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# Calculation of the Spatial Envelope Correlation Between Two Antennas in Terms of the System Scattering Parameters Including Conducting Losses

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**Abstract**— A simple method for calculating the envelope correlation for two element antenna array using the scattering parameters was introduced in 2003. This provides a major simplification compared with the conventional method using the antenna radiation field patterns. In this paper we propose a method for including the losses. This approach has the advantage of simplifying the antenna design, especially when low envelope correlation is needed. It also offers a better prediction of the spatial envelope correlation and good understanding of the effects of the mutual coupling. The accuracy of the proposed method is illustrated by two examples.

**Keywords**- antenna diversity, radiated power, envelope correlation, scattering parameters.

## I. INTRODUCTION

Mobile communication systems where there is only one antenna at both the transmitter and the receiver are known as Single Input Single Output (SISO) systems. SISO system capacity is limited by the Shannon Nyquist criterion [1]. In order to increase the capacity of the SISO systems to meet the high bit rate transmission demanded by the modern mobile communications the bandwidth and/or the power have to increase significantly. Fortunately using MIMO systems (Multiple Input, Multiple Output) has the potential to increase the capacity of the wireless system without the need to increase the transmission power or the bandwidth. On the other hand, mutual coupling between the antennas degrades the antenna system diversity performance; therefore designers try to minimize the mutual coupling of the antenna system while maintaining the matching requirements. MIMO systems are required to deliver maximum capacity with minimum bit error rate (BER). To achieve that, the antenna arrays should have high gain, narrow lobe patterns, and reasonable separation between the elements. In mobile communication the antenna spacing is usually small, thus the impact of the mutual coupling will be not negligible. Mutual coupling increases the spatial correlation between the array elements. Also, it deforms the radiation pattern of each array element, which affects the diversity gain.

There are three different methods used to calculate the antenna spatial correlation. The first method is based on the far field pattern [2], the second is based on the scattering parameters at the antenna terminals [3], a third method is based on Clarke's formula [4]. Calculating the envelope correlation using the radiation field pattern is a time consuming process, whether it is obtained by simulation or from experimental data. A simple approach for calculating the envelope correlation for two antenna elements using the scattering parameters was introduced in [5]. This approach caught the attention of researchers on antenna diversity [6-7] and it was generalized to N antenna element in [8].

The two methods in [5] and [8] don't take into account any power losses in the antenna structure, which is the main difference between the spatial envelope correlation as calculated by this method and the one computed from the radiation field pattern of two antenna elements [9], i.e.:

$$\rho_e = \frac{\left| \iint_{4\pi} d\Omega \mathbf{F}_1(\theta, \phi) \cdot \mathbf{F}_2(\theta, \phi) \right|^2}{\iint_{4\pi} d\Omega |\mathbf{F}_1(\theta, \phi)|^2 \iint_{4\pi} d\Omega |\mathbf{F}_2(\theta, \phi)|^2} \quad (1)$$

where  $\mathbf{F}_i(\theta, \phi) = \mathbf{F}_\theta^i(\theta, \phi)\hat{\mathbf{e}}_\theta + \mathbf{F}_\phi^i(\theta, \phi)\hat{\mathbf{e}}_\phi$  is the radiation field of the  $i^{\text{th}}$  antenna.

In the present paper, the calculation of the envelope correlation in (1) for a lossy two antenna port will be estimated in terms of the scattering parameters and the realistic power losses of the antenna structures. The power loss is presented in a matrix formulation, in order to match the presentation in [5, 8]. The result of this approach represents a considerable simplification over the complex calculations required to evaluate (1) using radiation field pattern data. Two illustrative examples are presented and discussed.

## II. SUMMARY OF THE METHOD

Consider the electromagnetic geometry shown in fig. 1. The total power is written in terms of the power radiated and lost internally:

$$P_{\text{total}} = P_{\text{rad}} + P_{\text{loss}} \quad (2)$$

where  $P_{\text{rad}}$  and  $P_{\text{loss}}$  are the radiated and the loss power respectively;  $P_{\text{total}}$  is also referred to as the accepted power and can be computed in terms of the incident ( $\mathbf{a}$ ) and reflected ( $\mathbf{b}$ ) wave amplitude vectors  $\mathbf{a} = [a_1, a_2]^T$  and  $\mathbf{b} = [b_1, b_2]^T$  by  $(\mathbf{a}^\dagger \mathbf{a} - \mathbf{b}^\dagger \mathbf{b})$ , where  $^T$  is the transpose and  $^\dagger$  the Hermitian transpose. The power loss is computed in terms of the currents on the antenna structure (see Fig. 1) as follows:

