and $0.015$ ($6\%$ annualised) respectively. The reason for these differences comes from different value settings of the depreciation rate and, most importantly, the model here is calibrated in an aggregated level with the presence of bankruptcy.

The steady-state leverage ratio, $B/K$, is endogenously determined by the proposed models, the figure for each model is about 40\%. This is consistent with the report from Taggart Jr (1986) that the leverage ratio for the after war period is between 30\% and 47\% [quoted by Alessandrini (2003)].
Table 4.7.1: Steady state values
This table summarises the steady state values of dividends ($D$), consumption ($C$), output ($Y$), capital ($K$), labour ($L$), investment ($I$), debt($B$), the risk-free rate ($R_f$), the gross risky interest rate ($R$), the share price ($Q$), the benchmark point of bankruptcy ($X$), the marginal product of capital ($MPK$), the marginal product of labour ($MPL$), the output-capital ratio ($Y/K$), the consumption-output ratio ($C/Y$) and the consumption-capital ratio ($C/K$) for four models. The first model, Benchmark, is a model in which its utility function is without habit formation. Model 1, 2 and 3 are models with internal habit formation. Model 1 and Model 2 are with additive habit formation (i.e. $C_t - \xi C_{t-1}$, $\xi$ is the parameter for habit persistence) and Model 3 is with non-additive habit formation (i.e. $C_t/(\xi C_{t-1})$).

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<th>$Y$</th>
<th>$K$</th>
<th>$L$</th>
<th>$I$</th>
<th>$B$</th>
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<td>12.4166</td>
<td>0.2976</td>
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<tr>
<td>Model 1</td>
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<td>0.9381</td>
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<td>0.3316</td>
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<td>5.5403</td>
</tr>
<tr>
<td>Model 2</td>
<td>0.0398</td>
<td>0.4047</td>
<td>0.5279</td>
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<td>0.1430</td>
<td>0.1134</td>
<td>2.3897</td>
</tr>
<tr>
<td>Model 3</td>
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<td>0.0024</td>
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<table>
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<th>$Q$</th>
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<td>1.0108</td>
<td>8.1905</td>
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</tr>
<tr>
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<td>1.0108</td>
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<td>1.0108</td>
<td>3.9371</td>
<td>0.3999</td>
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<tr>
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<td>2.3992</td>
<td>0.0885</td>
<td>0.7665</td>
<td>0.0678</td>
</tr>
<tr>
<td>Model 1</td>
<td>0.0310</td>
<td>2.3992</td>
<td>0.0885</td>
<td>0.7665</td>
<td>0.0678</td>
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<td>2.3992</td>
<td>0.0885</td>
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<td>0.0678</td>
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<tr>
<td>Model 3</td>
<td>0.0310</td>
<td>2.3992</td>
<td>0.0885</td>
<td>0.7665</td>
<td>0.0678</td>
</tr>
</tbody>
</table>
4.7.2 Cyclical variability

Table 4.7.2 displays moments of aggregate dividends, consumption and other main variables. The results provide significant information that aggregate dividends move in a less volatile manner with habit persistence than without habits, and so do other real variables. In addition, the model generates relatively low variability in the labour market through habit persistence in the utility of consumption.

Comparing three models which have different momentary utility functions with habit persistence, I find that Model 1 and Model 2 have similar volatility of aggregate dividends even though the specification of the momentary utility function is different. Model 3 generates results which are closer to the benchmark apart from much smoother consumption.

The relative standard deviation of consumption is lower in with-habit models rather than in the no-habit model because of the effect of habit persistence. This is confirmed by the fact that the household prefers a smoother consumption path in response to a shock.

There is a puzzle when I compare the results of volatilities of aggregate output and consumption between the no-habit model (the benchmark) and the habit model, Model 3. In Model 3, the variability of aggregate output is as great as that in the no-habit model but the variability of aggregate consumption in Model 3 is much lower than that in the no-habit mode. In the no-habit model, aggregate dividends fluctuate greatly while the volatility of consumption is high. This does not provide consistent results with the other models. One possible reason to explain the above results is the use of a ratio form for habit formation. Thus I question the effectiveness of modelling habit through the ratio of past and current consumption.

Labour is less volatile in with-habit models. The variability of the labour market is smaller for habit consumers rather than for no-habit consumers. It tells us that individuals having consumption habits prefer a stable state of the labour market. In particular, much smoother labour is generated by a model with habit persistence in utility of consumption and low risk aversion. That is, Model 2 with a log utility for
Table 4.7.2: A summary of macroeconomics statistics of the benchmark and three models
This table reports the cyclical statistics for dynamic stochastic general equilibrium (DSGE) models, a benchmark model without habit formation and three models with internal habit. Model 1 and Model 2 are with additive habit formation (i.e. $C_t - \xi C_{t-1}$, $\xi$ is the parameter for habit persistence) and Model 3 is with non-additive habit formation (i.e. $C_t/(\xi C_{t-1})$). The results are summarised into two sections. Section I includes the relative standard deviation (RSD) of each variable (dividends ($D$), consumption ($C$), output ($Y$), capital ($K$), labour ($L$), investment ($I$), debt($B$), the risk-free rate ($R_f$), the gross risky interest rate ($R$), the share price ($Q$), the benchmark point of bankruptcy ($X$), the marginal product of capital ($MPK$) and the marginal product of labour ($MPL$). Section II reports the relative movement of each variable to output by computing the ratio of each variable’s RSD to the RSD of output.

### Section I: Relative standard deviations (RSD) in percent of economic variables

<table>
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<tr>
<th></th>
<th>$\sigma_D$</th>
<th>$\sigma_C$</th>
<th>$\sigma_Y$</th>
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<th>$\sigma_L$</th>
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<tr>
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<td>0.99</td>
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<td>0.21</td>
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<td>0.21</td>
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<tr>
<td>Model 2</td>
<td>3.56</td>
<td>0.18</td>
<td>0.81</td>
<td>0.21</td>
<td>0.31</td>
<td>3.19</td>
<td>0.21</td>
</tr>
<tr>
<td>Model 3</td>
<td>4.99</td>
<td>0.15</td>
<td>1.06</td>
<td>0.30</td>
<td>0.71</td>
<td>4.40</td>
<td>0.30</td>
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<table>
<thead>
<tr>
<th></th>
<th>$\sigma_{Rf}$</th>
<th>$\sigma_R$</th>
<th>$\sigma_Q$</th>
<th>$\sigma_X$</th>
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<td>0.03</td>
<td>0.30</td>
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<td>Model 1</td>
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<td>0.02</td>
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<td>0.85</td>
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<td>0.02</td>
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<td>0.82</td>
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### Section II: Relative standard deviations (RSD) of each economic variable to RSD of output

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<th>$\frac{\sigma_C}{\sigma_Y}$</th>
<th>$\frac{\sigma_Y}{\sigma_Y}$</th>
<th>$\frac{\sigma_K}{\sigma_Y}$</th>
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<tr>
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<td>Model 1</td>
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<td>0.25</td>
<td>0.43</td>
<td>3.83</td>
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<td>0.39</td>
<td>3.94</td>
<td>0.26</td>
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<tr>
<td>Model 3</td>
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<td>1.00</td>
<td>0.28</td>
<td>0.67</td>
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<table>
<thead>
<tr>
<th></th>
<th>$\frac{\sigma_{Rf}}{\sigma_Y}$</th>
<th>$\frac{\sigma_R}{\sigma_Y}$</th>
<th>$\frac{\sigma_Q}{\sigma_Y}$</th>
<th>$\frac{\sigma_X}{\sigma_Y}$</th>
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<tr>
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<td>0.03</td>
<td>0.03</td>
<td>0.24</td>
<td>0.03</td>
<td>1.02</td>
<td>0.25</td>
</tr>
<tr>
<td>Model 1</td>
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<td>1.01</td>
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</tr>
<tr>
<td>Model 2</td>
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<td>Model 3</td>
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<td>0.03</td>
<td>0.28</td>
<td>0.03</td>
<td>1.03</td>
<td>0.37</td>
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</table>
consumption and habit in consumption develops a low standard deviation of labour.

My next findings are about the analysis of the relative strength of the volatility of aggregate dividends and other main variables to the volatility of output. (Table 4.7.2, Section II).

Aggregate dividends and investment have the highest relative volatility to output volatility when individuals have habits in consumption in the economy. The volatility of dividends is about three and half times that of output in the benchmark, while it is nearly four times that of output in models with habit formation. The standard deviation of investment is roughly four times that of output in cases with habit formation. That investment fluctuates much more than output is more obvious in with-habit models.

The magnitude of fluctuations in hours worked to output is 79% in the benchmark. In with-habit models, it becomes much lower. It shows the labour is much smoother than the output in models with habit formation.

The standard deviation of debt is about a quarter of that of the output in the benchmark. With habit formation, debt becomes slightly more volatile. This is because initially debt would absorb the short-term fluctuations of investment and now, with habit formation, the volatility of investment becomes greater which causes debt to be more volatile rather than in the model without habit formation.

**4.7.3 Co-movement**

Table 4.7.3 lists each variable’s correlation with output (first section), dividends (second section), consumption (third section) and debt (fourth section) respectively with and without habit formation.

The results show that dividends are countercyclical. Aggregate dividend policy is strongly and negatively correlated with output when habit persistence exists in each model. Intuitively, we would think that the higher the output, the more cash available for dividend payments. That is, aggregate dividend policy should be procyclical to business cycles (see Section 2.2.8 of Chapter 2). This is not the case, however, as corporations would prefer to hold more cash flows for potential investment projects when the economy is booming. The higher the output, the lower the proportion of
Table 4.7.3: Matrix correlations

This table reports a summary of correlations between dividends ($D$), consumption ($C$), output ($Y$), capital ($K$), labour ($L$), investment ($I$), debt ($B$), technology shock ($Z$), the risk-free rate ($Rf$), the gross risky interest rate ($R$), the share price ($Q$), the benchmark point of bankruptcy ($X$), the marginal product of capital ($MPK$) and the marginal product of labour ($MPL$) for four DSGE models. The first model is a benchmark model without habit formation and the other three models (named Model 1, 2 and 3) are with different internal habit formation. Model 1 and Model 2 are with additive habit formation (i.e. $C_t - \xi C_{t-1}$, $\xi$ is the parameter for habit persistence) and Model 3 is with non-additive habit formation (i.e. $C_t / (\xi C_{t-1})$). The simulated results are grouped into four sections. Section I includes the cross-correlation of output with others. Section II includes the cross-correlation of dividends with others. Section III reports the cross-correlation of consumption with other variables. The final section includes the cross-correlation of debt with other variables.

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<tr>
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</tr>
</tbody>
</table>
4.7. RESULTS AND DISCUSSION (I)

cash flows left for dividend payments. Therefore, the correlation between aggregate dividend policy and output is negative.

The correlation of dividends and consumption, \( \text{corr}(D, C) \), becomes less negative in models with habit formation. The correlation is -0.67 in no-habit model, while it is -0.30 in model 1, -0.23 in Model 2 and a nearly zero correlation in Model 3 (-0.06). It is expected that dividends and consumption have a less strong relationship. Models with habit persistence generate smooth consumption while dividends still fluctuate greatly over periods. As a result, the paths of dividends and consumption deviate greatly from each other. The correlation between them is less strong. I also find that aggregate labour has a strong negative correlation with aggregate dividends. Investors with habit formation maximise consumption and labour and intend to have smooth consumption over periods. The counter-correlation between dividend income and labour income can help them to smooth consumption, as in the bull market the increase in labour income increases and the decrease in dividend payouts can balance out any changes in the agent’s cash flows.

Consumption is a strong procyclical variable in the case without habit formation, that is, consumption has a positive and significant correlation with output. But, models with habit formation show that consumption has a less systematic relationship to the business cycle. This is consistent with the hypothesis that consumption moves smoothly when consumer’s utility is partly determined by past consumption. Because of the effect of habit formation, the strength of co-movement between consumption and output is less strong.

So far, results tell us that habit persistence makes consumption smooth. Because of that, consumption has a less strong relationship with other macroeconomic variables. The price of equity and the amount of risk-free bonds, however, are exceptions. Models with habit formation show that the price of equity is highly correlated with consumption \( \text{corr}(C, Q) \), around 97% to 99%), rather than in the model without habits (the figure of \( \text{corr}(C, Q) \) is 80%). And the correlation between consumption and debt is 97% in with-habit models and 67% in the no-habit model. I successfully present models which can show that equity and bond markets are more sensitive to consumers’ habit motive.
If there is no habit formation, the distribution of the household's revenues is independent from one period to another. Because of the effect of consumption habit, households like to have smooth consumption rather than having volatile consuming behaviour. In a booming economy, there is more cash going to equity and bond markets. In a recession, there is less money for investing in equities and bonds. Therefore, the price of equity and the amount of bonds are more sensitive and have greater correlation to consumption when individuals have habit motives.

### 4.7.4 Impulse responses to a technology shock

Figures 4.7.1, 4.7.2, 4.7.3 and 4.7.4 display results of impulse responses of macroeconomic variables to the stochastic technology shock for four models (one is without habit formation (the benchmark) and three models are with habit formation) respectively. These graphs show explicitly the differences of each real variable's reaction to a technology shock in DSGE models with multiplicative (Model 1) and additively separable momentary utility functions (Model 2 and Model 3) individually.

Comparing figures of Model 1 and 2, they illustrate the reaction of both dividends and consumptions to a positive technology shock. Model 1, with multiplicative utility function, demonstrates a larger deviation of dividends from the steady state in the early years after the shock than in Model 2. Consumption in Model 1 exhibits a big jump in the first ten years after the shock.

The additively separable utility function (Model 2) reduces the reaction of real variables, particularly productivity inputs, to a positive technology shock. Graphs of output, capital, labour, investment, debt and equity price in Model 2 are comparatively less volatile than in Model 1.

Another effect of non-additive habit formation in DSGE models is that consumption and aggregate dividends tend to respond to the technology shock passively. Figure 4.7.4 displays the impulse response function of consumption, dividends, investment and debt to the stochastic technology shock. In the early years, consumption deviates slightly from the steady state; the figure is approximately $0.25 \times 10^{-5}$. Aggregate dividends also show small changes from the steady state ($-2.5 \times 10^{-5}$) in the following years when the
Figure 4.7.1: Impulse responses to a technology shock for the benchmark
Figure 4.7.2: Impulse responses to a technology shock for Model 1
Figure 4.7.3: Impulse responses to a technology shock for Model 2
Figure 4.7.4: Impulse responses to a technology shock for Model 3
shock hits the market.

Overall, this section demonstrates DSGE models with habit formation in three cases: multiplicative utility functions with separable consumption habit formation, additively separable utility function with separable consumption habit formation, and additively separable utility function with the ratio of habit formation.

The results emphasise that a multiplicative utility function provides a strong effect on dividend policy in the general equilibrium process (see figures of Model 1, Figure 4.7.2). Aggregate dividends respond heavily to the technology shock in the first ten years. In contrast, dividends, output, investment and consumption in the additively separable utility function are less volatile. With this type of utility function, most real variables have less ability to respond the stochastic technology shock. The reasons is that the parameter of risk aversion is set equal to one, which means the household in Model 2 is less risk averse than in Model 1 because the coefficient of risk aversion is higher than one in Model 1 (implying a lower elasticity of intertemporal substitution in consumption). Model 2, by contrast, has investors who are less risk averse. That causes most real variables to respond less severely to the technology shock. Model 3, a DSGE model with a ratio of habit formation, causes consumption to be less volatile and dividends to fluctuate greatly.

The purpose of my next study intends to demonstrate that market-wide dividend policy plays an important role in helping investors to smooth consumption. I take an aggregated approach to corporate payouts, which contrasts with the more usual firm-level analysis that is standard in the corporate finance literature. Using a macroeconomic general equilibrium approach, the results show that dividend volatility increases, while consumption volatility decreases, as the consumption smoothing motive of investors gets stronger. This is consistent with the hypothesis that investors use the cash pay- outs from well diversified portfolios to help smooth consumption.
4.8 Calibration (II)

Consistent with the hypothesis, the expectation is that the smoother the consumption, the more volatile the aggregate dividends. All the models generate a low variability of the consumption growth rate through habit persistence in the utility of consumption. In particular, the non-additive habit formation brings much smoother consumption and the volatility of aggregate dividends is lowest among three habit-models. I therefore use the previous section’s Model 1 as this section’s model framework. The major and only change is the value for habit persistence. In this section, the habit persistence level is given from 0 to 0.9 with a range of 10%. Other parameter values remain the same and are reported in Table 4.6.1\(^\text{10}\).

The simulated results will be compared to the observed moments of the US economy. Details of data description have been provided in Chapter 3, Section 3.5.

4.9 Results and discussion (II)

The main finding of this calibration is that aggregate payout volatility increases while consumption volatility declines as the consumption habit motive of investors increases. Optimal aggregate payout policy is therefore shown to be sensitive to agents’ habit persistence. Moreover, dividends keep a strong negative correlation with most real variables (output, labour and investment) at each level of habit motive. Habit formation is therefore an important link between consumption smoothing and optimal corporate finance policy.

4.9.1 Variability of aggregate dividends and consumption smoothing

Table 4.9.1 gives a summary of relative standard deviations (RSD) of each macroeconomic variable. \(\xi\) indicates investors’ habit motive; the greater \(\xi\), the higher the

\(^{10}\text{Note that: ignore parameters of disutility of labour and the coefficient of relative risk aversion for labour as they are not applicable in this section, they are parameters for Model 2}
Table 4.9.1: Relative standard deviation (RSD) in percent for the equilibrium model and US data

This table reports relative standard deviations of aggregate dividends ($D$), consumption ($C$), labour ($L$), output ($Y$), capital ($K$), investment ($I$) and debt ($B$) of the extended equilibrium model. This model is the Model 1 introduced in calibration I. The cyclical statistics are grouped into three by giving different values of habit persistence in consumption ($\xi$), 0, 0.5 and 0.9 (the larger the value, the stronger the habit persistence). The data source for capital, consumption, labour and investment is from Hansen (1985). They are quarterly data from 1955.3-1984.1 detrended with a Hodrick-Prescott (HP) filter. The US quarterly data source for dividends and debt is Alessandrini (2003). Alessandrini (2003) gets the quarterly data from 1955-2001 from US National Income and Product Accounts (NIPA) statistics for dividends and from the US Flow of Funds statistics for debt. The multiplier of the HP filter aims to adjust the sensitivity of short-term fluctuations. Hodrick and Prescott suggest to set the value of the multiplier as 1600 for quarterly data. All data are detrended by the HP filter and are quoted in percent.

<table>
<thead>
<tr>
<th></th>
<th>Extended equilibrium model</th>
<th>US data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\xi = 0$</td>
<td>$\xi = 0.5$</td>
</tr>
<tr>
<td>$\sigma_D$</td>
<td>2.42</td>
<td>2.92</td>
</tr>
<tr>
<td>$\sigma_C$</td>
<td>0.30</td>
<td>0.27</td>
</tr>
<tr>
<td>$\sigma_L$</td>
<td>0.38</td>
<td>0.38</td>
</tr>
<tr>
<td>$\sigma_Y$</td>
<td>0.85</td>
<td>0.85</td>
</tr>
<tr>
<td>$\sigma_K$</td>
<td>0.18</td>
<td>0.19</td>
</tr>
<tr>
<td>$\sigma_I$</td>
<td>2.66</td>
<td>2.90</td>
</tr>
<tr>
<td>$\sigma_B$</td>
<td>0.18</td>
<td>0.19</td>
</tr>
</tbody>
</table>

influence of past consumption on investors’ current utility.

My calibrated results show that the variability of aggregate payouts increases from 2.42% to 3.69% with growing habit persistence while the volatility of aggregate consumption moves down from 0.30% to 0.14% (Table 4.9.1, Rows 1 and 2) as $\xi$ moves from 0 to 0.9. That consumption growth becomes smoother with increased habit motive is consistent with previous research; for example, Constantinides (1990). His study emphasises that a high level of habit persistence, about 80 percent, generates a low variability in consumption growth. Furthermore, a significant finding shows that the lower the variability of consumption the greater the fluctuation of aggregate payouts.

My contribution here is to demonstrate that this is associated with an increase in volatility of the optimal aggregate dividend payout. This is further illustrated by Figure 4.9.1, where the ratio $\sigma_D/\sigma_C$ for values of $\xi \in \{0, ..., 0.9\}$ is plotted.
This figure shows that, in aggregate, the relative standard deviation of dividends to consumption rises dramatically as the habit motive becomes large.

To further understand why a decrease in consumption volatility with increased habit formation is associated with a rise in aggregate dividend volatility, Table 4.9.2 reports the correlations between each of the macroeconomic variables with consumption, dividends and output.

From this, it is clear that, as the habit motive increases, the correlation between consumption and output, $\text{corr}(Y_t, C_t)$, drops dramatically from 0.97 when $\xi = 0$ to 0.37 when $\xi = 0.9$. Again, this is intuitively consistent with an increased desire for consumption smoothing. The question arises as to how investors manage to make their consumption smoother and less dependent on output. In this model, consumption is constructed from labour income, investing in the risk-free asset and dividends.

For all values of $\xi$, labour income and output have a very high correlation (Table 4.9.2). As a consequence, for $C_t$ to become less dependent on $Y_t$ as the habit motive
grows, so the cash flows that an investor generates from financial assets must become increasingly countercyclical to offset the procyclicality of their salaries. In the case of equity income, this is achieved by both the dividend and investment becoming more volatile (Table 4.9.1) as $\xi$ increases. In addition, dividends are strongly negatively correlated to labour, investment and output (Table 4.9.2). Aggregate dividend policy is thus important to investors as it helps agents keep a smoothness of consumption through its countercyclicality.

It should be noted that the highly negative correlation between dividends and output that is a feature of this model contrasts with the positive observed correlation reported by Hansen (1985). However, it should also be noted that “dividends” here are the net equity cash flows from the firm to the investor, which are more accurately interpreted as net free cash flows after the retained earnings and investment. Amdur (2010) calibrates a DSGE model in which the net equity payouts are not estimated under a residual dividend policy. His model therefore predicts a procyclicality of equity payouts and a countercyclicality of debt issues.

Figures 4.9.2 and 4.9.3 illustrate the impulse responses of the expected future path of consumption, aggregate dividends, investment and debt in two cases, without habit formation and with strong habit formation ($\xi = 0.9$). These response functions further indicate that consumption growth is smoother when habit persistence is higher. That is, consumption with high habit motive of $\xi = 0.9$ has lower deviations in the first ten years after a shock (the range is [below 0.0005, 0.002]) than in the case of no-habit motive of $\xi = 0.0$ ([0.002, 0.0025]). The figure further shows that aggregate dividends fluctuate more in the habit formation case than in the no-habit formation case.

### 4.9.2 Variability of other macroeconomic variables

Aggregate investment, $I_t$ has similar characteristics to $D_t$ both with and without habit in terms of its volatility and the absolute levels of its correlations with consumption and income. In particular, the optimal investment path becomes more volatile as the habit motive increases and investment volatility is close to aggregate payout volatility in each of the different habit persistence cases (Table 4.9.1, Row 6). This is, again, what we
4.9. RESULTS AND DISCUSSION (II)

Table 4.9.2: Cross-correlations of real variables

This table reports a summary of correlations between output \((Y)\) with capital \((K)\), consumption \((C)\), labour \((L)\), investment \((I)\), dividends \((D)\) and debt \((B)\) by varying the habit persistence \((\xi)\) parameter value from 0.0 (without habit) to 0.9 (high level of habit persistence) for a DSGE model with internal habit (Model 1 in the calibration I). The results are grouped into three sets. The first group is the cross-correlation of consumption with other variables. The second group includes the cross-correlation of dividends with other variables. The final group shows the observed cross-correlation of gross national product (GNP) with other real variables and theoretical cross-correlation of output with other variables. The data source for capital, consumption, labour and investment is from Hansen (1985). They are quarterly data from 1955.3-1984.1 detrended with a Hodrick-Prescott filter. The US quarterly data source for dividends and debt is Alessandrini (2003). Alessandrini (2003) gets the quarterly data from 1955-2001 from US National Income and Product Accounts (NIPA) statistics for dividends and from the US Flow of Funds statistics for debt. All data are detrended by the Hodrick-Prescott filter (HP filter) and are quoted in percent.

