

## **The University of Bradford Institutional Repository**

<http://bradscholars.brad.ac.uk>

This work is made available online in accordance with publisher policies. Please refer to the repository record for this item and our Policy Document available from the repository home page for further information.

To see the final version of this work please visit the publisher's website. Where available access to the published online version may require a subscription.

Author(s): Loizou, A., Qi, H. S. and Day, A. J.

Title: Analysis of heat partition ratio in vehicle braking processes.

Publication year: 2009

Conference title: Braking 2009.

Conference No: c 672.

Publisher: IMechE.

Link to publisher's site: <http://events.imeche.org/EventView.aspx?code=C672>

Citation: Loizou, A., Qi, H. S. and Day, A. J. (2009). Analysis of heat partition ratio in vehicle braking processes. In: Braking 2009: Institution of Mechanical Engineers, Automobile Division Conference, St Williams College, York, 9-10 June, 2009. London: IMechE.

Copyright statement: © 2009 IMechE. Reproduced in accordance with the publisher's self-archiving policy.

---

# Analysis of heat partition ratio in vehicle braking processes

A. LOIZOU, H.S. QI AND A.J. DAY

School of Engineering, Design & Technology, University of Bradford, West Yorkshire  
BD7 1DP, UK

## ABSTRACT

An examination of the heat partition ratio between the friction surfaces of a disc braking system is presented using finite element analysis (FEA). This includes a 2D static analysis of two semi-infinite bodies in contact with and without an interface layer which represents the interface tribo-layer (ITL). An analytical approach with a finite difference solution was used for cross-comparison with the static FE models. Results from the static model have provided the boundary conditions for a 2D dynamic model, where one rectangular block slides on another (fixed) rectangular block. The effects of normal loads and real contact area have also been studied.

## NOTATION

$k$	Thermal conductivity (W/mK)	<b>Superscripts</b>	
$k^*$	Virtual thermal conductivity (W/mK) (equal to $h_c d$ )	$t$	Present time
$k_m$	Interface layer thermal cond.(W/mK)	$t+1$	Next time increment
$h_c$	Surface heat transfer coeff. (W/m <sup>2</sup> K)	$t-1$	Previous time increment
$T$	Temperature (°C)	<b>Subscripts</b>	
$T_\infty$	Surrounding temperature (°C)	1	Refers to pad material
$\delta$	Space increment (m)	2	Refers to disc material
$n$	Number of nodes in interface layer	J	Position on disc contact surface
$\delta x$	Interface layer total thickness (m)	J+1	Next space increment
$q$	Heat flux (W/m <sup>2</sup> )	J-1	Previous space increm.
$\rho$	Density (kg/m <sup>3</sup> )	0	Position at bound. layers
$c$	Specific heat capacity (J/kgK)	0-1	Space increment before boundary layer
$\delta t$	Time increment (s)		
$\phi_1$	Heat partition ratio into the pad (%)		
$\lambda$	Stability coefficient (equal to $k\delta t/\rho c\delta^2$ )		

## 1 INTRODUCTION

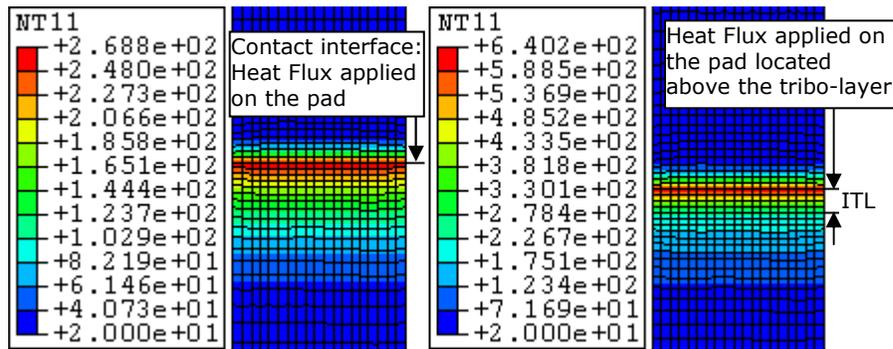
The heat partition ratio at the friction interface of a brake pad and disc is an important parameter in the study of brake disc/pad interface temperatures, and affects vehicle friction brake performance in many ways, e.g. disc thermal crack formation and fatigue failure, temperature-related frictional performance (e.g. fade), and thermally induced friction instability effects. In conventional braking thermal analysis the heat partition ratio is often assumed to be constant, even though it is reported as varying in transient thermal states [1], and is known to be affected by parameters such as sliding speed [2].