$$J_{s1} = \frac{1}{\sqrt{R_S}} (a_1 J_{s11} + a_2 J_{s21}) \quad (3)$$

$$J_{s2} = \frac{1}{\sqrt{R_S}} (a_1 J_{s12} + a_2 J_{s22}) \quad (4)$$

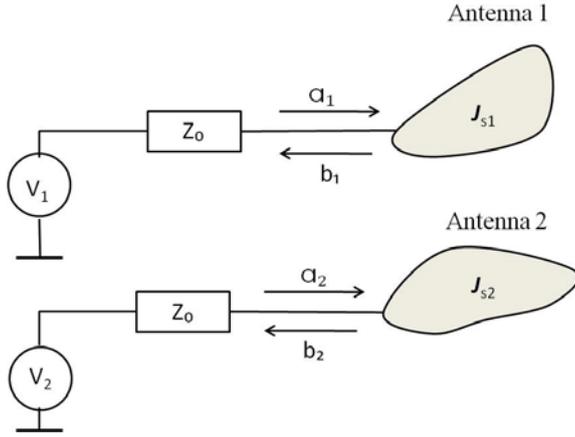


Fig. 1 Basic geometry for two antenna element system.

The currents are expressed in terms of the incident waves  $a_1$  and  $a_2$ .  $J_{s1}$  and  $J_{s2}$  are the total currents on the structures 1 and 2 respectively.  $J_{s11}$  and  $J_{s21}$  are the normalized currents on structure 1 and 2 due to the incident waves  $a_1$ , similarly  $J_{s12}$  and  $J_{s22}$  are the normalized currents on structures 1 and 2 due to the incident wave  $a_2$ , and  $R_S$  is the surface impedance of the antenna, therefore the power loss on structure 1 and 2 can be calculated by (5) and (6) respectively:

$$P_{1l} = (a_1 J_{s11} + a_2 J_{s21}) \cdot (a_1 J_{s11} + a_2 J_{s21})^* \quad (5)$$

$$P_{2l} = (a_1 J_{s12} + a_2 J_{s22}) \cdot (a_1 J_{s12} + a_2 J_{s22})^* \quad (6)$$

Expanding the above equations will give:

$$P_{1l} = |a_1|^2 J_{s11} J_{s11}^* + a_1 a_2^* J_{s11} J_{s21}^* + a_2 a_1^* J_{s21} J_{s11}^* + |a_2|^2 J_{s21} J_{s21}^* \quad (7)$$

$$P_{2l} = |a_1|^2 J_{s12} J_{s12}^* + a_1 a_2^* J_{s12} J_{s22}^* + a_2 a_1^* J_{s22} J_{s12}^* + |a_2|^2 J_{s22} J_{s22}^* \quad (8)$$

Hence, (7) and (8) can be expressed in a matrix notation as follows:

$$P_{\text{loss}} = (\mathbf{a}, \mathbf{L}\mathbf{a}) = \mathbf{a}^\dagger \mathbf{L}\mathbf{a} \quad (9)$$

Where,

$$\mathbf{L} = \mathbf{L}' + \mathbf{L}'' = \begin{pmatrix} L'_{11} & L'_{12} \\ L'_{21} & L'_{22} \end{pmatrix} + \begin{pmatrix} L''_{11} & L''_{12} \\ L''_{21} & L''_{22} \end{pmatrix} \quad (10)$$

The elements  $L'_{ij}$  are obtained from the power loss calculation and can be written as  $J_{s11} J_{s11}^*$ ,  $J_{s11} J_{s21}^*$ ,  $J_{s21} J_{s11}^*$  and  $J_{s21} J_{s21}^*$  similarly the elements  $L''_{ij}$  can be written as  $J_{s12} J_{s12}^*$ ,  $J_{s12} J_{s22}^*$ ,  $J_{s22} J_{s12}^*$  and  $J_{s22} J_{s22}^*$ . From the Hermitian inner product properties it follows that  $L_{ij} = L_{ij}^*$ .

Now, Equ. (2) can be expressed in terms of the incident waves  $a_1$  and  $a_2$  as follow:

$$\mathbf{a}^\dagger (\mathbf{1} - \mathbf{S}^\dagger \mathbf{S}) \mathbf{a} = \mathbf{a}^\dagger \mathbf{L}\mathbf{a} + \mathbf{a}^\dagger \mathbf{R}\mathbf{a} \quad (11)$$

where the scattering matrix  $\mathbf{S}$  of the two-antenna system, including the power loss.  $\mathbf{R}$  is the  $2 \times 2$  matrix defined in [5] as,

