

# Maintenance Scheduling for Railway Tracks under Limited Possession Time

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## ABSTRACT

Maintenance planning for busy railway systems is challenging since there is growing pressure on increasing operation time, which reduces the infrastructure-accessible time for maintenance. This paper proposes an optimization model that is aimed at finding the best maintenance schedule for multiple components in a railway track to minimize the total cost in the planning horizon. One distinct and practical feature of the model is that the track accessible time for maintenance is limited. We formulate all relevant costs in the component's life-cycle, including maintenance cost, fixed track-closure (possession) cost, social-economic cost related to the effects of maintenance time on the train operation, and service-life shortening cost due to the shifting of activities. Generally, it is beneficial to cluster and maintain several components in a single possession as this helps reduce the cost by occupying the track only once. However, the decision must depend on the available possession time. A sensitivity analysis is performed to highlight the effects of available possession time on the number of required possessions as well as the total cost incurred.

## INTRODUCTION

Maintenance of infrastructure is crucial for a safe and well-functioning railway system. The process of deciding what, when, and how infrastructure maintenance should best be performed is

a vital part in ensuring efficient operation. However, this is a complex planning problem since these decisions need to take several factors into consideration, such as track degradation, traffic condition, available resources, and other tangible and intangible issues (Uzarski and McNeil 1994). In large and extensively used railway networks, such as those in the US, UK, and continental Europe, maintenance planning is more challenging since a great amount of railway infrastructure is a mix of old and recently built assets that are often associated with a high demand for maintenance and are under pressure to increase operation time.

Railway infrastructure maintenance activities can be classified into two types: routine or ordinary maintenance, such as regular inspections and minor repairs, and major maintenance and renewal, such as rail grinding, ballast tamping, and renewal. These maintenance activities are performed in a track closure period called possession (Cheung et al. 1999; Higgins 1998; Budai 2009; Lidén 2015). Ordinary maintenance activities require a relatively short time to execute and are often scheduled in a minor possession. Major maintenance and renewal activities require a longer time to execute and are thus scheduled in a major possession. Planning of a minor possession is not difficult since minor inspections and repairs can be done at night or in a period between two consecutive trains, which usually does not affect the train operation (Lidén 2015). However, planning a major possession is more complex since it affects the train operation and involves several parties, including the rail infrastructure manager, the train operating company, traffic control, and maintenance contractors. Thus, multiple and long possessions may have severe impacts on regular train timetables, and major maintenance and renewal jobs are often combined or clustered to reduce the total costs. This paper mainly focuses on the scheduling of major maintenance and renewal activities of components in a rail track system.

In this paper, the railway track is regarded as a system consisting of several components such as rail, ballast, sleepers, and switches, and our maintenance planning model considers all of these components simultaneously. Maintenance includes preventive maintenance (PM) and renewal. The former includes activities that restore the track components to a better condition such as rail grinding, ballast cleaning, and tamping, etc., and the latter is regarded as the replacement

of components when maintaining them is no longer practical and economical (Levy 2012). A PM activity can help restore the component to a better condition, and it is often cheaper than a renewal of that component. Very frequently, a PM activity can be done several times on the same component prior to a renewal to utilize the cost advantage of PM. An example of PM is "rail surfacing", which can help to improve the rail surface condition at a specific maintenance cost, but after a certain number of surfacing activities, the rail thickness can no longer sustain another surfacing, and a renewal at a much higher cost is needed. Generally, components can have different intervals of performing PM activities, and renewal is needed when a fixed number of PM activities have been performed. In this paper, we assume that the component has a worn-out stock which can be represented by the number of PM activities applied on it. After a PM, the component is in a good condition but not "as good as new", and the stock for the number of remaining PM activities to the next renewal decreases. After a renewal, the component is assumed to be "as good as new" and the number of remaining PM activities is reset to the maximum number, i.e. the same life-cycle is repeated.

In a highly utilized transportation system, the task of assigning a busy track for maintenance and for train operation is a critical issue (Lidén and Joborn 2016). The infrastructure accessible time for maintenance is often tightened up to a specified window. For example, in Hong Kong and Singapore, the maintenance window is 3 to 5 hours at night; in the Netherlands, ordinary maintenance activities are performed at night within a maintenance window of 4 to 5 hours. Major possessions are often limited to one or two days depending on how busy the track location is, and they are scheduled at a time when the impact on the train operation is less severe, e.g. weekends and holidays.

In this paper, we present a maintenance scheduling optimization model for multiple components in a railway track and formulate it as a binary integer programming (IP) model. The novelty of the proposed model is that the time needed for maintenance is incorporated in the railway maintenance problem and the limitation of possession time is modeled for the first time. This is, nowadays, very critical for busy railway systems, where the demand for train operation is very high and where

there is growing pressure to increase operation time, thus reducing the infrastructure accessible time for maintenance. In addition, two new ways of modeling the cost related to the impact of possession on operation that values the customers and the service-life shortening resulting from clustering maintenance activities are proposed. The model can help maintenance managers in determining the most cost-effective maintenance and renewal schedule for several components in the same railway track.

## LITERATURE REVIEW

The existing literature on railway infrastructure maintenance scheduling can be classified into three categories. The first category concentrates on railway component degradation modeling and maintenance interval determination for a single component or a single type of maintenance activity. Ballast tamping is referred to as a preventive maintenance activity in a number of studies (Zhao et al. 2006; Andrade and Teixeira 2011; Caetano and Teixeira 2016; Vale et al. 2012). A life-cycle cost approach for ballast tamping and renewal is presented in Zhao et al. (2006). Vale et al. (2012) and Caetano and Teixeira (2016) use the track geometry degradation and tamping recovery quality modeling to determine the tamping schedule. Andrade and Caetano (2011) study the same problem but with a multi-objective optimization approach of minimizing both the costs of maintenance and train delay. The overhead contact line is analyzed to identify the best inspection frequency with a consideration of the criticality of components and area as well as the availability of inspection teams (Zorita A. L. et al. 2010). Although the models in this category can provide useful information and maintenance plans for a single type of component, there is a critique that these models have considered neither the track system as a whole nor the advantages of maintaining multiple components together.