When Huang and Chen [3] modelled a 1/38<sup>th</sup> static model of a ventilated disc brake to investigate the cooling performance of disc brakes, they assumed that heat flux is constant, and that about 88% of the braking power is consumed and changed into heat of the disc. This was the braking power that has been converted to heat (thermal energy) and entered the disc side. Apte and Ravi [4] included the calculation of heat flux from empirical equations which was then applied on the entire disc rubbing surface. This may keep the simulation simple and produce acceptable predictions of overall brake disc temperature, but ignoring the effects of braking friction pair heat partition and their variability means that frictional performance (which depends on friction interface temperature) and all associated phenomena (e.g. non uniform stresses and temperatures around the disc that can lead to cracks, hot spots and permanent deformation) cannot be studied. Qi and Day [5] presented a method to advance the understanding of disc and pad interface temperatures and friction, that combined experimental and FEA approaches. This had the advantage that it simulated and investigated the pad surface variation and real contact area ratio. The disc was only represented as a boundary condition on the pad friction surface, and the heat partition was assumed to have a constant value. Majcherczak, Dufrenoy and Nait-Abdelaziz [6] focused on the heat generation and temperatures on the disc and pad, and the results of their axial symmetric FE model showed that an interface layer between the disc and pad had an effect on the pad temperatures rather than the disc temperatures. They also concluded that significant interface layer properties could include thickness and thermal conductivity. That model could be improved by the incorporation of the variation of the partition ratio and the non-uniformity of pressure distribution. Komanduri and Hou [7] developed analytical solutions for the temperature rise due to friction. They re-introduced the following two fundamental theories from Blok and Jaeger [8, 9]. The first theory assumes having a uniform heat partition, but with the contact temperatures not matching. In the second theory the temperatures of the two surfaces are matched by varying the heat partition. These theories were later developed specifically for brake friction pairs by Newcomb [10]. It can be argued that none of the two theories represents reality in a brake friction pair. This demonstrates the limitations of some analytical methods, leading to the seeking of other methodologies or a combination of existing methodologies.

The current work does not assume neither the temperature, nor the heat flux (or heat flux partition) constant, but attempts to identify their real behaviour. Research into heat partitioning in a brake friction pair is made, based on 2D static and dynamic thermal models using FEA techniques. In the first 2D model, the scenario of two semi-infinite bodies in contact with and without an interface layer was modelled using ABAQUS Finite Element (FE) analysis, which was then verified using a separate study based on a finite difference approach. The effect of interface thermal conductance [11] in the interface tribo-layer on the heat partition ratio has been studied, and an equivalent value for the thermomechanical properties of the interface tribo-layer has been established. Results from an initial (static) 2D simulation set the conditions for a second (dynamic) 2D model, in which the scenario of one 2-D rectangular block sliding on another 2-D rectangular block, with friction heat generation at the contact interface, was simulated. Factors including the ratio of real to apparent contact area (surface roughness effect), the normal force applied (piston effect), and relative motion were included. The heat partition ratios were obtained by measuring the heat transfer into each block as a vector output, and the results demonstrate the percentage of heat partition that enters the pad for a range of operating conditions.

## 2 2-D STATIC TRANSIENT THERMAL ANALYSIS

Two semi-infinite bodies in contact and with no relative movement to each other (static) were modelled using the finite element method. A model with and a model without an interface layer was used for the analysis (Figure 1).



