Decision Analytics Journal 8 (2023) 100265

Contents lists available at ScienceDirect

Decision Analytics Journal

journal homepage: www.elsevier.com/locate/dajour

A fuzzy data-driven reliability analysis for risk assessment and decision making using Temporal Fault Trees

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ARTICLE INFO

Keywords:
Reliability
Uncertainty
Temporal Fault Trees
Fuzzy set theory
Sensitivity
Triangular and trapezoidal fuzzy numbers

ABSTRACT

Fuzzy data-driven reliability analysis has been used in different safety-critical domains for risk assessment and decision-making where precise failure data is non-existent. Expert judgements and fuzzy set theory have been combined with different variants of fault trees as part of fuzzy data-driven reliability analysis studies. In such fuzzy fault tree analyses, different people represented failure data using different membership functions for the fuzzy set, and different parameters were set differently in the expert opinion elicitation process. Due to the availability of a wide variety of options, it is possible to obtain different outcomes when choosing one option over another. This article performed an analysis in the context of fuzzy data-based temporal fault tree analysis to investigate the effect of choosing different membership functions on the estimated system reliability and criticality ranking of different failure events. Moreover, the effect of using different values for the *relaxation factor*, a parameter set during the expert elicitation process, was studied on the system reliability and criticality evaluation. The experiments on the fuel distribution system case study show system reliability did not vary when triangular and trapezoidal fuzzy numbers were used with the same upper and lower bounds. However, it was seen that the criticality rankings of a couple of events were changed due to choosing different membership functions and different values of relaxation factor

1. Introduction

Reliability is one of the crucial non-functional properties of a system which represents the probability that the system will perform its intended functionalities for a specified period of time under predefined environmental conditions. For any safety-critical system, it is necessary to demonstrate a high level of reliability to keep the risk associated with system failure at an acceptable level. Consequently, systems are analysed for their reliability very early when they are designed. Over the years different approaches such as Reliability Block Diagrams, Fault Tree Analysis, Failure Modes and Effects Analysis (FMEA), Markov Chains, Petri Nets, Bayesian Networks, etc. have been utilised for system reliability evaluation.

Among the different available approaches, FTA has been utilised broadly in a large number of areas. In FTA, system failures (i.e., hazardous events) and their potential causes are expressed graphically as fault trees, which show the logical links between causes (e.g. component failures) and their effects (system failure). For this logical representation, the classic fault tree usually uses Boolean AND and OR gates. Since the classic fault tree has limitations in capturing the dynamic failure behaviour of systems, it has been extended by proposing new logic gates to introduce different variants such as Dynamic Fault Trees (DFTs) [1] and Pandora Temporal Fault Trees(TFTs) [2].

Pandora TFT was initially proposed to enable qualitative failure behaviour analysis of dynamic systems. In addition to featuring the Boolean AND and OR gates, Pandora TFT retained the Priority-AND (PAND) gate of the DFT and proposed Priority-OR (POR) and Simultaneous-AND (SAND) gates. This method is unified with the model-based system design and analysis paradigm, which allows Pandora TFTs to be generated semi-automatically from computerised system models that are annotated with dynamic failure behaviour [3]. A TFT can be minimised for qualitative analysis to obtain minimal cut sequences (MCSQs), which are the smallest sequences of events that can cause a system failure. Although the Pandora TFT was primarily proposed for qualitative analysis, over the years different approaches such as an analytical method [4], a Petri Net-based method [5,6], and a Bayesian Network-based method [7,8] were proposed for the quantification of Pandora TFTs. Like other reliability analysis approaches, e.g., traditional FTA and DFTs, the above-mentioned approaches rely on components' failure data, e.g., failure probabilities to compute system reliability. Hence, these approaches take the availability of quantitative data as guaranteed. Nonetheless, in real applications, it is sometimes impossible to have concise failure data. The issue of unavailability of data and uncertainty associated with data are well recognised by academia and industry. Different efforts have been made by the reliability research community to address the issues with data

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https://doi.org/10.1016/j.dajour.2023.100265
Received 30 April 2023; Received in revised form 8 June 2023; Accepted 12 June 2023
Available online 14 June 2023

uncertainty during reliability evaluation. Among these efforts, the application of fuzzy set theory (FST) together with different reliability analysis methods for quantifying system reliability with imprecise data becomes increasingly popular.

Since FST has a strong mathematical foundation to handle uncertainty, it has been utilised to propose different variants of fuzzy fault tree analysis (FFTA) by different researchers. The earliest work on FFTA could be traced back to the 1980s [9]. Afterwards, a significant amount of research has been performed with FFTA and different approaches were developed over the years. Some approaches that utilised classical FST with FTA include but are not limited to [10–14]. Applications of FFTA can be found in a diverse range of areas. Recently, Bai et al. [15] utilised FFTA for reliability analysis of multi-state robotic systems. Reliability analysis of a digital control system in a nuclear power plant was conducted by considering different levels of redundancies in [16]. FFTA has been utilised in [17] for the reliability evaluation of wind energy systems. In [18], fire and explosion risk in the maritime industry was studied using FFTA; whereas Zheng et al. [19] studied fire explosion risk in the process industry. In addition to classic FST, intuitionistic fuzzy set theory (IFST) has been utilised with classic FTA in [20–23]. The most recent research based on IFST is reported in [24]. In a recent review [25], the improvements of FTA, including the research on FFTA, from 2011 to 2021 were presented. A comprehensive list of other FFTA approaches can be found in [26,27]. It is worth mentioning that the application of FST is confined not only to classic fault trees. Dynamic variants of fault trees also utilised FST to evaluate system reliability under the conditions of data uncertainty. In their review, Zhu et al. [28] discussed how FST has been integrated with dynamic fault trees by different researchers. For instance, FST has been used with DFTs in [29–32] and TFTs in [33,34].

In FST-based FTA, failure probabilities and/or failure possibilities of basic events (BEs), which usually represent component failures, are generally represented in *linguistics terms*. Using linguistic terms, failure data are represented as sentences or words in natural language. For instance, the probability of the occurrence of an event could be represented as *low, medium, high, very high,* etc. The linguistic variables are then mapped to corresponding fuzzy numbers in the form of membership functions. Triangular and trapezoidal are the commonly used membership functions. A group of experts are then engaged to collect judgements about the failure possibility/probability of BEs. The diverse judgements of different experts are aggregated to get a single judgement for each of the events. In the aggregation process, the weight of each expert, which is calculated based on different parameters such as their work experience (in years), professional position (i.e., job title) and educational qualifications, is considered to reflect their level of expertise in the decisions made. As part of this process, a *relaxation factor* (α or β is used interchangeably) is used to make a trade-off between the weight of an expert and the relative agreement of that expert with other experts. The value of this relaxation factor could be anything in the range $[0, 1]$, where the higher the value is the more importance is given to expert weighting over the relative agreement degree. After obtaining the failure data of the BEs in fuzzy format, the top event (TE) probability of the FT is calculated based on them as a fuzzy number. This number represents the probability/possibility of system failure as a range of values from lowest to highest. A crisp value is also computed based on the fuzzy number to represent the system failure probability/possibility as a single value. The criticality of different events that contribute to the occurrence of failure is also computed based on fuzzy data.

While the process of using expert judgements together with FST to obtain fuzzy failure data itself is subjective in nature, the availability of the option to choose different membership functions to represent fuzzy data could lead to variations in the outcome of the analysis. Moreover, the value chosen for the relaxation factor can significantly change the fuzzy failure data, which will eventually result in a variation in the outcome of the analysis. Therefore, it is necessary to investigate how sensitive the outcome of reliability evaluation is due to a different choice of membership functions and using different values for the relaxation factor. Different membership functions were used in [35,36] to represent component failure probability to compute the reliability of systems. Sun, Zhang, and Mu [37] analysed how different fuzzy membership functions can affect the reliability estimations of an underground wall. A study was performed in [38] to evaluate the effects of using different fuzzy membership functions on the outcome of risk assessment. Yazdi [39] performed a sensitivity analysis in intuitionistic fuzzy set-based risk assessment by varying the value of the relaxation factor. Kumar and Singh [40] determined the impact of the relaxation factor in the static fault tree analysis based on a case study in the Chlor-Alkali industry. Impact of experts' relative weight in the outcomes of FTA was studied in [41].

