The mean-variance relation: A 24-hour story

Abstract

This paper investigates the mean-variance relation during different time periods within trading days. We reveal that there is a positive mean-variance relation when the stock market is closed (i.e., overnight), but the positive relation is distorted when the market is open (i.e., intraday). The evidence offers a new explanation for the weak risk-return tradeoff in stock markets.

Keywords: Mean-variance relation; Overnight return; Risk-return tradeoff

JEL codes: G12; G14; G41

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1. Introduction

Standard asset pricing theories posit a positive mean-variance relation, i.e., the risk-return tradeoff; empirical evidence, however, is inconclusive, with extant literature evidencing positive (French et al., 1987; Pástor et al., 2008, etc.), negative (Campbell, 1987; Brandt and Kang, 2004, etc.), and mixed (Turner et al., 1989) relation.[1] A variety of arguments are developed to explain the weak risk-return tradeoff, such as techniques filtering conditional variance (Glosten et al., 1993; Harvey, 2001) and investor sentiment (Yu and Yuan, 2011; Wang, 2018). Yu and Yuan (2011), for example, posit that retail investors are likely to misestimate variance of returns and prefer taking long positions to short positions, so an elevated level of retail investors over high-sentiment periods would therefore distort the positive mean-variance relation while over low-sentiment periods, the positive relation would be observed, which is supported by their empirical findings.

We extend this literature by probing the mean-variance relation during different time periods within trading days and in particular, distinguishing non-trading hours (i.e., overnight) and trading hours (i.e., intraday), we find that the mean-variance relation depends on whether the market is open or closed. When the stock market is closed, there is a positive mean-variance relation; in contrast, the positive relation is distorted when the market is open.

This paper proposes a new explanation for the weak risk-return tradeoff in stock markets, and the reported market-level risk-return tradeoff overnight offers support to the most recent, stock-level evidence documented in Hendershott et al. (2020) that stock returns are positively related to beta, as a measure of systematic risk, overnight, but negatively intraday.

2. Data and methodology

[1] See, Yu and Yuan (2011), for a summary.

2.1 Excess returns

We source daily and monthly NYSE market indices and risk-free rates over the period 1981 to 2018 from Datastream[2] and Kenneth French Data Library, respectively. We examine three excess returns including the close-to-open return (CTO, the overnight return), the open-to-close return (OTC, the intraday return), along with the close-to-close return (CTC, the total return). Following Lou et al. (2019) and Hendershott et al. (2020), the CTO on day *t*, R_t^{CTO}, is constructed following,

$$R_t^{CTO} = (1 + R_t^{CTC})/(1 + R_t^{OTC}) - 1, \tag{1}$$

where $R_t^{CTC} = (Close_t - Close_{t-1})/Close_{t-1}$ and $R_t^{OTC} = (Close_t - Open_t)/Open_t$. The monthly CTO, R_T^{CTO}, is defined as the sum of the daily CTO of month *T*, and the monthly CTC and OTC follow $R_T^{CTC} = (Close_T - Close_{T-1})/Close_{T-1}$ and $R_T^{OTC} = (Close_T - Open_T)/Open_T$, respectively.[3] Table 1 shows that compared with the OTC and the CTC, the CTO is lower and less volatile. The mean of realized volatility is, by definition, close to the variance of excess returns, and the difference is due to Jensen's inequality (Ghysels et al., 2005).

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[2] We cross-check NYSE market indices with Bloomberg and the NYSE website for the quality control.

[3] Aboody et al. (2018) compute the weekly CTO as the average daily CTO for that week multiplied by 5, while for the monthly CTO in our paper, we use the sum of the daily CTO of month *T*, i.e., the average daily CTO multiplied by the actual number of trading days for that month. However, using the average daily CTO for that month multiplied by 22, does not affect our empirical results. Also, applying this sum approach to obtain the monthly CTC and OTC does not affect our empirical results, either.

Table 1: Monthly excess returns and realized variance.

	Excess returns (I)				Realized variance (II)			
	μ (×10^2)	σ^2 (×10^2)	*Skew.*	*Kurt.*	μ (×10^2)	σ^2 (×10^4)	*Skew.*	*Kurt.*
CTO	0.009	0.011	1.653	24.940	0.013	0.002	8.276	96.594
OTC	0.636	0.175	–0.752	3.045	0.230	0.280	8.138	79.428
CTC	0.639	0.176	–0.741	3.025	0.238	0.272	8.346	83.185

This table presents the summary statistics of monthly excess returns (Column I) and realized variance (Column II) of the CTO, the OTC, and the CTC, including the mean (μ), the variance (σ^2), the skewness (*Skew.*), and the kurtosis (*Kurt.*). Realized volatility is computed from within-month daily returns.