**Figure 1** 2D static models with (Right) and without (Left) interface layer

### 2.1 FEA Setup

The effect of the interface thermal conductance in the interface tribo-layer and the effects of model parameters including the mesh size and the time increments were investigated. The top box always represents the pad and the bottom box the disc, based on the material properties assigned to each (see Table 1). Both boxes were 50mm square and the interface tribo-layer had a thickness ( $\delta x$ ) of 5mm. In reality the interface tribo-layer would be much thinner, in the scale of  $\mu\text{m}$  but modelling the layer in the order of mm had a purpose. It allowed a better investigation on the heat flow and temperature distribution "in the layer". This is since it gave us the opportunity to use more rows of elements, as opposed to if it were to be modelled in the order of  $\mu\text{m}$  (which would also require a denser mesh). If the space increments between the nodes are equal, then  $\delta x = \delta(n-1)$ . In addition a layer in the order of  $\mu\text{m}$  was attempted for comparison purposes, and it is found that is not viable and introduces software and hardware problems (at least with the current PC configuration). The principle and the effects of the interface tribo-layer are adequately investigated with this design. Heat flux was applied on the pad contact surface, as friction is assumed to be generated there by adhesion, abrasion and deformation [12].

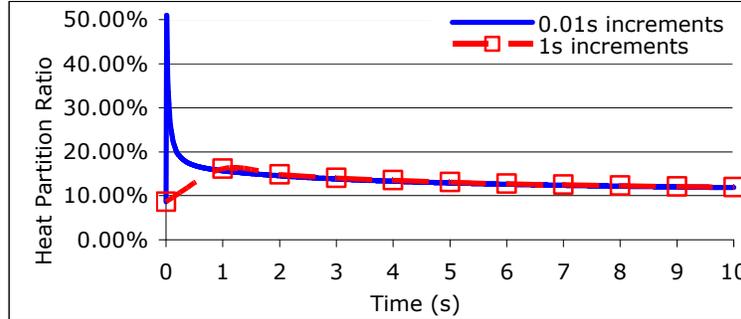
**Table 1** Thermal properties of the materials used for static analysis

	Pad [13]	Interface Layer [14]	Disc [13]
Thermal Conductivity (W/mK)	0.9	10	48
Density (Kg/m <sup>3</sup> )	1550	0.1	7800
Specific heat capacity (J/kgK)	1200	1000	452

To examine the effect of the mesh size two mesh grades were created. For the model without an interface tribo-layer (Figure 1 left) 1080 elements were needed for the coarser mesh model and 2080 for the finer mesh model. For the model with the interface tribo-layer (Figure 1 right), 1188 and 2496 elements were used for the coarser and finer models respectively. The simulations were carried out with time increments of 1 second and 0.01 seconds. The combination of two models (with or without a tribo-layer), with two mesh grades (coarser and finer), and two time increments (0 and 0.01 seconds), provided eight sets of simulations results.

Figure 1 shows an example of the difference in temperature distribution with and without the interface tribo-layer, as predicted by the coarser mesh model with time

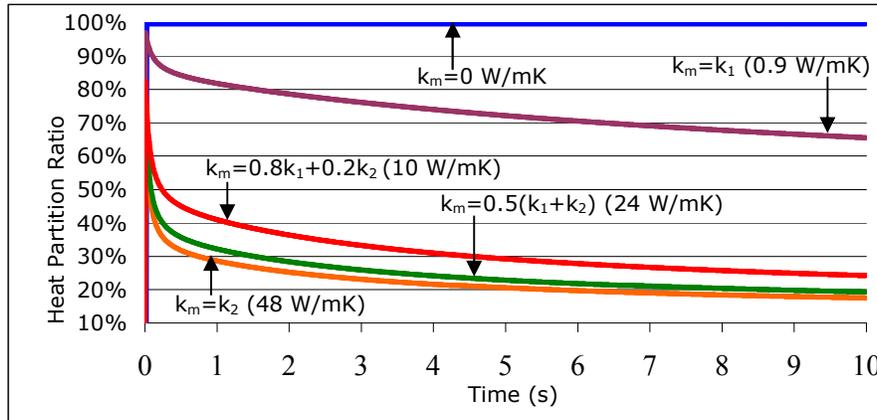
increments of 1 second. Figure 2 compares the heat partition ratio (percentage of the total heat entering the pad), of the same model with the time increments of 1 and 0.01 seconds respectively. The predictions from the two mesh designs, showed no significant difference between the finer mesh which consumes much more time and computing power than the coarser mesh. Smaller time increments stabilised the system faster (Figure 2). So based on these findings, the effect of the interface thermal conductance in the interface tribo-layer on the heat partition was modelled with the coarse mesh model with time increments of 0.01 seconds (section 2.2).