To the best of the author's knowledge, no research has been performed for uncertainty sensitivity analysis in dynamic and temporal fault tree-based risk assessment. Considering the importance of uncertainty sensitivity analysis, this article investigates the following questions in the context of temporal fault tree analysis based on FST.

- How does the width of the unreliability bounds change if a trapezoidal membership function is chosen over a triangular membership function to represent the fuzzy occurrence possibility of basic events?
- How does the crisp unreliability value change due to different choice of membership functions?
- Does the choice of different membership functions have any effect on the criticality of the system components?
- How sensitive are the component failure possibilities and system reliability to the choice of the value of relaxation factor in the expert opinion aggregation process?

2. Overview of fuzzy data-based temporal fault tree analysis

Pandora TFT extended the classical FT by defining PAND, POR, and SAND gates (shown in Fig. 1) in addition to the Boolean gates AND and OR. Utilising these gates Pandora enables capturing of sequence-dependent dynamic failure behaviour and combinatorial static failure behaviour into a single fault tree model. The definition of these gates can be found in [2,42]. PAND is not a new gate; it is also featured in DFTs. A PAND gate is a variant of an AND gate. This gate's output will be true if all inputs occur in a sequence and no input occurs concurrently. In a logical expression, '$\lhd$' is usually used to symbolise a PAND gate. In the POR gate, there is a priority event and for the POR gate's output to be true only the priority event needs to occur and the occurrence of non-priority events is optional. However, if any of the non-priority events occur it has to occur after the priority event. A POR gate is usually symbolised by '|'. The SAND gate is used to model the concurrent occurrence of inputs; consequently, the output of a SAND can only be true when all of its inputs occur simultaneously. From a probabilistic point of view, the chance of multiple statistically independent events occurring simultaneously is almost zero. Hence, in the quantitative analysis of TFTs, SAND gates are usually excluded. In qualitative analysis, the SAND gate is symbolised by '&'. For Boolean AND and OR gates, symbols '$\cdot$' and '+' are used respectively.

FST has been used with Pandora TFTs in [33,43]. In these approaches, the occurrence possibilities of BEs were denoted by triangular fuzzy numbers in the form of (l, m, h) with membership function $\mu_A(x) : \mathbb{R} \to [0, 1]$ as follows.

$$\mu_A(x) = \begin{cases} \dfrac{x-l}{m-l}, & for\ l < x \le m \\ \dfrac{h-x}{h-m}, & for\ m < x < h \\ 0, & otherwise. \end{cases} \tag{1}$$

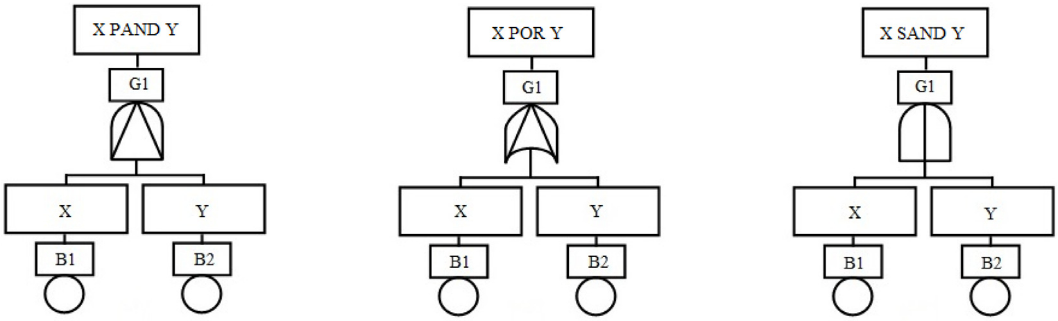

Fig. 1. Dynamic logic gates featured in Pandora TFT.

The $\mu_A(x)$ of a trapezoidal fuzzy number $A = (l, m_1, m_2, h)$ is specified as:

$$\mu_A(x) = \begin{cases} \frac{x-l}{m_1-l}, & for\ l \le x \le m_1, \\ 1, & for\ m_1 \le x \le m_2, \\ \frac{x-h}{m_2-h}, & for\ m_2 \le x \le h, \\ 0, & otherwise. \end{cases} \tag{2}$$

Failure possibilities were translated to failure probabilities using the following formula [44].

$$\rho = \begin{cases} \frac{1}{10^K}, & \gamma \neq 0, \\ 0, & \gamma = 0. \end{cases} \tag{3}$$

where ρ and γ are the failure probability and possibility, respectively and

$$K = \left(\frac{1-\gamma}{\gamma}\right)^{\frac{1}{3}} \times 2.301.$$

To enable the quantification of TFTs based on these fuzzy occurrence probabilities, the algebraic formulas [45–48] to quantify AND, OR, PAND, and POR gates with N statistically independent inputs as seen in Eqs. (4)–(7) were adapted to form the Eqs. (8)–(11).

$$\rho\{BE_1 \cdot BE_2 \cdot BE_3 \cdot \ \cdots \ \cdot BE_{N-1} \cdot BE_N\}(t) = \prod_{i=1}^{N} \rho_{BE_i}(t) \tag{4}$$

where $\rho_{BE_i}(t)$ is the occurrence probability of the BE_i at time t.

$$\rho\{BE_1 + BE_2 + BE_3 + \cdots + BE_{N-1} + BE_N\}(t) = 1 - \prod_{i=1}^{N}\left(1 - \rho_{BE_i}(t)\right) \tag{5}$$

$$\rho\{BE_1 \lhd BE_2 \lhd BE_3 \lhd \ \cdots \ \lhd BE_{N-1} \lhd BE_N\}(t) = \prod_{i=1}^{N} \lambda_i \sum_{k=0}^{N} \left[\frac{e^{(\phi_k t)}}{\prod_{j=0, j\neq k}^{N} (\phi_k - \phi_j)}\right] \tag{6}$$

where λ_i is the failure rate of BE_i, $\phi_0 = 0$ and $\phi_\psi = -\sum_{j=1}^{\psi} \lambda_j\ for\ \psi > 0$.

$$\rho\{BE_1 | BE_2 | BE_3 | \cdots | BE_{N-1} | BE_N\}(t) = \frac{\lambda_1\left(1 - \left(e^{-(\sum_{i=1}^{N} \lambda_i)t}\right)\right)}{\sum_{i=1}^{N} \lambda_i} \tag{7}$$

$$\rho_{ANDF} = AND_F\{\rho_{BE_1}(t), \rho_{BE_2}(t), \ldots, \rho_{BE_N}(t)\} = \left\{\prod_{i=1}^{N} l_i(t), \prod_{i=1}^{N} m_i(t), \prod_{i=1}^{N} h_i(t)\right\} \tag{8}$$

where (l_i, m_i, h_i) is the fuzzy occurrence probability of a basic event i.

$$\rho_{ORF} = OR_F\{\rho_{BE_1}(t), \rho_{BE_2}(t), \ldots, \rho_{BE_N}(t)\} = \left(1 - \prod_{i=1}^{N}(1 - l_i(t)), 1 - \prod_{i=1}^{N}(1 - m_i(t)), 1 - \prod_{i=1}^{N}(1 - h_i(t))\right) \tag{9}$$

$$\rho_{PANDF} = \left\{ \prod_{i=1}^{N} l_i \sum_{k=0}^{N}\left[\frac{e^{(\phi_{1k} t)}}{\prod_{j=0, j\neq k}^{N} (\phi_{1k} - \phi_{1j})}\right], \prod_{i=1}^{N} m_i \sum_{k=0}^{N}\left[\frac{e^{(\phi_{2k} t)}}{\prod_{j=0, j\neq k}^{N} (\phi_{2k} - \phi_{2j})}\right], \prod_{i=1}^{N} h_i \sum_{k=0}^{N}\left[\frac{e^{(\phi_{3k} t)}}{\prod_{j=0, j\neq k}^{N} (\phi_{3k} - \phi_{3j})}\right] \right\} \tag{10}$$

$$\rho_{PORF} = \left\{ \frac{l_1\left(1 - \left(e^{-(\sum_{i=1}^{N} l_i)t}\right)\right)}{\sum_{i=1}^{N} l_i}, \frac{m_1\left(1 - \left(e^{-(\sum_{i=1}^{N} m_i)t}\right)\right)}{\sum_{i=1}^{N} m_i}, \frac{h_1\left(1 - \left(e^{-(\sum_{i=1}^{N} h_i)t}\right)\right)}{\sum_{i=1}^{N} h_i} \right\} \tag{11}$$

In [33], expert opinions were used to obtain fuzzy occurrence possibility data for the basic events. For expert opinion elicitation, linguistic variables (see Fig. 2) were used. Since the engaged experts may have different levels of educational qualifications, backgrounds, work experience, and professional positions, their opinions were not treated equally. To provide appropriate weights to the opinions of the experts based on their level of expertise, the relative quality of the experts was measured as their weights based on the above-mentioned parameters. Once a group of experts provided their opinion about a BE, an aggregated opinion was obtained by following the steps shown in Fig. 3. For a detailed explanation of these steps, readers can refer to [33].