<table>
<thead>
<tr>
<th>(\xi)</th>
<th>(\text{Corr}(., C_t))</th>
<th>(\text{Corr}(., D_t))</th>
<th>(\text{Corr}(., Y_t))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.52, 1.00, 0.97, 0.93, 0.94, -0.91, 0.40</td>
<td>-0.11, -0.91, -0.98, -1.00, -0.99, 1.00, 0.03</td>
<td>0.04, 0.85, 1.00, 0.76, 0.92, 0.34, 0.29</td>
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<tr>
<td>0.1</td>
<td>0.55, 1.00, 0.96, 0.92, 0.93, -0.88, 0.43</td>
<td>-0.10, -0.88, -0.98, -0.99, -0.99, 1.00, 0.04</td>
<td>0.29, 0.97, 1.00, 0.99, 1.00, -0.98, 0.16</td>
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<tr>
<td>0.2</td>
<td>0.59, 1.00, 0.94, 0.89, 0.90, -0.83, 0.48</td>
<td>-0.09, -0.83, -0.97, -0.99, -0.99, 1.00, 0.04</td>
<td>0.30, 0.96, 1.00, 0.99, 1.00, -0.98, 0.17</td>
</tr>
<tr>
<td>0.3</td>
<td>0.64, 1.00, 0.92, 0.87, 0.86, -0.78, 0.53</td>
<td>-0.09, -0.78, -0.96, -0.99, -0.99, 1.00, 0.05</td>
<td>0.31, 0.94, 1.00, 0.99, 0.99, -0.97, 0.18</td>
</tr>
<tr>
<td>0.4</td>
<td>0.69, 1.00, 0.89, 0.83, 0.81, -0.71, 0.59</td>
<td>-0.09, -0.71, -0.96, -0.98, -0.99, 1.00, 0.05</td>
<td>0.32, 0.92, 1.00, 0.99, 0.99, -0.96, 0.19</td>
</tr>
<tr>
<td>0.5</td>
<td>0.75, 1.00, 0.84, 0.78, 0.75, -0.63, 0.66</td>
<td>-0.08, -0.63, -0.95, -0.98, -0.99, 1.00, 0.05</td>
<td>0.34, 0.89, 1.00, 0.99, 0.99, -0.96, 0.21</td>
</tr>
<tr>
<td>0.6</td>
<td>0.81, 1.00, 0.78, 0.70, 0.66, -0.54, 0.74</td>
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<td>0.35, 0.84, 1.00, 0.99, 0.99, -0.95, 0.23</td>
</tr>
<tr>
<td>0.7</td>
<td>0.89, 1.00, 0.70, 0.60, 0.56, -0.43, 0.83</td>
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<td>0.36, 0.78, 1.00, 0.99, 0.98, -0.95, 0.24</td>
</tr>
<tr>
<td>0.8</td>
<td>0.96, 1.00, 0.57, 0.45, 0.42, -0.30, 0.92</td>
<td>-0.09, -0.30, -0.95, -0.98, -0.99, 1.00, 0.03</td>
<td>0.37, 0.70, 1.00, 0.99, 0.98, -0.95, 0.26</td>
</tr>
<tr>
<td>0.9</td>
<td>1.00, 1.00, 0.37, 0.21, 0.25, -0.16, 0.99</td>
<td>-0.12, -0.16, -0.98, -1.00, -1.00, 1.00, -0.01</td>
<td>0.33, 0.37, 1.00, 0.99, 0.99, -0.98, 0.23</td>
</tr>
</tbody>
</table>
4.9. RESULTS AND DISCUSSION (II)

Figure 4.9.2: The figure plots the impulse responses of optimal consumption \( (C) \), aggregate dividends \( (D) \), investment \( (I) \) and debt \( (B) \) in the case of without habit persistence \( (\xi = 0) \).

Figure 4.9.3: The figure plots the impulse responses of optimal consumption \( (C) \), aggregate dividends \( (D) \), investment \( (I) \) and debt \( (B) \) in the case of with habit persistence \( (\xi = 0.9) \).
would expect. As investors’ habit motive gets stronger, the firm must adapt its own
investment plans to ensure that the firm helps maximise the expected utility of the
agent. This results in more volatile investment for the firm but smoother consumption
for the investor as $\xi$ increases. The correlation between investment and consumption
also decreases as the latter is smoothed, which is consistent with the absolute rela-
tionship between dividends and consumption. Investment is nearly perfectly correlated
with output for all values of $\xi$ because of the high persistence of technology change;
$\psi = 0.95$. High output this year signals high future profitability, giving firms a strong
incentive to increase investment levels. In addition, as with dividends, the volatility of
the amount of bonds $B_t$ increases with $\xi$. This suggests that firm financing is also
affected by the utility function of investors.

Labour supply is almost perfectly correlated with output. The explanation for this
is the same as for investment ($I_t$); firm profitability is very highly persistent, so high
output makes the firm want to employ more labour. However, if the agent supplies
more hours when times are good, then the more procyclical their salary becomes. This
then creates difficulties for them to smooth consumption. To counteract this, the
volatility of labour, $\sigma_L$, decreases as the habit motive increases. Again, as expected,
the correlation between labour and consumption decreases as the habit motive gets
stronger.

In this general equilibrium setting, the habit motive of investors is shown to have
a significant influence not only on the investors’ labour and consumption decisions but
also on the investment, financing and payout policy of the firm. This suggests that
there is a significant interaction between corporate policy and investor utility.

4.9.3 Sensitivity to various levels of gamma and habit per-
sistence

This section studies the cross-sensitivity of the coefficient of risk aversion ($\gamma$) and habit
persistence ($\xi$) to the aggregate payout volatility and others. From this research, I find
a relationship between the aggregate dividend policy with investors’ risk aversion and
Dividend policy, in an aggregated market, becomes more volatile when investors are willing to take more risk on their revenues. This hypothesis has been supported by my model (Table 4.9.3). At each level of habit persistence, the greater the elasticity of intertemporal substitution for consumption, the higher the variability of aggregate dividend policy. Moreover, until habit motive is equal to 0.8, the higher the habit motives investors hold, the greater the volatility of aggregate payouts, no matter what the level of risk aversion. Further result shows that aggregate dividends are less volatile when habit motive is set close to one and $\gamma = 0.5$. Little existing work has discussed this issue. Exploring this relationship is left for future research.

I have followed Lettau and Uhlig (2000) and find that a low variability of consumption growth is generated through a stochastic general equilibrium model with a certain high level of habit persistence (about 80 percent) and low risk aversion (Table 4.9.3, Section $\sigma_C$).

As to movements of other real macroeconomic variables, the results demonstrate that output, investment, capital and labour are less volatile when investors are more risk averse at every level of habit persistence.

Overall, the sensitivity of business cycles to the coefficient of risk aversion implies that habit persistence helps consumers smooth their consumption, and so does the factor of risk aversion. The results show that both factors of habit persistence and risk aversion can affect the variability of aggregate dividend policy. Under a value of 0.8 of habit persistence, I find that an economy has the most volatile aggregate payouts if investors are less risk averse and willing to use cash payouts to help smooth consumption.
Table 4.9.3: Sensitivity of real variables to various gamma and habit persistence for Model 1

This table reports a summary of the relative standard deviations of dividends \((D)\), consumption \((C)\), labour \((L)\), output \((Y)\), capital \((K)\), investment \((I)\), and debt \((B)\) for a range of habit persistence in consumption \((\xi)\) and four levels of coefficient of relative risk aversion. The results are obtained by simulating a dynamic stochastic general equilibrium (DSGE) model (Model 1, introduced in calibration I). This table shows the sensitivity of each variable's movement in various habit persistence parameter values from 0.0 (without habit) to 0.9 (high level of habit persistence) in four categories of 0.5, 1, 2 and 10 for the coefficient of risk aversion \((\gamma)\). The greater the coefficient of relative risk aversion, the stronger the individual's risk aversion level.

<table>
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<tr>
<th></th>
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<th>0.0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
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<td>(\sigma_D)</td>
<td>(\gamma = 0.5)</td>
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<td>4.12</td>
<td>4.14</td>
<td>4.16</td>
<td>4.19</td>
<td>4.23</td>
<td>4.26</td>
<td>4.30</td>
<td>4.31</td>
<td>4.16</td>
</tr>
<tr>
<td></td>
<td>(\gamma = 1)</td>
<td>3.18</td>
<td>3.22</td>
<td>3.27</td>
<td>3.33</td>
<td>3.41</td>
<td>3.50</td>
<td>3.61</td>
<td>3.75</td>
<td>3.88</td>
<td>3.90</td>
</tr>
<tr>
<td></td>
<td>(\gamma = 2)</td>
<td>2.42</td>
<td>2.49</td>
<td>2.57</td>
<td>2.66</td>
<td>2.78</td>
<td>2.92</td>
<td>3.09</td>
<td>3.30</td>
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<tr>
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<td>1.82</td>
<td>1.95</td>
<td>2.12</td>
<td>2.31</td>
<td>2.55</td>
<td>2.84</td>
<td>3.19</td>
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<td>0.21</td>
<td>0.21</td>
<td>0.21</td>
<td>0.21</td>
<td>0.20</td>
<td>0.20</td>
<td>0.19</td>
<td>0.17</td>
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</tr>
<tr>
<td></td>
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<td>0.25</td>
<td>0.24</td>
<td>0.24</td>
<td>0.24</td>
<td>0.24</td>
<td>0.23</td>
<td>0.22</td>
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<td>0.13</td>
</tr>
<tr>
<td></td>
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<td>0.30</td>
<td>0.29</td>
<td>0.28</td>
<td>0.28</td>
<td>0.27</td>
<td>0.26</td>
<td>0.24</td>
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<tr>
<td></td>
<td>(\gamma = 10)</td>
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<td>0.36</td>
<td>0.35</td>
<td>0.34</td>
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<td>0.58</td>
<td>0.57</td>
<td>0.57</td>
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</tr>
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<td>0.38</td>
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</tr>
<tr>
<td></td>
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<td>0.28</td>
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<tr>
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<td>0.98</td>
<td>0.98</td>
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<tr>
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<td>(\gamma = 10)</td>
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<td>(\sigma_K)</td>
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<td>0.18</td>
<td>0.18</td>
<td>0.19</td>
<td>0.19</td>
<td>0.20</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>(\gamma = 10)</td>
<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
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<td>0.15</td>
<td>0.16</td>
<td>0.17</td>
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</tr>
<tr>
<td>(\sigma_I)</td>
<td>(\gamma = 0.5)</td>
<td>3.77</td>
<td>3.78</td>
<td>3.78</td>
<td>3.79</td>
<td>3.80</td>
<td>3.81</td>
<td>3.82</td>
<td>3.83</td>
<td>3.80</td>
<td>3.64</td>
</tr>
<tr>
<td></td>
<td>(\gamma = 1)</td>
<td>3.16</td>
<td>3.18</td>
<td>3.20</td>
<td>3.23</td>
<td>3.27</td>
<td>3.31</td>
<td>3.36</td>
<td>3.42</td>
<td>3.48</td>
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<tr>
<td></td>
<td>(\gamma = 2)</td>
<td>2.66</td>
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<td>2.72</td>
<td>2.77</td>
<td>2.83</td>
<td>2.90</td>
<td>2.98</td>
<td>3.09</td>
<td>3.21</td>
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</tr>
<tr>
<td></td>
<td>(\gamma = 10)</td>
<td>2.12</td>
<td>2.16</td>
<td>2.22</td>
<td>2.28</td>
<td>2.36</td>
<td>2.46</td>
<td>2.59</td>
<td>2.75</td>
<td>2.94</td>
<td>3.12</td>
</tr>
<tr>
<td>(\sigma_B)</td>
<td>(\gamma = 0.5)</td>
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<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
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</tr>
<tr>
<td></td>
<td>(\gamma = 1)</td>
<td>0.21</td>
<td>0.21</td>
<td>0.21</td>
<td>0.21</td>
<td>0.21</td>
<td>0.22</td>
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<td>0.22</td>
<td>0.22</td>
<td>0.23</td>
</tr>
<tr>
<td></td>
<td>(\gamma = 2)</td>
<td>0.18</td>
<td>0.18</td>
<td>0.18</td>
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<td>0.20</td>
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</tr>
<tr>
<td></td>
<td>(\gamma = 10)</td>
<td>0.15</td>
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<td>0.16</td>
<td>0.17</td>
<td>0.18</td>
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<td>0.21</td>
</tr>
</tbody>
</table>
4.10 Summary

In this chapter, I studied the relationship between aggregate dividend policy and investor utility; an issue that previously has rarely been discussed in the standard corporate finance and macroeconomics literature.

From the first calibration, I showed that separable habit formation in both multiplicative and additively separable utility functions generates similar results for movements of aggregate dividends. Another result from my models was that aggregate dividend policy is countercyclical to business cycles. Corporations would prefer to leave earnings for potential positive NPV investment plans rather than pay dividends, particularly when the economy is booming. Thus, the correlation between aggregate dividends and output is negative.

The second calibration exploited the fact that investors’ desire to smooth consumption across time is directly linked to their internal habit motive. Therefore, by changing the impact that previous consumption has on current utility, I examined the relationship between optimal corporate finance activity and investor welfare.

As expected, the results showed that, as the habit motive becomes stronger, consumption growth becomes smoother. My primary contribution was to demonstrate that dividends play an important role in helping to isolate investors from the volatility of the business cycle. In particular, I showed that the optimal dividend payout policy becomes more volatile and stays negatively correlated with output as the habit motive increases. This volatile and counter-cyclical stream of financial income helps counterbalance the highly procyclical nature of labour income in this economy.

I further found that the form of investors’ utility impacts upon a number of other aspects of financial activity. In particular, I found that as the habit motive increases, the volatility of real investment rises, as does the volatility of debt financing. Agents will also change their labour availability as their consumption habits change.
4.11 Appendices

4.11.1 Procedure for solving DSGE models with habit formation

**Step 1: Solving the firm’s optimisation problem**

The optimisation function of the firm’s cash flow is:

\[
\max_{L_t^d, K_t, B_t} D_t + E_t \left[ \sum_{h=1}^{\infty} M_{t,h} D_{t+h} \right]
\]  

subject to

\[
D_t = (1-\tau) \left[ K_{t-1}^\alpha (Z_t L_t^d)^{1-\alpha} - W_t L_t^d - R_{t-1} B_{t-1} \right] - \tau B_{t-1} + B_t - K_t + (1-\eta) K_{t-1}
\]  

(4.11.2)

Setting the partial derivative of the objective function with respect to \( L = 0 \), \( K = 0 \), \( B = 0 \) reveals the following three constraints for the firm:

\[
W_t = (1-\alpha) K_{t-1}^\alpha (Z_t L_t^d)^{1-\alpha} L_t^{-\alpha} = (1-\alpha) \frac{Y_t}{L_t}
\]  

(4.11.3)

\[
1 = E_t \left\{ M_{t,1} \left[ [(1-\tau)\alpha K_{t+1}^{\alpha-1} (Z_{t+1} L_{t+1})^{1-\alpha}] + 1 - \eta \right] \right\}
\]  

(4.11.4)

\[
1 = E_t \left\{ M_{t,1} [(1-\tau) R_t + \tau] \right\}
\]  

(4.11.5)

the above equations 4.11.3, 4.11.4 and 4.11.5 are the firm’s optimal conditions.

**Step 2: Solving the household’s optimisation problem**

To solve the household’s problem, it starts form the Lagrangian of the household’s
optimisation problem based on page 47:

\[
\mathcal{L} = E_t \left\{ \sum_{h=0}^{\infty} \beta^h \left[ \right. U(C_{t+h}, C_{t+h-1}, 1 - L_{t+h}^*) - \lambda_{t+h}(C_{t+h} + Q_{t+h}N_{t+h+1} \\
+ B_{t+h}^H - W_{t+h}L_{t+h}^* - Q_{t+h}N_{t+h} - D_{t+h}N_{t+h} - R_{f(t+h-1)}B_{t+h-1}^H) \left. \right] \right\}
\]

(4.11.6)

where \(\lambda_{t+h}\) is a Lagrangian multiplier. I solve the optimisation problem by computing partial derivatives of the Lagrangian with respect to each of the control variables and setting each equal to zero. The following equations are yielded according to \(\partial \mathcal{L}/\partial C_t = 0\), \(\partial \mathcal{L}/\partial L_t^* = 0\), \(\partial \mathcal{L}/\partial N_t = 0\) and \(\partial \mathcal{L}/\partial B_t^H = 0\) respectively:

\[
\lambda_t = \frac{\partial U(C_t, C_{t-1}, 1 - L_t^*)}{\partial C_t} + \beta \frac{\partial U(C_{t+1}, C_t, 1 - L_{t+1}^*)}{\partial C_t} \quad (4.11.7)
\]

\[
\frac{\partial U(C_t, C_{t-1}, 1 - L_t^*)}{\partial L_t^*} + \lambda_t W_t = 0 \quad (4.11.8)
\]

\[
Q_t = \beta E_t \left[ \left( \frac{\lambda_{t+1}}{\lambda_t} \right) (Q_{t+1} + D_{t+1}) \right] \quad (4.11.9)
\]

\[
\lambda_t = \beta E_t [\lambda_{t+1} R_{f,t}] \quad (4.11.10)
\]

The above four equations can be summarised into three major closed forms for the household’s optimisation. First, by substituting \(\lambda_t\) from equation 4.11.7 to equation 4.11.8:

\[
W_t = - \frac{\partial U(C_t, C_{t-1}, 1 - L_t^*)}{\partial L_t^*} \left[ \frac{\partial L_t^*}{\partial C_t} + \beta \frac{\partial U(C_{t+1}, C_t, 1 - L_{t+1}^*)}{\partial C_t} \right] \quad (4.11.11)
\]

Second, the standard Euler equation follows directly from equation 4.11.10:

\[
1 = \beta E_t \left[ \left( \frac{\lambda_{t+1}}{\lambda_t} \right) R_{f,t} \right] \quad (4.11.12)
\]

This defines the intertemporal marginal rate of substitution (IMRS) with respect
4.11 APPENDICES

To consumption

\[ M_{t,h} = \beta^h \frac{\lambda_{t+h}}{\lambda_t} = \beta^h \left( \frac{\partial U(C_{t+h}, C_{t+h-1}, 1 - L_{t+h}^s)}{\partial C_{t+h}} \frac{\partial U(C_{t+h}, C_{t+h}, 1 - L_{t+h+1}^s)}{\partial C_{t+h}} \right) + \beta \left( \frac{\partial U(C_{t+h+1}, C_{t+h}, 1 - L_{t+h+1}^s)}{\partial C_{t}} \right) \]  

(4.11.13)

To rewrite equation 4.11.9

\[ Q_t = E_t [M_{t,1}(Q_{t+1} + D_{t+1})] \]  

(4.11.14)

Equations 4.11.11, 4.11.12, 4.11.13 and 4.11.14 are the household’s optimal conditions.

**Step 3:** Equilibrium conditions from firm and household

To merge the firm and the household’s optimal conditions for a general equilibrium, we can obtain five equilibrium conditions:

\[ (1 - \alpha) \frac{Y_t}{L_t} = -\frac{\partial U(C_t, C_{t-1}, 1 - L_t^s)}{\partial C_t} \frac{\partial L_t^s}{\partial C_t} + \beta \frac{\partial U(C_{t+1}, C_t, 1 - L_{t+1}^s)}{\partial C_t} \]  

(4.11.15)

\[ 1 = E_t [M_{t,1}R_{f,t}] \]  

(4.11.16)

\[ R_{ft} = E_t [(1 - \tau)\alpha K_t^{\alpha-1}(Z_{t+1}L_{t+1})^{1-\alpha} + 1 - \eta] \]  

(4.11.17)

\[ R_t = \left( \frac{1}{1 - \tau} \right) [R_{f,t} - \tau] \]  

(4.11.18)

\[ Q_t = E_t [M_{t,1}(Q_{t+1} + D_{t+1})] \]  

(4.11.19)

where \( Y_t \) is determined by the Cobb-Douglas production function. \( U(.) \) is the utility function. This chapter adopts three types of habit formation utility functions. \( M_{t,1} \) is identified from the representative household’s optimisation (equation 4.11.13).

**Step 4:** Market clearing conditions
Finally, by calibrating equations 4.11.15, 4.11.16, 4.11.17, 4.11.18, 4.11.19 from step 3 with aforementioned (in chapter 3) two conditions in the banking sector (3.3.12 and 3.3.14) together with the market clearing conditions (equations 3.4.10, 3.4.11, 3.4.12, 3.4.15 and 3.4.16), an expected dynamic stochastic general equilibrium can be obtained with respect to the technology shock $Z$ which is assumed to follow an AR(1) process.

### 4.11.2 Steady-state growth

Table 4.7.1 shows that the steady state output-capital, consumption-output and consumption-capital ratios are constant. The information implies that variables of technology, capital, output and consumption are growing at a constant rate. For instance, the ratios of

\[ \frac{K_{t}}{K_{t-1}}, \frac{Z_{t+1}}{Z_{t}}, \frac{Y_{t+1}}{Y_{t}}, \text{ and } \frac{C_{t+1}}{C_{t}} \]

are the same as the constant growth rate of $G$.

I examine the steady-state growth rate in which technology, capital, output and consumption all grow at a constant rate ($G$) by an analytical approach. The capital accumulation function is

\[
K_{t} = (1 - \eta)K_{t-1} + I_{t}
\]

\[
\frac{K_{t}}{K_{t-1}} = 1 - \eta + \frac{I_{t}}{K_{t-1}}
\]

where $I = I^{a}$ when it is valued in aggregate level and given that $I^{a} = Y^{a} - C^{a}$. The steady state condition of the above equation is given

\[
G = 1 - \eta + \frac{Y^{a}}{K} - \frac{C^{a}}{K}
\]

Next I substitute in the parameter value of $\eta$ and the results shown in Table 4.7.1, the growth rate is

\[
G = 1 - 0.019 + 0.0885 - 0.0678 = 1.0017
\]
The growth rate can be computed in log form as

\[ g = \ln(G) = \ln\left(\frac{K_t}{K_{t-1}}\right) \approx 0.002 \]

where the value is a quarterly value, the annual rate is 0.8%. Details about the steady-state growth can be referred from Campbell (1994).

### 4.11.3 The nature of multiplicative utility function with internal habits

This section follows the study of Seckin (2001) to investigate the nature of the multiplicative utility function with habits.

#### 4.11.3.1 Intratemporal marginal rate of substitution

By using an algorithm, the intratemporal marginal rate of substitution \( (MRS_{\text{intra}}) \) between current consumption and current leisure with internal habits can be defined.

The general momentary utility function includes three variables that control the household’s optimal preferences, current leisure, current and past consumption. Previously, Model 1 showed that the representative household has a multiplicative utility function with internal habits given a form of

\[
U(C_t, C_{t-1}, 1 - L_t) = \frac{[(C_t - \xi C_{t-1})^\rho (1 - L_t)^{1-\rho}]^{1-\gamma}}{1 - \gamma} \quad (4.11.20)
\]

where \( \gamma, \xi \) and \( \rho \) are parameters and between zero and one. \( \gamma \) is the coefficient of relative risk aversion, \( \xi \) determines habit persistence in consumption, and \( \rho \) measures the proportion of time devoted to market activities.

Model 1 with habit formation displays a decrease in the intratemporal marginal rate of substitution between consumption and leisure \( (MRS_{\text{intra}}) \). To calculate \( MRS_{\text{intra}} \), we first calculate the marginal utility of leisure \( (1 - L_t) \) which is

\[
MU_{(1-L_t)} = (1 - \rho)[(C_t - \xi C_{t-1})^\rho (1 - L_t)^{1-\rho}]^{-\gamma}[(C_t - \xi C_{t-1})^\rho (1 - L_t)^{-\rho}]^{-\gamma} \quad (4.11.21)
\]
and the net marginal utility of consumption:

\[ MU_{C_t} = \rho[(C_t - \xi C_{t-1})^\rho(1 - L_t)^{1-\rho}]^{1-\rho} - \rho\xi\beta[(C_{t+1} - \xi C_t)^\rho(1 - L_{t+1})^{1-\rho}]^{1-\rho} \]

(4.11.22)

The \( MRS^{\text{intra}} \) in (\( \xi > 0 \)) is obtained by

\[
MRS^{\text{intra}}_{|\xi>0} = \frac{MU_{(1-L_t)}}{MU_{C_t}} = \frac{(1-\gamma)^{1-\rho}}{C_{t-\xi C_{t-1}}^{1-\rho}} U(C_t, C_{t-1}, 1 - L_t) - \frac{\rho\xi\beta(1-\gamma)^{1-\rho}}{C_{t+1-\xi C_{t+1}}^{1-\rho}} E_t U(C_{t+1}, C_t, 1 - L_{t+1})
\]

\[
= \left(\frac{1-\rho}{\rho}\right) \left[ \frac{C_t}{1 - L_t} - \frac{\xi C_{t-1}}{1 - L_t} \right] \left[ \frac{1}{1 - \xi \beta(C_{t+1-\xi C_{t+1}}^{1-\rho} E_t U(C_{t+1}, C_t, 1 - L_{t+1}))} \right]
\]

\[
= \left[ MRS^{\text{intra}}_{|\xi=0} - \left(\frac{1-\rho}{\rho}\right) \left(\frac{\xi C_{t-1}}{1 - L_t} \right) \right] \ast \left[ \frac{1}{1 - \xi \beta(C_{t+1-\xi C_{t+1}}^{1-\rho} E_t U(C_{t+1}, C_t, 1 - L_{t+1}))} \right]
\]

(4.11.23)

On the right-hand side of the equation, the first is positive and the second is assumed to be greater than one. This equation implies that the individual investor with habits has stronger marginal rate of substitution between current leisure and current consumption. The elasticity of substitution is therefore smaller than the case without habits. The investor is less willing to alter his/her preferences for leisure and consumption units at the current time.