**Figure 2** Heat partition ( $\phi_1$ ) ratio with time increments of 1s and 0.01s

### 2.2 Interface tribo-layer properties study

The current study on the interface tribo-layer proposes that its properties are a result of a combination of the disc and pad material properties, defined as a ratio of each material in the composition. To study the importance of thermal conductivity of the interface tribo-layer, Figure 3 shows the heat partition entering the pad ( $\phi_1$ ), for an interface tribo-layer of  $k_m = 0$ ,  $k_1$ ,  $k_2$  and  $0.5(k_1 + k_2)$ . The heat partition for the thermal conductivity of 10 W/mK (as in Table 1) [14] for the interface tribo-layer is also shown (resulting from  $0.8k_1 + 0.2k_2$ ). For the following sections, material properties are as in Table 1.



**Figure 3** Heat partition ( $\phi_1$ ) for various thermal conductivities of interface layer

#### 2.2.1 Equivalent thermal conductance

The interface layer is reported to be much thinner than the one modelled for the 2D static analysis ( $1 \sim 10 \mu\text{m}$ ) [15]. One way of simulating the interface layer in a realistic thickness was by an equivalent thermal conductance which represents the

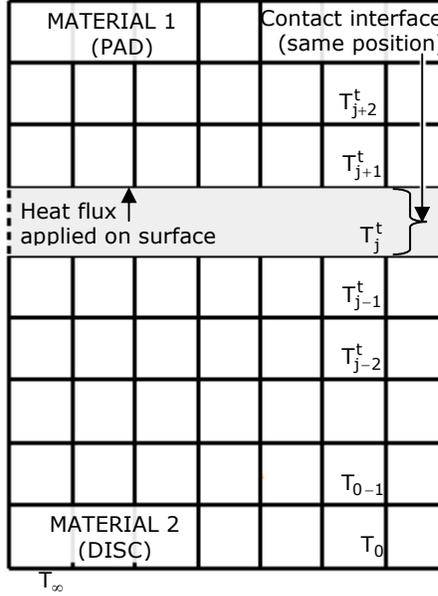
thermal properties of the layer. A similar approach was presented by Homani and Farhang [16]. They obtained equivalent values with a combination of dynamometer test data and predictive simulation, and adjusted the conductance until a maximum temperature was reached at a known point.

In the current work, calculation of the equivalent values was completed and verified using the static 2D models (Figure 5), in which the line where the equivalent layer properties are modelled has been shifted because the actual tribo-layer dimensions do not exist in the model. The equivalent layer properties are the thickness of the layer ( $\delta x$ ) and the thermal conductivity  $k_m$ . The equivalent thermal conductance established by the interface layer is equal to  $k_m/\delta x$ . When the equivalent thermal conductance value line is shifted 0.005m (as indicated by the arrow in Figure 5) it matches the line of the pad region of the FEA model. Thus the thermal effects of tribo-layer are replicated.

### 2.3 Finite difference method (FDM) setup

An analytical solution solved by the Finite Difference method was used for cross comparison with the FE models. The theory is described using a coarse mesh model (Figure 4), but during the validation identical mesh and time increments to that presented in section 2.1 was used.

Equations 1 and 2 represent the heat balance on the two contact surfaces. Solving for  $T_j^{t+1}$  and  $T_{j+1}^{t+1}$  in these equations, the temperatures at this point at time  $t+1$  are found. Equation 3 represents the energy balance at the boundary layers (temperature is found by solving for  $T_0^{t+1}$ ) and equation 4 is used to find the temperatures at the middle nodes [17]. These finite difference parabolic equations were used to describe the time-variable heat transfer problem as recommended in [18]. Figure 5 shows the comparison of FEA and FDM results.