After obtaining the fuzzy occurrence probabilities of BEs, the mathematical equations for quantifying different logic gates were used to calculate the top event probability as a fuzzy number. The fuzzy values in the form of $\tilde{A} = (l, m, h)$ and $\tilde{A} = (l, m_1, m_2, h)$ were defuzzified using the Eqs. (12) and (13), respectively to obtain a crisp value, respectively. Moreover, the criticality of BEs was measured by calculating and comparing the fuzzy importance measure (FIM) of the BEs. In [33], the FIM of a BE was calculated by computing the Euclidean distance between $\tilde{\rho}_{T_i=1}$ and $\tilde{\rho}_{T_i=0}$, where $\tilde{\rho}_{T_i=1}$ is top event probability when BE i is considered to be fully available and $\tilde{\rho}_{T_i=0}$ is top event probability when BE i is considered to be failed. If these two values are presented as $\tilde{\rho}_{T_i=1} = (l^1,\ m^1\ , h^1)$ and $\tilde{\rho}_{T_i=0} = (l^0,\ m^0\ , h^0)$ then FIM was calculated

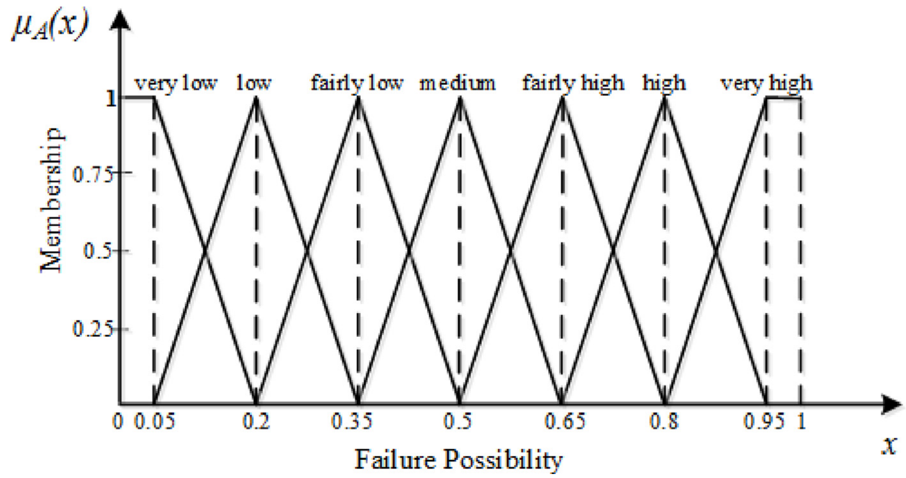

Fig. 2. Examples of linguistic variables.

using (14).

$$X = \frac{\int x\mu_{\tilde{A}}(x)dx}{\int \mu_{\tilde{A}}(x)dx} = \frac{\int_l^m \frac{x-l}{m-l}x\,dx + \int_m^h \frac{h-x}{h-m}x\,dx}{\int_l^m \frac{x-l}{m-l}dx + \int_m^h \frac{h-x}{h-m}dx} \tag{12}$$

$$= \frac{1}{3}(l+m+h)$$

$$X = \frac{\int_l^{m_1} \frac{x-l}{m_1-l}x\,dx + \int_{m_1}^{m_2} x dx + \int_{m_2}^h \frac{h-x}{h-m_2}x\,dx}{\int_l^{m_1} \frac{x-l}{m_1-l}dx + \int_{m_1}^{m_2} dx + \int_{m_2}^h \frac{h-x}{h-m_2}dx} \tag{13}$$

$$= \frac{(h+m_2)^2 - hm_2 - (l+m_1)^2 + lm_1}{3(h+m_2-m_1-l)}$$

$$FIM(BE_i) = ED[\tilde{p}_{T_i=1}, \tilde{p}_{T_i=0}] = \sqrt{(l^1-l^0)^2 + (m^1-m^0)^2 + (h^1-h^0)^2} \tag{14}$$

3. Methodology for uncertainty sensitivity analysis

To answer the questions listed in Section 1, this article follows the workflow shown in Fig. 4. During modelling and computation within this workflow, it was assumed that system components are non-repairable, failure events are statistically independent, and component failures follow an exponential distribution. The workflow has the following broad steps:

A. TFT modelling
B. Failure Data Collection using Existing Fuzzy TFT Method
C. Reliability Quantification
D. Criticality Sensitivity Analysis

Step A. TFT modelling

The first step in the proposed workflow is the TFT modelling. The first task within this step is to define the scope of the analysis. As part of this, we will identify a system and its constituents subsystems and components. Then part(s) of the system that will be analysed and the part(s) that will be excluded will be identified. For the analysed part(s) of the system, a TE (i.e., a system failure condition) and the failure modes that will be considered for different components will be defined. In other words, a boundary will be set for the level of performed analysis. Once the scope is defined, a temporal fault tree will be developed based on the failure behaviour of the system. This will be done by placing the TE at the top of the tree and then iteratively decomposing the TE and the intermediate events to grow the tree downward until the BEs are reached. To logically connect different events, the logic gates described in Section 2 will be used. When the TFT development is complete, it will be analysed qualitatively to derive the MCSQs. Each of these MCSQs will represent the smallest sequence of events that can cause the TE to occur.

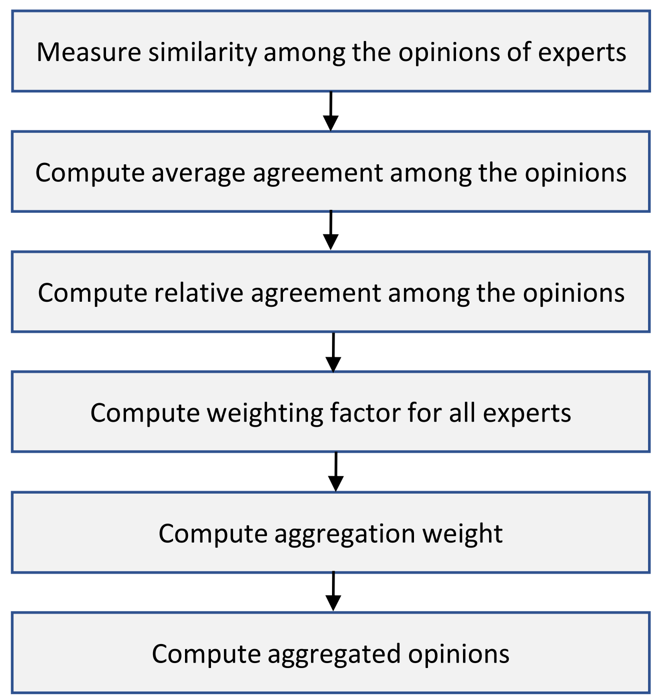

Fig. 3. Steps for aggregating expert opinions.