$$\mathbf{R} = \begin{pmatrix} \frac{D_1}{4\pi} \iint_{4\pi} d\Omega |\mathbf{F}_1(\theta, \phi)|^2 & \frac{\sqrt{D_1 D_2}}{4\pi} \iint_{4\pi} d\Omega \mathbf{F}_1(\theta, \phi) \cdot \mathbf{F}_2(\theta, \phi) \\ \frac{\sqrt{D_1 D_2}}{4\pi} \iint_{4\pi} d\Omega \mathbf{F}_2(\theta, \phi) \cdot \mathbf{F}_1(\theta, \phi) & \frac{D_2}{4\pi} \iint_{4\pi} d\Omega |\mathbf{F}_2(\theta, \phi)|^2 \end{pmatrix} \quad (12)$$

where  $D_i$  is the  $i^{\text{th}}$  maximum directivity of the  $i^{\text{th}}$  antenna. Therefore, from (11) the equivalent elements of (12) can be expressed as follows:

$$\frac{D_1}{4\pi} \iint_{4\pi} d\Omega |\mathbf{F}_1(\theta, \phi)|^2 = 1 - (|S_{11}|^2 + |S_{21}|^2) - (L'_{11} + L''_{11}) \quad (13)$$

$$\frac{\sqrt{D_1 D_2}}{4\pi} \iint_{4\pi} d\Omega \mathbf{F}_1(\theta, \phi) \cdot \mathbf{F}_2(\theta, \phi) = -(S_{11}^* S_{12} + S_{21}^* S_{22}) - (L'_{12} + L''_{12}) \quad (14)$$

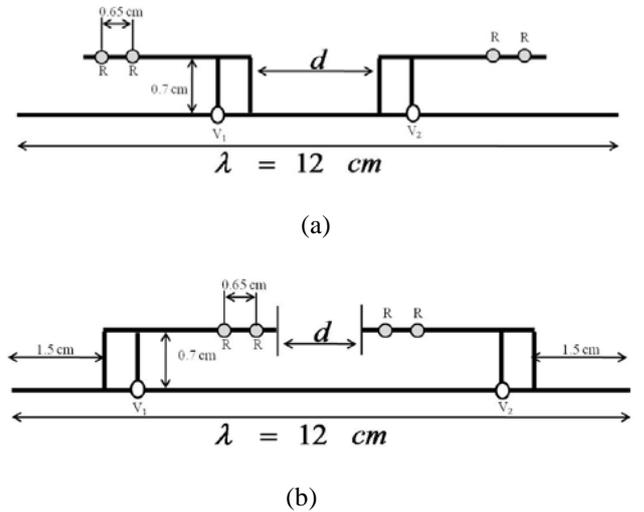


Fig. 2 Example under test: structure (a) and structure (b).

Thus, the envelope correlation for the two-antenna system geometry shown in Fig. 1, can be expressed in terms of the

scattering parameters, and the intrinsic power losses, as follows:

$$\rho_e = \frac{|-(S_{11}^* S_{12} + S_{21}^* S_{22}) - (L_{12}^* + L_{12}^{\prime\prime})|^2}{\{1 - (|S_{11}|^2 + |S_{21}|^2) - (L_{11}^* + L_{11}^{\prime\prime})\} \{1 - (|S_{22}|^2 + |S_{12}|^2) - (L_{22}^* + L_{22}^{\prime\prime})\}} \quad (15)$$

### III. SIMULATION AND RESULTS

In order to verify (15), the envelope correlation has been computed for an array of two Planar Inverted F-antennas (PIFA) in free space, as a function of their separation distance 'd'. The antenna far fields and scattering parameters were obtained using NEC [10]. The wire radius used to create the whole system was set to 1 mm. Two different sources of losses were considered for the purpose of validation. In the first test, both antennas were loaded by lumped resistive loads each of 10  $\Omega$  as shown in Figs. 2a and 2b; whereas in the second a conducting surface resistivity was considered on both wires.

The difference between the proposed method and the lossless approach was checked by simulation. The spatial envelope correlation calculated using the far field parameters, versus the antenna separation distance d for the example shown in Fig. 2a, is given in Fig. 3 for both lossless and lossy case together with the result from the scattering parameters with loss.

There is good agreement between the proposed method and the results calculated from (1) for the lossy case. The envelope correlation based on the proposed method takes values slightly lower than that calculated using the method in [5] without loss. It is interesting to note that the  $S_{11}$  value varies between -0.2 and -1 dB. Fig 2a show the two planar antennas with their feed ends adjacent, whilst in Fig. 2b they are the other way round. For Fig. (2a) a strong mutual coupling will presumably be introduced between the two antennas created by the high currents passing through the short feed pins. At almost all antenna separations the envelope correlation takes values higher than the  $S_{21}$  values.