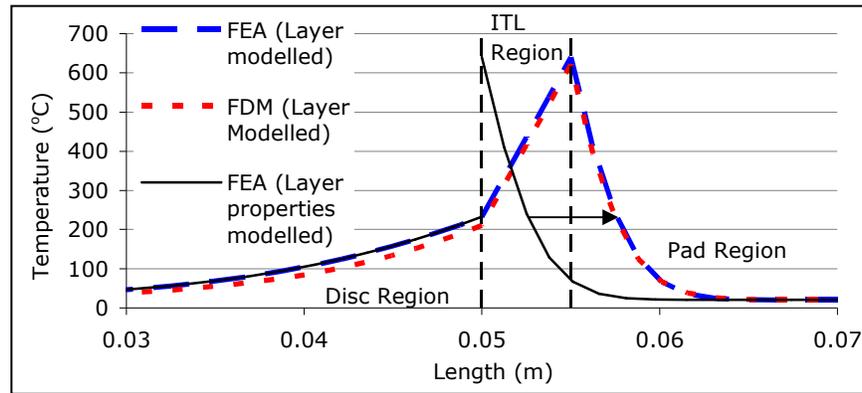
**Figure 4** Model for demonstrating the analytical method

$$k_2 \frac{T_{j-1}^{t+1} - T_j^{t+1}}{\delta} + k^* \frac{T_{j+1}^{t+1} - T_j^{t+1}}{\delta} + q = \left( \rho_2 c_2 \frac{\delta}{2} \right) \frac{T_j^{t+1} - T_j^t}{\delta t} \quad \text{Equation 1}$$

$$k_1 \frac{T_{j+2}^{t+1} - T_{j+1}^{t+1}}{\delta} + k^* \frac{T_j^{t+1} - T_{j+1}^{t+1}}{\delta} = \left( \rho_1 c_1 \frac{\delta}{2} \right) \frac{T_{j+1}^{t+1} - T_{j+1}^t}{\delta t} \quad \text{Equation 2}$$

$$h_w (T_\infty - T_0^{t+1}) + k \frac{T_{0+1}^{t+1} - T_0^{t+1}}{\delta} = \rho c \frac{\delta}{2} \frac{T_0^{t+1} - T_0^t}{\delta t} \quad \text{Equation 3}$$

$$T_j^{t+1} = \frac{T_j^t + \lambda (T_{j-1}^{t+1} + T_{j+1}^{t+1})}{1 + 2\lambda} \quad \text{Equation 4}$$

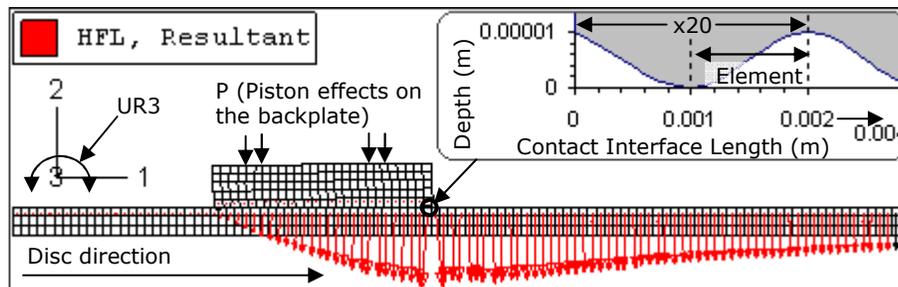


**Figure 5** Temperatures with two different methods and equivalent properties

### 3 2D DYNAMIC FRICTIONAL HEAT FEA ANALYSIS

Following the static 2-D FE and FD study, frictional heating was introduced into a dynamic frictional heat FE analysis (instead of applying directly heat flux on the pad surface as in section 2) which used two 2-dimensional blocks as shown in Figure 6. The top (smaller) block represented the pad with dimensions 40x5 mm, and the bottom (larger) block the disc with dimensions 200x5 mm. An interface tribo-layer thickness of 5 $\mu$ m was specified and taking advantage of the equivalent thermal conductance calculations ( $k_m / \delta x = 10.32 / 5E-6 = 2.064E6$ ).

The heat partition ratios were calculated from the heat transfer into each block as a vector output (Figure 6). For comparison purposes all values were measured at the same point and time instant. For example, at a specific location on the contact interface the heat flux vector entering the pad and disc are  $q_1$  and  $q_2$  respectively. The percentage of heat partition that enters the pad ( $\phi_1$ ) can be found from  $(q_1 / (q_1 + q_2)) \times 100\%$ .



**Figure 6** Heat flux entering each block as a vector output for rough surface model

#### 3.1 FEM Setup

For the perfect initial contact model, the parameters in sections 3.1.1 and 3.1.2 were considered.