In a special case of the value of \( \gamma = 1 \), the utility function turns to be the one in model 2 which is

\[ U(C_t, C_{t-1}, 1 - L_t) = \log(C_t - \xi C_{t-1}) + A \frac{(1 - L_t)^{1-\gamma_L}}{1 - \gamma_L} \]

The \( MRS^{\text{intra}}_{|\xi>0} \) is calculated as
\[ MRS_{\xi>0}^{\text{intra}} = \frac{MU(1-L_t)}{MU_C} \]
\[ = MRS_{\xi=0}^{\text{intra}} \ast \left[ \frac{1}{C_t(\frac{1}{C_t} - \xi)} - \xi \frac{1}{C_{t+1} - \xi} \right] \] (4.11.24)

Suppose the habit persistence (\( \xi \)) is increasing, the higher the \( \xi \), the greater the second term of equation 4.11.24. It indicates that \( MRS_{\xi>0}^{\text{intra}} \) is greater than \( MRS_{\xi=0}^{\text{intra}} \). The message is that the intratemporal elasticity of substitution between consumption and leisure is low when the household has a strong habit motive. The household would take more leisure and prefer to consume less consumption.

### 4.11.3.2 Intertemporal marginal rate of substitution

To further study the intertemporal marginal rate of substitution (\( MRS_{\xi>0}^{\text{inter}} \)) between current and future leisure, \( MRS_{\xi>0}^{\text{inter}} \) can be analysed by using an algorithm approach:

\[ MRS_{\xi>0}^{\text{inter}} = \frac{MU(1-L_t)}{MU(1-L_{t+1})} \]
\[ = \frac{(1-\rho)(C_t - \xi C_{t-1})\rho(1-L_t)^{1-\rho} - \gamma)(C_t - \xi C_{t-1})\rho(1-L_t)^{1-\rho}}{(1-\rho)(C_{t+1} - \xi C_t)^\rho(1-L_{t+1})^{1-\rho} - \gamma)(C_{t+1} - \xi C_t)^\rho(1-L_{t+1})^{1-\rho}} \]
\[ = \frac{C_t - \xi C_{t-1}^{\rho(1-\gamma)}}{C_{t+1} - \xi C_t^{\rho(1-\gamma)}} \ast \left[ \frac{1}{1-L_{t+1}} \right]^{(1-\rho)(-\gamma)^{1-\rho}} \]
\[ = MRS_{\xi=0}^{\text{inter}} \ast \left[ \frac{1-\xi C_{t-1}^{\rho(1-\gamma)}}{C_{t+1} - \xi C_t^{\rho(1-\gamma)}} \right] \]
\[ = MRS_{\xi=0}^{\text{inter}} \ast \left[ \frac{C_t - \xi C_{t-1}^{\rho(1-\gamma)}}{C_{t+1} - \xi C_t^{\rho(1-\gamma)}} \right] \]

Suppose that the ratio of \( \frac{C_{t+1}}{C_t} \) is positive and the ratio of \( \frac{C_{t+1} - \xi C_t}{C_{t+1} - \xi C_t} \) is smaller than one but positive, we could then analyse the elasticity of current and future leisure.

1. If the household is risk averse, the value of \( \gamma > 1 \). Thus \( MRS_{\xi>0}^{\text{inter}} \) is greater than \( MRS_{\xi=0}^{\text{inter}} \) when the habit motive is increasing. This provides an important message that the elasticity between consuming leisure today and the future is
smaller in the case of habit formation than in the case of without habits. When the habit persistence gets stronger (\( \xi \uparrow \)), the value of \( MRS_{\xi>0} \) is bigger. The household is less willing to take more leisure in the future, and prefers taking more of leisure today. Note that, this finding is restricted on this multiplicative utility function.

2. It is presumed that the household is not risk neutral. This is because the multiplicative utility function will transfer to an additively separable utility function of consumption and leisure when \( \gamma \) is equal to one. Then the utility function is the one used in model 2. Under an additively separable utility function, the consumption habit persistence does not influence the utility of leisure. Therefore, the intertemporal elasticity of substitution between current and future leisure remains the same in both cases of with and without habits.

4.11.4 DYNARE code for the Log-LD DSGE model with effective labour (benchmark)

% Economic growth: labor augmenting technical change (effective labor units)
% LOG-LD
% no habits
% similar to file name: testutility00__testnew6
% similar to test0904__testnew6 from \mystuff\20090904
% utility function: type 1 - log-power utility (gamma_l = 0) => log-ld
% similar to testnew6 from \mystuff\20090810 but A=2.85 here
% testnew6 : similar to testnew3. now trying to put aggregate equilibrium for d and (c=y-i)
% (not partial b and k of r, put the equation in, then can run.)
% file name: testnew5.mod
% \mystuff\20090810__startingfromDynareSummerSchool
%
% Purpose: Extended RBC model: 13 variables
% with indivisible labor, dividends, debt, risk-free rate,
% interest rate and default probability estimate.
%
% 0. Housekeeping
% close all;
% 1. Defining variables
% 2. Calibration

A = 2.85;

alpha = 0.35; % capital share / Uhlig & Hansen: 0.36, Alessandrini: 0.35
beta = 0.99; % subjective time discount factor
delta = 0.019; % depreciation share / Uhlig & Hansen: 0.025, Alessandrini: 0.019
psi = 0.95; % autocorrelation of technology shock
sigma = 0.00712; % standard deviation of technology shock units
tau = 0.06; % tax rate
theta = 0.92; % recovery rate
U = 1.225; % U = 1 + 0.3*epi
L = 0.475; % L = 1 - 0.7*epi
epi = 0.75; % specific shock

% 3. Model

model;
1 = beta* (c/c(+1))*rf;
rf = (1-tau)*alpha*(exp(z(+1))^(1-alpha))*(k^alpha)*(l^(-alpha));
A*c = (1-l)^gamma_l*(1-alpha)*(exp(z)^(1-alpha))*(k(-1)^alpha)*(l^(-alpha));
y = (exp(z)^(1-alpha))*(k(-1)^alpha)*(l^(-alpha));
k = i + (1-delta)*k(-1);
c = (1 - (alpha*(1-theta)*(x-L)/epi))*y - ((U-x)/epi)*i;
r = (1/(1-tau))*rf-
q = beta* (c/c(+1)) * (q(+1)+ d(+1) );
x = b(-1)*r(-1) / (alpha*y + (1-delta)*k(-1));
d = ((U-x)/epi)*((1-tau)*(alpha*y - r(-1)*b(-1)) -tau*b(-1) - i + b);
z = psi*z(-1) + e; fau = x/(alpha*y + (1-delta)*k(-1));
mpk = alpha*y/k(-1);
mpl = (1-alpha)*y/l;
end;

% 4. Computation

initval;
k = 2;
4.11. APPENDICES

\[ \begin{align*} 
c &= 1.33; 
l &= 0.31; 
b &= 0.6; 
x &= 0.3; 
z &= 0; 
e &= 0; 
\end{align*} \]

end;
steady;
shocks; var e = sigma^2; end;

\[ \text{stoch\_simul(hp\_filter = 1600, order = 1);} \]

% 5. Some Results

\[ \begin{align*} 
\text{statistic1} &= 100*\sqrt{\text{diag}(\text{oo\_.var}(1:15,1:15))}/\text{oo\_.mean}(1:15); 
\text{dyntable('Relative standard deviations in %',strvcat('VARIABLE','REL. S.D.'),} 
\text{M\_.endo\_names(1:15,:),statistic1,15,10,4)} 
\text{statistic2} &= 100*\sqrt{\text{diag}(\text{oo\_.var}(1:15,1:15))}; 
\text{dyntable('standard deviations in %',strvcat('VARIABLE','S.D.'),} 
\text{M\_.endo\_names(1:15,:),statistic2,15,8,4);} 
\end{align*} \]

4.11.5 \textbf{DYNARE code for the DSGE model with habit formation and effective labour (Model 1)}

\[ \begin{align*} 
\% \text{file: Paper1001com0 (under Mystuff\RBCexamples\20100210_paper1)} 
\% \text{file: Atestutility22} 
\% \text{file: Paper1001xi0gamma1 (under Mystuff\RBCexamples\20100210) for running} 
\% \text{different gammas} 
\% \text{technical change in labor} 
\% \text{non-separable model with habit formation} 
\% \text{similar to file name: testutility22} 
\% \text{utility function: non-additively separable utility in consumption and leisure (rho =} 
\% \text{0.36, gamma=2, xi}=0.80} 
\% \text{with habit formation} 
\% \text{0. Housekeeping} 
\% \text{close all;} 
\% \text{1. Defining variables} 
\% \text{var k, c, y, l, i, d, b, r, q, rf, x, z, fau,mpk, mpl; %fau} 
\% \text{output, consumption, capital, labor, interest, investment, technology, q: the price of} 
\% \text{the equity} 
\% \text{varexo e;} 
\% \text{parameters alpha, beta, delta, psi, sigma, tau, theta, U, L, epi, rho, gamma, xi; } 
\% \text{2. Calibration} \end{align*} \]
%A = 2.85; % 2.85 self-estimated by Hansen (1985)’s parameter results (not use here)
alpha = 0.35; % capital share / Uhlig & Hansen: 0.36, Alessandrini: 0.35
beta = 0.99; % subjective time discount factor
delta = 0.019; % depreciation share / Uhlig & Hansen: 0.025, Alessandrini: 0.019
psi = 0.95; % autocorrelation of technology shock
sigma = 0.00712; % standard deviation of technology shock units
tau = 0.06; % tax rate
theta = 0.92; % recovery rate
U = 1 + 0.3*epi;
L = 1 - 0.7*epi;
epi = 0.75; % specific shock
rho = 0.36;
gamma = 2;
xi = 0.80; % habit persistence

3. Model

%--------------------------------------------------
model;
1 = beta*( (1 - xi*beta*((c(+2)-xi*c(+1))/(c(+1)-xi*c) )^(rho-gamma*rho-1) *((1-
l(+2))/(1-l(+1)))^(1-gamma*(1-rho)-rho))/ ( ((c-xi*c(-1))/(c(+1)-xi*c))^ (rho-gamma*rho-1) *((1-l)/(1-l(+1)))^(1-gamma*(1-rho)-rho) - xi*beta) *rf );
rf = (1-tau)*(alpha*(exp(z(+1))^(1-alpha))*(k^(alpha-1))*(l(+1)^(1-alpha))) + 1 -delta;
c = xi*c(-1)+((rho/(1-rho))*(1-alpha)*(exp(z)^(1-alpha))*(k(-1)^alpha)* (l^(1-alpha))*(1-l)/l* ( 1 - xi*beta*((1-l(+1))/(c(+1)-xi*c))^(1+rho*(gamma-1))* ((1-l)/(c-xi*c(-1)))^(rho*(1-gamma)-1)*((1-l)/(1-l(+1)))^gamma));
y = (exp(z)^(1-alpha))*(k(-1)^alpha)/((1-alpha));
k = i + (1-delta)*k(-1);
c = (1 - (alpha*(1-theta)*(x-1+0.7*epi)/epi))/y - ((1+0.3*epi-x)/epi)*i;
r = (1/(1-tau))*(rf-tau);
q = beta*( (1 - xi*beta*((c(+2)-xi*c(+1))/(c(+1)-xi*c) )^(rho-gamma*rho-1) *((1-
l(+2))/(1-l(+1)))^(1-gamma*(1-rho)-rho))/ ( ((c-xi*c(-1))/(c(+1)-xi*c))^ (rho-gamma*rho-1) *((1-l)/(1-l(+1)))^(1-gamma*(1-rho)-rho) - xi*beta) *(q(+1)+ d(+1)));
x = b(-1)*r(-1) / (alpha*y + (1-delta)*k(-1));
d = ((1+0.3*epi-x)/epi)*((1-tau)*(alpha*y - r(-1)*b(-1)) -tau*b(-1) - i + b);
z = psi*z(-1) + e;
fau = x/(alpha*y + (1-delta)*k(-1));
mpk = alpha*y/k(-1);
mpl = (1-alpha)*y/l;
end;
%--------------------------------------------------

4. Computation

%--------------------------------------------------
initalval;
k = 2;
4.11.6 DYNARE code for the DSGE model with habit formation and effective labour (Model 2)

% file: Atestutility37
% technical change in labor
% additively separable model with separable habit formation
% (gamma_c=1, gamma_l=5 xi=0.80) => log-power (with separable habit)
% file name: testutility30
% utility function: additively separable utility in consumption and leisure
% — habit formation —
% 0. Housekeeping
%-----------------------------------------------
close all;
%-----------------------------------------------
% 1. Defining variables
%-----------------------------------------------
var k, c, y, l, i, d, b, rf, r, q, x, fau,mpk, mpl; %fau
% output, consumption, capital, labor, interest, investment, technology, q: the price of the equity
varexo e;
parameters A, alpha, beta, delta, psi, sigma, tau, theta, U, L, epi, xi, gamma_c, gamma_l;
% 2. Calibration
%-------------------------------------
A = 2.85; % 2.85 self-estimated by Hansen (1985)'s parameter results
alpha = 0.35; % capital share / Uhlig & Hansen: 0.36, Alessandrini: 0.35
beta = 0.99; % subjective time discount factor
delta = 0.019; % depreciation share / Uhlig & Hansen: 0.025, Alessandrini: 0.019
psi = 0.95; % autocorrelation of technology shock
sigma = 0.00712; % standard deviation of technology shock units
tau = 0.06; % tax rate
theta = 0.92; % recovery rate
U = 1 + 0.3*epi;
L = 1 - 0.7*epi;
epi = 0.75; % specific shock
xi = 0.80; % habit persistence
gamma_c = 1;
gamma_l = 5;
%-------------------------------------
% 3. Model
%-------------------------------------
model;
1 = beta*(((1-xi*beta*(((c(+1)-xi*c)/(c+1)-xi*c(+1)))^(gamma_c))) /
( (((c+1)-xi*c)/(c-xi*c(-1)))^(gamma_c))/xi*beta))*rf);
rf = (1-tau)*(alpha*(exp(z(+1))^alpha)*(k^(alpha-1))*(l(+1)^alpha)**(1-alpha)) + 1 - delta;
A* ((1/(c-xi*c(-1)))^(gamma_c))-xi*beta*((1/(c+1)-xi*c))^(gamma_c))^-1 =
(1-alpha)*((1-l)^(gamma_l))*(exp(z)^(1-alpha))*(k(-1)^alpha)*l^(1-alpha));
y = (exp(z)^(1-alpha))*(k(-1)^alpha)*l^(1-alpha));
k = i + (1-delta)*k(-1);
c = (1 - (alpha*(1-theta)*(x-1+0.7*epi)/epi))*y - ((1+0.3*epi-x)/epi)*i;
r = (1/(1-tau))*rf-tau);
r = (b*rf - ((theta/(2*epi))*(alpha*y(+1)+(1-delta)*k))*(x(+1)^2 -
(1-0.7*epi)^2)*epi/(b*(1+0.3*epi-x(+1))));
q = beta*(((1-xi*beta*(((c+1)-xi*c)/(c+2)-xi*c(+1)))^(gamma_c))) /
( (((c+1)-xi*c)/(c-xi*c(-1)))^(gamma_c))/xi*beta )) * (q(+1)+ d(+1));
x = b(-1)*r(-1) / (alpha*y + (1-delta)*k(-1));
d = ((1+0.3*epi-x)/epi)((1-tau)*(alpha*y - r(-1)*b(-1)) -tau*b(-1) - i + b);
z = psi*z(-1) + e;
fau = x/(alpha*y + (1-delta)*k(-1));
mpk = alpha*y/k(-1);
mpl = (1-alpha)*y/l;
end;
%-------------------------------------
% 4. Computation
%-------------------------------------
initval;
k = 2;
c = 1.33;
l = 0.31;
b = 0.6;
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x = 0.3;
fau = 0.03;
z = 0;
e = 0;
end;
steady;
shocks;
var e = sigma^2;
end;

\texttt{stoch\_simul(hp\_filter = 1600, order = 1);}
\texttt{
\% 5. Some Results
\%}
\texttt{statistic1 = 100*sqrt(diag(oo_.var(1:15,1:15)))./oo_.mean(1:15);
dyntable('Relative standard deviations in \%',strvcat('VARIABLE','REL. S.D.'),
M_.endo_names(1:15,:),statistic1,15,10,4)
statistic2 = 100*sqrt(diag(oo_.var(1:15,1:15)));
dyntable('standard deviations in \%',strvcat('VARIABLE','S.D.'),
M_.endo_names(1:15,:),statistic2,15,8,4);
}

4.11.7 DYNARE code for the DSGE model with nonseparable habit formation and effective labour (Model 3)

\texttt{\% file: Atestutility35}
\texttt{\% technical change in labor}
\texttt{\% additively separable model with nonseparable habit formation (gamma\_c=2, gamma\_l=5}
\texttt{\% xi=0.80)
\texttt{\% similar file name: testutility35}
\texttt{\% utility function: additively separable utility in consumption and leisure}
\texttt{\% — habit formation — ratio way — (c\_t/xi*c\_t-1)
\%}
\texttt{\% 0. Housekeeping
\%}
\texttt{close all;}
\texttt{
\% 1. Defining variables
\%}
\texttt{var k, c, y, l, i, z, d, b, rf, r, q, x, fau,mpk,mpl; %fau
\% output, consumption, capital, labor, interest, investment, technology, q: the price
\% of the equity
\% varexo e;
\texttt{parameters A, alpha, beta, delta, psi, sigma, tau, theta, U, L, epi, xi, gamma\_c,
\% gamma\_l;}
\texttt{
\% 2. Calibration
\%}
A = 2.85; % 2.85 self-estimated by Hansen (1985)'s parameter results
alpha = 0.35; % capital share / Uhlig & Hansen: 0.36, Alessandri: 0.35
beta = 0.99; % subjective time discount factor
delta = 0.019; % depreciation share / Uhlig & Hansen: 0.025, Alessandri: 0.019
psi = 0.95; % autocorrelation of technology shock
sigma = 0.00712; % standard deviation of technology shock units
tau = 0.06; % tax rate
theta = 0.92; % recovery rate
U = 1 + 0.3*epi;
L = 1 - 0.7*epi;
epi = 0.75; % specific shock
xi = 0.80; % habit persistence
gamma_c = 2;
gamma_l = 5;

% 3. Model

model;
1 = beta*( ( ( (c(+1)/c)/(c/c(-1)))^(-gamma_c)*(1/(c/c(-1)))* ((1-beta*((c(+2)/c(+1))/(c(+1)/c))^(1-gamma_c)))/(1-beta*((c(+1)/c)/(c/c(-1)))^(1-gamma_c))))*rf;
rf = (1-tau)*(alpha*(exp(z(+1))^(1-alpha))*(k^(alpha-1))*l(+1)^(1-alpha)) + 1 - delta;
A = (1/(c^gamma_c))*(1-alpha)*(exp(z)^(1-alpha))*(k(-1)^alpha)*(l^(-alpha))*(1-l)^(gamma_l)*(1/(xi*c(-1)))^(1-gamma_c)* (1- beta* ((c(+1)/c)/(c/c(-1)))^(1-gamma_c));
y = (exp(z)^(1-alpha))*(k(-1)^(alpha-1))*(l^(1-alpha));
k = i + (1-delta)*k(-1);
c = (1 - (alpha*(1-theta)*(x-1+0.7*epi)/epi))y - ((1+0.3*epi-x)/epi)*i;
r = (1/(1-tau))*(rf-tau);
r = (b*rf - ((theta/(2*epi))*(alpha*y(+1)+(1-delta)*k)*(x(+1)^2 - (1-0.7*epi)^2)))*(epi/(b*(1+0.3*epi-x(+1))));
q = beta* ( ( ( (c(+1)/c)/(c/c(-1)))^(-gamma_c)*(1/(c/c(-1)))*(( 1-beta* ((c(+2)/c(+1))/(c(+1)/c))^(1-gamma_c)))/(1-beta*((c(+1)/c)/(c/c(-1)))^1-gamma_c)))^ (1-gamma_c)) * (q(+1) + d(+1) + 1);
x = b(-1)*r(-1) / (alpha*y + (1-delta)*k(-1));
d = ((1+0.3*epi-x)/epi)*((1-tau)*(alpha* y - r(-1)*b(-1) - tau*b(-1) - i + b);
z = psi*z(-1) + e;
fau = x/(alpha*y + (1-delta)*k(-1));
mpk = alpha*y/k(-1);
mpl = (1-alpha)*y/l;
end;

% 4. Computation

% initval;
k = 2;
c = 1.33;
l = 0.31;
b = 0.6; 
x = 0.3; 
fau = 0.03; 
z = 0; 
e = 0; 
end; 
steady; 
shocks; 
var e = sigma^2; 
end; 
stoch_simul(hp_filter = 1600, order = 1); 

% 5. Some Results 

statistic1 = 100*sqrt(diag(oo_.var(1:15,1:15)))./oo_.mean(1:15); 
dyntable('Relative standard deviations in %',strvcat('VARIABLE','REL. S.D.'), 
M_.endo_names(1:15,:),statistic1,15,10,4) 
statistic2 = 100*sqrt(diag(oo_.var(1:15,1:15))); 
dyntable('standard deviations in %',strvcat('VARIABLE','S.D.'), 
M_.endo_names(1:15,:),statistic2,15,8,4);
CHAPTER 5

THE IMPLICATION OF COSTLY CAPITAL ADJUSTMENT COSTS FOR THE VARIABILITY OF DIVIDEND SMOOTHING AND STOCK PRICES
5.1 Introduction

In this chapter, I examine the impact of frictions in the production process on aggregate dividends. In particular, capital adjustment costs as well as habit formation are included within the DSGE models. External financing and default, however, are not considered in this chapter for reasons of analytical tractability.

Several papers have previously taken both capital adjustment costs and habit formation into account in DSGE models [Jermann (1998), Boldrin et al. (2001), and Gershun (2010)]. Their focus, however, is to address asset pricing issues by using the DSGE methodology but they have not discussed optimal aggregate dividend behaviour.

This objective of this chapter is to examine the variability of optimal aggregate dividends when there are frictions generated from the corporate production system. The topic of dividend smoothing has been investigated over many years [Lintner (1956), Kumar (1988), Allen and Michaely (2002) and Fuller and Goldstein (2005)]. There are, however, few studies to examine the determinants of aggregate dividend smoothing. The papers that do report evidence of smooth dividends [for example, Kang and Kumar (1991) and Leary and Michaely (2010)], are based on empirical observations, rather than studied using a macroeconomic approach. Shiller (1981) raises the issue that the stock returns are far too volatile within a standard discounted dividend model given the low volatility of aggregate dividends. Motivated by prior work, in this chapter, I aim to investigate simultaneous interactive movements of optimal aggregate dividends and stock prices in the presence of capital adjustment costs (CAC) and habit formation in a production economy. This field has not been examined in depth by previous papers.

My first calibration is to construct a DSGE model in which the labour is fixed and productivity shocks are extremely high [Jermann (1998)]. I find that, when the economy has both frictions of capital adjustment costs and habit formation, it becomes difficult for corporations to reduce consumption fluctuations via adjusting the production plan. This result is consistent with the claim by Jermann (1998) that a joint effect from capital adjustment costs and habit formation reduces consumption smoothing. However, an increase in the level of CAC increases the volatility of consumption if and only if the
labour input is restricted to equal one. In addition, in this labour-fixed model, the variability of the countercyclical aggregate dividends in the case of high capital adjustment costs is not very different to that in the case of low capital adjustment costs.

My second calibration extends the Jermann model to include an endogenous labour with standard exogenous technology shocks in the economy. Jermann (1998) assumes that the labour supply is fixed and the economy is hit by high technology shocks. These assumptions limit (a) the model’s ability to explain labour supply and (b) the equilibrium is influenced heavily by stochastic shocks. After adjusting the specifications for the model, I find that optimal aggregate dividends become more sensitive to (a) the determinants of firms’ production functions and (b) the strength of capital adjustment costs. This work examining the impact on optimal aggregate dividends is performed by adjusting the capital adjustment costs within a wide range in the DSGE model. Two parameters are taken to determine the strength of capital adjustment costs. The first value determines very low capital adjustment costs [Gershun (2010)] and the second parameter gives very costly capital adjustment costs [Jermann (1998)]. My results show that optimal aggregate dividend volatility is very small when it is costly for the firm to change its capital base. I also find that aggregate dividends are smooth while the variability of stock prices increases in equilibrium in an economy with production frictions. These findings help to resolve Shiller’s (1981) that stock index movements are volatile while market-wide dividends are smooth.