Step B. Failure data collection using existing fuzzy TFT method

In this step, fuzzy failure data will be collected. To do this, the BEs will be identified from the list of MCSQs. The fuzzy failure data for these basic events will be collected following the steps described in [33]. The fuzzy data collection will involve defining linguistic terms

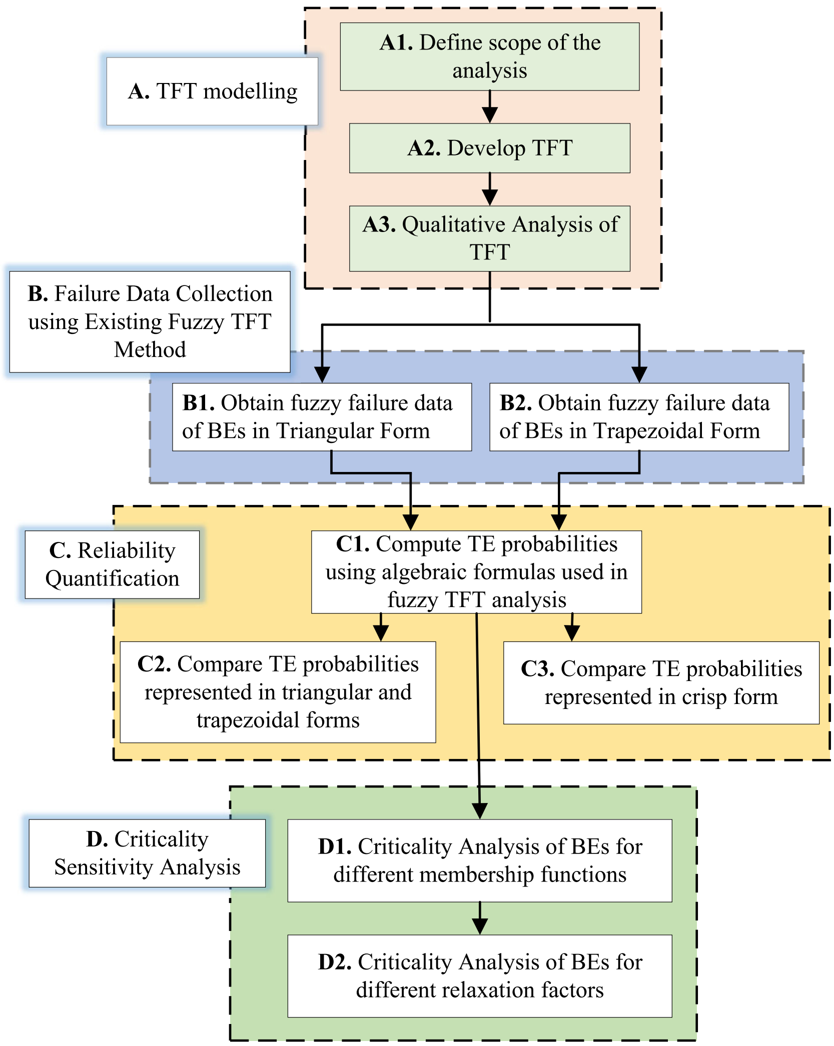

Fig. 4. Workflow of the performed study.

and their associated numerical mapping to represent the occurrence possibility of BEs. Experts will be used to provide judgements about the BEs' occurrence possibility. Since experts' opinions will differ from each other, we will utilise the aggregation process shown in Fig. 3 to obtain the aggregated outcomes. Since the overall aggregation process is explained in [33], this article omitted the detailed explanation due to brevity. Note that in [33], the failure data were obtained in the triangular format. However, to be able to find the answers to the questions defined in Section 1, we will collect the data both in triangular and trapezoidal formats. Moreover, during the expert opinion elicitation, the relaxation factor was considered to be 0.5 in [33]. To be able to perform multiple experiments to investigate the effects of the selection of the value of the relaxation factor on the system reliability and components' criticality ranking, in this article, we will obtain data for a range of values of the relaxation factor.

Step C. Reliability quantification

When the fuzzy failure data is ready, in this step, the TFT will be quantified to determine the TE occurrence possibility, i.e., system unreliability. For the quantification, the MCSQs derived in step A and fuzzy data collected in step B will be utilised. The triangular and trapezoidal fuzzy data representing the failure possibilities of the BEs will be used in Eqs. (8), (10), and (11) and their trapezoidal variants, respectively to quantify each of the MCSQs. Note that depending on the logical expression (i.e., logic gates involved) of each MCSQ, the quantification will involve a different set of equations. After computing the occurrence possibilities of all MCSQs, Eq. (9) will be used to obtain the TE occurrence possibility. As a result, we will get the system's unreliability both in triangular and trapezoidal forms. These values will be compared to understand how the width of the unreliability bounds changes if a trapezoidal membership function is chosen over a triangular membership function to represent the fuzzy failure possibility of system components. Moreover, the crisp unreliability values will be compared to see how the crisp unreliability value changes due to different choices of membership functions. In this step, we will also use different sets of values of BEs possibilities that were obtained for different values of relaxation factor to evaluate the TE event occurrence possibilities for different values of relaxation factor. These TE occurrence possibilities will then be compared to observe the effects of the relaxation factor on the system's unreliability.

Step D. Criticality sensitivity analysis

In this final step, a criticality analysis will be performed to rank the BEs according to their contributions to the occurrence of the TE. To do this, FIM of each BE will be calculated using Eq. (14). Firstly, the BEs will be ranked for both triangular and trapezoidal data, and the ranking will be compared to see if there is an effect on the criticality due to the choice of different membership functions. Secondly, the criticality of the BEs will be measured for data with different relaxation factors. Comparing these criticality rankings will reveal the effects of the different choices of relaxation factors on the BEs' criticality.

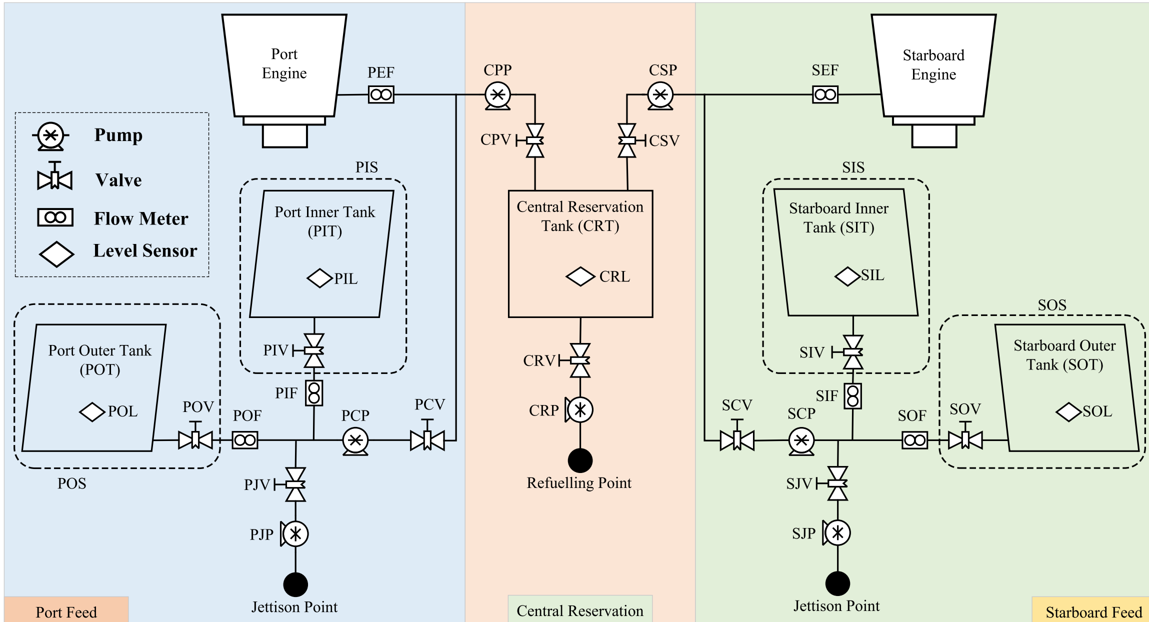

Fig. 5. Schematic diagram of a simplified aircraft fuel distribution system.