In the second category, maintenance scheduling is considered as a “resource allocation” or “resource assignment” problem where maintenance jobs and resources, such as maintenance manpowers and available time slots, are the inputs, and maintenance schedulers need to assign the jobs to available resources as much as possible. Higgins (1998) considers maintenance scheduling as a job-allocation problem in which the maintenance activities should be assigned to a given set of

crews. Meanwhile, Cheung et al. (1999) regard railway maintenance as a time-allocation problem; that is, they aim to assign as many maintenance jobs as possible to the available time slots. Focusing on maintenance crew scheduling, Gorman and Kanet (2010) formulate it as a job-shop scheduling problem. In a more recent paper, Peng and Ouyang (2014) present a special approach for modeling the railway maintenance planning problem. It is modeled as a vehicle routing problem, where the maintenance crews are considered as “vehicles”, and there is a set of projects with several jobs in each project being regarded as “routes”. A vehicle, i.e. maintenance crew, needs to travel to several routes, i.e. jobs in the projects, in such a way that all the routes are covered and the total traveling cost is minimized. A drawback of the models in this category is that the number of maintenance tasks, crews, and the available time slots in the allocation problem are fixed. Thus, these models are difficult to be extended to a long-term planning problem where a maintenance task can be performed at any time in the planning horizon.

In the last category, the aim of railway maintenance scheduling is to find the time periods to perform maintenance activities for components with different maintenance intervals. In general, maintenance requires several set-up activities, and it is more economical to maintain several components at the same time to reduce the number of possessions and spend the set-up cost only once. The problem of maintenance planning for rail-track components belongs to a general maintenance class of multi-component systems with economic dependence (Dekker et al. 1997; Nicolai and Dekker 2008; Dao et al. 2014). Budai et al. (2006) formulate a preventive maintenance scheduling problem for repetitive routine works and renewal projects. A mathematical programming model is presented to minimize the total maintenance and possession costs in a finite planning horizon. Pouryousef et al. (2010) extend Budai’s model by considering the planning of multiple segments at the same time. Zhao et al. (2009) and Pargar et al. (2017) study the maintenance scheduling problem and consider multiple types of cost savings due to joint maintenance and renewal activities at multiple segments. The types of saving depend on the number of adjacent segments and the share of special machineries for maintenance. Focusing on the renewal of track components, Caetano and Teixeira (2013) optimize the components’ life-cycle cost and the

unavailability of track when it is available for maintenance.

In this paper, we study the last category of railway maintenance scheduling problem and address a situation where the possession time is limited. To our best knowledge, no paper in this category considers the limitation of possession time and its effects on the maintenance scheduling problem. By investigating the limitation of possession time, this paper can provide a solution for infrastructure managers of busy railway networks, where there is an increasing demand of using the track for train-path operation and where a shrinking time window is available for infrastructure maintenance. In addition, the existing maintenance planning models assume that a fixed possession cost is incurred when at least one activity is performed in a time period regardless of the number of activities to be clustered in that period. In fact, the possession cost varies depending on the possession time as well as the “social-economic” impacts, i.e. the expected number of customers and the cost per customer per hour, of the track location. Besides, when different maintenance activities are clustered in the same time period, it is often seen that a component is maintained or renewed in a period that is earlier than its recommended time. In this case, the service-life of the component is shortened compared to the service-life when its recommended maintenance interval is used. Thus, we also consider a service-life shortening cost due to early maintenance of components.

The maintenance scheduling problem is considered in a finite planning horizon using a rolling-horizon approach (Wildeman et al. 1997). In the rolling-horizon approach, the decision is made at the horizon starting time. At the end of the current horizon, the same planning horizon can be repeated, or if there is an information update, the scheduling model generates a new horizon using updated input, and so on. This approach is practical when a fixed-term planning is required and the information may be updated, or when the necessity of another planning term needs to be taken into consideration.

## **PROBLEM FORMULATIONS**

In this section, we first provide a general problem statement of the railway maintenance scheduling problem under limited possession time. Then, we formulate four cost factors including maintenance and renewal cost, fixed possession cost, social-economic cost, and service-life shortening

cost. Several constraints that a maintenance scheduler needs to consider are also discussed.

### **Problem statement**

In the railway maintenance scheduling problem, we have to schedule PM and renewal activities for a track system consisting of  $N$  components in a discrete planning horizon  $T$  with a time index  $t = 1, 2, \dots, T$ . Each component  $i, i = 1, 2, \dots, N$ , in the system has its recommended PM interval, i.e. the maximum number of time periods between two consecutive PMs, denoted by  $\tau_{p,i}$ . A renewal is required on the component after a maximum number of PMs, denoted by  $N_{p,i}$ , have been done. In the railway maintenance scheduling problem, both  $\tau_{p,i}$  and  $N_{p,i}$  are assumed to be known. Other input data include the time and cost of each maintenance activity, the cost of shifting activities to an earlier period, the number of time periods elapsed from the last PM, the number of PM activities from the last renewal, and the available possession time. We need to determine the time periods to perform activities so that the total cost incurred in the planning horizon is minimized. A summary of input and output of the railway maintenance scheduling problem in this paper is presented in Table 1.

*Table 1. Input and output of the railway maintenance scheduling problem*

The problem is formulated as a binary IP model. The main reason that binary IP is selected for modeling the problem is that it is appropriate for the type of railway maintenance scheduling problem studied in the paper. In this study, the objective is to find the time period to perform a preventive maintenance or renewal activity. By defining binary decision variables based on whether to do an activity at each time period or not, a solution that contains binary values for all periods in the planning horizon can be easily transferred to a maintenance schedule as required for the problem. The formulation of the railway maintenance scheduling problem is focused mainly on different cost elements and constraints in the planning horizon which are further explained in the remaining parts of this section.