My third investigation in this chapter is that, since capital adjustment functions take different forms in different papers, I present another model (called the control model) in which the alternative form of capital adjustment costs used in Collard and Dellas (2006) is adopted. Comparing these results to my previous models, I find that results in the control model are similar to the previous model with low capital adjustment costs. Optimal aggregate dividends, as hypothesised, are volatile in the models with low capital adjustment costs, rather than with high capital adjustment costs. This implies that corporate dividend policy is determined to a large degree by costly capital based frictions.

In this chapter, my fourth calibration is to construct a DSGE model in which both
(a) the firm is utility-maximising (UM) instead of value-maximising (VM) and (b) habit formation is considered, but with no capital adjustment costs. While the model is without any capital frictions, the equilibrium economy of firms with risk aversion is similar to firms with capital adjustment costs. This work is inspired by Cárceles Poveda (2003) but she has not taken habit formation into account and does not examine aggregate dividends in detail. This is an interesting topic for investigation because an individual firm’s investment cash flows become inelastic when the firm is risk averse to uncertain future cash flows, and consequently aggregate dividend policy is less volatile. This is similar to the result that high CAC smooths aggregate dividends.

I depict this analogy by modelling these two cases: individual firms with CAC versus with risk aversion. The results are consistent with the finding by Cárceles Poveda (2003). I find that optimal aggregate dividend payouts are very smooth in an economy in which the firm is risk averse. This finding gives the interesting result that, if the individual firm is utility-maximising oriented, rather than value-maximising, optimal aggregate dividends become nearly smooth in the long run while share prices fluctuate greatly per output unit. However, it is difficult to tell whether the individual corporation is risk averse or wishes to maximise its expected cash flows. It is then not straightforward to examine which approach is more accurate to represent the real economy.

The remainder of this chapter is organised as follows. Section 5.2 provides a detailed literature review on capital adjustment costs in DSGE models. Section 5.3 presents the fixed-labour DSGE model with capital adjustment costs and discusses the results. Section 5.4 performs an endogenous-labour DSGE model and examines the results. Section 5.5 demonstrates a DSGE model in which firms and households are utility-maximising but with no capital adjustment costs. A series of results and discussion are also presented in this section. Section 5.6 reports the results of impulse responses to the technology shock for each model of this chapter. Section 5.7 concludes. Section 5.8 provides appendices.
5.2 Literature review: capital adjustment costs (CAC)

Capital adjustment costs (CAC) are the spending that corporations need to take account of when they plan to change the flow of the capital stock between periods. CAC have been discussed extensively in the literature on investment. In general, CAC are considered in the capital accumulation function in order to study the impact of CAC on investment decisions. The leading pioneer is Jermann (1998). Several popular papers, Boldrin et al. (2001), Danthine and Donaldson (2002), Lettau (2003), Bouakez et al. (2005), Cárceles-Poveda (2009), Santoro and Wei (2010) and Gershun (2010), consider CAC in their models for different research objectives.

5.2.1 Why are CAC important in DSGE models?

Jermann (1998) brings up the issue that a macroeconomic model can successfully explain the observed equity premium. He claims that the essence of modelling a theoretical economy is to consider both habit formation and capital adjustment costs. A finding from Rouwenhorst (1995) makes a key point: researchers have overcome the problem of exogenous consumption in the endowment economy by taking the methodology of DSGE modelling to study asset prices because this model allows consumption to be determined endogenously. This method, however, entails another problem: Endogenous consumption in the production economy moves smoothly over time. The less volatile consumption, the smaller the equity premium. A model with endogenous consumption still cannot solve the equity premium puzzle due to an underestimated risk premium.

Jermann (1998) then takes advantage of the fact that CAC makes the firm reluctant to adjust its investment plan for the next period since it is costly. Consequently, the corporation cannot easily adjust the production plan to reduce fluctuations in consumption. That is, CAC increases consumption volatility. The model, as a result, is able to generate an equity premium which is consistent with the historic average. The paper of Jermann (1998) thus becomes a guidebook to academics subsequently studying the
asset pricing implications of RBC-based DSGE models.

Adjustment costs not only play an important role in asset pricing implications, but also in explaining economic cyclical facts. Christiano and Fisher (1995) apply adjustment costs to Tobin’s q and show that CAC are related to the cyclical properties of equity prices and investment goods. Therefore, it is important to take the issue of adjustment costs into account when studying macroeconomic and financial market facts.

### 5.2.2 Specification of CAC

The purpose of considering CAC in the model is to reduce the household’s ability to smooth consumption and further to estimate a high equity premium. CAC can directly affect the firm’s investment decision and indirectly influence the market’s equilibrium. The capital accumulation function with CAC used in Jermann (1998) is given by:

\[ K_t = (1 - \eta)K_{t-1} + g\left(\frac{I_t}{K_{t-1}}\right)K_{t-1} \]

The function \( g(\cdot) \) captures capital adjustment costs and is a function of the investment and capital ratio:

\[ g\left(\frac{I_t}{K_{t-1}}\right) = \frac{b}{\zeta} - \frac{1}{\zeta} \left(\frac{I_t}{K_{t-1}}\right)^{1 - \frac{1}{\zeta}} + c \]

where \( b \) and \( c \) are parameters. \( \zeta \) is defined as the elasticity of the investment and capital ratio. There is no explicit specification of \( b \) and \( c \) in Jermann (1998). However, it is not difficult to define the parameter values for \( b \) and \( c \). If there is no CAC in the economy (i.e. \( \zeta = \infty \)), the capital accumulation function is \( K_t = (1 - \eta)K_{t-1} + bI_t + cK_{t-1} \), which is the same as in the previous chapters if and only if \( b = 1 \) and \( c = 0 \). The role of \( b \) and \( c \) is to maintain the same steady state values over periods in the economy with CAC, which means the steady state values are unchanged for any given value of \( \zeta \). When \( \zeta = \infty \), there is no capital adjustment costs in the economy. With the assumption that \( 0 < \zeta < \infty \), the smaller the value, the greater CAC.

Table 5.2.1 on page 156 displays a summary of specifications of CAC corresponding
5.2. LITERATURE REVIEW: CAPITAL ADJUSTMENT COSTS (CAC)


The behaviour of macroeconomic variables should be likewise in any form of CAC hypothetically, which means investment tends to vary less and consumption varies more over time. In section 5.4, I provide an explicit discussion among the results of both specifications.

There is no one standard value for the parameter of $\zeta$ since the CAC formulation varies from one paper to another. However, there is a guideline: Jermann (1998) reports a range of $\zeta$ as $[0.16: \infty]$ and he applies the value of 0.23 in his model. Gershun (2010) however uses a value of 40 which implies much lower adjustment costs than Jermann (1998)’s.

5.2.3 Studies of DSGE models with CAC

5.2.3.1 Asset pricing implications

Jermann (1998) influences most researchers who study asset pricing implications through RBC-based DSGE models. He contributes one central point for better understanding the equity premium. In his production economy, the household’s habit formation causes smooth consumption. In order to solve this problem, he introduces CAC. This vital factor is called capital adjustment costs (CAC). Once the theoretical economy incorporates this friction, aggregate investment is no longer as flexible as an economy without CAC. The higher the level of CAC, the greater the volatility of consumption. The agent’s consumption fluctuates over time because of inelastic dividends, which are derived from the costly investment decision. Boldrin et al. (2001), Cárceles Poveda (2003), and Santoro and Wei (2010) borrow the specification of CAC from Jermann (1998) and provide
Table 5.2.1: A summary of literature building DSGE models with capital adjustment costs

This table reports two capital accumulation functions with different specifications used in the investment decision literature. Specification I includes a proportional-type of capital adjustment costs. The capital adjustment costs are a part of a function of the investment-capital ratio. Specification II includes an additive-form of capital adjustment costs. Capital adjustment costs are the additional expenses after the capital accumulation process.

Specification I

\[ K_t = (1 - \eta)K_{t-1} + g\left(\frac{I_t}{K_{t-1}}\right)K_{t-1} \]

a. where function \( g\left(\frac{I_t}{K_{t-1}}\right) \) is simply defined as \( \frac{b}{1 - \frac{\eta}{\zeta}}\left(\frac{I_t}{K_{t-1}}\right)^{1-\frac{\eta}{\zeta}} + c \)


b. where function \( g\left(\frac{I_t}{K_{t-1}}\right) = \frac{(\exp(\bar{x})-1+\eta)}{1-\frac{\eta}{\zeta}}\left(\frac{I_t}{K_{t-1}}\right)^{1-\frac{\eta}{\zeta}} + \frac{1-\eta-\exp(\bar{x})}{\zeta-1} \)

Paper: Boldrin et al. (2001)

(\( \bar{x} \) denotes mean value of the persistence of technology (shown on p.151 in Boldrin et al. (2001))

c. where function \( g\left(\frac{I_t}{K_{t-1}}\right) = \frac{\eta}{1-\frac{\eta}{\zeta}}\left(\frac{I_t}{K_{t-1}}\right)^{1-\frac{\eta}{\zeta}} + \frac{\eta}{1-\zeta} \)

Papers: Gershun (2010)

Specification II

\[ K_t = (1 - \eta)K_{t-1} + I_t - \frac{1}{2}\zeta\left(\frac{I_t}{K_{t-1}} - \eta\right)^2K_{t-1} \]


Nomenclature:

\( K \) - capital; \( I \) - investment; \( \eta \) - the depreciation rate; \( g(.) \) - a capital adjustment cost function; \( b \) and \( c \) - parameters; \( \bar{x} \) - a mean value of the persistence of technology ; \( \zeta \) - a value for capital adjustment costs.
further detailed information about calibration.

Boldrin et al. (2001) emphasise that a two-sector RBC model is better than a one-sector model to account for asset returns and business cycles simultaneously. Influenced by Boldrin et al. (2001), Gershun (2010) presents two models (one-sector and two sectors) with habit formation and CAC to account for financial market facts under a self-fulfilling expectation assumption. In her paper, she assumes agents’ behaviour is based on expectation which results in multiple equilibria. The result shows that one-sector DSGE models with self-fulfilling expectation fail to explain business cycle stylised financial facts. The two-sector DSGE model generates a low risk-free rate but is unable to explain the equity premium (i.e. it is lower than the historical value). Even though these two papers have different findings, both of them have a common objective, which is to investigate the asset pricing implications in DSGE models by incorporating habit formation and capital adjustment costs.

Furthermore, researchers consider several frictions to examine the business cycles and study asset pricing implications in an incomplete market. Danthine and Donaldson (2002) apply CAC and a labour market friction in the DSGE model. With these additional frictions, their model can well account for the equity premium when the risk aversion is low and agents are heterogeneous. Their CAC formulation is another standard specification (see table 5.2.1, specification II). As mentioned in their paper, CAC cause variable asset returns through volatile dividends. From the equity pricing formula, the gross return on equity displayed in Danthine and Donaldson (2002, p.45) is:

\[ R_{t-1,t}^{e} = \frac{P_t + D_t}{P_{t-1}} = \frac{K_{t+1} + D_t}{K_t} \]

Intuitively, adjustment costs cause a small standard deviation of capital. Then the variation of the market return on equity is expected to depend on high fluctuation in dividends. Thus, equity premium puzzle can be explained when the economy is with the friction of capital adjustment costs.

Another extension of Jermann (1998) is provided by Santoro and Wei (2010). The
authors study the implication of corporate dividend taxation on asset pricing by using Jermann (1998)’s RBC framework and incorporating corporate taxation issues. They conclude that corporate income taxes greatly influence firm value and the volatility of dividends, investment and consumption in a general equilibrium model.

5.2.3.2 CAC and risk aversion

The effect of CAC on business cycles and financial markets has been discussed extensively. To corporations, CAC smooth the deviation of capital and investment inputs but these are not the only factor influencing the movement of investment. Risk aversion could as well cause inelastic investment. A risk-averse firm is less willing to gamble its profits for any uncertain gains/losses. As a result, the firm has inelastic investment plans, and over time, aggregate investment moves smoothly. Cárceles Poveda (2003) has made an important contribution to this field. His paper verifies that there is a supportive relationship between CAC and corporate risk aversion in DSGE models. What he proves is that the equilibrium of a DSGE model with CAC is the equivalent of another optimal economy with a representative risk-averse firm and without CAC.

No matter if the economy is changed from one with a risk-neutral firm and CAC to another one with a risk-averse firm but no CAC, the steady states of macroeconomic variables stay completely identical. The firm’s objective in the former economy is value maximising (VM) and in the latter it is called utility maximising (UM). It is an absolutely instinctive idea because both frictions (CAC and risk aversion) have one character in common, that is, to make aggregate investment less variable.

5.2.3.3 Monetary economics with CAC and others

Apart from building RBC-based DSGE models, there is another stream called New Keynesian (NKS) based DSGE models working with CAC. NKS models are another school of DSGE models. These models are tailored for analysing monetary and fiscal policy when prices are assumed to be sticky, which is different to RBC models (i.e. the economy in RBC models is under the assumption of flexible prices).
Bouakez et al. (2005) focus on the effect of nominal monetary stock in investor’s preferences and further examine the persistent effects of monetary shocks on output and consumption. They take the method of maximum likelihood and use quarterly data to estimate a DSGE model which incorporates habit formation, CAC and sticky prices. They emphasise that habit formation interacts with CAC to enlarge the propagation of monetary shocks. With the presence of costly adjustment costs to capital, investment and output respond weakly to shocks. This implies that the volatility of investment is small. As a result, the net cash flows are not very variable over time.

However, the model fails to capture the behaviour of labour demand and the inflation rate. Collard and Dellas (2006), in a study using a DSGE model with price rigidity for inflation stability, assess the property of inflation in a NKS model incorporating several production and monetary frictions including adjustment costs, money demand and government expenditures. They conclude that the question of inflation stabilisation in the economy remains unsolved. The impact of the capital accumulation on the deviation of output with other frictions is, however, related to the case for inflation stabilisation. In particular, it is successful in generating reasonable estimates for inflation if the model is with low CAC and other strong monetary frictions (e.g. monetary shock).

Besides the study of monetary economics with CAC, Cooper and Haltiwanger (2006) concentrate on the nature of CAC and the relationship between investment and profitability. Their empirical results show that a model mixing convex and non-convex adjustment cost functions can match observed investment behaviour and profitability.

### 5.3 The value-maximising model with fixed labour supply

The objective of this chapter is to demonstrate macroeconomic models by incorporating production friction for the study of the variability of aggregate dividend policy. In this section I present how I develop a DSGE model with the friction of CAC.

Compared to previous chapters, the major difference is that the firm’s objective function is in a utility form. This framework allows us to examine two cases: (1)
firms are value maximising (VM) when they are risk neutral and (2) firms are utility maximising (UM) when they are risk averse.

Within both VM and UM models, a representative firm that has a technology shock, a quarterly trend in labour augmenting technical change, and capital adjustment cost function are involved. In the investor’s utility specification, the objective function has a momentary utility function involving consumption, internal consumption habits and leisure. The market will achieve equilibrium when both of their optimisation problems are solved simultaneously.

This section introduces a VM-based DSGE model with fixed labour supply to simplify the optimisation process. In the next section, I display another VM-based DSGE model but allow the labour supply to be variable. As to the calibration of the UM-based model, details are presented in section 5.5.

### 5.3.1 Specification of the Social Planners

**The firm** The representative firm aims to maximise its present value of future cash flows \( D \) by solving the optimal labour \( L^d \), capital \( K \) and investment \( I \) inputs:

\[
\max_{L^d_t, K_t, I_t} E_t \left[ \sum_{h=0}^{\infty} \phi_t h U_F(D_{t+h}) \right]
\]

(5.3.1)

where \( U_F(D_{t+h}) \) represents the firm’s utility function of its future cash flows. \( \phi \) denotes a discount factor and \( D \) denotes the firm’s net cash flows. The superscript \( d \) on \( L^d \) is meant to indicate “demand” in the labour market.

It is assumed that the firm’s objective function is based on a power form of the utility function:

\[
U_F(D_{t+h}) = \frac{D_{t+h}^{1-\gamma_F}}{1-\gamma_F}
\]

(5.3.2)

where \( \gamma_F \) is the coefficient of the firm’s risk aversion. In general, the firm in the standard DSGE model is assumed to be risk neutral. That is, \( \gamma_F = 0 \) in the above equation stands for a value-maximising (VM) firm in the economy. The firm’s objective function is to maximise its present value of future cash flows. When \( \gamma_F > 0 \), it implies
that the representative firm is risk averse to its future cash flows.

Dividend payouts made after the investment decision are defined as:

\[ D_t = Y_t - W_t L_t^d - I_t \] (5.3.3)

where \( W \) is the wage rate. It is assumed that a representative firm produces according to a Cobb-Douglas production technology which is given by:

\[ Y_t = Z_t f(K_{t-1}, xL_t^d) = Z_t K_{t-1}^{\alpha} (xL_t^d)^{1-\alpha} \] (5.3.4)

where \( Y, Z, f(\cdot) \) and \( x \) denote output, a stochastic technology shock, a type of Cobb-Douglas production function of capital and labour inputs and the deterministic trend in labour augmenting technical change respectively\(^1\). \( \alpha \) and \( 1 - \alpha \) are capital and effective labour ratios respectively. The technology shock process is assumed to follow a first order autoregressive process (AR(1)) in logs and evolves exogenously (see chapter 3). All the subscript notations \( t \) and \( t - 1 \) are at the point in time at which the input actually happens.

The capital accumulation process is given by

\[ K_t = (1 - \eta) K_{t-1} + g\left(\frac{I_t}{K_{t-1}}\right) K_{t-1} \] (5.3.5)

where \( \eta \) denotes the capital depreciation rate. \( g(\frac{I}{K}) \) is a concave capital adjustment cost function. It captures the costs the firm needs to spend if it adjusts the stock of capital for each period. Because of the concavity of the function of CAC, it is costlier to adjust the investment-to-capital ratio upwards than adjust the ratio downwards. This type of capital adjustment costs have been used in Danthine and Donaldson (2002), Canzoneri et al. (2005) and Collard and Dellas (2006)\(^2\).

\(^1\)The specification of the firm’s production function is borrowed from Jermann (1998).

\(^2\)Capital adjustment costs are not always considered in the function of capital accumulation but are considered in the goods clearing condition. This is because some scholars assert the influence of capital adjustment costs is on the amount of output, not the amount of capital. For the case of not putting capital adjustment costs in capital accumulation form, see Danthine and Donaldson (2002). For papers considering capital adjustment costs in the function of the capital accumulation, see Jermann (1998), Boldrin et al. (2001), Cárceles Poveda (2003), Gershun (2010). I follow the latter one as I believe that capital adjustment costs influence the level of new capital input. Thus, the capital adjustment costs should be formed in the function of capital accumulation.
5.3. THE VALUE-MAXIMISING MODEL WITH FIXED LABOUR SUPPLY

The investor

The representative investor solves an optimisation problem:

\[
\max_{C_t, L_t^s, N_t} E_t \left[ \sum_{h=0}^{\infty} \beta^h U(C_{t+h}, C_{t+h-1}, 1 - L_{t+h}^s) \right]
\]  

subject to

\[
C_t + Q_t N_{t+1} = W_t L_t^s + (Q_t + D_t) N_t
\]

where \( L^s \) is the quantity of the labour supplied by the household. For simplification of the household’s specification, I assume that there are no transactions on bonds.

5.3.2 Market clearing conditions

The market clearing is achieved when three market conditions hold. Firstly, in the goods market:

\[
Y_t = C_t + I_t
\]

The second market clear condition is in the labour market:

\[
L^d_t = L^s_t = L_t
\]

Thirdly, the equity holdings are normalised:

\[
N_t = 1
\]

5.3.3 Model implementation

This section outlines the required conditions and specifications of the equilibrium including the representative investor and the representative firm together with the parameter setting for implementing the model to capture a dynamic stochastic general equilibrium economy.

The optimisation problems of the representative firm and investor (equations 5.3.1 and 5.3.6) are solved simultaneously and simulated to reach an equilibrium status. Carrying out the simulation through MATLAB and DYNARE for DSGE models requires
five first order conditions (FOCs) of the optimisation problems. To obtain the main
FOCs, each decision maker’s problem is solved by maximising the key variables. The
agent’s optimisation process is to maximise his/her utility of consumption, labour sup-
ply and the equity holdings. It is similar to the representative firm, in that the firm
maximises its utility based on capital, labour and investment inputs. The main FOCs
are in the form of:

$$W_t = (1 - \alpha) \frac{Y_t}{L_t}$$  \hspace{1cm} (5.3.11)

$$\frac{1}{g'_t(\frac{L_t}{K_{t-1}})K_{t-1}} = E_t\{\phi D_t^{-\gamma} \frac{\partial f}{\partial K_t} [Z_{t+1} \frac{1}{g'_t(\frac{L_t}{K_{t-1}})K_{t-1}} + 1 - \eta + g(\frac{L_{t+1}}{K_t}) + g_K(\frac{L_{t+1}}{K_t})K_t] \}$$  \hspace{1cm} (5.3.12)

$$R_{t+1} = Z_{t+1} \frac{\partial f}{\partial K_t} + \frac{1 - \eta + g(\frac{L_{t+1}}{K_t}) + g_K(\frac{L_{t+1}}{K_t})K_t}{g'_t(\frac{L_t}{K_{t-1}})K_{t-1}}$$  \hspace{1cm} (5.3.13)

$$W_t = \frac{\frac{\partial U(C_t, C_{t-1}, 1 - L_t^s)}{\partial L_t^s}}{\frac{\partial U(C_t, C_{t-1}, 1 - L_t^s)}{\partial C_t} + \beta \frac{\partial U(C_{t+1}, C_{t+1}, 1 - L_{t+2}^s)}{\partial C_t}}$$  \hspace{1cm} (5.3.14)

$$Q_t = E_t[M_{t+1}(Q_{t+1} + D_{t+1})]$$  \hspace{1cm} (5.3.15)

where $$M_{t+1} = \beta\left(\frac{\partial U(C_{t+1}, C_{t+1}, 1 - L_{t+1}^s)}{\partial C_{t+1}} + \beta \frac{\partial U(C_{t+2}, C_{t+1}, 1 - L_{t+2}^s)}{\partial C_{t+1}}\right)$$  \hspace{1cm} (5.3.16)

$R$ is defined as the return on capital. $M$ represents the intertemporal marginal rate of
substitution in consumption from the representative household’s section.

Finally, the equilibrium can be established when these conditions and the market
clearing conditions (in section 5.3.2) are satisfied. The results including steady state
values, standard deviations and cross-correlations are displayed below.

To implement a general equilibrium for a VM model, we have to specify three
functions: (1) the discount factor from the firm’s utility function, $\phi$, (2) CAC and (3)
the household’s utility function. The mathematical process is reported below:

(1) To specify the firm’s discount factor:

In a value-maximising environment, the firm’s stochastic discount factor is assumed to be equal to the intertemporal substitution with respect to consumption \( M_{t+1} \) which is identified by the representative household’s optimisation process.

The FOC of capital (equation 5.3.12), for a VM-based DSGE model, is therefore calibrated and displayed as:

\[
\frac{1}{g(I/K_{t-1})K_{t-1}} = E_t \{ M_{t+1}[\alpha Z_{t+1}K_t^{\alpha-1}(xL_{t+1})]^{1-\alpha} + \frac{1 - \eta + g(I_{t+1}) + g_K(I_{t+1})K_t}{g(I/K_{t-1})K_{t-1}} \} \tag{5.3.17}
\]

(2) To specify the firm’s CAC function

The specification of the capital adjustment cost (CAC) function is borrowed from Gershun (2010) whose paper presents a detailed CAC function. The form is given by:

\[
g(I_t/K_{t-1}) = \frac{\eta^\frac{1}{\zeta}}{1 - \frac{1}{\zeta}} (\frac{I_t}{K_{t-1}})^{1 - \frac{1}{\zeta}} + \frac{\eta}{1 - \zeta} \tag{5.3.18}
\]

This specification of CAC is close to the form used in Jermann (1998). \( \eta \) is the depreciation ratio coming from the firm’s capital accumulation process. With this CAC function, the capital accumulation function (equation 5.3.5) is rewritten as:

\[
K_t = (1 - \eta)K_{t-1} + g(I_t/K_{t-1})K_{t-1} \tag{5.3.19}
\]

\[
\Rightarrow K_t = (1 - \eta)K_{t-1} + \left[ \frac{\eta^\frac{1}{\zeta}}{1 - \frac{1}{\zeta}} (\frac{I_t}{K_{t-1}})^{1 - \frac{1}{\zeta}} + \frac{\eta}{1 - \zeta} \right]K_{t-1} \tag{5.3.20}
\]

\[
\Rightarrow K_t = [1 - \eta + \frac{\eta^\frac{1}{\zeta}}{1 - \frac{1}{\zeta}} (\frac{I_t}{K_{t-1}})^{1 - \frac{1}{\zeta}} + \frac{\eta}{1 - \zeta}] K_{t-1} \tag{5.3.21}
\]

The above rearranged capital accumulation function clearly shows that the relationship between previous and current capital is determined by three factors: the investment and capital ratio, the depreciation ratio and the CAC parameter.