4. Illustrative example

To evaluate uncertainty sensitivity in fuzzy data-based TFT analysis, we consider the hypothetical aircraft fuel distribution system (AFDS) found in [4,5,49]. The architecture of the system is shown in Fig. 5. The AFDS' primary function is to store fuel in different tanks while refuelling and delivering fuel to Port Engine and Starboard Engine during the operation of the aircraft (i.e., in consumption mode). As can be seen, the whole system is divided into three parts–Port Feed, Starboard Feed, and Central Reservation. Port Feed and Starboard Feed have identical structures and they have the same set of identical components. In this article, we will consider the analysis of the failure of fuel supply to the Port Engine only. As the task of supplying fuel to the Port Engine is accomplished by the Port Feed and a part of the Central Reservation, we will consider the components in these two parts only. Port Feed has two subsystems namely Port Outer Subsystem (POS) and Port Inner Subsystem (PIS). POS consists of Port Outer Tank (POT), Port Outer Level (POL) sensor, and Port Outer Valve (POV). Similarly, PIS consists of Port Inner Tank (PIT), Port Inner Level (PIL) sensor, and Port Inner Valve (PIV). In addition to POS and PIS, Port Outer Flowmeter (POF), Port Inner Flowmeter (PIF), Port Connecting Pump (PCP), Port Connecting Valve (PCV), and Port Engine Flowmeter (PEF) from the Port Feed take part in the fuel delivery process to the Port Engine. From the Central Reservation, Central Reservation Tank (CRT), Central Reservation Level (CRL), Central Port Valve(CPV), and Central Port Pump (CPP) participates in the fuel delivery process to the Port Engine. Under normal operating conditions, the Port Engine gets fuel from the POS. If POS fails, PIS takes over the responsibility of delivering fuel to the Port Engine. If PIS fails, Port Engine draws fuel from the CRT.

4.1. TFT modelling and qualitative analysis

As mentioned above, the scope of the analysis is to find the causes of the failure of fuel supply to the Port Engine. The absence of fuel flow to the Port Engine will be reflected in the omission of flow through the PEF. Hence, the TE of the TFT will be O−PEF, i.e., *omission of the flow of fuel through PEF*. For this TE, the TFT shown in Fig. 6 is developed. Note that during TFT development, only AND, OR, PAND, and POR gates were used. The SAND gate was not used as the probability of concurrent occurrence of multiple statistically independent events is almost zero. The BEs of this TFT and their description are shown in Table 1. Following the approach presented in [3], the TFT was analysed to obtain the MCSQs. While the minimisation of the TFT produces a large number of MCSQs, for simplicity, a logical cut-off was applied to keep the MCSQs with three or fewer events. The resultant MCSQS are: 1. I-CPP.I-PCV; 2. I-CPP.I-PCP; 3. I-CRL.I-PCV; 4. I-CRL.I-PCP; 5. I-CPV.I-PCV; 6. I-CPV.I-PCP; 7. Hi-POF◁I-POV.I-CPP; 8. Hi-POF◁I-POV.I-CRL; 9. Hi-POF◁I-POV.I-CPV; 10. Hi-POF◁I-POL.I-CPP; 11. Hi-POF◁I-POL.I-CRL; 12. Hi-POF◁I-POL.I-CPV; 13. Hi-PIF.I-POV.I-CPP; 14. Hi-PIF.I-POV.I-CRL; 15. Hi-PIF.I-POV.I-CPV; 16. I-POL.I-PIL.I-CPP; 17. I-POL.I-PIL.I-CRL; 18. I-POL.I-PIL.I-CPV; 19. Hi-PIF.I-POL.I-CPP; 20. Hi-PIF.I-POL.I-CPV; 21. I-PIV◁I-POV|I-POL; 22. I-PIV◁I-POL|I-POV.

Table 1
Description of BEs of TFT in Fig. 6.

Event tags	Description
I-PCP	Internal failure of pump PCP
I-CPP	Internal failure of pump CPP
I-POV	Internal failure of valve POV
I-PIV	Internal failure of valve PIV
I-CPV	Internal failure of valve CPV
I-PCV	Internal failure of valve PCV
I-CRL	Internal failure of level sensor CRL
I-PIL	Internal failure of level sensor PIL
I-POL	Internal failure of level sensor POL
Hi-POF	An erroneous high-value reading by flowmeter POF
Hi-PIF	An erroneous high-value reading by flowmeter PIF
Hi-PEF	An erroneous high-value reading by flowmeter PFF

To obtain the occurrence possibilities of the BEs in triangular and trapezoidal forms, linguistic terms with seven levels as shown in Table 2 were used. The linguistic terms and their corresponding numerical conversion scales were derived from [12,33]. Note that in both forms, the lower and upper bounds for each level were kept the same. Moreover, the weighting scores used in [12] were used to calculate the weights and weighting factors of the experts. For this experimental study, six experts with weighting factors 0.206, 0.191, 0.176, 0.147, 0.162, and 0.118 were used. Table 3 shows the judgements of the experts in the linguistic form regarding the occurrence

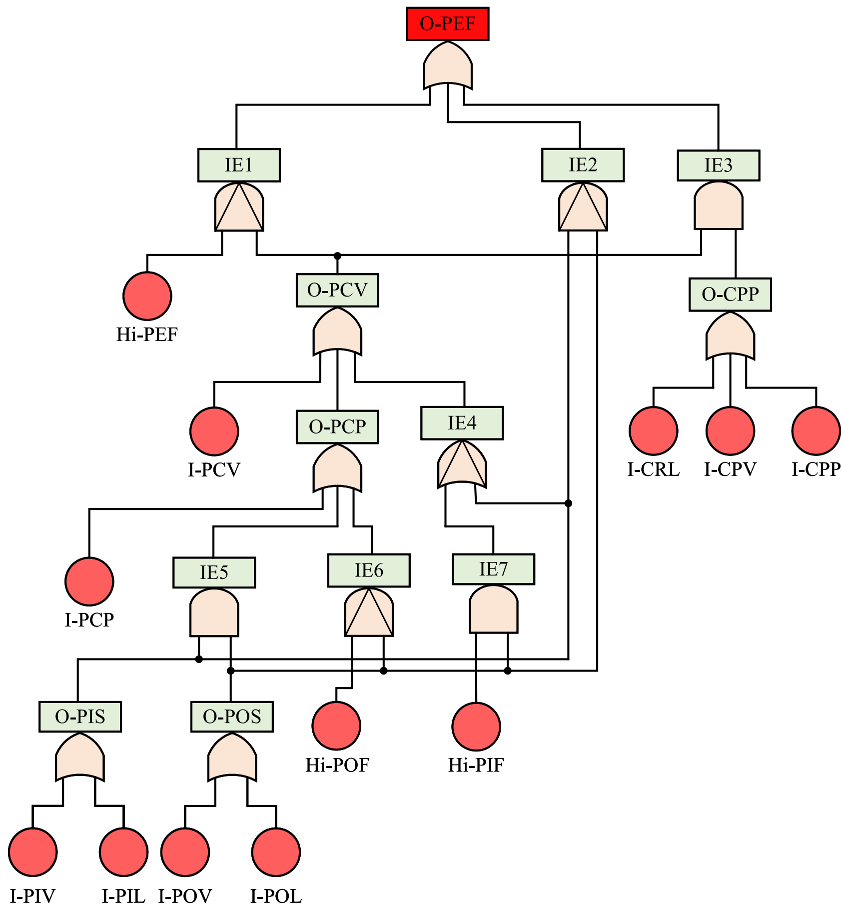

Fig. 6. TFT of the failure of fuel delivery to port engine.

Table 2
Conversion scales of linguistic variables [33,50].

Linguistic variables	Triangular form			Trapezoidal form			
	l	m	h	l	m_1	m_2	h
Very low (VL)	0	0.04	0.08	0	0.03	0.058	0.08
Low (L)	0.07	0.13	0.19	0.07	0.11	0.15	0.19
Fairly low (FL)	0.17	0.27	0.37	0.17	0.25	0.29	0.37
Medium (M)	0.35	0.5	0.65	0.35	0.46	0.54	0.65
Fairly high (FH)	0.62	0.73	0.82	0.62	0.7	0.75	0.82
High (H)	0.81	0.87	0.93	0.81	0.86	0.89	0.93
Very high (VH)	0.92	0.96	1.00	0.92	0.95	0.97	1.00

Table 3
Experts' opinion on the occurrence possibilities of BEs.