## Maintenance and renewal cost and time

In the planning horizon  $T$ , two binary variables are defined as follows:

$$x_{i,t}^m = \begin{cases} 1 & \text{if component } i \text{ is maintained in period } t \\ 0 & \text{otherwise} \end{cases}, \quad \forall i = 1, 2, \dots, N \quad (1)$$

$$x_{i,t}^r = \begin{cases} 1 & \text{if component } i \text{ is renewed in period } t \\ 0 & \text{otherwise} \end{cases}, \quad \forall i = 1, 2, \dots, N \quad (2)$$

It is seen that the maintenance schedule for the track system with  $N$  components can be realized if the set  $\{x_{i,t}^m, x_{i,t}^r\}$  is explicitly determined with  $\forall i = 1, 2, \dots, N$ . In a time index  $t$ , a component cannot be maintained or renewed more than one time. Let  $c_{i,t}^m$  and  $c_{i,t}^r$  be the maintenance and renewal costs of component  $i$  in period  $t$  respectively. The total costs of maintenance and renewal for  $N$  components in  $T$  time periods are simply the summation of all individual component maintenance costs in each period. They are shown in (Eqs. 3 and 4).

$$C_M = \sum_{i=1}^N \sum_{t=1}^T c_{i,t}^m \times x_{i,t}^m \quad (3)$$

$$C_R = \sum_{i=1}^N \sum_{t=1}^T c_{i,t}^r \times x_{i,t}^r \quad (4)$$

Similarly, denote  $t_{i,t}^m$  and  $t_{i,t}^r$  as the maintenance and renewal times of component  $i$  in period  $t$ . It is assumed that the maintenance and renewal activities are performed sequentially in a possession. Then, the possession time for maintenance of all components in period  $t$  is calculated as in (Eq. 5).

$$T_t = \sum_{i=1}^N (t_{i,t}^m \times x_{i,t}^m + t_{i,t}^r \times x_{i,t}^r) \quad (5)$$

## Possession and social-economic costs

In period  $t$ , if a maintenance or a renewal is performed on at least one component, a track possession is needed to ensure that the track is available for maintenance and no trains can enter

the maintenance area. We define the following binary variable.

$$x_t^p = \begin{cases} 1 & \text{if there is a possession in period } t \\ 0 & \text{otherwise} \end{cases}, \forall i = 1, 2, \dots, N \quad (6)$$

We have:

$$x_t^p = \max_{\forall i} \{x_{i,t}^m, x_{i,t}^r\} \quad \text{or} \quad \{x_t^p \geq x_{i,t}^m \text{ and } x_t^p \geq x_{i,t}^r\}, \forall i = 1, 2, \dots, N \quad (7)$$

In a period, if the track is needed for maintenance, i.e. at least one maintenance or renewal activity is performed, a fixed possession cost,  $c_p^0$ , is incurred (in this notation,  $p$  stands for possession).  $c_p^0$  represents the cost of having a track section for maintenance, which is thus not available for train service. This concept is similar to the definition of possession cost in (Budai et al. 2006), and it includes the cost of isolating the track section such as possession booking, re-timetabling, and all relevant set-up costs that are needed before the maintenance crew can work on the track. In addition, there is a social-economic impact that depends on the duration of possession, which can be modeled as a social-economic cost, i.e. a variable cost in the possession. Let the expected number of customers in period  $t$  and the cost per customer per unit time be ( $N_t^C$ ) and ( $c_p^e$ ) respectively. The cost per customer per unit time is independent from component maintenance cost and it is a new concept in this paper. This cost takes into account the effect of the train service disruption on customers, i.e. they have to re-route and need extra hours to travel; it may also include "indirect" costs such as the reputation lost, decreased customer satisfaction, and losses of future customers due to disruptions. The total fixed possession cost -  $C_p^f$  and social-economic cost -  $C_p^e$  in the entire planning horizon are calculated as in Eqs. (8) and (9) respectively.

$$C_p^f = \sum_{t=1}^T c_p^0 \times x_t^p \quad (8)$$

$$C_p^e = \sum_{t=1}^T c_p^e \times N_t^C \times \sum_{i=1}^N T_i = \sum_{t=1}^T c_p^e \times N_t^C \times \sum_{i=1}^N (t_{i,t}^m \times x_{i,t}^m + t_{i,t}^r \times x_{i,t}^r) \quad (9)$$

It is also noted that not all costs are easy to gather. However, a great deal of money is involved in rail infrastructure maintenance (Lidén 2015), and it is worthwhile to spend resources and acquire the cost data in practice. Regarding the possession cost, it can be obtained by general accounting data sharing between railway infrastructure manager, traffic planning, and railway train operators. On the other hand, the cost per customer per unit time greatly depends on how the railway companies value their customers and the infrastructure manager has to discuss this with railway train services in order to quantify this value.

### **Service-life shortening cost**

Each component  $i$  in the system has its recommended PM interval, i.e. the maximum number of time periods between two consecutive PMs,  $\tau_{p,i}$ , and the maximum number of PM activities before its renewal,  $N_{p,i}$ . If the component is considered individually, its PM and renewal activities should be performed exactly in the due time period. However, when components are considered simultaneously, it is more economical to cluster and jointly maintain several components in the same time period. As a result, PM and renewal activities of a component may be performed earlier than its individual due time, and this will reduce the service-life of the component compared to its service-life in the individual schedule.

In this paper, we define an original schedule as the schedule obtained by directly using the individual PM intervals and the maximum number of PM activities to schedule maintenance and renewal of each component. Since  $\tau_{p,i}$  and  $N_{p,i}$  are given, the set of  $\{x_{i,t}^{m0}, x_{i,t}^{r0}\}$ , which corresponds to the decision to do PM and renewal activities in the original schedule, can be explicitly determined for all  $i = 1, 2, \dots, N$  and  $t = 1, 2, \dots, T$ .