Since the depreciation ratio is given (which is constant) and the ratio of investment
and capital is stable in equilibrium, the major factor that causes fluctuations in capital is the CAC parameter. The computational test shows that the greater CAC (which means the smaller the CAC parameter), the less volatile the capital accumulation process.

With the given CAC function, the partial derivatives of the function $g(\cdot)$ at $K_t$, $I_t$ and $I_{t+1}$ can be obtained and used for the FOC of capital, which are:

$$g_K'(\frac{I_{t+1}}{K_t}) = (-1)\eta^\frac{1}{2}(\frac{I_{t+1}}{K_t})^{-\frac{1}{2}}\frac{I_{t+1}}{K_t^2} \quad (5.3.22)$$

$$g_I'(\frac{I_t}{K_{t-1}}) = \eta^\frac{1}{2}(\frac{I_t}{K_{t-1}})^{-\frac{1}{2}}\frac{1}{K_{t-1}} \quad (5.3.23)$$

Applying the above equations to equation 5.3.17 gives a FOC of capital for the general equilibrium economy as:

$$\frac{1}{\eta^\frac{1}{2}(\frac{I_{t+1}}{K_{t-1}})^{-\frac{1}{2}}} = E_t\{M_{t+1}[\alpha Z_{t+1}K_t^{\alpha-1}(xL_{t+1})^{1-\alpha} + \frac{1-\eta + \frac{\eta}{\zeta}}{\eta^\frac{1}{2}(\frac{I_{t+1}}{K_t})^{-\frac{1}{2}}} + (\frac{1}{\zeta-1})(\frac{I_{t+1}}{K_t})]\} \quad (5.3.24)$$

The formula placed in middle brackets $[]$ on the right-hand of the above equation 5.3.24 can be defined as the expected gross return on capital:

$$R_{t+1} = \alpha Z_{t+1}K_t^{\alpha-1}(xL_{t+1})^{1-\alpha} + \frac{1-\eta + \frac{\eta}{\zeta}}{\eta^\frac{1}{2}(\frac{I_{t+1}}{K_t})^{-\frac{1}{2}}} + (\frac{1}{\zeta-1})(\frac{I_{t+1}}{K_t}) \quad (5.3.25)$$

In the case that a DSGE model incorporates CAC, the gross return is derived not only by output but also by the CAC parameter and the investment and capital ratio. Consequently, the model would generate a volatile gross rate of return on capital. The more CAC, the greater the standard deviation of the gross return. This is because it is less feasible for the firm to adjust the investment plan, and the consumer has to face more uncertainties regarding the rate he/she invests at.

(3) To specify the investor's utility function

This chapter utilises the previous chapter's specification of the momentary utility
function to define the agent’s preferences. It is a multiplicative form of consumption and leisure and consists of a linear (or separable) internal habit:

\[
U(C_t, C_{t-1}, 1 - L_t) = \left[\frac{(C_t - \xi C_{t-1})^\rho (1 - L_t)^{1-\rho}}{1 - \gamma}\right]^{1 - \gamma} \tag{5.3.26}
\]

Given that the consumer’s utility is determined only by the choice of consumption when \(\rho = 1\), the hours worked from the labour supply market could not be decided simultaneously in equilibrium. Jermann (1998)’s model is calibrated with this specification that there is no optimal labour in the general economy. The first model of this chapter presents how the economy reacts in this kind of situation, where the value for \(\rho\) is set equal to one.

### 5.3.4 Calibration

Now I provide parameter values for an example of simulating this model. The model is parameterised on a set of values for technology and preferences. Apart from capital adjustment costs, I use standard values for all other parameters which are borrowed from Jermann (1998). Table 5.3.1 on the next page presents the parameter values.

**Technical change and technology shock** The capital elasticity of output \(\alpha\) is given a value of 0.36. Accordingly, the labour share for the overall output is 0.64. The depreciation rate \(\eta\) is set equal to a quarterly value of 0.025 (10% at an annual rate). A quarterly trend in labour augmenting technical change, \(x\), is assumed to be 1.005 implying a quarterly growth rate of 0.005 (2% at an annual rate). The economy is under a stochastic technology shock: \(Z\) is assumed to follow an autoregressive process of order 1 ((AR(1)) with the persistence parameter \(\psi = 0.99\) and the standard deviation \(\sigma_\epsilon = 0.01\).

**Capital adjustment costs** In order to examine the sensitivity of real aggregated variables to the level of capital adjustment costs, I vary the value of CAC parameter. The first value, suggested in Gershun (2010), is given as \(\zeta_L = 40\). This implies
Table 5.3.1: Parameter values for the VM model with fixed labour supply

This table lists the parameter values used to solve and simulate the dynamic stochastic general equilibrium (DSGE) value-maximising (VM) model with fixed labour. All the parameters are categorised into two groups. The first group, Technology, includes parameters in the firm’s production function and capital adjustment cost (CAC) function. The value of the CAC parameter, $\zeta_L = 40$, is taken from Gershun (2010) which implies low capital adjustment costs. The value of the CAC parameter, $\zeta_H = 0.23$, is given by Jermann (1998) and leads to high capital adjustment costs. The second group, Preferences, is calibrated for the firm’s and the investor’s utility function. A value-maximising firm does not compute its profits based on utility, and therefore the coefficient of relative risk aversion for the firm is zero. The individual household has an internal habit formation. The value for the habit persistence is taken from Constantinides (1990). The model is with an exogenous labour which is $L = 1$. Hence, the individual’s utility is only on consumption as the given assumption that the value for determining the time allocated to market activities (consumption) is one.

<table>
<thead>
<tr>
<th>Technology</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital share to output</td>
<td>$\alpha$</td>
<td>0.36</td>
</tr>
<tr>
<td>Depreciation ratio</td>
<td>$\eta$</td>
<td>0.025</td>
</tr>
<tr>
<td>Trend in labour augmenting technical change</td>
<td>$\chi$</td>
<td>1.005</td>
</tr>
<tr>
<td>Persistence of the technology shock</td>
<td>$\psi$</td>
<td>0.99</td>
</tr>
<tr>
<td>Standard deviation of the technology shock</td>
<td>$\sigma_\epsilon$</td>
<td>0.01</td>
</tr>
<tr>
<td>A value leads to low capital adjustment costs</td>
<td>$\zeta_L$</td>
<td>40</td>
</tr>
<tr>
<td>A value leads to high capital adjustment costs</td>
<td>$\zeta_H$</td>
<td>0.23</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Preferences</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient of relative risk aversion - the firm</td>
<td>$\gamma_F$</td>
<td>0</td>
</tr>
<tr>
<td>Subjective discount factor - the investor</td>
<td>$\beta$</td>
<td>0.99</td>
</tr>
<tr>
<td>Coefficient of relative risk aversion - the investor</td>
<td>$\gamma$</td>
<td>5</td>
</tr>
<tr>
<td>Habit persistence</td>
<td>$\xi$</td>
<td>0.82</td>
</tr>
<tr>
<td>Value for determining the time allocated to market activities</td>
<td>$\rho$</td>
<td>1.00</td>
</tr>
</tbody>
</table>
low capital adjustment costs. The second value, given by Jermann (1998), is set to $\zeta_H = 0.23$ which leads to high capital adjustment costs.

**Firm’s preferences** The coefficient of the risk aversion for the firm, $\gamma_F$, is set equal to zero, implying the economy is with a value-maximising firm which is risk neutral to its future cash flows.

**Investor’s preferences** The quarterly discount rate, $\beta$, is equal to 0.99. The relative risk aversion $\gamma$ is set to 5 in both of the models. The estimate of the agent’s internal consumption habit persistence parameter $\xi$ is the equivalent of 0.82. The weighted ratio of time devoted to consumption ($\rho$) in the momentary utility function is assumed to be one implying that the agent’s expected utility totally depends on consuming goods. In terms of this parameter value, the investor’s preferences in the model are analogous to that of Jermann (1998).

### 5.3.5 Solving methods

The moments of the economic variables are computed and evaluated by running simulations with two techniques: MATLAB and DYNARE.

### 5.3.6 Results and discussion

The dynamic stochastic general equilibrium model with costly capital adjustment costs provides the important finding that the fluctuation of aggregate dividends decreases while the variance of stock prices increases. This result gives a message that the equity market is volatile in the presence of smoothing aggregate dividend policy in the economy.

Considering the friction of CAC in a DSGE model therefore enables us to examine the feasibility of aggregate dividend policy. The higher the CAC, the more inelastic the aggregate dividend policy.

Table 5.3.2 presents the simulated and observed results of (1) volatility of each variable and (2) volatility of each variable relative to the volatility of output.
5.3.6.1 The statistics for the US economy


Hansen (1985) uses quarterly US data including real gross national product (GNP), total consumption expenditures, gross private domestic investment, nonresidential equipment and structures for the capital stock, and the total hours for persons at work in non-agricultural industries for the work hours. Other US time series in different observed periods are from two sources. I use the online data provided by Shiller (1992) for data of dividends, consumption and stock prices. GNP and private fixed investment for output and investment respectively are from Federal Reserve Economic Data (FRED). All real data are detrended with a Hodrick-Prescott filter.

5.3.6.2 Cyclical variability

The volatility of dividends, $\sigma_D$, is decreasing from 6.16\% to 4.56\% when CAC are greater. This is intuitively consistent with an inflexible investment decision. Therefore, dividend policy is inelastic over time. Adjustment costs make the firm less willing to alter its capital accumulation process. This consequence influences the variability of dividend payouts. Since the firm is not flexible to alter the investment decision, dividends determined by its residual earnings after the investment decision become less volatile when it comes to higher capital adjustment costs.

The observed volatility of personal dividend income on the period 1985.1-2009.4 is 2.01\% which is far below to the simulated result. However, the variability of aggregate dividends to output, $\frac{\sigma_D}{\sigma_Y}$, in the high CAC case, is 3.44 which is close to the observed statistics, 3.19. Both theoretical and empirical results imply that aggregate dividends fluctuate roughly three times more than output does.

The volatility of consumption, $\sigma_c$, in the high CAC framework is nearly two times

\textsuperscript{3}The number after the decimal is the number of quarters in a year. For example, 1955.3 is defined as the third quarter, which is from July to September, of the year 1955.
Table 5.3.2: Fluctuations of economic variables for the VM model with fixed labour supply

This table reports the relative standard deviation (RSD) of each variable (dividends ($D$), consumption ($C$), output ($Y$), capital ($K$), labour ($L$), investment ($I$), the gross return on capital ($R$) and the share price ($Q$)) and each RSD to the RSD of output in fixed-labour value-maximising (VM) models under cases of no capital adjustment costs (CAC), low CAC, high CAC and habits. For the US data, four data sets are provided. a: The first US quarterly data for the period 1955.3-1984.1 is borrowed from Hansen (1985). He uses quarterly US data of real gross national product (GNP), total consumption expenditures, gross private domestic investment, the total of nonresidential equipment and structures for the capital stock, and the total hours for persons at work in non-agricultural industries for the work hours. b, c and d: I compute quarterly fluctuations of the US economy for the periods 1955.3-1984.1, 1985.1-2009.4 and 1959.1-2009.4. The data for US time series in different observed periods are from two sources: I use the online data provided by Shiller (1992) for data of dividends, consumption and stock prices; GNP and private fixed investment for output and investment respectively are from Federal Reserve Economic Data (FRED). All real data are detrended with a Hodrick-Prescott filter.

### Section I: Relative standard deviations (RSD) in percent of economic variables

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_D$</th>
<th>$\sigma_C$</th>
<th>$\sigma_Y$</th>
<th>$\sigma_K$</th>
<th>$\sigma_I$</th>
<th>$\sigma_R$</th>
<th>$\sigma_Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No CAC; with habits</td>
<td>4.63</td>
<td>0.76</td>
<td>1.33</td>
<td>0.31</td>
<td>4.09</td>
<td>4.12</td>
<td>4.56</td>
</tr>
<tr>
<td>Low CAC ($\zeta_L = 40$); with habits</td>
<td>6.16</td>
<td>0.57</td>
<td>1.317</td>
<td>0.34</td>
<td>4.20</td>
<td>0.15</td>
<td>0.37</td>
</tr>
<tr>
<td>High CAC ($\zeta_H = 0.23$); with habits</td>
<td>4.56</td>
<td>1.02</td>
<td>1.325</td>
<td>0.21</td>
<td>3.16</td>
<td>13.78</td>
<td>13.92</td>
</tr>
<tr>
<td>US quarterly data (1955.3 - 1984.1)$^a$</td>
<td>-</td>
<td>1.29</td>
<td>1.76</td>
<td>0.63</td>
<td>8.60</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>US quarterly data (1955.3 - 1984.1)$^b$</td>
<td>1.62</td>
<td>0.51</td>
<td>1.12</td>
<td>-</td>
<td>2.76</td>
<td>-</td>
<td>7.22</td>
</tr>
<tr>
<td>US quarterly data (1985.1 - 2009.4)$^c$</td>
<td>2.01</td>
<td>0.38</td>
<td>0.63</td>
<td>-</td>
<td>2.22</td>
<td>-</td>
<td>8.27</td>
</tr>
<tr>
<td>US quarterly data (1959.1 - 2009.4)$^d$</td>
<td>1.72</td>
<td>0.45</td>
<td>0.89</td>
<td>-</td>
<td>2.49</td>
<td>-</td>
<td>7.78</td>
</tr>
</tbody>
</table>

### Section II: Relative standard deviations (RSD) of each variable to RSD of output

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_D/\sigma_Y$</th>
<th>$\sigma_C/\sigma_Y$</th>
<th>$\sigma_Y/\sigma_Y$</th>
<th>$\sigma_K/\sigma_Y$</th>
<th>$\sigma_I/\sigma_Y$</th>
<th>$\sigma_R/\sigma_Y$</th>
<th>$\sigma_Q/\sigma_Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No CAC; with habits</td>
<td>3.48</td>
<td>0.57</td>
<td>1.00</td>
<td>0.23</td>
<td>3.08</td>
<td>3.10</td>
<td>3.42</td>
</tr>
<tr>
<td>Low CAC ($\zeta_L = 40$); with habits</td>
<td>4.68</td>
<td>0.43</td>
<td>1.00</td>
<td>0.26</td>
<td>3.19</td>
<td>0.11</td>
<td>0.28</td>
</tr>
<tr>
<td>High CAC ($\zeta_H = 0.23$); with habits</td>
<td>3.44</td>
<td>0.77</td>
<td>1.00</td>
<td>0.16</td>
<td>2.38</td>
<td>10.40</td>
<td>10.51</td>
</tr>
<tr>
<td>US quarterly data (1955.3 - 1984.1)$^a$</td>
<td>-</td>
<td>0.73</td>
<td>1.00</td>
<td>0.36</td>
<td>4.89</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>US quarterly data (1955.3 - 1984.1)$^b$</td>
<td>1.44</td>
<td>0.45</td>
<td>1.00</td>
<td>-</td>
<td>2.46</td>
<td>-</td>
<td>6.45</td>
</tr>
<tr>
<td>US quarterly data (1985.1 - 2009.4)$^c$</td>
<td>3.19</td>
<td>0.60</td>
<td>1.00</td>
<td>-</td>
<td>3.52</td>
<td>-</td>
<td>13.10</td>
</tr>
<tr>
<td>US quarterly data (1959.1 - 2009.4)$^d$</td>
<td>1.94</td>
<td>0.51</td>
<td>1.00</td>
<td>-</td>
<td>2.81</td>
<td>-</td>
<td>8.76</td>
</tr>
</tbody>
</table>
that of the low CAC case (1.02% vs. 0.57%). The message of this finding is as follows: Since dividends vary slightly, the investor has to stay with a volatile consumption to balance its budget constraint over periods.

In equilibrium, the result shows that the level of CAC do not have a significant impact on the magnitude of fluctuations in output. From an analytical point of view, this is consistent with a higher volatility of consumption and a lower volatility of investment deriving a flat output over time if the equilibrium condition of the goods market holds. The total value of products is equal to the sum of consumption and investment in the market.

In section II of Table 5.3.2, the results of values of the relative standard deviation of each variable to the standard deviation of output (e.g. \( \frac{\sigma_D}{\sigma_Y} \), \( \frac{\sigma_C}{\sigma_Y} \), \( \frac{\sigma_I}{\sigma_Y} \), \( \frac{\sigma_Q}{\sigma_Y} \)) help us evaluate the sensitivity of the real economic variable to output. With low CAC, the variability of dividends is about five times that of output. This figure moves down to, roughly, three and half times that of output when CAC increases. The reason is that dividends become less volatile while output remains the same. The same happens to capital and investment.

Consumption fluctuates much more in the high CAC framework than in the low CAC one. They are, respectively, around 43 and 77 percent of the output’s fluctuation in the low and high CAC cases. The standard deviation of the gross return on capital gets greater when CAC are higher. It is obvious that consumption absorbs the variability of output when the firm’s investment and dividend payout decision are invariable over time.

Overall, aggregate capital adjustment costs substantially affect the fluctuations of dividends, consumption and investment but have less impact on aggregate output. The aggregate investment decision is not independent of aggregate consumption. This model verifies the theorem in Jermann (1998), that aggregate investment breaks the scheme of smoothing consumption in the DSGE model if there are CAC in the production economy. Consumption volatility increases when capital adjustment costs are higher. In addition, the more volatile consumption, the smaller the volatility of aggregate dividends (details in chapter 3). Thus the standard deviation of aggregate
dividends becomes small because high CAC influence the variability of consumption. The more costly the CAC, the more stable the aggregate dividend policy.

5.3.6.3 Cross-correlations

Table 5.3.3 presents the co-movements between dividends, output, investment, equity prices, consumption and capital.

The model shows that dividends are negatively related to economic fluctuations (countercyclical variable), which is not in line with the empirical results of Chapter 2 that have shown the observed net cash flows have a positive relationship with Gross National Product during the sample period (see Section 2.2.8). The reason that the simulated result is not consistent with the observed data may be because theoretical models apply several assumptions, such as the residual dividend policy, capital adjustment cost function and habit formation, leading to differences in the results for some variables (e.g. dividends) from the empirical result.

The value, $corr(D, Y)$, from -0.87 drops to -0.46 while the economy is with high CAC instead of low CAC. It is apparent that the firm with high adjustment costs is less flexible on adjusting the dividend policy.

The price of equity is significantly associated with aggregate dividends when there are huge CAC. The value of $corr(D, Q)$ from -0.31 (low CAC) to -0.88 (high CAC). This implies that there is a strong and negative connection between the equity market and the aggregate dividend policy when there is high CAC in the economy.

Row 4 of Table 5.3.3 shows the result that there is no strong change in relationship between dividends and consumption when the level of CAC changes. Consumption is strongly procyclical. The model simulates a significantly positive correlation with the overall state of the economy when CAC are increasing. The value of $corr(C, Y)$ is 0.83 in the case of high CAC and 0.67 of low CAC. The results are as expected, investment, capital and equity prices are positively related to the business cycle in both situations of high and low CAC (last three rows of Table 5.3.3).
5.4 The advanced value-maximising DSGE model

That the utility function only depends on the choice of consumption in the economy is debatable. The investor’s utility should involve not only consumption but also leisure (the substitution of working). Since Jermann (1998)’s model does not maximise the household’s objective function with respect to labour, it is still an issue whether the variation of consumption is greater or unchanged by the friction of CAC in the general equilibrium economy. Will the theory, that the model with habits and CAC increases fluctuations in consumption, still hold if the optimal labour is solved simultaneously in both the firm’s and the household’s objective functions?

This section revises the previous model by allowing for the choice of leisure in the representative household’s utility function. In addition, the given parameter value for the shock persistence is too high, so that the productivity has to take some time and wait for the shock to disappear accordingly. Therefore, this section is to demonstrate another
model, called “the advanced VM model” afterwards, by adjusting three parameter values to improve its effectiveness in the real business cycle: (1) an endogenous labour by adjusting the fraction of time devoted to consumption; (2) the shock persistence; and (3) the standard deviation of the technology shock. The results display different implications of CAC to the fluctuation in optimal aggregate payouts.

5.4.1 Calibration and solving method

The previous setting up for the parameter $\rho$, $\rho = 1$ implies that the household does not take his/her choice of leisure into account in determining utility. If there is no leisure in the representative investor’s utility function, it is impossible to solve the investor’s optimisation problem through maximising the labour supply (e.g. hours worked). As a result, the general equilibrium economy is unable to obtain an optimal level of labour.

In order to obtain an optimal level of labour from both decision makers (the firm and the household), the parameter for the fraction of time devoted to consumption is set to 0.36. The investor’s momentary utility function therefore involves both consumption and leisure under this assumption. Given that the time endowment between leisure time and the labour market is normalised to one, the expected utility is solved by maximising the labour supply. The more time an employee spends working, the less time left for the activity of leisure. Hours worked certainly influence the investor’s utility accordingly.

The value for shock persistence, another parameter used in the previous model, is also adjusted here. The last section used a value of 0.99 as given in Jermann (1998). But I find that this figure is close to the upper bound of a benchmark range [0.95, 1.00]. Hansen (1985) measures the production function residual by using US data on output, capital and labour inputs and obtains an autocorrelation coefficient for the technology shock persistence of about 0.95. If the parameter value used in the previous model is adjusted to a lower value, $\psi = 0.95$, it is then expected that the model will obtain a reasonable procyclical labour.

Meanwhile, the standard deviation of the technology shock might also have been too high in the previous calibration. According to Hansen (1985), the standard deviation of the technology shock lies in the interval [0.007, 0.01]. His paper set a value of 0.00712.
The purpose is to satisfy the condition that the theoretical mean standard deviation of output is equal to the observed US GNP for the economy, which is $\sigma_Y = 1.76\%$. I set the value of the standard deviation of the technology shock as 0.00712. In addition, the observed standard deviations of GNP for the US economy in the most recent five decades (from 2nd quarter of 1959 to 4th quarter of 2009) and a shorter period (from 1st quarter of 1985 to 4th 2009) are computed subsequently.

Overall, the table below summarises the changes between the previous model and the new model.

<table>
<thead>
<tr>
<th>Parameter values</th>
<th>$\rho$</th>
<th>$\psi$</th>
<th>$\sigma_\epsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Previous setting up</td>
<td>1.00</td>
<td>0.99</td>
<td>0.01</td>
</tr>
<tr>
<td>Revised setting up</td>
<td>0.36</td>
<td>0.95</td>
<td>0.00712</td>
</tr>
</tbody>
</table>

Parameter $\rho$ is a value for determining the time allocated to market activities, e.g. consumption, $\psi$ is for the value of the technology shock persistence with a given standard deviation of the shock $\sigma_\epsilon$.

**Solving methods** To solve the model, I provide a complete DYNARE code for this calibration in Appendix 5.8.1 on page 201.

### 5.4.2 Results and discussion

This subsection reports examples of simulating the advanced VM model with the different set of parameter values. I develop four scenarios which display the economy is (1) without habits and CAC (benchmark 1); (2) with habits but no CAC (benchmark 2); (3) with habits and low CAC; and (4) with habits and high CAC respectively. Table 5.4.1 provides a summary of the criteria that each scenario fulfils.
Table 5.4.1: Criteria for each scenario of the advanced VM model
This table lists the similarities and differences of four models in this chapter. Two benchmark models are not calibrated with capital adjustment costs (CAC) and one of them does not have habit formation. Two value-maximising (VM) models are with capital adjustment costs and habits but one is with costly capital adjustment costs and another is with a lower level of capital adjustment costs.

<table>
<thead>
<tr>
<th>Benchmark 1: no CAC and no habits</th>
<th>CAC</th>
<th>habits</th>
<th>Parameter values(^a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark 2: no CAC but habits</td>
<td>-</td>
<td>v</td>
<td>(\zeta \to \infty; \xi = 0.82)</td>
</tr>
<tr>
<td>Advanced VM model - case 1: low CAC and habits</td>
<td>v</td>
<td>v</td>
<td>(\zeta = 40; \xi = 0.82)</td>
</tr>
<tr>
<td>Advanced VM model - case 2: high CAC and habits</td>
<td>v</td>
<td>v</td>
<td>(\zeta = 0.23; \xi = 0.82)</td>
</tr>
</tbody>
</table>

\(^a\)The parameter \(\zeta\) determines the level of capital adjustment costs (CAC), the higher the number, the lower the level of CAC. \(\xi\) decides the strength of habit motive.