Failure modes	Expert opinion					
	Ex1	Ex2	Ex3	Ex4	Ex5	Ex6
I-PCP	H	FH	M	M	H	FH
I-CPP	FH	H	H	FH	M	H
I-POV	FH	M	L	FL	M	FL
I-PIV	M	FL	FL	L	M	FH
I-CPV	FL	M	M	FH	H	FH
I-PCV	M	FL	FH	H	FH	M
I-CRL	F	FH	M	L	FH	M
Hi-POF	FH	H	H	FL	M	FH
Hi-PIF	H	M	FH	H	M	FH
I-PIL	L	FL	M	VL	M	FL
I-POL	M	L	FH	FL	M	VL

possibilities of the BEs. As expert opinions vary, we aggregated opinions to obtain a single fuzzy number for each BE. In the aggregation process, at first, we set the relaxation factor β's value to 0.5. The values obtained for this particular setting are shown in Table 4. Using these values and the Eqs. (8)–(11), the possibilities of the MCSQs both in triangular and trapezoidal forms were calculated and reported in Table 5. For these fuzzy values, the TE possibilities were obtained in triangular and trapezoidal forms as $(0.938807, 0.991791, 0.999577)$ and $(0.938773, 0.985165, 0.995666, 0.999576)$, respectively. Based on these values, it can be said that the width of the unreliability bounds does not change much due to choosing a trapezoidal membership function over a triangular membership function when the lower and upper bounds of the fuzzy numbers representing each linguistic term were kept the same. The crisp values corresponding to these two fuzzy values obtained by using Eqs. (12) and (13) were 0.976725 and 0.977298, which also showed that the crisp values are close to each other. Now, for criticality analysis with $\beta = 0.5$, as per the steps stated in Section 2, we take one BE at a time and then calculate the TE possibilities by setting the BE possibilities to 1 and 0, respectively. Table 6 shows the calculated values for all BEs in Trapezoidal form. For brevity, the corresponding data in the triangular form is not shown in this article. FIM for each BE was calculated using Eq. (14). Table 7 shows the FIM values for all BEs for trapezoidal and triangular data. As can be seen, while FIM values for all BEs change due to the choice of membership function, the rankings remain the same for most of the BEs. Only the rankings of I-CPV and I-PCV changed (highlighted in the table) due to the change in the membership function. Hence, it can be said that the criticality ranking of the BEs may change when we choose the trapezoidal membership function over the triangular membership function to represent the fuzzy occurrence possibilities of BEs.

To find the answer to the final question mentioned in Section 1, we varied the value of the relaxation factor β within the interval [0,1] by starting with 0 and increasing its value by 0.1 at a time and obtained BE occurrence possibilities for the different values of β. Note that the

Table 4
Aggregation of experts' judgements in triangular and trapezoidal fuzzy forms for the basic events' failure possibilities for $\beta = 0.5$.

Basic events	Triangular fuzzy number			Trapezoidal fuzzy number			
	l	m	h	l	m_1	m_2	h
I-PCP	0.599791	0.705168	0.803919	0.599717	0.678874	0.731515	0.803875
I-CPP	0.674953	0.765853	0.849771	0.674944	0.744381	0.788833	0.849765
I-POV	0.294517	0.407358	0.516767	0.294543	0.378587	0.434464	0.516791
I-PIV	0.277458	0.390814	0.501619	0.277484	0.362337	0.418067	0.501644
I-CPV	0.475033	0.589959	0.698679	0.474938	0.561103	0.617119	0.698616
I-PCV	0.482901	0.597386	0.704929	0.482805	0.568529	0.624104	0.704866
I-CRL	0.368917	0.483247	0.590763	0.368934	0.453212	0.509908	0.590778
Hi-SOF	0.582873	0.680282	0.770756	0.582794	0.657036	0.703482	0.770703
Hi-SIF	0.594423	0.700757	0.800611	0.594349	0.674169	0.727397	0.800566
I-PIL	0.185121	0.285125	0.385129	0.185121	0.260039	0.310212	0.385129
I-POL	0.265746	0.370366	0.471843	0.265776	0.342859	0.396362	0.471873

Table 5
Failure possibilities of the MCSQs for t = 4320 h.

MCSQs	Triangular fuzzy number			Trapezoidal fuzzy number			
	l	m	h	l	m_1	m_2	h
I-CPP.I-PCV	0.325935	0.457510	0.599028	0.325866	0.423202	0.492314	0.598970
I-CPP.I-PCP	0.404831	0.540055	0.683147	0.404775	0.505341	0.577043	0.683105
I-CRL.I-PCV	0.178150	0.288685	0.416446	0.178123	0.257664	0.318236	0.416419
I-CRL.I-PCP	0.221273	0.340770	0.474926	0.221256	0.307674	0.373005	0.474912
I-CPV.I-PCV	0.229394	0.352433	0.492519	0.229302	0.319003	0.385146	0.492431
I-CPV.I-PCP	0.284921	0.41602	0.561681	0.284828	0.380918	0.451432	0.561600
Hi-POF⊲I-POV.I-CPP	0.047548	0.080599	0.128815	0.047544	0.071138	0.091002	0.128807
Hi-POF⊲I-POV.I-CRL	0.025989	0.050857	0.089553	0.025988	0.043312	0.058825	0.08955
Hi-POF⊲I-POV.I-CPV	0.033464	0.062088	0.105911	0.033456	0.053623	0.071193	0.105896
Hi-POF⊲I-POL.I-CPP	0.044062	0.074508	0.118232	0.04406	0.065709	0.084161	0.118227
Hi-POF⊲I-POL.I-CRL	0.024084	0.047014	0.082195	0.024084	0.040007	0.054403	0.082194
Hi-POF⊲I-POL.I-CPV	0.031011	0.057396	0.09721	0.031004	0.049531	0.065841	0.097198
Hi-PIF.I-POV.I-CPP	0.118162	0.21862	0.351575	0.118157	0.18999	0.249293	0.351569
Hi-PIF.I-POV.I-CRL	0.064585	0.137947	0.244416	0.064586	0.115674	0.161145	0.244420
Hi-PIF.I-POV.I-CPV	0.083163	0.168409	0.289064	0.083143	0.143211	0.195027	0.289035
I-POL.I-PIL.I-CPP	0.033204	0.080875	0.154421	0.033208	0.066367	0.096992	0.154429
I-POL.I-PIL.I-CRL	0.018149	0.051031	0.107354	0.018152	0.040407	0.062696	0.107363
I-POL.I-PIL.I-CPV	0.023369	0.062300	0.126964	0.023367	0.050026	0.075879	0.126961
Hi-PIF.I-POL.I-CPP	0.106619	0.198767	0.321012	0.106617	0.17206	0.227430	0.321012
Hi-PIF.I-POL.I-CPV	0.075039	0.153116	0.263935	0.075023	0.129696	0.177923	0.263913
I-PIV⊲I-POV|I-POL	0.041632	0.065636	0.096587	0.041637	0.058951	0.072443	0.096595
I-PIV⊲I-POL|I-POV	0.039001	0.061248	0.089425	0.039006	0.054995	0.067602	0.089433

Table 6
Fuzzy top event possibilities by setting BE possibilities to 0 and 1.