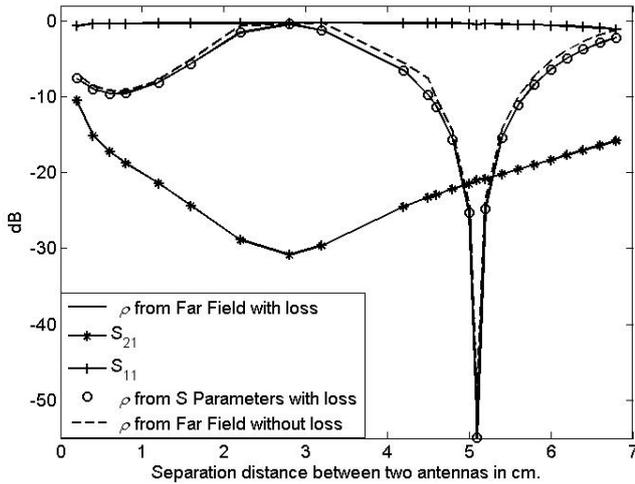


Fig. 3 Computed spatial envelope correlations and S-parameters for two PIFA antennas against their separated distance 'd' in centimetres.

Fig. 4 illustrates variations in the spatial envelope correlations versus the antennas separation distance for the antenna array structure shown in Fig. 2b. Again, the result shows a good agreement between the proposed method and the results calculated from (1) for the lossy case, it is noticeable from Figs. 3 and 4 that the envelope correlation values computed by the proposed method and from Equ. (1) for the lossy case for antenna separations above 1 cm has a 1 dB difference. Note that for this configuration the  $S_{11}$  is much lower.

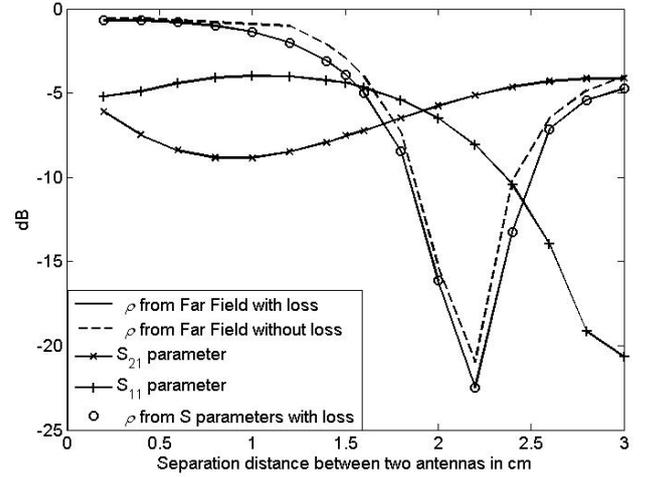


Fig. 4 Computed spatial envelope correlations and S-parameters for two PIFA antennas against their separated distance 'd' in centimetres.

Fig. 5 illustrates the variations in the spatial envelope correlation versus the surface conductivities for the structure shown in Fig. 2a. The antenna separation was kept constant at 3.2 cm. It can be seen in Fig. 5 that the envelope correlation values are closer to the  $S_{11}$  values than to the  $S_{21}$ , there is no clear effect of the surface conductivity on the envelope correlation values.

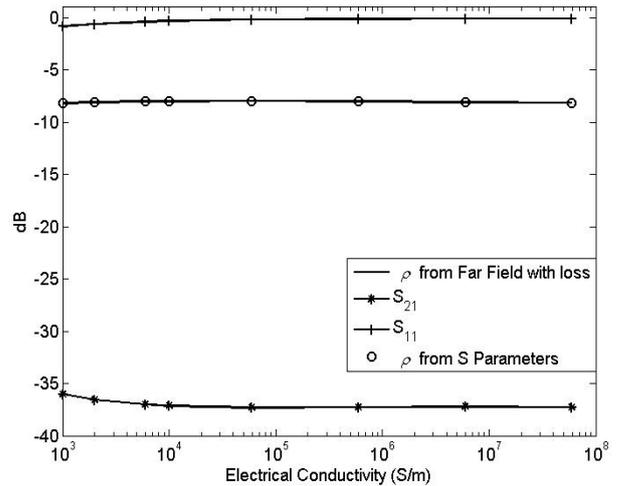


Fig. 5 Computed spatial envelope correlations and S-parameters for two PIFA antennas against surface electric conductivity on both antennas.

#### IV. CONCLUSIONS

A simple method for calculating the spatial envelope correlation of two antennas system including the losses, using the system scattering parameters has been proposed. The proposed method would reduce the complexity in evaluating and predicting the spatial envelope correlation also it will simplify and direct the effort of the antenna design where envelope correlation low is required. Two examples applying the proposed method were presented, the results shows a good agreement between the proposed method and the calculation using the far field radiation pattern data.

#### V. ACKNOWLEDGEMENTS

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