##### 3.1.1 Normal loads

**Case 1 (Uniform pad pressure):** Actuation was applied to the model in the form of uniformly applied pressure on the entire top surface of the pad. The interface contact surfaces between the two parts of the model were assumed to be in full contact with a friction coefficient of 0.4.

**Case 2 (Piston effects & pad backplate):** When actuation force is applied via a hydraulic piston, a circular locus of contact actuation force is applied to the pad backplate. Since the model was two-dimensional this circular locus was modelled as two small areas of actuation pressure as shown in Figure 6, adjusted so the same load was applied as in Case 1.

**Table 2** Heat flux and heat partition ratios ( $\phi_1$ ) for Case 1 and Case 2

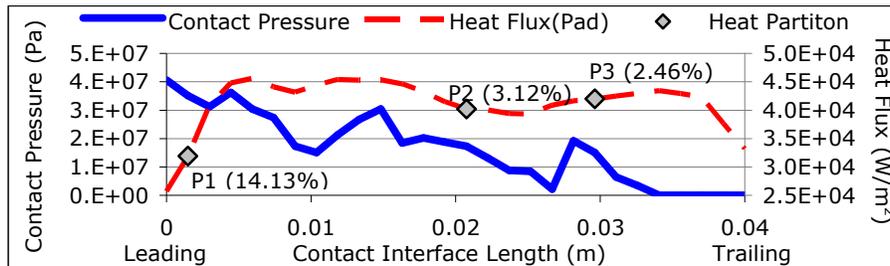
Case 1 (Uniform pressure)			
	Leading	Middle	Trailing
Temp. disc / pad ( $^{\circ}\text{C}$ )	24.2 / 52.3	51.7 / 62.4	66.4 / 78.3
Disc Heat Flux ( $\text{W}/\text{m}^2$ )	133626	901429	1324570
Pad Heat Flux ( $\text{W}/\text{m}^2$ )	15820.1	25126.5	37423
Heat Partition (%)	10.59	2.71	2.75

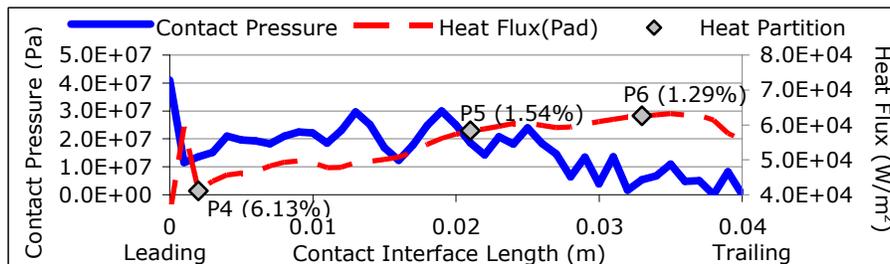
Case 2 (Piston effects and back plate)			
	Leading (P1)	Middle (P2)	Trailing (P3)
Temp. disc / pad ( $^{\circ}\text{C}$ )	26.4 / 85.3	62.9 / 90.1	79.8 / 95.3
Disc Heat Flux ( $\text{W}/\text{m}^2$ )	193569	1248880	1667680
Pad Heat Flux ( $\text{W}/\text{m}^2$ )	31864.9	40232.7	42027
Heat Partition (%)	14.13	3.12	2.46

### 3.1.2 Real contact area

The real disc / pad contact area is not equal to the total (apparent) pad contact surface area, due to the roughness of the surfaces. The effect of real contact area was examined by modifying the Case 2 model. The pad surface was altered by creating twenty repeated peaks of 0.01mm normal to the already existing even pad surface. The wavelength from peak to peak was 2 mm (see Figure 6). The total number of elements in the pad has not been changed, but the elements on the last row (where contact takes place) have been modified. The element size was 1.67x1mm and a total of 90 elements have been used for the pad. The disc surface was not altered during this test. Comparison results are shown in Figures 7 and 8.



**Figure 7** Contact pressure and heat flux at interface (Perfect initial contact model)



**Figure 8** Contact pressure and heat flux at interface (Rough contact model)

## 4 DISCUSSION

The variation of heat partition ratio, both in time and space (different location on the contact interface) has been considered. It is found that it is difficult to predict the instantaneous heat partition ratio since it is affected by many factors. A two step approach (first step being the static analysis and second the dynamic analysis) is presented in this paper. This allows the phenomenon to be fundamentally investigated more efficiently and later this to be extended to the specific application.