Denote  $c_i^s$  as the service-life shortening cost per time period of component  $i$ , i.e. a cost of  $c_i^s$  is realized if a maintenance activity is performed a period earlier than the recommended period. This cost takes into account of utilizing the track until its end of service life as much as possible and preventing too early maintenance and renewal of rail-track components. Considering all components in the system, the total service-life shortening cost of a current schedule with  $\{x_{i,t}^m, x_{i,t}^r\}$

is presented in Eq. (10).

$$C_S = \sum_{i=1}^N c_i^s \left\{ \tau_{p,i} \sum_{t=1}^T (x_{i,t}^m - x_{i,t}^{m0}) + \sum_{t=T-\tau_{p,i}+1}^T (T-t)(x_{i,t}^m - x_{i,t}^{m0}) + \sum_{t=T-\tau_{p,i}+1}^T (T-t)(x_{i,t}^r - x_{i,t}^{r0}) \right\} \quad (10)$$

Eq. (10) implies that the total service-life shortening cost of each component is equal to a product of the service-life shortening cost per time period and the total number of periods shortened due to the maintenance and renewal of that component. The first term in the curly bracket represents the number of periods shortened if the number of PM actions on component  $i$  in the current schedule is greater than that in the original schedule. This newly introduced term is critical in reducing the possibilities of having additional PM activities when the planning horizon is shorter than the asset's life-cycle. The second and third terms take into consideration the number of periods in the last PM cycle, i.e. from  $T - \tau_{p,i} + 1$  to  $T$ , and offset the difference between the times to do the last activity in the two schedules. These two terms can play a similar role as in Budai et al. (Budai et al., 2006) to value the last PM interval and to schedule the last PM and renewal activity as late as possible.

### Total cost in the planning horizon

In summary, the total cost in the whole planning horizon, i.e. the objective function to be minimized, is presented as in Eq. (11).

$$C = C_M + C_R + C_P^f + C_P^e + C_S \quad (11)$$

### Sets of constraints in the railway maintenance scheduling problem

The following constraints are considered in this paper.

- Latest possible time to do preventive maintenance activities: This set of constraints ensures that the PM activities are performed within the specified PM interval.
- Latest possible time to do renewal activities: This set of constraints ensures that the a renewal activity is performed once the number of PMs reaches its limit.
- Maximum available time in each possession: This set of constraints guarantees that the

activities are performed within the available time of blocking a railway section for maintenance, and

- Variables constraints.

In the first and second sets of constraints, the first preventive maintenance and renewal activities need to be performed earlier than the regular interval if the asset is not new at the start of the planning horizon, i.e. time  $t = 0$ . Let  $\tau_{p,i}^0$  and  $N_{p,i}^0$  be the number of periods and number of PM activities elapsed from the last PM/renewal of component  $i$ . The first maximum interval of doing PM and renewal for component  $i$  is presented as in Eqs. (12) and (13) respectively.

$$\tau_{p,i}^1 = \tau_{p,i} - \tau_{p,i}^0 \quad (12)$$

$$\tau_{r,i}^1 = \tau_{p,i} \times (N_{p,i} - N_{p,i}^0) - \tau_{p,i}^0 \quad (13)$$

The available possession time constraint is critical for a busy railway system where maintenance planners have to coordinate with train operators to reserve the track for maintenance. Since the demand for operation is very high, too long possessions are not desirable, and the train operation usually limits the time for possession on the railway track, for example one day a week or two days a month.

The mathematical formulations of each constraint is presented in the following section.

### The railway maintenance scheduling optimization model

The railway maintenance scheduling problem under a limitation of available possession time is formulated as a IP model. Details are as follows:

$$\begin{aligned} \text{Min } C = & \sum_{i=1}^N \sum_{t=1}^T c_{i,t}^m \times x_{i,t}^m + \sum_{i=1}^N \sum_{t=1}^T c_{i,t}^r \times x_{i,t}^r + \sum_{t=1}^T c_P^0 \times x_t^p + \sum_{t=1}^T c_P^e \times N_t^C \times \sum_{i=1}^N (t_{i,t}^m \times x_{i,t}^m + t_{i,t}^r \times x_{i,t}^r) \\ & + \sum_{i=1}^N c_i^s \left\{ \tau_{p,i} \sum_{t=1}^T (x_{i,t}^m - x_{i,t}^{m0}) + \sum_{t=T-\tau_{p,i}+1}^T (T-t)(x_{i,t}^m - x_{i,t}^{m0}) + \sum_{t=T-\tau_{p,i}+1}^T (T-t)(x_{i,t}^r - x_{i,t}^{r0}) \right\} \end{aligned} \quad (14)$$

Subject to:

$$\sum_{t=1}^{\tau_{p,i}^1} x_{i,t}^m \geq 1, \forall i = 1, 2, \dots, N \quad (15)$$

$$\sum_{t=1+k}^{\tau_{p,i}+k} x_{i,t}^m \geq 1, \forall i = 1, 2, \dots, N; k = 1, 2, \dots, T - \tau_{p,i} \quad (16)$$

$$\sum_{t=1}^{\tau_{r,i}^1} x_{i,t}^r \geq 1, \forall i = 1, 2, \dots, N \quad (17)$$

$$\sum_{t=1+k}^{\tau_{r,i}+k} x_{i,t}^r \geq 1, \forall i = 1, 2, \dots, N; k = 1, 2, \dots, T - \tau_{r,i}; \tau_{r,i} = \tau_{p,i} \times N_{p,i} \quad (18)$$

$$\sum_{i=1}^N (t_{i,t}^m \times x_{i,t}^m + t_{i,t}^r \times x_{i,t}^r) \leq T_t^0, \forall t = 1, 2, \dots, T \quad (19)$$

$$x_t^p \geq x_{i,t}^m, \forall i = 1, 2, \dots, N; t = 1, 2, \dots, T \quad (20)$$

$$x_t^p \geq x_{i,t}^r, \forall i = 1, 2, \dots, N; t = 1, 2, \dots, T \quad (21)$$

$$x_t^p, x_{i,t}^m, x_{i,t}^r \in \{0, 1\}, \forall i = 1, 2, \dots, N; t = 1, 2, \dots, T \quad (22)$$

The objective of the railway maintenance scheduling problem is to minimize the total cost,  $C$ , that is incurred in the planning horizon. Each element in the objective function has been previously described in this section.