5.4.2.1 Fluctuations and cyclical variability

Table 5.4.2 presents the performance of the advanced VM models in cases of low and high capital adjustment costs (CAC). The model successfully shows that CAC are a friction that strongly affects the production economy. Results show that high CAC reduce the volatilities of dividends and investment dramatically. With variable labour supply, the relative standard deviation of aggregate dividends reduces dramatically compared to the model with fixed labour supply. Results show that, in the equilibrium economy, aggregate dividend policy remains stable over time while stock prices are volatile in the presence of high costly CAC.

The optimal labour (hours worked) fluctuates greatly within an economy with high CAC. In addition, fluctuations in financial variables (the gross return) are increasing in the presence of high adjustment costs. This model delivers an important message that the level of CAC enormously affects the firm’s investment and dividend decisions.

In the case of high CAC, the volatility of dividends, output, capital and investment drops dramatically compared to the case of low CAC. When the level of capital adjustment costs is high, dividends fluctuate from 6.12% to 1.06% while the variability of the equity price, \(Q\), moves from 0.37% (low adjustment costs) to 3.42% (high adjustment costs). The firm having costly capital adjustment process hesitates to alter future investment plans and remain a smooth dividend policy over time. In fact, a smooth and stable path of dividend payments in the long run attracts households because there
Table 5.4.2: Fluctuations of economic variables for the advanced VM model

This table reports the relative standard deviation (RSD) of each variable (dividends ($D$), consumption ($C$), output ($Y$), capital ($K$), labour ($L$), investment ($I$), the gross return on capital ($R$) and the share price ($Q$)) and each RSD to the RSD of output in various variable-labour value-maximising (VM) DSGE models with/without capital adjustment costs (CAC) and habits. I also compute quarterly fluctuations of the US economy on periods 1955.3-1984.1, 1985.1-2009.4 and 1959.1-2009.4. The data for US time series in different observed periods are from two sources. I use the online data provided by Shiller (1992) for data of dividends, consumption and stock prices. GNP and private fixed investment for output and investment respectively are from Federal Reserve Economic Data (FRED). All real data are detrended with a Hodrick-Prescott filter.

### Section I: Relative standard deviations (RSD) in percent of economic variables

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_D$</th>
<th>$\sigma_C$</th>
<th>$\sigma_Y$</th>
<th>$\sigma_K$</th>
<th>$\sigma_L$</th>
<th>$\sigma_I$</th>
<th>$\sigma_R$</th>
<th>$\sigma_Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark 1 - no CAC; no habits</td>
<td>3.44</td>
<td>0.57</td>
<td>1.20</td>
<td>0.28</td>
<td>0.43</td>
<td>3.07</td>
<td>0.04</td>
<td>0.28</td>
</tr>
<tr>
<td>Benchmark 2 - no CAC; with habits</td>
<td>6.29</td>
<td>0.35</td>
<td>1.21</td>
<td>0.35</td>
<td>0.43</td>
<td>4.19</td>
<td>0.04</td>
<td>0.35</td>
</tr>
<tr>
<td>Advanced VM model:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>low CAC; with habits</td>
<td>6.12</td>
<td>0.348</td>
<td>1.18</td>
<td>0.35</td>
<td>0.39</td>
<td>4.08</td>
<td>0.14</td>
<td>0.37</td>
</tr>
<tr>
<td>high CAC; with habits</td>
<td>1.06</td>
<td>0.350</td>
<td>0.40</td>
<td>0.06</td>
<td>0.78</td>
<td>3.39</td>
<td>3.42</td>
<td></td>
</tr>
<tr>
<td>US quarterly data (1955.3 - 1984.1)</td>
<td>1.62</td>
<td>0.51</td>
<td>1.12</td>
<td></td>
<td></td>
<td>2.76</td>
<td>7.22</td>
<td></td>
</tr>
<tr>
<td>US quarterly data (1985.1 - 2009.4)</td>
<td>2.01</td>
<td>0.38</td>
<td>0.63</td>
<td></td>
<td></td>
<td>2.22</td>
<td>8.27</td>
<td></td>
</tr>
<tr>
<td>US quarterly data (1959.1 - 2009.4)</td>
<td>1.72</td>
<td>0.45</td>
<td>0.89</td>
<td></td>
<td></td>
<td>2.49</td>
<td>7.78</td>
<td></td>
</tr>
</tbody>
</table>

### Section II: Relative standard deviations (RSD) of each variable to RSD of output

<table>
<thead>
<tr>
<th></th>
<th>$\frac{\sigma_D}{\sigma_Y}$</th>
<th>$\frac{\sigma_C}{\sigma_Y}$</th>
<th>$\frac{\sigma_Y}{\sigma_Y}$</th>
<th>$\frac{\sigma_K}{\sigma_Y}$</th>
<th>$\frac{\sigma_L}{\sigma_Y}$</th>
<th>$\frac{\sigma_I}{\sigma_Y}$</th>
<th>$\frac{\sigma_R}{\sigma_Y}$</th>
<th>$\frac{\sigma_Q}{\sigma_Y}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark 1 - no CAC; no habits</td>
<td>2.86</td>
<td>0.47</td>
<td>1.00</td>
<td>0.23</td>
<td>0.36</td>
<td>2.55</td>
<td>0.04</td>
<td>0.23</td>
</tr>
<tr>
<td>Benchmark 2 - no CAC; with habits</td>
<td>5.20</td>
<td>0.29</td>
<td>1.00</td>
<td>0.29</td>
<td>0.35</td>
<td>3.46</td>
<td>0.04</td>
<td>0.29</td>
</tr>
<tr>
<td>Advanced VM model:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>low CAC; with habits</td>
<td>5.17</td>
<td>0.29</td>
<td>1.00</td>
<td>0.29</td>
<td>0.33</td>
<td>3.45</td>
<td>0.12</td>
<td>0.32</td>
</tr>
<tr>
<td>high CAC; with habits</td>
<td>2.61</td>
<td>0.86</td>
<td>1.00</td>
<td>0.16</td>
<td>2.29</td>
<td>1.92</td>
<td>8.36</td>
<td>8.44</td>
</tr>
<tr>
<td>US quarterly data (1955.3 - 1984.1)</td>
<td>1.44</td>
<td>0.45</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US quarterly data (1985.1 - 2009.4)</td>
<td>3.19</td>
<td>0.60</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US quarterly data (1959.1 - 2009.4)</td>
<td>1.94</td>
<td>0.51</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
is less uncertainty of variability of firms paying dividends. Investors can benefit from stable cash incomes and smooth their consumption. On the contrary, a firm becomes less flexible to utilise its free cash flows, some positive NPV investment projects are delayed. The stock market, as a consequence, is much volatile in the economy with high CAC.

The magnitude of the fluctuation in dividends to that of output drops from five times to two and a half times (see $\sigma_D / \sigma_Y$ in Table 5.4.2, Section II). There is less deviation between dividends and output in the general equilibrium economy when CAC are very high to the firm.

Interestingly, the impact of high CAC on the social planner’s consumption is weak. In both situations, results for the volatilities of consumption are similar, 0.348% and 0.350% in the low and high CAC case respectively. The reason why the variation of consumption does not change greatly is because of labour market fluctuations. When changing investment plans wastes more profits than adjusting labour, the firm prefers to alter the labour input. Therefore, labour becomes volatile when CAC are high.

As mentioned before, the individual’s working hours fluctuate over periods due to the sticky production plan; $\sigma_L = 0.93\%$ when CAC are high and $\sigma_L = 0.39\%$ when CAC are small. Given that the economy involves high level of CAC, every unit increase of the volatility of output is driven by roughly two and a half unit increases of the volatility of labour input (see $\sigma_L / \sigma_Y$ in Table 5.4.2, Section II).

However, since the variability of output is decreasing, the variability of consumption to output is comparatively greater in the economy with high capital adjustment costs rather than with low adjustment costs.

The theoretical standard deviation of output, $\sigma_Y$, is 1.18% in the low CAC economy and it is 0.40% in the high CAC economy. An assumption introduced in Hansen (1985) is that the theoretical result of $\sigma_Y$ should be equal to the observed value when the standard deviation of technology shock is $\sigma_\epsilon = 0.00712$. I compute the standard deviation of GNP taken from the Federal Reserve Bank of St. Louis and obtain a value of 0.89% for the most recent five decades (see $\sigma_Y$ in Table 5.4.2, Section I, Row 4). It is not equal, but close to that in the case of low CAC. Since the focus here is
not a precise value for the standard deviation of technology shock, I leave this finding for scholars who specialise in the research area of estimating parameters for general equilibrium models.

The variability of investment, $\sigma_I$, in the case of low CAC is 4.08% which is higher than the observed US data (1959.1 - 2009.4), 2.49%. In effect, this model (with low CAC) could be used to explain the variation of investment by introducing slightly more CAC. However, the case of high CAC, with the value of $\sigma_I$ equal to 0.78%, implies an underestimated variation in investment. Danthine and Donaldson (2002) point out that a dramatically high parameter value of adjustment costs ($\zeta$) causes an unacceptably low estimate of the variability of investment. The result from the model with high CAC verifies their viewpoint.

The gross return on capital becomes very volatile following high capital adjustment costs. Its volatility, respectively, is 0.14% for low CAC and 3.39% for high CAC. Corporations are less willing to alter physical capital plans and tend to adjust the amount of labour needed. This phenomenon makes the labour market significantly less stable over time.

5.4.2.2 Co-movements

In Table 5.4.3, it can be seen that as before aggregate dividends are countercyclical in the cases of without CAC or low CAC. Indeed, in a bull market, corporations prefer to have flexible cash flows for investment opportunities instead of paying dividends. Paying no dividends has a lesser effect on shareholders as long as the value of shares is increasing when corporations invest in projects with a NPV greater than zero.

However, the correlation between dividends (consumption) and output becomes weaker (stronger) when corporations have costly capital adjustment process. Labour is also negatively correlated with output in the presence of high frictions of capital adjustments. Net cash flows are inelastic and strongly correlated to capital input when the firm is unwilling to alter its investment plans because of costly capital adjustment process. The agent’s labour income is also influenced by inelastic cash flows of the firm. As a consequence, the agent’s consumption smoothing plan is distorted. This is
Table 5.4.3: Correlations of the benchmarks and the advanced VM models
This table displays the cross-correlation between output (Y), dividends (D), consumption (C), capital (K), labour (L), investment (I), the gross return on capital (R) and the share price (Q) in various models with/without (low/ high) capital adjustment costs (CAC) and habits. The advanced value-maximising (VM) model, different to previous fixed-labour VM models, includes an endogenous labour. The first advanced VM model is with low CAC and the parameter value for low CAC is taken from Gershun (2010). The second advanced VM model includes high CAC and its parameter value for high CAC is taken from Jermann (1998). The results are reported in three sections. Section I is the cross-correlation of output with other variables. Section II includes the cross-correlation of dividend with other variables. The final section presents the cross-correlation of consumption with others.

<table>
<thead>
<tr>
<th>Section I</th>
<th>$\text{Corr}(Y, \cdot)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark:</td>
<td>D</td>
</tr>
<tr>
<td>1 - no CAC; no habits</td>
<td>-0.99</td>
</tr>
<tr>
<td>2 - no CAC; with habits</td>
<td>-0.95</td>
</tr>
<tr>
<td>Advanced VM model:</td>
<td></td>
</tr>
<tr>
<td>low CAC; with habits</td>
<td>-0.95</td>
</tr>
<tr>
<td>high CAC; with habits</td>
<td>-0.20</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Section II</th>
<th>$\text{Corr}(D, \cdot)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark:</td>
<td>D</td>
</tr>
<tr>
<td>1 - no CAC; no habits</td>
<td>1.00</td>
</tr>
<tr>
<td>2 - no CAC; with habits</td>
<td>1.00</td>
</tr>
<tr>
<td>Advanced VM model:</td>
<td></td>
</tr>
<tr>
<td>low CAC; with habits</td>
<td>1.00</td>
</tr>
<tr>
<td>high CAC; with habits</td>
<td>1.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Section III</th>
<th>$\text{Corr}(C, \cdot)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark:</td>
<td>D</td>
</tr>
<tr>
<td>1 - no CAC; no habits</td>
<td>-0.97</td>
</tr>
<tr>
<td>2 - no CAC; with habits</td>
<td>-0.32</td>
</tr>
<tr>
<td>Advanced VM model:</td>
<td></td>
</tr>
<tr>
<td>low CAC; with habits</td>
<td>-0.32</td>
</tr>
<tr>
<td>high CAC; with habits</td>
<td>0.22</td>
</tr>
</tbody>
</table>

the reason that consumption is smooth in the only-with-habit model but not anymore in the with-CAC-and-habit model.

5.4.3 Robustness: the control model with an alternative CAC function
This section presents that a DSGE model with an alternative form of capital adjustment costs (CAC) as used in Collard and Dellas (2006). The results of the variability of the main macroeconomic variables - dividends, consumption, output and investment are
going to be compared with the advanced VM model. The hypothesis here is that the
previous advanced VM model should be supported by the model here even though the
CAC functions are different.

The objective of this study is to explore the similarities and differences between two
different CAC functions. This section replaces previous CAC function with that used by
Collard and Dellas (2006). The same or close business statistics capturing the general
equilibrium condition are expected.

The results show that the general equilibrium economy would stay similar regardless
of the capital accumulation process containing inseparable or separable capital adjust-
ment costs. However, the correlation of equity price is less related to the production
economy in the control model. This suggests the use of the inseparable CAC in the
DSGE model for the study of asset pricing implications.

5.4.3.1 Assumption

Collard and Dellas (2006) use a different function of capital adjustment costs. They
consider separate independent adjustment costs in the capital accumulation process,
which are:

\begin{equation}
K_t = (1 - \eta)K_{t-1} + I_t - \zeta \left( \frac{I_t}{K_{t-1}} - \eta \right)^2 K_{t-1}
\end{equation}

where the last term represents CAC. \( \zeta \) is defined as the CAC parameter. If \( \zeta = 0 \), there
are no capital adjustment costs and the economy is with a standard capital accumulation
process (i.e. \( K_t = (1 - \eta)K_{t-1} + I_t \)). In this model, the greater \( \zeta \), the more CAC in
the economy.

5.4.3.2 Model implementation

All the first-order conditions (FOCs) remain the same, except the one for solving the
aggregate capital. The FOC of \( K_t \) is revised because of a different CAC incorporated
into the model, which is:
1 = \mathbb{E}_t\{M_{t+1}[\alpha Z_{t+1}K_t^{\alpha-1}(xL_{t+1})^{1-\alpha} + 1 - \eta + \frac{\zeta}{2}(\frac{I_{t+1}^2}{K_t} - \eta^2)]\} \quad (5.4.2)

From the right-hand-side (RHS) of the above equation, we can obtain the gross return on capital:

\[ R_{t+1} = \alpha Z_{t+1}K_t^{\alpha-1}(xL_{t+1})^{1-\alpha} + 1 - \eta + \frac{\zeta}{2}(\frac{I_{t+1}^2}{K_t} - \eta^2) \quad (5.4.3)\]

The equilibrium can be solved by simulating all FOCs (equations 5.3.11, 5.3.14, 5.3.15 on page 163 and 5.4.2 on page 182) and market clearing conditions (equations 5.3.8, 5.3.9 and 5.3.10 on page 162) simultaneously.

5.4.3.3 Calibration and solving method

The parameter value for the CAC, \( \zeta \), is set equal to 10 as given by Collard and Dellas (2006). The rest of the parameter values are the same as in the previous advanced VM model. For example, the production process is assumed to have a standard technology shock following the persistence of \( \psi = 0.95 \) and the standard deviation of \( \sigma_\epsilon = 0.00712 \). The investor’s momentary utility function involves both consumption and leisure inputs.

A complete DYNARE code for calibrating the control model is provided in Appendix 5.8.2 on page 202.

5.4.3.4 Results and discussion

Table 5.4.4 presents the results of the advanced VM model in low and high CAC cases and the model with the CAC function of Collard and Dellas (2006) (called “the control model” afterwards). It is expected that regardless of the DSGE model having an inseparable or separable CAC function to the capital accumulation function, the model should display the same or close business statistics.

The results successfully indicate that the control model is able to produce similar results, including variability and cross-correlations to VM model does in the low level CAC case.
### Table 5.4.4: Fluctuations of economic variables - DSGE Models with different CAC functions

This table reports cyclical statistics for two dynamic stochastic general equilibrium (DSGE) models, including the relative standard deviation (RSD) of each variable (dividends \(D\), consumption \(C\), output \(Y\), capital \(K\), labour \(L\), investment \(I\), the gross return on capital \(R\)) and the share price \(Q\)) and each RSD to the RSD of output. The first model, called the advanced value-maximising (VM) model, is calibrated with (a) low capital adjustment costs (CAC), suggested by Gershun (2010), and (b) high CAC suggested by Jermann (1998) (note: Jermann (1998) and Gershun (2010) use the same CAC function). The control VM model is constructed with a different CAC function used by Collard and Dellas (2006). Section I reports the first set of statistics, the relative standard deviation, and Section II includes each variable’s movement with output.

#### Section I: Relative standard deviations (RSD) in percent of economic variables

<table>
<thead>
<tr>
<th></th>
<th>(\sigma_D)</th>
<th>(\sigma_C)</th>
<th>(\sigma_Y)</th>
<th>(\sigma_K)</th>
<th>(\sigma_L)</th>
<th>(\sigma_I)</th>
<th>(\sigma_R)</th>
<th>(\sigma_Q)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>The advanced VM model</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>with low CAC</td>
<td>6.12</td>
<td>0.35</td>
<td>1.18</td>
<td>0.35</td>
<td>0.39</td>
<td>4.08</td>
<td>0.14</td>
<td>0.37</td>
</tr>
<tr>
<td>with high CAC</td>
<td>1.06</td>
<td>0.35</td>
<td>0.40</td>
<td>0.06</td>
<td>0.93</td>
<td>0.78</td>
<td>3.39</td>
<td>3.42</td>
</tr>
<tr>
<td><strong>The control VM model</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>with Collard and Dellas (2006)’s CAC</td>
<td>6.58</td>
<td>0.36</td>
<td>1.25</td>
<td>0.37</td>
<td>0.50</td>
<td>4.36</td>
<td>0.07</td>
<td>0.42</td>
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</table>

#### Section II: Relative standard deviations (RSD) of each variable to RSD of output

<table>
<thead>
<tr>
<th></th>
<th>(\frac{\sigma_D}{\sigma_Y})</th>
<th>(\frac{\sigma_C}{\sigma_Y})</th>
<th>(\frac{\sigma_Y}{\sigma_Y})</th>
<th>(\frac{\sigma_K}{\sigma_Y})</th>
<th>(\frac{\sigma_L}{\sigma_Y})</th>
<th>(\frac{\sigma_I}{\sigma_Y})</th>
<th>(\frac{\sigma_R}{\sigma_Y})</th>
<th>(\frac{\sigma_Q}{\sigma_Y})</th>
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</thead>
<tbody>
<tr>
<td><strong>The advanced VM model</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>with low CAC</td>
<td>5.17</td>
<td>0.29</td>
<td>1.00</td>
<td>0.29</td>
<td>0.33</td>
<td>3.45</td>
<td>0.12</td>
<td>0.32</td>
</tr>
<tr>
<td>with high CAC</td>
<td>2.61</td>
<td>0.86</td>
<td>1.00</td>
<td>0.16</td>
<td>2.29</td>
<td>1.92</td>
<td>8.36</td>
<td>8.44</td>
</tr>
<tr>
<td><strong>The control VM model</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>with Collard and Dellas (2006)’s CAC</td>
<td>5.24</td>
<td>0.28</td>
<td>1.00</td>
<td>0.29</td>
<td>0.40</td>
<td>3.48</td>
<td>0.06</td>
<td>0.33</td>
</tr>
</tbody>
</table>
There, nevertheless, are slight differences between these two models. The reason is that CAC in the control model are additional costs which are separable to the standard capital accumulation function. However, the disparities of business fluctuations between the advanced VM with low CAC and the control model are very small. They are, respectively, 0.0046 for dividends, 0.0011 for labour fluctuations, 0.0007 for output, 0.0026 for investment 0.0007 for gross return and 0.0005 for the equity price.

Table 5.4.5 reports the results of correlations across real variables for the control model and the advanced VM models in the low and high CAC cases. The results support the hypothesis that the inseparable CAC and the separable CAC display similar results of correlations. Aggregate dividends are countercyclical while labour and investment are procyclical. Consumption is procyclical when CAC are high in the economy.

My simulated results in this section, therefore, imply that these two CAC functions (Gershun (2010) and Collard and Dellas (2006)) respectively can derive highly similar general equilibria. The interesting finding, however, is the disparity of the relationship of the gross capital return and the equity price. In the advanced VM model (with low CAC), the equity price has a positive but moderately weak correlation with the gross return (the figure is 0.4). In the control model, there is no economically significant relationship between them (the figure is -0.10). Accordingly, the correlations of the equity price to dividends, output, investment and labour individually in the control model are different to that in the advanced VM model.

To sum up, this section suggests that for studying the asset pricing implications of DSGE models the CAC function used by Jermann (1998), Boldrin et al. (2001), Cárceles Poveda (2003), Gershun (2010) is a better choice rather than the form of equation (5.4.1). The reason is that the type of CAC that Collard and Dellas (2006) used in their paper shows a weak relationship between production and financial economics. The equity price does not have as strong an association with output as in the advanced VM model. Its correlations with dividends, labour, investment and return on capital are extremely low.
### Table 5.4.5: Cross-correlations of the advanced VM models

This table reports the cross-correlation of dividends ($D$), consumption ($C$), output ($Y$), capital ($K$), labour ($L$), investment ($I$), the gross return on capital ($R$) and the share price ($Q$). Section I shows the results simulated from the advanced value-maximising (VM) models with low capital adjustment costs (CAC). The parameter for low CAC is taken from Gershun (2010). Section II includes results simulated from the same advanced VM model but with expensive CAC. The value for the CAC parameter is taken from Jermann (1998). The specification of the CAC function in the advanced VM model is used in Jermann (1998) and Gershun (2010). Section III presents the results from the model with an alternative capital adjustment cost function used by Collard and Dellas (2006).