2.2 Volatility models

We apply three approaches, including the rolling window (RW), GARCH, and GJR-GARCH models to measure conditional volatility, in that the mean-variance relation is dependent on the choice of volatility models (Ghysels et al., 2005). The RW model follows,

$$Var_t(R_{t+1}) = \sigma_t^2 = \frac{22}{N_t}\sum_{d=1}^{N_t} r_{t-d}^2, \tag{2}$$

where $Var_t(R_{t+1})$ is conditional volatility for forecasting next-month market returns R_{t+1}; σ_t^2 is realized volatility in month t; r_{t-d} is the demeaned daily return in month t, computed by subtracting the within-month mean daily return from the daily raw returns; N_t is the actual number of trading days in month t, and 22 is the commonly adopted number of trading days in one month (Yu and Yuan, 2011). For GARCH and GJR-GARCH models, we estimate the mean equation,

$$r_{t+1} = \mu + \varepsilon_{t+1}, \tag{3}$$

where r_{t+1} is the daily market return at day (t + 1); μ is the conditional mean of the daily return. The daily conditional volatility of market returns is then filtered from,

$$\sigma_{t+1}^2 = \omega + \alpha\varepsilon_t^2 + \beta\sigma_t^2, \tag{4}$$

$$\sigma_{t+1}^2 = \omega + \alpha_1 \varepsilon_t^2 + \alpha_2 I_t \varepsilon_t^2 + \beta \sigma_t^2, \quad (5)$$

for GARCH and GJR-GARCH models, respectively. The term I_t in Eq. (5) is the dummy variable for good news to account for the leverage effect (Glosten et al., 1993). We store the daily conditional volatility series from the two specifications and apply them to the following,

$$Var_t(R_{t+1}) = E_t(\sum_{d=1}^{N_t} \sigma_{t+d}^2), \quad (6)$$

where the monthly conditional volatility, $Var_t(R_{t+1})$, is computed as the linear sum of daily conditional volatility (Engle, 2001).

3. Empirical results

To test the mean-variance relation, we estimate,

$$R_{t+1} = \alpha + \beta Var_t(R_{t+1}) + \xi_{t+1}, \quad (7)$$

where β is the estimated mean-variance relation, and as per the traditional financial framework, we expect it to be positive, i.e., that high (low) risk is rewarded by high (low) returns.

Table 2 reveals that the mean-variance relation does vary during different time periods. In particular, a positive mean-variance relation is found for the CTO, which is consistent across three volatility models, supporting the theoretical risk-return tradeoff and empirical evidence in Pástor et al. (2008), etc.[4] For example, as per the RW model, a 1% increase (decrease) in conditional volatility would lead to a 3.694% increase (decrease) in the CTO. Oppositely, the risk-return tradeoff crashes for the OTC and the CTC, and there is a significantly negative mean-variance relation in several cases, in line with Brandt and Kang (2004), etc. Revealed by the RW model, for a same magnitude of movement in conditional volatility, i.e., a 1%

[4] In many months the CTO is very small, so we replicate the test based on a reduced sample that excluding such months. Results remain consistent.

increase (decrease), a 0.958% and a 1.062% decrease (increase) would be expected for the OTC and the CTC, respectively. Prior literature tests the mean-variance relation largely on the basis of total returns (i.e., the CTC), while our results show that day and night stories are completely different, and using total returns only could, thus, be misleading.

Table 2: Monthly excess returns against conditional variance.

	CTO (I)		OTC (II)		CTC (III)	
Method	***β***	***prob.***	***β***	***prob.***	***β***	***prob.***
RW	3.694	(0.002)[a]	–0.958	(0.010)[a]	–1.062	(0.005)[a]
GARCH	3.700	(0.004)[a]	–0.597	(0.182)	–0.779	(0.109)
GJR-GARCH	3.631	(0.000)[a]	–0.641	(0.123)	–0.861	(0.059)[c]

This table presents the mean-variance relation for CTO (Column I), OTC (Column II), and CTC (Column III), across three volatility models.
[a] and [c] represent statistical significance at the 1% and 10% level, respectively.

Our findings add a new explanation for the weak risk-return tradeoff in stock markets and show that the mean-variance relation is not constant within a trading day but varies with the clock. Also, our market-level results are in line with the stock-level evidence of Hendershott et al. (2020) documenting a positive return-beta relation overnight while a negative one during trading hours.

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