Constraints (15) and (16) ensure that the PM activities are performed within the allowed interval,  $\tau_{p,i}$ . Constraints (17) and (18) ensure that the renewal activities are performed on component  $i$  when the number of PM activities reach  $N_{p,i}$ . In Constraints (16) and (18), index  $k$  starts from 1 instead of 0 because when  $k = 0$  these constraints are very similar to (15) and (17), except the upper limit of the sum on the left-hand side. Following the reasoning from (12), the upper limit in (15) is smaller than that in (16). Thus, Constraint (16) is automatically satisfied for  $k = 0$ . In another words, Constraint (15) already covers (16) for the case  $k = 0$  and thus the index in (16) starts from 1. It is noted that Constraints (15) and (17) have taken the current ages of components into consideration

for the first maintenance and renewal. Constraint (19) presents the effect of possession time, which guarantees that the total maintenance times of all components in a possession must be within the available possession time in each period,  $T_i^0$ . The available possession time,  $T_i^0$ , is given and the evaluation of possession time in each period is shown in Eq.(5). Constraints (20) and (21) ensure that a possession is required whenever a maintenance or renewal activity takes place which is equivalent to Eq. (7). Constraint (22) sets the binary conditions of decision variables.

### ILLUSTRATIVE EXAMPLE

In this section, we present an example for a railway track system consisting of  $N = 5$  components to illustrate the maintenance scheduling problem in a finite planning horizon with  $T = 12$  time periods. The input data on PM intervals, the maximum number of PMs in each life-cycle, the cost and time to do a PM/renewal, the unit service-life shortening cost, and the time elapsed from the last maintenance are given in Table 2. Other input data on the fixed possession cost, the social-economic cost, and the available possession time are provided in Table 3.

*Table 2. Input data of components*

*Table 3. Possession cost and available possession time*

In Table 2, the number of periods elapsed from the last PM/renewal activities is the duration from the last PM/Renewal to the starting time of the planning horizon. It is defined to accommodate the fact that components are generally in use for certain periods of time at the start of the planning horizon, or some maintenance activities have been performed in the past.

Generally, the service-life shortening cost per unit of time (the sixth column in Table 2 -  $c_i^s$ ), depends on the component's renewal and maintenance costs as well as its recommended service-life. The later term can be determined based on component's PM interval,  $\tau_i^p$  and the maximum number of PMs in a renewal cycle  $N_{p,i}$ . We present the following equation to calculate  $c_i^s$ .

$$c_i^s = \frac{c_i^r + N_{p,i}c_i^m}{\tau_{p,i}(N_{p,i} + 1)} \quad (23)$$

The nominator of the right hand side in Equation ( 23) is the total maintenance and renewal

costs between two consecutive renewals of component  $i$ , and the denominator is the component's recommended service-life. The above formula implies that the service-life shortening cost of a component is an average of maintenance and renewal costs per period of use in its renewal cycle.

## RESULTS AND DISCUSSIONS

The results of maintenance scheduling models with and without the limitation on available possession time are investigated in terms of maintenance activities clustering, and the analysis is based on the cost and time breakdown as in the objective function ( 14) and constraint ( 19). In addition, the sensitivity analysis of the total cost and number of possessions over the available possession time is also performed.

In this paper, the binary linear IP model is solved using IBM CPLEX optimizer. The reason of using CPLEX is that it is able to find a true optimal solution for the linear IP model in this paper. CPLEX is a commercial software, but available for researchers to use, and it has been successfully applied to solve several similar linear programming models in the railway maintenance context (Budai et al. 2006; Vale et al. 2012; Vale and Ribeiro 2014; Vansteenwegen et al. 2016).

### Maintenance schedules with and without limitation of possession time

First, we solve the railway maintenance scheduling problem and analyze two maintenance schedules: A – when there is no limitation on the available possession time, i.e. without the limitation on possession time, and B – when there is a possession time limitation of  $T^0 = 24$  hours (1 day). Schedule B can be obtained with the given optimization model in the previous section. The optimization model to obtain Schedule A is obtained by removing constraint (19) out of the model. These two schedules are compared with the original schedule which is generated by assigning the latest possible time to do a maintenance/renewal activity as the planned time to do it. The three schedules are presented in Figs 1, 2, and 3 respectively.

*Fig. 1. The original maintenance schedule*

*Fig. 2. Maintenance schedule A - without limitation of possession time*

*Fig. 3. Maintenance schedule B - with 24 hrs limitation of possession time*

From Fig. 1, we can see that maintenance and renewal activities in the original schedule are spread out in the planning horizon. This is because the components have different preventive maintenance and renewal intervals. There are in total ten possessions, and activities are rarely performed together in the same period. On the other hand, many maintenance and renewal activities are clustered in Schedule A, i.e. when there is no limitation on the possession time (Fig. 2). There are in total five possessions in the planning horizon in periods 1, 3, 6, 7, and 11. Noticeably, three maintenance activities of components 1, 3, and 4 are clustered in period 3, and the renewal activity of component 4 is also clustered with two other maintenance activities of components 1 and 5 in period 11. When considering the limitation of available possession time, maintenance and renewal activities in Schedule B are also combined but not as much as in Schedule A. The four maintenance activities of components 1, 2, 4, and 5 cannot be clustered as in Schedule A since it takes 28 hours, i.e. more than  $T^0 = 24$  hours to perform these activities in the same time period. Also, each renewal of the two components 1 and 4 can only be combined with one more maintenance activity. There are, in total, six possessions with either two maintenance activities or a maintenance and a renewal activity to be clustered in Schedule B.

The clustering of maintenance activities in a schedule can be measured by two factors: the average number of activities in a possession and the total number of possessions in the schedule. The clustering of activities is high when the former factor is big and the later one is small. These factors are calculated and tabulated in a Table 4. We compare Schedule B with the original schedule and observe that the average number of activities in a possession clearly increases, by 1.7 times, and the number of possessions reduces from 10 (in the original schedule) to 6 possessions (in Schedule B).

*Table 4. Clustering of different maintenance schedules*

### **Cost and time analysis**

A cost analysis is performed to compare three different schedules: the original schedule, Schedule A (without limitation of possession time), and Schedule B (with limitation of possession time) to identify the similarities and differences. Four cost elements in the objective function,

i.e. maintenance, renewal, possession, and service-life shortening costs, are calculated for each schedule and presented in Table 5 and Fig. 4.