Basic events	Fuzzy top event possibility							
	BE occurrence possibility = 0				BE occurrence possibility = 1			
	l	m_1	m_2	h	l	m_1	m_2	h
I-PCP	0.815304	0.93003	0.970205	0.994191	0.980107	0.995708	0.998819	0.999892
I-CPP	0.779971	0.904314	0.953703	0.988341	0.97386	0.994298	0.998424	0.999853
I-POV	0.905792	0.969458	0.988399	0.998101	0.972345	0.99445	0.998588	0.999889
I-PIV	0.93352	0.983319	0.994988	0.999485	0.965561	0.993186	0.998288	0.999868
I-CPV	0.8568	0.944776	0.975781	0.994837	0.982988	0.996709	0.999175	0.999935
I-PCV	0.856614	0.949125	0.979633	0.996433	0.984556	0.996879	0.999193	0.999934
I-CRL	0.890423	0.962964	0.985509	0.997547	0.98454	0.99713	0.999307	0.99995
Hi-POF	0.924465	0.979308	0.993258	0.999182	0.969993	0.995074	0.998954	0.999941
Hi-PIF	0.902033	0.966455	0.986537	0.997566	0.956853	0.990545	0.997382	0.999757
I-PIL	0.933956	0.98257	0.994459	0.999357	0.957288	0.991145	0.997704	0.999814
I-POL	0.904004	0.968092	0.987529	0.997859	0.974277	0.995227	0.99885	0.999918

values were generated in both triangular and trapezoidal forms. Due to brevity, this article does not show all the data. Table 8 shows the BE possibilities in trapezoidal format for $\beta = 0$ and $\beta = 1$ only. Based on the reported data, it is seen that the BEs possibilities change due to the change in the values of β. Based on the trapezoidal fuzzy data, we have generated crisp occurrence possibilities for the BEs for different values of β. Fig. 7 shows a comparison between the crisp possibility values for $\beta = 0, 0.5$, and 1.0, respectively. As can be seen, for some BEs, the occurrence possibilities increase with the increase in β and for some BEs the opposite happens. We believe this is because of the different levels of trade-off made between the weight of experts and the relative agreement among them, together with the varied and subjective judgements of the experts about the occurrence possibilities of the BEs. In any case, the change in the BE possibility values due to the different choice of the value of the β will be propagated to change the TE possibility. To observe the change in TE possibilities, we have plotted the lower and upper bound of the trapezoidal fuzzy possibilities of the TE in Fig. 8. A downward trend in the upper and lower bounds of the TE was observed with the increase in the values of β. In other words, when more priority is given to the weights of the experts over their relative agreement degree, TE possibilities tend to decrease for this particular system. An important observation to make here is that the value we choose for the relaxation factor will have an effect on the TE possibility.

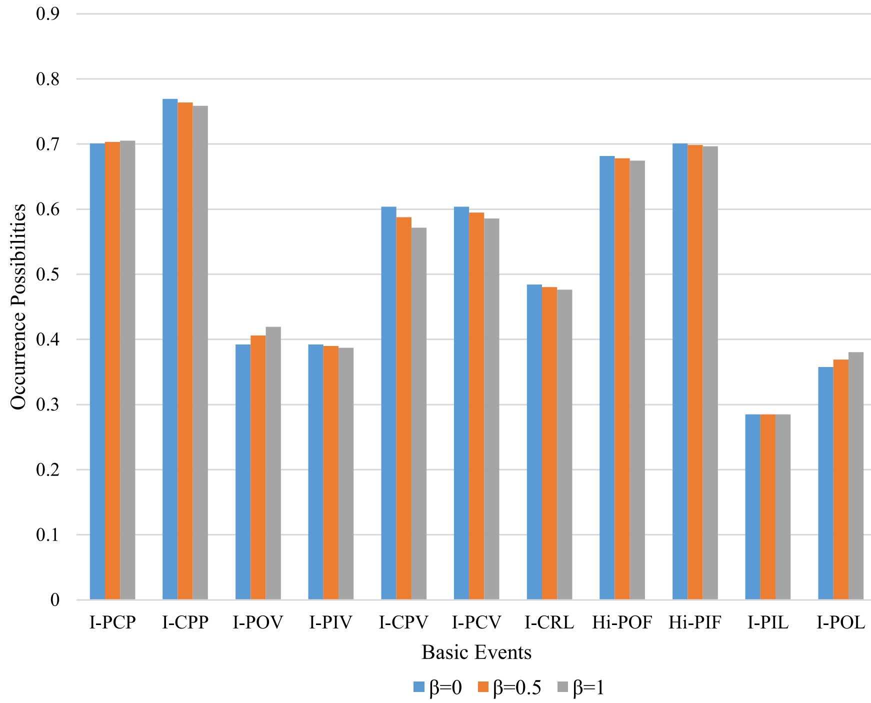

Fig. 7. Comparison of crisp BE possibilities for $\beta = 0, 0.05$, and 1.0.

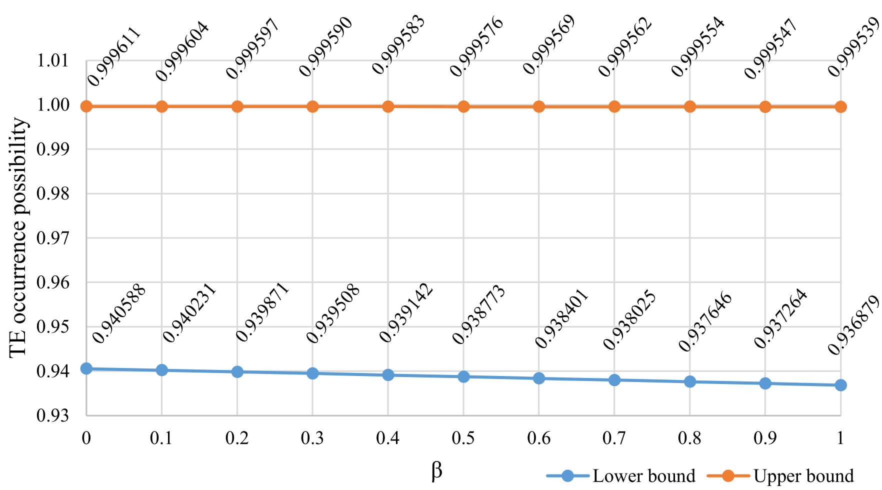

Fig. 8. Comparison of the lower and upper bounds of TE possibilities for different values of β.

Table 7
FIM of BEs for trapezoidal and triangular data, and the comparison of ranking of BEs.

BEs	$FIM(BE_i)$ for trapezoidal data	Rank	$FIM(BE_i)$ for triangular data	Rank
I-PCP	0.179791	2	0.170642	2
I-CPP	0.218684	1	0.204494	1
I-POV	0.071839	7	0.068513	7
I-PIV	0.033690	10	0.032556	10
I-CPV	0.138541	3	0.131160	4
I-PCV	0.138002	4	0.131709	3
I-CRL	0.101101	5	0.096691	5
I-PIL	0.025073	11	0.023945	11
I-POL	0.076203	6	0.072509	6
Hi-POF	0.048522	9	0.046538	9
Hi-PIF	0.060893	8	0.057270	8

To investigate whether the different values for β will have an effect on the criticality ranking of the BEs, we evaluated the FIM of all BEs for different values of β using Eq. (14). Table 9 shows the calculated values of FIM. Based on these values, the BEs were ranked and rankings were shown in Table 10. As seen in the table, the rankings of 9 BEs out of 11 BEs remain unchanged for different values of β. However, the criticality ranking of I-CPV and I-PCV got altered (see highlighted rows in the table) when the β has values in the range [0.4, 1.0]. Hence, from the current study, it is evident that the criticality of BEs can change depending on the value chosen for the relaxation factor. For different case studies, such an effect of the value of the relaxation factor on the criticality ranking of BEs might be observed.

5. Discussion and conclusion

Pandora TFT, like other variants of the classic FT, has the capability to model the dynamic failure behaviour of systems. While the qualitative analysis of a TFT enables us to identify the potential causes of system failure in the form of minimal cut sequences, a quantitative analysis is needed to determine the likelihood of system failure.

Table 8
Trapezoidal fuzzy failure possibilities of basic events with $\beta = 0$ and $\beta = 1$.