It is generally not easy to study the properties and behaviour of the interface tribo-layer, because if it is included in a dynamic analysis, computing power and time requirements are dramatically increased due to a very small element size that will be required, making it impossible even for a static analysis (if actual dimensions were used). The static models with semi-infinite bodies allowed the investigation of the interface tribo-layer thermal conductance. By understanding the effects of the interface tribo-layer it can be evaluated if it is important for accurate modelling. By using the 2D static models, it has been found that the interface tribo-layer directly affects the heat partition between two bodies in frictional sliding contact, and hence the temperatures of the two surfaces. The thermophysical properties of the interface tribo-layer were represented as a percentage of the disc and pad material properties. In this study the value of 10 W/mK initially taken from the literature can be compared with the value calculated from 80% pad and 20% disc (see Table 1). Higher thermal conductivity on the interface tribo-layer reduces the percentage heat partition value; therefore more heat flows into the disc and less to the pad. Since the pad material is softer and might be expected to contribute more to the interface tribo-layer composition, it could be possible to control the heat partition more easily by altering the pad material properties. The thermophysical properties of the interface tribo-layer were modelled with an equivalent thermal conductance value. This value takes into account the thermal conductivity and the thickness of the layer, which according to [6] are important parameters.

Moving to the dynamic modelling it has been shown that the heat partition between the contact surfaces does not only vary with time, but also across the length of the surface. The general trend is that there is a higher value of heat partition on the leading edge, where there is higher pressure. This contact pressure distribution was because of the boundary conditions set, where the back plate (on which the pad is tied), is allowed to rotate about axis UR3 (see Figure 6) to take into account flexibilities that exist in the disc brake assembly. As the moving disc (bigger block) was rubbed against the pad (smaller block), it forced this kind of pressure distribution. According to Figure 7 at P1 (pad leading) there is 14.13%, at P2 (pad middle) 3.12% and at P3 (pad trailing) 2.46% heat partition on the pad. It was observed that the higher the contact pressures are the more heat flux is generated on the smaller block. This means that with increased pressure more heat is generated on the pad (softer material), and that as the pressure increases the real interface contact area increases which leads to more heat transferred to the disc (hence the heat partition ratio increases). Comparing the two models in Table 2 there is a similar results pattern, but Case 2 is considered more consistent than Case 1. The reason is that, by not having the back plate to add rigidity on the pad it deforms excessively and the results cannot always be trusted. This shows the importance of the back plate on the model assembly, and for that reason it has been considered (Case 2) for the next step (rough surface model).

From the rough surface model, the main difference in the results was higher output values (heat flux, and temperatures) compared with the initial perfect contact model (Case 2) (see Figures 7 and 8). Lower heat partition values were predicted; at P4 (pad leading) was 6.13%, at P5 (pad middle) it was 1.54% and at P6 (pad

trailing) it was 1.29% (see Figure 8). Temperatures ranged from 34°C to 120°C on the disc (bigger block), and 100°C to 137°C on the pad (smaller block). The 2D rough area model provided valuable information on the effect of rough interface surfaces, which will allow making appropriate assumptions in the future.

## 5 CONCLUSIONS

- (1) The interface tribo-layer directly affects the heat partition ratio in a brake friction pair. It cannot be easily modelled in a dynamic analysis, so if a relationship could be established in a 2D static model, this would allow its effects to be included in 2D and 3D dynamic modelling.
- (2) The thermophysical properties of the interface tribo-layer can be related to the thermophysical properties of the disc and pad materials, but further evaluation of the thermophysical properties of the interface tribo-layer from first principles is required.
- (3) The contact pressure distribution affects the heat partition at the brake friction interface. This is influenced by parameters including the location of the pistons, the unevenness of contact patches at the disc/pad interface, and the stiffness of the backing plate.
- (4) The heat partition ratio is variable both in space (contact points), and time.
- (5) It has been confirmed that the temperatures at the apparent friction surfaces are not equal and the role of interface tribo-layer needs further scientific research.