*Table 5. Cost of alternative maintenance schedules*

*Fig. 4. Possession cost vs. service-life shortening cost*

From Table 5, we see that the original schedule has a significantly higher total cost in comparison with the other two schedules. Schedule A gives the least total cost of 84.05 cost units, while the total cost of Schedule B is higher at 86.48 cost units. The maintenance and renewal costs in the three schedules are similar since the number of maintenance and renewal activities are identical. The major differences are in the possession and service-life shortening costs. The possession cost significantly reduces when considering the clustering of maintenance activities. The original schedule requires the highest possession cost of 29.1 cost units. Schedules A and B have lower possession cost with approximately 34.5% and 27.5% less than the original schedule, respectively. This result can be explained by the reduced number of possessions in Schedules A (5 possessions) and B (6 possessions). Regarding the service-life shortening cost, Schedule B, with a limitation of possession time, results in slightly higher cost compared to Schedule A, which implies that the activities in Schedule A are shifted to earlier periods more often in the planning horizon.

In addition, it is noticed that a too long possession may not be possible due to the limitation of the available possession time. The possession times in the three schedules are calculated and compared to the available possession time as shown in Table 6 and Fig. 5.

*Table 6. Possession time comparisons*

*Fig. 5. Unused and overused possession time*

Although Schedule A requires the least number of possessions, it ignores the limitation of available possession time and is the only schedule that violates the limitation of possession time. Two out of five possessions in Schedule A exceed the time limitation,  $T^0$ , with total excessive time of 7 hours. Thus, Schedule A may not be applicable, or there will be a huge penalty when implementing it. Schedule A also has a short possession of 3 hours which is not desirable. The original schedule requires many possessions with short durations, e.g. 3 or 6 hours, which indicate

that there are opportunities for clustering and reducing the number of possessions. Meanwhile, the possession time in Schedule B is more equally distributed compared to the other two schedules.

The number of possessions and possession times results in a great disparity in average unused and overused possession times of the three schedules. On average, the original schedule does not use 14.9 hours out of 24 hours (62%), which indicates that this schedule has a very low maintenance time utilization, and to improve this schedule, activities should be clustered. Schedule A (without limitation of possession time) shows less unused possession time of 7.2 hours (30%). However, it is the only schedule that overuses the time available for maintenance with 1.4 hours per possession. The schedule with a consideration of the limitation of possession time is the best schedule in terms of available time utilization with only 4.8 hours unused per possession and zero hours of overused available possession time.

### **Sensitivity analysis of the available possession time**

We further investigate the effects of the available possession time –  $T^0$  on the maintenance scheduling problem. The two distinct factors characterizing a maintenance schedule, which are the total cost and the number of required possessions, are calculated when the available possession changes. In this experiment,  $T^0$  is set to vary from 18 to 28 hours, and the results are presented in Fig. 6. Here, it is not possible to get a feasible schedule when the available possession time is less than 18 hours; when the available possession time is greater than 28 hours, the solution does not change compared to the one with  $T^0$  of 28 hours. Generally, the corresponding total cost and number of possessions are negatively proportional with the available time for maintenance.

*Fig. 6. Sensitivity analysis of the available possession time*

From Fig. 6, it can be seen that both the total cost and the number of possessions tend to decrease as the available possession time,  $T^0$ , increases. However, the total cost gradually decreases while the number of possessions sometimes remains constant when  $T^0$  increases, e.g. between  $T^0 = 20$  and  $T^0 = 22$ ; between  $T^0 = 24$  and  $T^0 = 26$ . The reduction of the total cost indicates that a different maintenance schedule is obtained when the available possession time varies. The results of this analysis can help the maintenance decision makers in two ways:

- Select the best maintenance schedule with a specific available possession time: This is highly applicable for busy railway sections with limited flexibility on the available possession time, and
- Decide whether to modify the threshold on the available possession time: this is more practical for the case that there is some flexibility of available possession time. For example, it may be of interest to decrease the available possession time for the studied section from 24 hrs to 22 hrs but not from 22 hrs to 20 hrs because the additional expense in the first change (0.19 cost units) is notably less than that of the second change (1.53 cost units).

In this paper, due to the sensitivity of cost data, the cost data in our example are presented in terms of cost units, and we have tried to select the relative cost as close to practical data of rail track components (rails, ballast, switch, etc.) as possible. However, it is also noted that the cost for each type of component can vary depending on several factors, e.g. the sleeper's material, the switch's size and type of turn-out. The proposed model in this paper is a conceptual model that can be flexibly applied for different rail-track systems.

## **CONCLUSIONS**

A maintenance scheduling model for several rail infrastructure components considering a limitation of possession time is presented in this paper. This model is practical for busy railway networks associated with a high demand of train operation. Several costs factors related to railway maintenance and operation have been formulated and optimized. We have analyzed the effects of the available possession time on the maintenance decisions and the total cost in a finite planning horizon, and our main finding is that the limitation of available possession time can alter the resulting maintenance schedule and the total cost. Furthermore, it may be worthwhile to change the limitation of possession time for the studied rail-track section. However, it is also noted that the detailed course of actions vary depending on the actual input costs as well as the type of railway section under investigation.

The proposed new maintenance scheduling model in this paper is for different components in a

single track link. Based on the model, several research directions can be identified as follows.

- Study the maintenance scheduling problem in a railway network context: when considering multiple components in a railway network, it is necessary to model the relationships between different track links to guarantee a smooth operation of the network. For example, when a track link is possessed for maintenance, it is not possible to possess another link since the block of both tracks will cause severe disruptions to the network operation.
- Study the methods to obtain the solution: The linear integer optimization model is currently solved by CPLEX, a commercial software for solving linear optimization problems. With this, the computational time increases exponentially when the number of components and the number of time periods increase. Heuristics and evolutionary computing methods are suggested for large-size problems and are recommended for future study.