Section I: The advanced VM model - with low CAC

<table>
<thead>
<tr>
<th>Corr(,,)</th>
<th>$D$</th>
<th>$C$</th>
<th>$Y$</th>
<th>$K$</th>
<th>$L$</th>
<th>$I$</th>
<th>$R$</th>
<th>$Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
<td>1.00</td>
<td>-0.32</td>
<td>-0.95</td>
<td>-0.10</td>
<td>-0.97</td>
<td>-0.99</td>
<td>-1.00</td>
<td>-0.37</td>
</tr>
<tr>
<td>$C$</td>
<td>-0.32</td>
<td>1.00</td>
<td>0.61</td>
<td>0.95</td>
<td>0.53</td>
<td>0.44</td>
<td>0.37</td>
<td>0.98</td>
</tr>
<tr>
<td>$Y$</td>
<td>-0.95</td>
<td>0.61</td>
<td>1.00</td>
<td>0.41</td>
<td>0.99</td>
<td>0.98</td>
<td>0.96</td>
<td>0.64</td>
</tr>
<tr>
<td>$K$</td>
<td>-0.10</td>
<td>0.95</td>
<td>0.41</td>
<td>1.00</td>
<td>0.31</td>
<td>0.23</td>
<td>0.14</td>
<td>0.96</td>
</tr>
<tr>
<td>$L$</td>
<td>-0.97</td>
<td>0.53</td>
<td>0.99</td>
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<td>0.98</td>
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<td>$I$</td>
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<td>1.00</td>
<td>1.00</td>
<td>0.49</td>
</tr>
<tr>
<td>$R$</td>
<td>-1.00</td>
<td>0.37</td>
<td>0.96</td>
<td>0.14</td>
<td>0.98</td>
<td>1.00</td>
<td>1.00</td>
<td>0.41</td>
</tr>
<tr>
<td>$Q$</td>
<td>-0.37</td>
<td>0.98</td>
<td>0.64</td>
<td>0.96</td>
<td>0.56</td>
<td>0.49</td>
<td>0.41</td>
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Section II: The advanced VM model - with high CAC

<table>
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<tr>
<th>Corr(,,)</th>
<th>$D$</th>
<th>$C$</th>
<th>$Y$</th>
<th>$K$</th>
<th>$L$</th>
<th>$I$</th>
<th>$R$</th>
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</thead>
<tbody>
<tr>
<td>$D$</td>
<td>1.00</td>
<td>0.22</td>
<td>-0.20</td>
<td>0.47</td>
<td>0.83</td>
<td>-0.70</td>
<td>-0.75</td>
<td>-0.74</td>
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<tr>
<td>$C$</td>
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<td>1.00</td>
<td>0.91</td>
<td>0.87</td>
<td>-0.37</td>
<td>0.55</td>
<td>0.48</td>
<td>0.50</td>
</tr>
<tr>
<td>$Y$</td>
<td>-0.20</td>
<td>0.91</td>
<td>1.00</td>
<td>0.67</td>
<td>-0.72</td>
<td>0.84</td>
<td>0.80</td>
<td>0.81</td>
</tr>
<tr>
<td>$K$</td>
<td>0.47</td>
<td>0.87</td>
<td>0.67</td>
<td>1.00</td>
<td>-0.04</td>
<td>0.23</td>
<td>0.15</td>
<td>0.17</td>
</tr>
<tr>
<td>$L$</td>
<td>0.83</td>
<td>-0.37</td>
<td>-0.72</td>
<td>-0.04</td>
<td>1.00</td>
<td>-0.98</td>
<td>-0.99</td>
<td>-0.99</td>
</tr>
<tr>
<td>$I$</td>
<td>-0.70</td>
<td>0.55</td>
<td>0.84</td>
<td>0.23</td>
<td>-0.98</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>$R$</td>
<td>-0.75</td>
<td>0.48</td>
<td>0.80</td>
<td>0.15</td>
<td>-0.99</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>$Q$</td>
<td>-0.74</td>
<td>0.50</td>
<td>0.81</td>
<td>0.17</td>
<td>-0.99</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
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Section III: The control model - VM with Collard and Dellas (2006)'s CAC

<table>
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<th>$C$</th>
<th>$Y$</th>
<th>$K$</th>
<th>$L$</th>
<th>$I$</th>
<th>$R$</th>
<th>$Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
<td>1.00</td>
<td>-0.30</td>
<td>-0.95</td>
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<td>-0.98</td>
<td>-0.99</td>
<td>-1.00</td>
<td>0.11</td>
</tr>
<tr>
<td>$C$</td>
<td>-0.30</td>
<td>1.00</td>
<td>0.59</td>
<td>0.96</td>
<td>0.46</td>
<td>0.42</td>
<td>0.34</td>
<td>0.89</td>
</tr>
<tr>
<td>$Y$</td>
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<td>0.59</td>
<td>1.00</td>
<td>0.41</td>
<td>0.99</td>
<td>0.98</td>
<td>0.96</td>
<td>0.20</td>
</tr>
<tr>
<td>$K$</td>
<td>-0.11</td>
<td>0.96</td>
<td>0.41</td>
<td>1.00</td>
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<td>0.98</td>
</tr>
<tr>
<td>$L$</td>
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<td>0.46</td>
<td>0.99</td>
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<td>1.00</td>
<td>0.99</td>
<td>0.05</td>
</tr>
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<td>$I$</td>
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</tr>
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<td>0.99</td>
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</tr>
<tr>
<td>$Q$</td>
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<td>0.98</td>
<td>0.05</td>
<td>0.02</td>
<td>-0.08</td>
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</table>
5.5 The utility-maximising DSGE model

So far, this chapter has examined a DSGE model with CAC for the aggregated market. The advanced VM model shows that the firm is unwilling to alter its production plan because it is costly to do so. This information entails another discussion. There is another factor which possesses the same character of reducing the fluctuation of investment; risk aversion. When the firm is risk averse to future uncertainty, its investment plan becomes inelastic. Therefore, the variation of investment over periods is flatter and it results in more volatile consumption than in the case of a risk neutral firm.

The objective of the study in this section is to investigate whether the impact of a risk averse firm on equilibrium economy without CAC is equivalent to the influence of a risk neutral firm with CAC on the economy. The hypothesis is that both frictions, risk aversion and CAC, possess the function of smoothing the variation of investment. In particular, the steady states in both economies are supposed to be similar.

My calibration results support the claim by Cárceles Poveda (2003) that the steady state values for a standard DSGE model with CAC are equal to those for a model with a risk-averse firm but without CAC.

Interestingly, aggregate dividends are less volatile in the UM model than in the advanced VM model. The results imply that a firm’s risk-averse objective function helps to smooth aggregate dividends more than the friction of capital adjustment.

5.5.1 Assumption

The representative UM firm aims to maximise its expected utility of cash flows with two assumptions:

**Assumption 1:** The objective function is with a coefficient of risk aversion $\gamma_F > 0$.

The firm is risk averse to future uncertainties.

**Assumption 2:** There is no capital adjustment costs in the economy so the parameter for CAC in equation 5.3.18 is assumed to be $\zeta \rightarrow \infty$. 
5.5.2 Model implementation

By solving the optimal aggregate labour $L^d$, capital $K$ and investment $I$, the firm maximises its utility on profits $(D)$:

$$\max_{L^d_t,K_t,I_t} E_t \left[ \sum_{h=0}^{\infty} \beta F(D_{t+h}) \right]$$

subject to

$$K_t = (1 - \eta)K_{t-1} + I_t$$

$$Y_t = Z_t K_{t-1}^{\alpha} (xL^d_t)^{1-\alpha}$$

$$\log Z_t = \psi \log Z_{t-1} + \epsilon_t$$

where $U_F(D_{t+h}) = \frac{D_{t+h}^{1-\gamma_F}}{1-\gamma_F}, \gamma_F > 0$

As to the investor’s optimisation, it remains the same as in equation 5.3.6. For viewing convenience, it is displayed again here:

$$\max_{C_t,L^s_t,N_t} E_t \left[ \sum_{h=0}^{\infty} \beta^h U(C_{t+h},C_{t+h-1},1-L^s_{t+h}) \right]$$

subject to

$$C_t + Q_t N_{t+1} = W_t L^s_t + (Q_t + D_t) N_t$$

where $U(C_t,C_{t-1},1-L_t) = \frac{[\xi(C_t - \xi C_{t-1})^{\rho}(1-L_t)^{1-\rho}]^{1-\gamma}}{1-\gamma}, \xi,\rho,\gamma > 0$

The first order conditions (FOC) are obtained by solving both representatives’ optimisation problems. FOCs for the labour demand, the labour supply and the equity price are the same as those of advanced VM models, which are equations 5.3.11, 5.3.14 and 5.3.15. The only and main difference is the FOC for capital because the model in this section is with a risk-averse firm and no capital adjustment costs. A revised FOC for aggregate capital is given by:

$$1 = E_t[\beta F\left(\frac{D_t}{D_{t+1}}\right)^{\gamma_F} (Z_{t+1} \frac{\partial f}{\partial K_t} + 1 - \eta)]$$

The above formula can also be expressed as an asset pricing formula $1 = E(mR)$ where $m = \beta F\left(\frac{D_t}{D_{t+1}}\right)^{\gamma_F}$ and the expected return on capital is $R_{t+1} = Z_{t+1} \frac{\partial f}{\partial K_t} + 1 - \eta$ on the
period \([t, t + 1]\).

Equilibrium can finally be reached when these FOC are satisfied along with goods, labour and equity market clearing conditions (equations 5.3.8, 5.3.9 and 5.3.10).

5.5.3 Calibration for the UM firm’s preferences

Table 5.5.1 on the following page summarises parameter values for the UM DSGE model. For simplification, I take an assumption from Cárceles Poveda (2003) to define the firm’s stochastic discount factor as a subjective discount factor: \(\phi \equiv \beta_F\) for the UM model. The values of the firm’s discount factor and the risk aversion coefficient are taken from Cárceles Poveda (2003): \(\beta_F = 0.99\) and \(\gamma_F = 1.44\) in the firm’s preferences.

The values for the technology shock persistence and the standard deviation are set to be the same as in the advanced VM model \((\psi = 0.95\) and \(\sigma_\epsilon = 0.00712\)). The investor’s momentary utility is determined by both consumption and leisure. Therefore, the parameter value for deciding the time allocated to market activities is set to be the same as before \((\rho = 0.36)\). Other related parameter values for the UM model remain the same as those in the advanced VM model.

For a complete DYNARE code for this calibration, refer to Appendix 5.8.3 on page 204.

5.5.4 Results and discussion

5.5.4.1 Steady state values

Cárceles Poveda (2003) proves that the equilibrium for the economy of a standard DSGE model with CAC and that for the economy with a representative risk-averse firm but no capital installation costs should have the same impact on the steady state of each macroeconomic variable. Table 5.5.2 lists the steady state values of the advanced VM models and the UM model. There are two cases (low and high CAC) in the advanced VM models. The results successfully demonstrate that both VM and UM economies achieve a similar equilibrium in terms of the economic behaviour.

The capital output ratio is 10.26 when the economy is in the optimal equilibrium.
This table lists the parameter values used to solve and simulate the dynamic stochastic general equilibrium (DSGE) utility-maximising (UM) model. In the UM model, the firm maximises its profits with a utility function and there are no capital adjustment costs. All the parameters are categorised into two groups. The first group, Technology, includes parameters in the firm’s production function. The second group, Preferences, is calibrated for the firm’s and the investor’s utility function. A utility-maximising firm has a coefficient of relative risk aversion. The individual household has an internal habit formation. The value for the habit persistence is taken from Constantinides (1990). The model is with an endogenous labour. Hence, the individual’s utility is both on consumption and leisure. The value for determining the time allocated to market activities (consumption) is taken from Campbell (1994).

### Technology

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>Capital share to output</td>
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</tr>
<tr>
<td>Depreciation ratio</td>
<td>$\eta$ 0.025</td>
</tr>
<tr>
<td>Trend in labour augmenting technical change</td>
<td>$X$ 1.005</td>
</tr>
<tr>
<td>Persistence of the technology shock</td>
<td>$\psi$ 0.95</td>
</tr>
<tr>
<td>Standard deviation of the technology shock</td>
<td>$\sigma_e$ 0.00712</td>
</tr>
</tbody>
</table>

### Preferences

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subjective discount factor - the firm</td>
<td>$\beta_F$ 0.99</td>
</tr>
<tr>
<td>Subjective discount factor - the investor</td>
<td>$\beta$ 0.99</td>
</tr>
<tr>
<td>Coefficient of relative risk aversion - the firm</td>
<td>$\gamma_F$ 1.44</td>
</tr>
<tr>
<td>Coefficient of relative risk aversion - the investor</td>
<td>$\gamma$ 5</td>
</tr>
<tr>
<td>Habit persistence</td>
<td>$\zeta$ 0.82</td>
</tr>
<tr>
<td>Value for determining the time allocated to market activities</td>
<td>$\rho$ 0.36</td>
</tr>
</tbody>
</table>
Table 5.5.2: The steady state values in the advanced VM and UM models

This table summarises results of the steady state values of dividends ($D$), consumption ($C$), output ($Y$), capital ($K$), labour ($L$), investment ($I$), the gross return on capital ($R$) and the share price ($Q$) for the value-maximising (VM) and the utility-maximising (UM) model. The VM model is with capital adjustment costs (CAC) which make the firm less willing to change its investment decision. The UM model considers a risk-averse firm in the economy and hence the investment flow is inelastic. In the VM model, two calibrations are made. One is with low CAC (Gershun (2010)) and another is with high CAC (Jermann (1998)). Cárceles Poveda (2003) shows that the equilibrium of the VM model is analogous to the UM model, which means the steady state values are the same.

<table>
<thead>
<tr>
<th>Steady state value</th>
<th>Type of DSGE models</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>VM</td>
</tr>
<tr>
<td></td>
<td>low CAC</td>
</tr>
<tr>
<td>D</td>
<td>0.13</td>
</tr>
<tr>
<td>C</td>
<td>0.93</td>
</tr>
<tr>
<td>Y</td>
<td>1.25</td>
</tr>
<tr>
<td>K</td>
<td>12.83</td>
</tr>
<tr>
<td>L</td>
<td>0.34</td>
</tr>
<tr>
<td>I</td>
<td>0.32</td>
</tr>
<tr>
<td>R</td>
<td>1.01</td>
</tr>
<tr>
<td>Q</td>
<td>12.83</td>
</tr>
</tbody>
</table>

This figure demonstrates that the firm needs to increase capital by roughly ten units when it wants to produce one unit of output. Meanwhile, every output needs nearly one third unit of the labour input (the labour output ratio is 0.27). In general equilibrium, the ratio of consumption and output is 0.74, which means that one unit of aggregate consumption is equal to 74 percent of one unit of aggregate output. Aggregate dividends are about 10 percentage of one unit of output.

5.5.4.2 Cyclical variability

Even though the steady states are the same in both the VM and UM models, the business cycle fluctuations are not identical. Table 5.5.3 presents the results of macroeconomic statistics of the VM (low and high CAC) and the UM models.

My simulated results show that the variability of optimal aggregate dividend payouts, $\sigma_D$, in the UM model is 0.05%, in the low (high)-CAC VM model is 6.12% (1.06%), which imply that aggregate payouts are much more sensitive to the firm’s utility compared to the case with costly capital adjustment process. The UM model calibrates a representative risk-averse firm in the economy, which means the firm is
5.5. THE UTILITY-MAXIMISING DSGE MODEL

Table 5.5.3: A summary of macroeconomic statistics for VM and UM models
This table reports cyclical statistics for value-maximising (VM) and utility-maximising (UM) models. The VM model is with capital adjustment costs which make the firm less willing to change its investment decision. The UM model considers a risk-averse firm in the economy. In the VM model, two calibrations are made. One is with low CAC (Gershun (2010)) and another is with high CAC (Jermann (1998)). The simulated results are categorised into two sections. Section I includes the relative standard deviations (RSD) of aggregate dividends ($D$), consumption ($C$), output ($Y$), capital ($K$), labour ($L$), investment ($I$), the gross return on capital ($R$) and the share price ($Q$). Section II includes the relative standard deviations of each variable to that of output.

Section I: Relative standard deviations (RSD) in percent of economic variables

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_D$</th>
<th>$\sigma_C$</th>
<th>$\sigma_Y$</th>
<th>$\sigma_K$</th>
<th>$\sigma_L$</th>
<th>$\sigma_I$</th>
<th>$\sigma_R$</th>
<th>$\sigma_Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Advanced VM (low CAC)</td>
<td>6.12</td>
<td>0.35</td>
<td>1.18</td>
<td>0.35</td>
<td>0.39</td>
<td>4.08</td>
<td>0.14</td>
<td>0.37</td>
</tr>
<tr>
<td>Advanced VM (high CAC)</td>
<td>1.06</td>
<td>0.35</td>
<td>0.40</td>
<td>0.06</td>
<td>0.93</td>
<td>0.78</td>
<td>3.39</td>
<td>3.42</td>
</tr>
<tr>
<td>UM</td>
<td>0.05</td>
<td>0.30</td>
<td>0.35</td>
<td>0.06</td>
<td>1.19</td>
<td>0.48</td>
<td>0.01</td>
<td>3.84</td>
</tr>
</tbody>
</table>

Section II: Relative standard deviations (RSD) of each economic variable to RSD of output

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_D/\sigma_Y$</th>
<th>$\sigma_C/\sigma_Y$</th>
<th>$\sigma_K/\sigma_Y$</th>
<th>$\sigma_L/\sigma_Y$</th>
<th>$\sigma_I/\sigma_Y$</th>
<th>$\sigma_R/\sigma_Y$</th>
<th>$\sigma_Q/\sigma_Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Advanced VM (low CAC)</td>
<td>5.17</td>
<td>0.29</td>
<td>1.00</td>
<td>0.29</td>
<td>0.33</td>
<td>3.45</td>
<td>0.12</td>
</tr>
<tr>
<td>Advanced VM (high CAC)</td>
<td>2.61</td>
<td>0.86</td>
<td>1.00</td>
<td>0.16</td>
<td>2.29</td>
<td>1.92</td>
<td>8.36</td>
</tr>
<tr>
<td>UM</td>
<td>0.14</td>
<td>0.87</td>
<td>1.00</td>
<td>0.17</td>
<td>3.41</td>
<td>1.37</td>
<td>0.03</td>
</tr>
</tbody>
</table>

unwilling to change investment decisions and try to avoid taking any risk as much as possible. Since the firm is utility-maximising, similar to the case of investor’s risk aversion, it has a similar pattern of smoothing its cash flows over time. I find that, ceteris paribus, optimal aggregate output, investment and dividends are very smooth compared to models with capital adjustment costs. Dividend smoothing to risk-averse firms is as consumption smoothing to investors.

The UM model shows that optimal aggregate dividends are nearly as much as volatile as optimal aggregate output ( $\sigma_D/\sigma_Y = 0.14$, Table 5.5.3, Section II) while share prices are dramatically volatile than aggregate output volatility ( $\sigma_Q/\sigma_Y = 11.02$, Table 5.5.3, Section II). This calibration, compared to existing investigations, particularly demonstrates that both smooth optimal aggregate dividends and volatile share prices exist in the economy. This result is consistent with my previous calibration of models with capital adjustment costs.

Interestingly, apart from dividends and investment, these two types of models (VM and UM) produce similar fluctuations in consumption, labour, output, capital and the
equity price. The magnitude of the movements of consumption to output in the UM model is the same as that in the high CAC VM model. The outcomes of the high CAC VM and the UM models show that every increase in the standard deviation (s.d.) of consumption is roughly eighty to ninety percent of the s.d. of aggregate output. Investment in the UM model, however, varies about 1.37 times output’s fluctuation. It moves much more smoothly than when it is in the VM models.

The variance of the equity price is about 3.84% in the UM model, which is close to the result in the high CAC VM model (3.42%). However, it seems that the equity price movements in the UM economy are stronger than in the VM models. Every increase of the s.d. of the equity price is approximately eleven times that of output in the UM model, which is much higher than that in the VM model.

Table 5.5.4 reports the correlations between variables for the UM model. Compared to results of the advanced VM model, the correlation between aggregate dividends and stock prices is much weaker in the case of the UM model. However, aggregate labour is countercyclical in the UM model. This is inconsistent to the observed data.

The UM model simulates procyclical dividends, which is different to previous calibrations in this thesis. The result is consistent with the observed data, shown in Section 2.2.8 of Chapter 2, that net cash flows are procyclical. If gross dividends are the major or the only part of net cash flows, net cash flows are very much equal to gross dividends. As gross dividends are positively correlated to Gross National Product (GNP), net cash flows are therefore expected to have a positive correlation with GNP. Therefore, the UM model simulates procyclical dividends which are much closer to observed gross dividends. The cyclicity of dividends is, to some extent, influenced by the firm’s characteristic, i.e. in this case, a firm is risk averse to its future cash flows.

In summary, the equilibrium in the economy with CAC is the same as in the economy with a risk-averse firm but without CAC. This has been proved by the results of steady states in both VM and UM models. Optimal equilibrium around steady states is the same but each model produces different business cycle fluctuations. In the UM economy, the firm prefers secured dividend policy and investment decision. As a result, investment and dividends both become smooth. Therefore, the variations of aggregate investment
5.6. IMPULSE RESPONSES TO A TECHNOLOGY SHOCK

Table 5.5.4: Cross-correlations for the UM model
This table displays the cross-correlation (Corr(.,.)) between dividends (D), consumption (C), output (Y), capital (K), labour (L), investment (I), the gross return on capital (R) and the share price (Q) for the utility-maximising (UM) model. This model has a risk-averse firm and a risk-averse investor.

<table>
<thead>
<tr>
<th>Corr(.,.)</th>
<th>D</th>
<th>C</th>
<th>Y</th>
<th>K</th>
<th>L</th>
<th>I</th>
<th>R</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>1.00</td>
<td>0.70</td>
<td>0.69</td>
<td>0.76</td>
<td>-0.12</td>
<td>0.67</td>
<td>0.57</td>
<td>0.22</td>
</tr>
<tr>
<td>C</td>
<td>0.70</td>
<td>1.00</td>
<td>1.00</td>
<td>0.26</td>
<td>-0.26</td>
<td>1.00</td>
<td>0.98</td>
<td>0.41</td>
</tr>
<tr>
<td>Y</td>
<td>0.69</td>
<td>1.00</td>
<td>1.00</td>
<td>0.25</td>
<td>-0.27</td>
<td>1.00</td>
<td>0.99</td>
<td>0.41</td>
</tr>
<tr>
<td>K</td>
<td>0.76</td>
<td>0.26</td>
<td>0.25</td>
<td>1.00</td>
<td>0.50</td>
<td>0.22</td>
<td>0.08</td>
<td>-0.44</td>
</tr>
<tr>
<td>L</td>
<td>-0.12</td>
<td>-0.26</td>
<td>-0.27</td>
<td>0.50</td>
<td>1.00</td>
<td>-0.27</td>
<td>0.36</td>
<td>-0.99</td>
</tr>
<tr>
<td>I</td>
<td>0.67</td>
<td>1.00</td>
<td>1.00</td>
<td>0.22</td>
<td>-0.27</td>
<td>1.00</td>
<td>0.99</td>
<td>0.41</td>
</tr>
<tr>
<td>R</td>
<td>0.57</td>
<td>0.98</td>
<td>0.99</td>
<td>0.08</td>
<td>-0.36</td>
<td>0.99</td>
<td>1.00</td>
<td>0.49</td>
</tr>
<tr>
<td>Q</td>
<td>0.22</td>
<td>0.41</td>
<td>0.41</td>
<td>-0.44</td>
<td>-0.99</td>
<td>0.41</td>
<td>0.49</td>
<td>1.00</td>
</tr>
</tbody>
</table>

and dividends are much smaller than in the VM economy. Moreover, since the UM economy faces a flat production process, the variations of the labour and the equity price then become slightly more volatile to balance smooth dividends, the s.d. of labour (equity price) is about four (eleven) times the s.d. of output. There is not a great difference with regard to an investor’s consumption pattern between these two VM (high CAC) and UM economies.

5.6 Impulse responses to a technology shock

Figures 5.6.1 and 5.6.2 displays two models: one is without both consumption habit formation and CAC, another is with habits but no CAC. In the previous chapter, I examined models with habit formation and showed that they produce smooth consumption and volatile dividends. Consistent with the previous findings, one can see that aggregate dividends respond to the technology shock greatly in the model with habit rather than the model without habit. Consumption becomes smooth in the early years after the shock when the individual has a consumption habit.

Figures 5.6.3, 5.6.4, 5.6.5 and 5.6.6 present the deviation of the real variable from its steady state in terms of the impulse responses to a technology shock. Several findings can be easily picked up by viewing the visual results.

1. Dividends fluctuate less when the economy has a high level of CAC (compare
Figure 5.6.3 to Figure 5.6.4). However, the variation of consumption remains the same in the first ten years after the technology shock. There is a difference regarding the fluctuations in consumption in the long run, but it is small. High CAC smooth not only investment but also capital over periods. Interestingly, the greater CAC the more volatile the gross return on capital. The fluctuation of the gross return on capital is possibly influenced by the volatile equity price in the early-years stage after the shock.

2. Even though my advanced VM model demonstrates that aggregate dividends vary less in the case of high CAC, the UM model shows more inelastic dividends in the long run (compare Figure 5.6.4 to Figure 5.6.6). Through the calibration, this chapter provides evidence that the case of a risk-averse firm in the DSGE model supports the DSGE model with a high level of CAC rather than a low level of CAC. Their equilibrium economies are the same.

3. The labour charts in Figure 5.6.4 and Figure 5.6.6 convey the information that when there are CAC in the production process or the firm is risk averse in the economy, labour is much more volatile than in the case of low CAC. This implies the firm prefers to adjust the labour input for the production process instead of the costly capital installation.

4. Comparing Figure 5.6.3 to Figure 5.6.5, economic fluctuations in the control model are very close to those in the case of low CAC in the advanced VM model. The main difference is the variation of the gross return. The control model with a separable form of CAC in the capital accumulation function demonstrates a smooth gross return. It is less efficient than the advanced VM model with an inseparable form of CAC in terms of providing a volatile return to equity.
Figure 5.6.1: Impulse responses to a technology shock for the advanced VM model - Benchmark 1 (no CAC; no habits)

Figure 5.6.2: Impulse responses to a technology shock for the advanced VM model - Benchmark 2 (no CAC; with habits)
Figure 5.6.3: Impulse responses to a technology shock for the advanced VM model in the low CAC case (with habits)

Figure 5.6.4: Impulse responses to a technology shock for the advanced VM model in the high CAC case (with habits)
Figure 5.6.5: Impulse responses to a technology shock for the control model

Figure 5.6.6: Impulse responses to a technology shock for the UM model
5.7 Summary

The objective of this chapter is to explore the impact of the level of capital adjustment costs on aggregate dividend policy. CAC result in the firm smoothing investment variation, thereby the fluctuation in consumption is much higher than in the economy without CAC. That the smoothness of consumption is reduced due to the CAC friction, brought up by the pioneer paper by Jermann (1998), has been proven by my first calibration in this chapter. From this simulation, I notice a disadvantage of this type of calibration, that is, the household does not maximise his/her flows of labour. That is, the model does not generate an optimal level of labour when both the firm and the household maximise their objectives simultaneously. My second calibration, called the advanced value-maximising (VM) model, then extends this to include endogenous labour. The results show that the level of CAC does not influence the fluctuation of consumption but makes dividends high smooth when CAC are high. This is the first main finding in this chapter.