## 6 FUTURE WORK

Future planned work includes creating a symmetric 3D FE model to develop the technique presented here. This model will include a half-thickness disc and a single pad with backing plate, which will allow more realistic predictions to be made since the actual rotational motion of the disc will be simulated. This will also allow the investigation of the heat partition not only in the circumferential, but also in the radial direction. Based on the 3D FEA model, a design of experiments (DoE) approach will be implemented to investigate the effect of various parameters on the heat partition ratio; the parameters will be identified from the conclusions drawn from the 2D static and dynamic models. FEA results will be validated with experimental work from a brake friction pair test rig with the same geometry as the 3D simulation.

## Acknowledgments

The authors would like to express their appreciation to the Institution of Mechanical Engineers (IMechE) for sponsoring this research.

## REFERENCES

1. Kennedy, T.C., C. Plengsaard, and R.F. Harder, *Transient heat partition factor for a sliding railcar wheel*. *Wear*, 2006. **261**(7-8): p. 932-936.
2. Bonnet, C., et al., *Identification of a friction model - Application to the context of dry cutting of an AISI 316L austenitic stainless steel with a TiN coated carbide tool*. *International Journal of Machine Tools & Manufacture*, 2008. **48**(11): p. 1211-1223.

3. Huang, Y.M. and S.H. Chen, *Analytical Study of Design Parameters on Cooling Performance of a Brake Disk*, SAE World Congress. 2006, SAE International: Detroit.
4. Apte, A.A. and H. Ravi, *FE Prediction of Thermal Performance and Stresses in a Disc Brake System*, in *Commercial Vehicle Engineering Congress and Exhibition*. 2006, SAE International.
5. Qi, H.S. and A.J. Day, *Investigation of disc/pad interface temperatures in friction braking*. *Wear*, 2007. **262**(5-6): p. 505-513.
6. Majcherczak, D., P. Dufrenoy, and M. Nait-Abdelaziz, *Third body influence on thermal friction contact problems: Application to braking*. *Journal of Tribology-Transactions of the Asme*, 2005. **127**(1): p. 89-95.
7. Komanduri, R. and Z.B. Hou, *Analysis of heat partition and temperature distribution in sliding systems*. *Wear*, 2001. **250**: p. 925-938.
8. H. Blok, *Theoretical study of temperature rise at surfaces of actual contact under oiliness lubricating conditions*, in *Proceedings of the General Discussion on Lubrication and Lubricants*. 1937, Instn. Mech. Engrs: London. p. 222-235.
9. Jaeger, J.C., *Moving sources of heat and the temperature at sliding contacts*. 1942, Proc. Royal Soc. NSW 76. p. 203-224.
10. Newcomb, T.P., *Transient temperatures attained in disk brakes*. *British Journal of Applied Physics*, 1959. **10**: p. 339-340.
11. Tirovic, M. and G.P. Voller, *Interface pressure distributions and thermal contact resistance of a bolted joint*. *Proceed. of the Royal Soc. a-Mathematical Physical and Engineering Sciences*, 2005. **461**(2060): p. 2339-2354.
12. Day, A.J., *Friction and Friction Materials*, in *Braking of Road Vehicles 2008*, A.J. Day and B.R. Shilton, Editors. 2008, University of Bradford.
13. Day, A.J. and T.J. Newcomb, *The Dissipation of Frictional Energy from the Interface of an Annular Disc Brake*. *Proc.Instn.Mech.Engrs. Vol.198D No.11*, 1984: p.201-209.
14. Day, A.J., *Energy Transformation at the Friction Interface of a Brake*. 1983, Ph.D. Thesis, The Loughborough University of Technology.
15. Osterle, W., et al., *Towards a better understanding of brake friction materials*. *Wear*, 2007. **263**(7-12 SPEC ISS): p. 1189-1201.
16. Homani, A. and K. Farhang. *A model for prediction of temperature in disk pair in dry friction contact*. in *Proceedings of the Asme International Mechanical Engineering Congress and Exposition 2007, Vol 8, Pts a and B - Heat Transfer, Fluid Flows, and Thermal Systems*. 2008. New York: Amer Soc Mechanical Engineers.
17. Rosala, G.F., *Computer Applications of Numerical Methods lecture notes*. 2006-07, University of Bradford: UK.
18. Chapra, S.C. and R.P. Canale, *Numerical methods for engineers*. 5th ed, McGraw-Hill. 2006.