## APPENDIX I. NOTATION

*The following symbols are used in this paper:*

- $C$  = Total cost;
- $c_{i,t}^m$  = Cost of maintaining component  $i$  in period  $t$ ;
- $c_{i,t}^r$  = Cost of renewing component  $i$  in period  $t$ ;
- $c_i^s$  = Service-life shortening cost per time period of component  $i$  ;
- $c_P^0$  = Fixed set-up cost occurred once per possession;
- $c_P^e$  = Social-economic cost per customer per unit time;
- $C_M$  = Maintenance cost;
- $C_P^e$  = Variable social-economic cost;
- $C_P^f$  = Fixed possession cost;
- $C_R$  = Renewal cost;
- $C_S$  = Service-life shortening cost;
- $i$  = Component index;
- $N$  = Number of components;
- $N_{p,i}$  = Number of PM activities between two renewals of component  $i$ ;
- $N_{p,i}^0$  = Number of PM activities elapsed from the last renewal of component  $i$ ;
- $N_t^C$  = Expected number of customers per unit time in period  $t$ ;
- $t$  = Time index;
- $t_{i,t}^m$  = Time of maintaining component  $i$  in period  $t$ ;
- $t_{i,t}^r$  = Time of renewing component  $i$  in period  $t$ ;
- $T$  = Maximum time in the planning horizon;
- $T_t$  = Total maintenance and renewal time for all components in period  $t$ ;
- $T_t^0$  = Available possession time in period  $t$ ;
- $\tau_{p,i}$  = Maximum number of time periods between two PMs of component  $i$ ; and
- $\tau_{p,i}^0$  = Number of time periods elapsed from the last PM of component  $i$ .

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## List of Tables

1	Input and output of the railway maintenance scheduling problem . . . . .	26
2	Input data of components . . . . .	27
3	Possession cost and available possession time . . . . .	28
4	Clustering of different maintenance schedules . . . . .	29
5	Cost of alternative maintenance schedules (cost units) . . . . .	30
6	Possession time comparisons (in hours) . . . . .	31

**TABLE 1.** Input and output of the railway maintenance scheduling problem

Input	Output
Planning horizon	Time to perform PM(s)
List of components with	of each component
- PM intervals	Time to perform renewal(s)
- PM cost and time	of each component
- Number of PMs between	Total estimated cost
two renewals	
- Renewal cost and time	
- Current components usages	
- Service life shortening cost	
Possession costs	
Available possession time	

**TABLE 2.** Input data of components

Component ( $i$ )	PM int. (periods)	Number of PMs ( $N_{p,i}$ )	Main. cost (cost units)	Ren. cost (cost units)	Ser. shortening cost (cost units)	Main. time(hrs)	Renewal time (hrs)	Elapsed periods	Elapsed number of PMs
1	4	9	2	6	0.6	9	18	1	3
2	6	8	6	15	1.17	6	16	4	3
3	9	7	5.5	20	0.81	8	23	5	2
4	8	10	4	18	0.66	10	16	4	5
5	5	6	4	25	1.4	3	12	4	2

**TABLE 3.** Possession cost and available possession time

Fixed possession cost (cost units)	Cost per customer (cost units)	Number of customers	Avail. Possession time (hrs)
2	0.001	100	24

**TABLE 4.** Clustering of different maintenance schedules

Schedule	Number of possessions	Ave. number of activities per possession
Original	10	1.10
A	5	2.20
B	6	1.83

**TABLE 5.** Cost of alternative maintenance schedules (cost units)

Schedule	Maintenance cost	Renewal cost	Possession cost	Service-life shortening cost	Total cost
Original	37.5	24	29.1	-	90.6
A	37.5	24	19.1	3.45	84.05
B	37.5	24	21.1	3.88	86.48

**TABLE 6.** Possession time comparisons (in hours)

Schedule	Possession number										Excess $T^0$ ?
	1	2	3	4	5	6	7	8	9	10	
Original	3	6	9	10	8	3	18	6	12	16	No
A	9	27	3	24	28	-	-	-	-	-	<b>Yes</b>
B	9	19	11	24	19	9	-	-	-	-	No

"-": not applicable

**List of Figures**

1 The original maintenance schedule . . . . . 33

2 Maintenance schedule A - without limitation of possession time . . . . . 34

3 Maintenance schedule B - with 24 hrs limitation of possession time . . . . . 35

4 Possession cost vs. service-life shortening cost . . . . . 36

5 Unused and overused possession time . . . . . 37

6 Sensitivity analysis of the available possession time . . . . . 38

<i>Component</i>													
<i>1</i>			M				R				M		
<i>2</i>		M						M					
<i>3</i>					M								
<i>4</i>				M								R	
<i>5</i>	M					M					M		
	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>7</i>	<i>8</i>	<i>9</i>	<i>10</i>	<i>11</i>	<i>12</i>	<i>Time</i>
			M	<i>Maintenance</i>				R	<i>Renewal</i>				

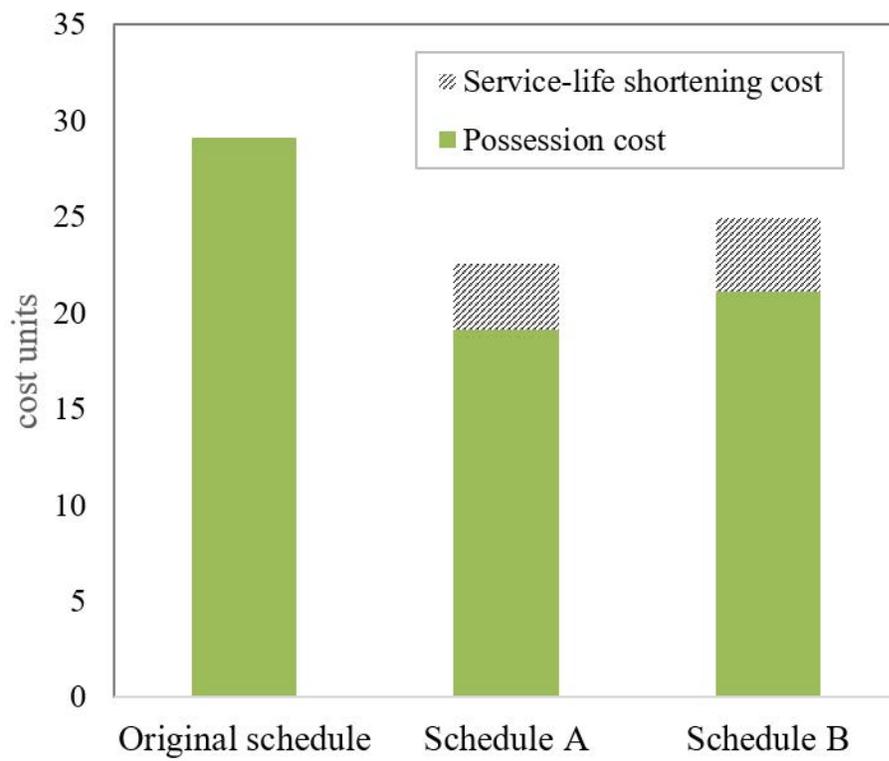
**Fig. 1.** The original maintenance schedule