Basic events	Trapezoidal fuzzy failure possibility							
	$\beta = 0$				$\beta = 1$			
	l	m_1	m_2	h	l	m_1	m_2	h
I-PCP	0.596934	0.676572	0.729501	0.802309	0.6025	0.681176	0.733529	0.805441
I-CPP	0.681358	0.749938	0.793696	0.853647	0.668529	0.738824	0.783971	0.845882
I-POV	0.280557	0.365115	0.420693	0.503876	0.308529	0.392059	0.448235	0.529706
I-PIV	0.280557	0.365115	0.420693	0.503876	0.274412	0.359559	0.415441	0.499412
I-CPV	0.491052	0.577205	0.633503	0.714584	0.458824	0.545000	0.600735	0.682647
I-PCV	0.491052	0.577205	0.633503	0.714584	0.474559	0.559853	0.614706	0.695147
I-CRL	0.370810	0.456423	0.514522	0.596849	0.367059	0.450000	0.505294	0.584706
Hi-POF	0.585589	0.660248	0.707111	0.774641	0.580000	0.653824	0.699853	0.766765
Hi-PIF	0.596934	0.676572	0.729501	0.802309	0.591765	0.671765	0.725294	0.798824
I-PIL	0.184948	0.260224	0.309983	0.385259	0.185294	0.259853	0.310441	0.385000
I-POL	0.255082	0.331748	0.384635	0.459922	0.276471	0.353971	0.408088	0.483824

Table 9
FIM of BEs for different values of β.

Basic events	FIM										
	$\beta = 0$	$\beta = 0.1$	$\beta = 0.2$	$\beta = 0.3$	$\beta = 0.4$	$\beta = 0.5$	$\beta = 0.6$	$\beta = 0.7$	$\beta = 0.8$	$\beta = 0.9$	$\beta = 1.0$
I-PCP	0.177911	0.178284	0.178659	0.179035	0.179412	0.179791	0.180171	0.180553	0.180937	0.181322	0.181709
I-CPP	0.212311	0.213565	0.214830	0.216104	0.217388	0.218684	0.219988	0.221304	0.222631	0.223968	0.225316
I-POV	0.068789	0.069227	0.069668	0.070108	0.070551	0.071839	0.071439	0.071885	0.072332	0.072780	0.073229
I-PIV	0.031402	0.031852	0.032305	0.032762	0.033224	0.033690	0.034160	0.034635	0.035114	0.035598	0.036086
I-CPV	0.137632	0.137811	0.137992	0.138173	0.138356	0.138541	0.138726	0.138913	0.139101	0.139291	0.139482
I-PCV	0.139244	0.138982	0.138727	0.138478	0.138236	0.138002	0.137774	0.137552	0.137337	0.137129	0.136927
I-CRL	0.097890	0.098521	0.099158	0.099800	0.100447	0.101101	0.101759	0.102424	0.103094	0.103770	0.104452
Hi-POF	0.046006	0.046504	0.047005	0.047508	0.048013	0.048522	0.049031	0.049544	0.050059	0.050576	0.051096
Hi-PIF	0.057116	0.057863	0.058614	0.059369	0.060128	0.060893	0.061661	0.062435	0.063212	0.063994	0.064781
I-PIL	0.023725	0.023992	0.024260	0.024529	0.024800	0.025073	0.025347	0.025623	0.025901	0.026180	0.026461
I-POL	0.073117	0.073564	0.074012	0.074461	0.074911	0.076203	0.075817	0.076271	0.076727	0.077184	0.077642

Table 10
Comparison of the BE criticality rankings for different values of β.

Basic events	Criticality ranking										
	$\beta = 0$	$\beta = 0.1$	$\beta = 0.2$	$\beta = 0.3$	$\beta = 0.4$	$\beta = 0.5$	$\beta = 0.6$	$\beta = 0.7$	$\beta = 0.8$	$\beta = 0.9$	$\beta = 1.0$
I-PCP	2	2	2	2	2	2	2	2	2	2	2
I-CPP	1	1	1	1	1	1	1	1	1	1	1
I-POV	7	7	7	7	7	7	7	7	7	7	7
I-PIV	10	10	10	10	10	10	10	10	10	10	10
I-CPV	4	4	4	4	3	3	3	3	3	3	3
I-PCV	3	3	3	3	4	4	4	4	4	4	4
I-CRL	5	5	5	5	5	5	5	5	5	5	5
Hi-POF	9	9	9	9	9	9	9	9	9	9	9
Hi-PIF	8	8	8	8	8	8	8	8	8	8	8
I-PIL	11	11	11	11	11	11	11	11	11	11	11
I-POL	6	6	6	6	6	6	6	6	6	6	6

Moreover, quantitative analysis enables us to identify the critical components of a system. All this information can aid the decision-maker in assessing the risk posed by the system and putting preventive measures to minimise the likelihood of the system failure.

For a quantitative risk assessment, it is necessary to have numerical data associated with system components such as failure rate or failure probability of components. It is accepted by the reliability community both in academia and industry that it may not be possible to get such precise data in many real-world cases. As a mechanism to tackle the reliability evaluation problem with unavailable precise data, fuzzy set theory has been widely used with different reliability analysis models. Since fuzzy set theory-based approaches are reliant on expert opinions, the outcome produced by those approaches could be subjective. Moreover, the opportunity to choose different membership functions to represent fuzzy data and to provide different levels of priority to expert weights can make the analysis sensitive.

In this paper, we performed an experimental study with a hypothetical fuel distribution system to investigate the effects of the choice of membership functions and the value of the relaxation factor on the estimated reliability of the system and the criticality ranking of the basic events. Based on the study, it was understood that if we choose triangular and trapezoidal membership functions to represent basic event occurrence possibilities with the same lower and upper bounds then the width of the unreliability bounds remain almost the same for both data formats. Consequently, the crisp unreliability values obtained for triangular and trapezoidal data are also very close. In the current study, we have noticed that the criticality ranking of two basic events was changed due to the change in membership function. That means, for other studies involving TFTs and fuzzy data, there is a chance that the criticality of components may change due to the choice of membership functions. Since criticality analysis identifies weak links in the system, it leads to different actions such as making a change in the system model, introducing redundant components, and replacing less reliable components with more reliable components. Therefore, accurately identifying the critical components is crucial. According to our experiments, criticality analysis can be sensitive to the choice of membership functions. Hence, one needs to be more cautious when identifying critical components using a fuzzy set theory-based approach since fuzzy data represented differently may produce different outcomes, which may lead to misleading rectification actions.

Finally, we generated basic events' occurrence possibility data with different values of relaxation factor. It is observed that basic events' occurrence possibilities change with the change in the value of the relaxation factor. Multiple experiments performed based on these different sets of data show that due to the change in relaxation factor the top event occurrence possibilities change and the criticality ranking of a couple of basic events changes. This gives an indication that for the same system, if we perform a fuzzy TFT analysis with two different values of relaxation factor, we may obtain two different outcomes. This will contribute to the subjectivity of fuzzy data-based TFT analysis and will also add a different level of uncertainty to an approach which was originally developed to tackle uncertainty associated with the availability of precise data.

Nevertheless, fuzzy set theory-based approaches are still useful in enabling reliability analysis of systems when precise failure data are scarce. It is worth emphasising that combining expert judgement with fuzzy set theory helps us to obtain at least some approximation of reliability and criticality ranking where precise failure data are nonexistent and it is impossible to apply classical reliability analysis approaches. However, we need to remember that the integration of fuzzy set theory with reliability analysis approaches will not create accuracy in uncertain situations and the outcome of the analysis can only be as dependable as the used fuzzy data. Hence, we can conclude that fuzzy set theory-based approaches could be useful in the early stages of the design processes where an analysis may need to be done without having solid quantitative failure data. It could work as a starting point to facilitate the design refinement based on the results at very early design phases and then can support further analysis in the later stages when precise data is available.

This study focused specifically on classical fuzzy set-based Pandora TFTs. Triangular and trapezoidal fuzzy numbers were considered in this article since the existing works on fuzzy set-based Pandora TFT analysis are based on these numbers. As fuzzy numbers in these forms have linear membership functions, it is worthwhile to explore the usage of other shapes for membership functions beyond linear functions in Pandora TFT analysis and then perform a similar sensitivity analysis. Moreover, in addition to the exponential failure distribution, other distributions such as the Weibull distribution are used in dynamic fault tree analysis [51,52]. Therefore, this work can be extended in the future by considering other distributions together with the exponential distribution. Since there exist other dynamic variants of fault trees such as DFT that utilise fuzzy set theory, this study can be extended to investigate the impact of the choice of membership functions and relaxation factor on those variants of fault tree analysis. Additionally, different versions of fuzzy set theory such as intuitionistic fuzzy set were used with different reliability analysis approaches. Therefore, it will be worth performing a similar study for those approaches.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request

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