This chapter further demonstrates another CAC function, a type of separable form to the capital accumulation function. The results of this control model are close to the advanced VM model in the case of low CAC. Even though most macroeconomic variables’ business statistics are close to the VM model, the major disparity is that the control model with a separable form of CAC has a less volatile gross return. This calibration shows that this type of CAC function may not be suitable for the study of asset pricing implications.

My fourth contribution is that I examine a hypothesis that the equilibrium with CAC can be generalised by another DSGE model in which there is no CAC but a risk-averse firm. In particular, the business fluctuations in the utility maximising firm model are close to the value maximising firm model with a high level of CAC. This is the second main finding and implies that if there is a representative risk averse firm in the economy, the general equilibrium market reacts analogously to the case of the economy having a very high level of CAC. While the steady state values are similar in both economies, optimal aggregate payouts are much smooth in the utility-maximising (UM) model.
This finding implies that there is a stronger impact on optimal aggregate dividends if firms are risk averse compared to firms facing costly adjustment expenses on capital. Corporations are reluctant to take uncertain risk and become conservative to their free cash flows across time, and consequently dividend payouts are smooth.

The VM model with the technology shock is found to have countercyclical aggregate dividends and the UM model shows that theoretical aggregate dividends are procyclical. A very recent paper by Jermann and Quadrini (2009) points out that the addition of financial shocks can better explain the movements of real and financial variables and their findings show that equity payouts are procyclical. Without considering financial shocks, my UM model simulates procyclical aggregate payouts. It implies that the cyclical behaviour of aggregate payouts is strongly related to the specifications of both firms’ and investors’ optimisation problems. In Chapter 2, both net cash flows and gross payouts are positively correlated to Gross National Product (GNP) during 1973 - 2000 and several non-overlapping sub-periods. In this chapter, two different types of DSGE models are presented and one (the VM model), involving a costly capital adjustment function, simulates countercyclical dividends while another model (the UM model), introducing risk-averse firms, calibrates procyclical dividends. It is interesting to see that either the firm’s production function and its capital accumulation process or the firm’s characteristic leading to maximise its future cash flows can affect the cyclicality of dividends.

Assuming that some empirical studies take gross payouts (the empirical data) as dividends, the fact that my UM model simulates the result of procyclical theoretical aggregate dividends is accordingly in line with the empirical result using data for gross dividends. In that case, results of dividends in the VM model then contradict the empirical data. However, if the so-called dividends are the net cash flows, it is found that an inverse relationship between net cash flows and GNP exists during some periods. The cyclicality of net cash flows varies more than gross payouts. Thus, net cash flows are not as positively related to GNP as gross payouts. Hence, as prior theoretical studies, countercyclical theoretical cash flows could explain some cyclical patterns of net cash flows instead of gross payouts.
Overall, high capital adjustment costs cause the investment plan and dividend policy to become less volatile. They also drive a volatile labour market. This chapter helps us understand the economy’s cyclical movements by analysing different frictions. As a possible extension, research could work on incorporating investment adjustment costs. Related research papers are Christiano et al. (2005) and Beaubrun-Diant and Tripier (2005) who use models with investment adjustment costs to study monetary policy and asset returns.
5.8 Appendices

5.8.1 DYNARE code for the VM model

```matlab
% the model in section of advanced calibration
% file name: CAC201 (means ["Capital" Adjustment Costs 201])
% try paper Carcedes Poveda (2003) "VM" firm objective with capital
% adjustment cost from Gershun(2009)

% 0. Housekeeping
close all;

% 1. Defining variables
% periods 20100;
var k, c, y, l, i, r, z, g, d, q;
% output, consumption, capital, labor, interest, investment, technology
varexo e;
parameters beta, delta, alpha, sigma, zeta, xi, psi, gamma, rho, x;

% 2. Calibration
% A = 2.85;
xi = 0.82; % habit persistence
beta = 0.99;
delta = 0.025; % depreciation share
alpha = 0.36; % capital share
psi = 0.95; % autocorrelation of technology shock (for 0.99 0.95)
sigma = 0.00712; % standard deviation of technology shock units (for 0.01 .00712)
zeta = 40; % the elasticity of investment and capital ratio 1/0.565=1.77; 0.23 (Jermann)
gamma = 5; % coefficient of risk aversion
rho = 0.36; % time endowment 0.36
x = 1.005; % the deterministic trend in labor augmenting technical change (from
% (run 1) psi=0.99 / sigma = 0.01 / rho =1
% (run 2) psi =0.95 / sigma = 0.00712/rho =0.36

% 3. Model
model;
1 /((delta^((1/zeta)))^((i/k(-1))^(1/zeta))) = beta* (1- xi*beta*((c(+2)-xi*c(+1))/
(c(+1)-xi*c) ) ^ (rho-gamma*rho-1)*((1-l(+2))/(1-l(+1)))^(1-gamma*(1-rho)-rho) -
xi*beta)*r(+1) ;
r=alpha*exp(z)*(k(-1)^(alpha-1))*(x^l)^(1-alpha)+ ((1-delta+(delta/(1-zeta))))/
```

---
((\delta^{(1/\zeta)}*(i/k(-1))^{(-1/\zeta)})+(1/(\zeta-1)) *(i/k(-1))) ;
\begin{align*}
c &= \xi*c(-1) + ((\rho/(1-\rho))*(1-\alpha)*(\exp(z))*(k(-1)^{\alpha}) *(1-l)/(c(-1)+\alpha)^{\gamma}*(1-l)/(c-xi*c(-1))^{\gamma}); \\
y &= \exp(z)*(k(-1)^{\alpha})*((x*l)^{(1-\alpha)}); \\
k &= (1-\delta)*k(-1)+ g*k(-1); \\
g &= ((\delta^{(1/\zeta)})/(1-(1/\zeta)))*((i/k(-1))^{(1-(1/\zeta))}) + (\delta/(1-\zeta)); \\
q &= \beta*( (1- xi*beta*(((c(+2)-xi*c(+1))/(c(+1)-xi*c) )^{\rho-gamma*rho-1} )* ((c-xi*c(-1))/(c(+1)-xi*c))^{\rho-gamma*rho-1} *((1-l)/(1-l(+1)))^{1-gamma*(1-rho)-rho} )/ ((c-xi*c(-1))/(c(+1)-xi*c))^{\rho-gamma*rho-1} *((1-l)/(1-l(+1)))^{1-gamma*(1-rho)-rho} - xi*beta) * (q(+1)+d(+1)) ;
\end{align*}
y = c+i; \\
d = \alpha*y - i; \\
z = \psi*z(-1) + e;
end;
%

% 4. Computation

% initial

k = 11; \\
c = 1.33; \\
l = 0.31; \\
i = 0.28; \\
g = 0.025; \\
z = 0; \\
e = 0; \\
q = 0.3; 
end;

steady; 
shocks; 
var e = sigma^2; 
end;

stoch_simul(hp_filter = 1600, order = 1);
%

% 5. Some Results


5.8.2 DYNARE code for the control model

% a model with Collard and Dellas (2006)'s capital adjustment cost function 
% file name: CAC005 (means ["Capital" Adjustment Costs 005])
5.8. APPENDICES

% 0. Housekeeping
% close all;

% 1. Defining variables
% periods 20100;
var k, y, l, i, r, g, d, q;
% output, consumption, capital, labor, interest, investment, technology
varexo e;
parameters beta, delta, alpha, sigma, zeta, psi, gamma, rho, x;

% 2. Calibration
xi = 0.82; % habit persistence
beta = 0.99;
delta = 0.025; % depreciation share
alpha = 0.36; % capital share
psi = 0.95; % autocorrelation of technology shock
sigma = 0.00712; % standard deviation of technology shock units
zeta = 10; % the elasticity of investment and capital ratio
gamma = 5; % coefficient of risk aversion
rho = 0.36; % time endowment
x = 1.005; % the deterministic trend in labor augmenting technical change (from
% Jermann (1998))

% 3. Model
model;
1 = beta*( (1- xi*beta*((c(+2)-xi*c(+1))/(c(+1)-xi*c) )^(rho-gamma*rho-1) *
*(1-l(+2))/(1-l(+1))) (1-gamma*(1-rho)-rho))/((c-wc(-1))/(c(+1)-xi*c))^-rho -(1-1)/(1-l(+1))^(1-gamma*(1-rho)-rho)
) *r(+1);
r = alpha*exp(z)*(k(-1)^(alpha-1))*(x*l)^(1-alpha) + (1-delta)+ 0.5*zeta *
*(g^-2)-(delta^-2));
c = xi*c(-1)+(xi*(c(-1)/(1-(1-rho)))*((1-alpha)*exp(z)*(1-alpha))*(k(-1)^(alpha)
*(1-alpha)*(1-l)/(1-l*(1-xi*beta*(1-l(+1))/(c(-1)-xi*c)))*gamma); y = exp(z)*(k(-1)^(alpha)*((x*l)^(1-alpha)); k = (1-delta)*k(-1) + i - 0.5*zeta*((g-delta)^2)*k(-1);
g = i/k(-1);
y = c+i;
d = alpha*y - i;
q = beta*( (1- xi*beta*((c(+2)-xi*c(+1))/(c(+1)-xi*c) )^(rho-gamma*rho-1) *
*(1-l(+2))/(1-l(+1))) (1-gamma*(1-rho)-rho))/((c-wc(-1))/(c(+1)-xi*c))^-rho -(1-1)/(1-l(+1))^(1-gamma*(1-rho)-rho)
) *q(+1)+d(+1));
end;
% 4. Computation

initval;
k = 11;
c = 1.33;
l = 0.31;
i = 0.28;
g = 0.025;
z = 0;
e = 0;
end;
steady;
shocks;
var e = sigma^2;
end;
stoch_simul(hp_filter = 1600, order = 1);

% 5. Some Results

statistic1 = 100*sqrt(diag(oo_.var(1:10,1:10)))./oo_.mean(1:10);
dyntable('Relative standard deviations in %',strvcat('VARIABLE','REL. S.D.'),
M_.endo_names(1:10,:),statistic1,10,10,4)
statistic2 = 100*sqrt(diag(oo_.var(1:10,1:10)));
dyntable('standard deviations in %',strvcat('VARIABLE','S.D.'),
M_.endo_names(1:10,:),statistic2,10,8,4);

5.8.3 DYNARE code for the UM model

% file name: CAC202 (means [risk averse firm without capital adjustment costs])
% try paper Carcedes Poveda (2003) "UM" firm objective (firm is risk averse/ DSGE is without capital adjustment costs)
% 0. Housekeeping
close all;
% 1. Defining variables
%periods 20100;
var k, c, y, l, i, r, z, g, d, q;
% output, consumption, capital, labor, interest, investment, technology
varexo e;
parameters beta, delta,alpha, sigma, xi, psi, gamma, rho, x, gammahat;
% 2. Calibration

xi = 0.82; % habit persistence
beta = 0.99; % beta = beta_F
delta = 0.025; % depreciation share
alpha = 0.36; % capital share
psi = 0.95; % autocorrelation of technology shock
sigma = 0.00712; % standard deviation of technology shock units
gamma = 5; % coefficient of risk aversion
rho = 0.36; % time devoted to consumption
x = 1.005; % the deterministic trend in labor augmenting technical change (from Jermann (1998))
gammaf=1.44; % the coefficient of firm’s risk aversion

% 3. Model
model;
1 = beta*( ((d/(d(+1)))^gammaf ) *r(+1) ) ;
r = alpha*exp(z)*k(-1)^alpha*exp((x*l)^(1-alpha)+1-delta);
c = xi*c(-1)+((rho/(1-rho))*(1-alpha)*(exp(z))*k(-1)^alpha)*((x*l)^(1-alpha))
*(1-l)/(1-xi*beta*(1-l(+1))/(c(+1)-xi*c)^(rho-gamma*rho-1) *((1-l)/(1-l(+1)))^gamma));
y = exp(z)*((k-1)^alpha)*((x*l)^(1-alpha));
k = (1-delta)*k(-1)+ g*k(-1);
g = i/k(-1);
y = c+i;
d = alpha*y - i;
q = beta*( (1- xi*beta*((c(+2)-xi*c(+1))/(c(+1)-xi*c) )^(rho-gamma*rho-1) *
((1-l(+2))/(1-l(+1)))^(1-gamma*(1-rho)-rho))/((c-xi*c(-1))/(c(+1)-xi*c))
*(rho-gamma*rho-1) *((1-l)/(1-l(+1)))^(1-gamma*(1-rho)-rho) - xi*beta)
*(q(+1)+d(+1)));
z = psi*z(-1) + e;
end;

% 4. Computation
initval;
k = 11;
c = 1.33;
l = 0.31;
i = 0.28;
g = 0.025;
d = 0.4;
z = 0;
e = 0;
end;
steady;
shocks;
var e = sigma^2;
end;
stoch_simul(hp_filter = 1600, order = 1);
% 5. Some Results

statistic1 = 100*sqrt(diag(oo_.var(1:10,1:10)))./oo_.mean(1:10);
dyntable('Relative standard deviations in %',strvcat('VARIABLE','REL. S.D.'),
M_.endo_names(1:10,:),statistic1,10,9,4)
statistic2 = 100*sqrt(diag(oo_.var(1:10,1:10)));
dyntable('Standard deviations in %',strvcat('VARIABLE','S.D.'),
M_.endo_names(1:10,:),statistic2,10,8,4);
CHAPTER 6

CONCLUSION
6.1 Introduction

In this thesis, I have used a macroeconomic, dynamic stochastic general equilibrium (DSGE) approach to study the simultaneous interaction between aggregate dividend policy and corporate and investor's activities; including investment decisions, production processes and consumption. Using this aggregated approach, which contrasts with the more usual firm-level analysis that is standard in the corporate finance literature, I have examined the role of market-wide dividend policy in the presence of market frictions. This issue has rarely been discussed in both the standard corporate finance and macroeconomic literature. In addition, I took a combined MATLAB and DYNARE technique to simulate results, which improves the speed of the simulation process and provides more robust results than the standard, second-order Taylor's series expansion approach.

This thesis was motivated by Marsh and Merton (1987) who suggest that it might be fruitful to investigate corporate and investor activities from a macroeconomic point of view. A number of existing theoretical and empirical papers have studied corporate finance issues such as taxation theory, asymmetric information, signalling theory, agency theory, incomplete contracts and transaction costs. Their work, however, is developed within a microeconomic, rather than aggregate economy, analysis. While several papers in the last recent decade have studied corporate finance from a macroeconomic perspective but their foci are on asset returns, dividend taxation, and the tax benefits on debt financing, rather than on the role and the variability of optimal aggregate payouts. My thesis contributes to this stream of literature by considering how dividend policy, investment decision and consumption, can be examined from an aggregated analysis.

In this chapter, I conclude this thesis with a summary of findings in section 6.2 and suggestions for future research in section 6.3.
6.2 Summary of findings

In chapter 2, I briefly reviewed two streams of literature: (a) micro-based dividend theory and empirical evidence and (b) business cycles and economic growth theory. It is helpful to review prior studies on standard corporate dividend issues from an individual firm’s perspective as it provides an opportunity to compare and contrast this approach against the macroeconomic angle that I take. For instance, dividend taxation makes individual agents reluctant to invest in dividend-paying stocks. In an aggregated market, however, investors, who wish to smooth consumption, need to use dividend income to balance out any changes in other source of income, such as salaries. In part (b), I reviewed the foundation of a general equilibrium model, which is my main methodology. The rest of the literature review in this thesis was placed in subsequent chapters and organised as follows. Chapter 3 calibrates a much-cited Hansen (1985) model plus two extensions, and provides a literature view on real business cycle based DSGE models. In chapter 4, I studied aggregate dividend policy volatility in the presence of habit formation in DSGE models, following a detailed literature view on DSGE models with habit formation. Chapter 5 examines the impact of capital adjustment costs on dividend policy, and includes a review of the literature on DSGE models with capital adjustment costs.

In chapter 3, I calibrated three models. The first model is the well-known Hansen model. It is a stochastic growth model with an endogenous probability of working. There is an optimisation problem solved from the representative household’s side. My results are analogous to those in the original paper and are also similar to observed economic statistics: output, capital, labour and investment. The standard Hansen model does not take dividend policy into account and therefore is not suitable to investigate the variability of dividends. I then extend it to that the case when the net cash flow is determined jointly with labour wages and investment decisions. Both my simulated standard deviations of aggregate dividend payouts and investment are very large. My third investigation in this chapter calibrated a DSGE model in a competitive market by taking external financing and a credit process into account. My results showed that dividends are much more volatile than the observed results. This analysis
led to my following studies in subsequent chapters, which I examined whether market frictions influence the variability of optimal aggregate payouts in equilibrium.

In chapter 4, I initially built three DSGE models with different internal habit formation utility functions. This is a substantive contribution as there are not many previous papers that study the simultaneous interaction of optimal aggregate payouts and the investor’s utility in a DSGE model. Comparing my three incorporated-habit DSGE models and one benchmark (no-habit) model, I found that:

1. The three with-habit models provided similar results in that the variability of optimal aggregate dividends to the variability of output in each with-habit model is greater than the no-habit model. It implies that aggregate dividend policy is influenced by investors’ consumption habit formation.

2. Simultaneously, the volatilities of aggregate consumption in with-habit models decrease in all three with-habit models compared to the figure of the no-habit model. These results are consistent with the hypothesis that agents prefer having smooth consumption.

3. Aggregate dividends are countercyclical while aggregate labour is procyclical. These findings can be explained from a conservative firm’s position. When the economy is booming, individual firms prefer to hold onto more cash flows for positive NPV projects. As a consequence, they decide to cut dividend payments. At the same time, companies recruit more labour in order to meet their growing sale targets across time. The marginal utility of consumption therefore becomes lower as labour income is high. My results show that there was a low correlation between consumption and labour in the with-habit models.

My second contribution in this chapter is that I captured changes in the variability of optimal aggregate payouts by changing the strength of the habit motive. There are two main findings discovered from this investigation:

1. My first finding is that the stronger the habit motive the more volatile (smoother) is optimal aggregate dividend growth (consumption). As expected, agents prefer
6.2. SUMMARY OF FINDINGS

to have steady consumption in the long run, and consequently they need to balance out changes in their incomes by adjusting their dividend and labour availability when their consumption habits change. In this equilibrium, dividends are countercyclical and labour is procyclical. Dividend policy plays an important role as a mechanism to help investors hedge themselves from the volatility of the business cycle.

2. I further found that the strength of investors’ habit motives influences corporate financial activity. When the habit motive gets stronger, there is a greater associated volatility in optimal real aggregate investment, and also of debt derived from external financing.

My last investigation in this chapter looked at the reaction of optimal aggregate dividend behaviour when the agent’s risk aversion changes. My simulated results showed that the volatility of optimal aggregate payouts decreases when the coefficient of risk aversion increases for all levels of habit motive. When the economy has investors who have extremely strong risk aversion, investors prefer having smooth dividend incomes as steady payouts help investor to stay away uncertain changes in their saving and spending plans. This is an interesting opposite effect to optimal dividend policy compared to the investor’s habit motive.

In chapter 5, I demonstrated that aggregate dividend policy is inelastic across time when the production process has high capital adjustment costs (CAC) and an internal habit in a DSGE model. My initial model is a fixed-labour DSGE model in which labour is an exogenous variable. My findings in terms of this basic model are summarised as follows:

1. Optimal aggregate dividend payouts do not vary greatly when the strength of capital adjustment costs changes. This is also true for real investment. The observed data is much smoother than these theoretical results.

2. Aggregate dividends remain as a countercyclical variable to the business cycle but have a weak relationship with aggregate output in the presence of high capital
adjustment costs. The correlation between optimal aggregate dividend payouts and share prices becomes stronger when capital adjustment costs increase.

3. My findings are consistent with Jermann (1998)'s result that the volatility of aggregate consumption in with-CAC-and-habit model is greater than that in the only-habit model. Jermann (1998) found that consumption smoothing is distorted when a DSGE model incorporates both habit formation and capital adjustment costs. This statement, however, exists if and only if (a) labour is an exogenous variable and (b) capital adjustment costs are very expensive.

I then extended my first calibration to release the assumption of fixed labour. My second investigation was based on a variable-labour model and I found that:

1. With an endogenous labour supply, my advanced model showed especially smooth optimal aggregate payouts in the presence of expensive capital adjustment costs. There is also a smaller associated volatility in real investment. These findings are consistent with my conjecture that capital adjustment costs impede corporations to use their cash flows in investment projects in a flexible way, and consequently net cash flows become less volatile across time. It is also consistent with prior empirical observations that dividend smoothing is a significant signal part of business performance.

2. Simultaneously, the volatility of share prices increases with costly capital adjustment fees. This is due to that the rigid production process and firms are unable to invest in positive NPV projects at all times.

I have also constructed an additional model (the control model) in which the capital adjustment cost function of Collard and Dellas (2006) is considered. This calibration helped to examine the variability of aggregate dividends to a different form of capital adjustment costs. I have compared these results to my previous models with high/low capital adjustment costs and found that the strength of capital adjustment costs in the control model is similar to the model with a costly capital adjustment function.

My final contribution in chapter 5 is that I examined whether an economy that has utility-maximising (UM) firms with risk aversion but without capital adjustment costs,
is equivalent to another economy in which there are value-maximising (VM) firms and capital adjustment costs. Cárcelès Poveda (2003) emphasises that these two economies have identical equilibrium behaviour around the steady states of variables. From my simulated results, I found that:

1. The UM model generated similar steady states of aggregate dividends, investments, consumption and other variables to those in the VM model. These two models, as hypothesised, have an analogous equilibrium.

2. As to the cyclical variability of each variable, both aggregate payouts and real investment are much smoother in the UM model, rather than in the VM model while aggregate consumption volatility is similar. In both economies, net cash flows are smooth across time since firms face inelastic investments. Optimal aggregate dividend policy and investment decisions are influenced by high capital adjustment costs or by risk aversion.

3. Interestingly, the UM model resulted in procyclical aggregate dividends. This is in line with the empirical data for gross dividends and in contrast to previous calibrated results of the VM model that show there is an inverse relationship between dividends and output. As presented in Chapter 2, during several periods, the empirical data for net cash flows displays a positive relationship with Gross National Product (GNP). It seems that a firm’s characteristic has an impact to the cyclicity of total payouts.

4. Firms are more cautious in their attitude to investing in uncertain investment projects. A smaller correlation coefficient between aggregate dividends and output in the UM model, than in the VM model, is found.

Overall, this thesis studied corporate dividend policy from a market-wide viewpoint. From an aggregated perspective, I showed that optimal aggregate dividends are greatly influenced by not only corporate but also investor activities.
6.3 Future development

In this thesis, I have shown that smooth consumption and the capital accumulation process are closely related to the variability of optimal aggregate payouts. I believe that this opens a large number of areas for future research as the results presented here are likely to be sensitive to the exact specification of the optimisation problem of the individual, the firm and the bank. This work could therefore be extended, for example, by (a) examining optimal aggregate dividend and investment policy when there is irreversible investment; (b) investigating the impact of taxation on cash dividends\(^1\) in a general equilibrium state, and the influence of firms’ financing and investment decisions on dividend tax reform. For a related discussion, see Hutton and Kenc (1998), Santoro and Wei (2009) and Gourio and Miao (2010); (c) to consider financial shocks in the models. A recent paper by Jermann and Quadrini (2009), finds that additional financial shocks make the model perform more in line with observed economic behaviour; particularly the movements of output and labour. In their report, they also demonstrate that a model with both frictions of technology and financial shocks result in optimal dividends being positively correlated with output while a model only with technology shocks has countercyclical dividends. This then helps to explain why models only with technology shocks (for example, this thesis) produce the result that dividends are negatively correlated with output.

In conclusion, this thesis indicates that corporate finance issues can be examined from an aggregated market dimension. It contributes to a deeper understanding of corporate finance activity by taking many factors and frictions into account in order to provide a substantial and integrated macroeconomic approach. This work should be of interest to both academics and practitioners with an interest in payout policy.

\(^1\)Suggested by participants at the 17th Annual Global Finance Conference which was hosted by Poznan University of Economics in Poznan, Poland, June 27-30, 2010.
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