<i>Component</i>													
<i>1</i>			M				R				M		
<i>2</i>	M						M						
<i>3</i>			M										
<i>4</i>			M								R		
<i>5</i>	M					M					M		
	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>7</i>	<i>8</i>	<i>9</i>	<i>10</i>	<i>11</i>	<i>12</i>	<i>Time</i>
			M	<i>Maintenance</i>				R	<i>Renewal</i>				

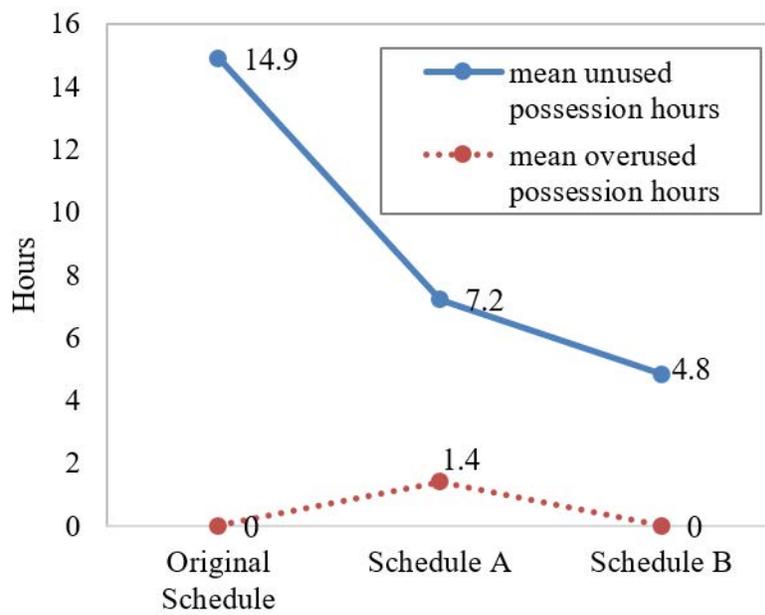
**Fig. 2.** Maintenance schedule A - without limitation of possession time

<i>Component</i>													
<i>1</i>			M				R				M		
<i>2</i>	M						M						
<i>3</i>					M								
<i>4</i>			M							R			
<i>5</i>	M				M					M			
	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>7</i>	<i>8</i>	<i>9</i>	<i>10</i>	<i>11</i>	<i>12</i>	<i>Time</i>
			M	<i>Maintenance</i>				R	<i>Renewal</i>				

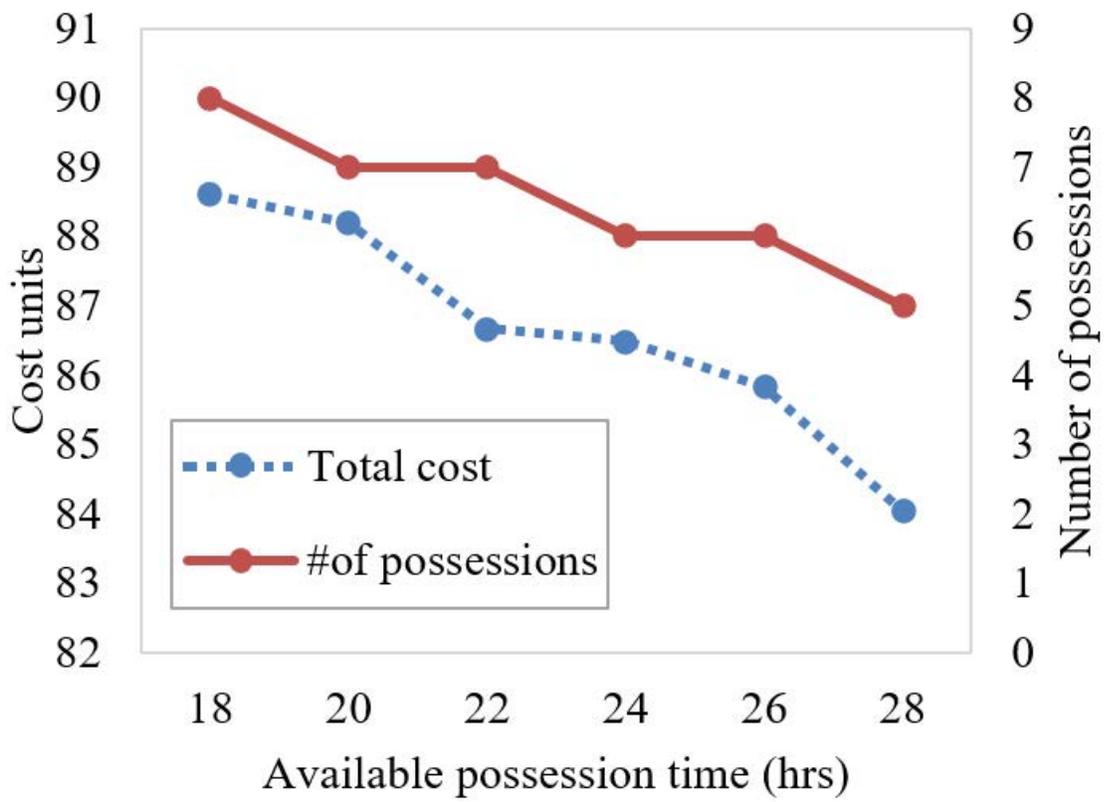
**Fig. 3.** Maintenance schedule B - with 24 hrs limitation of possession time



**Fig. 4.** Possession cost vs. service-life shortening cost



**Fig. 5.** Unused and overused possession time



**Fig. 6.** Sensitivity analysis of the available possession time