

# Maintenance Scheduling for Railway Tracks under Limited Possession Time

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## ABSTRACT

Maintenance planning for busy railway systems is challenging since there is growing pressure on increasing operation time, which reduces the infrastructure-accessible time for maintenance. This paper proposes an optimization model that is aimed at finding the best maintenance schedule for multiple components in a railway track to minimize the total cost in the planning horizon. One distinct and practical feature of the model is that the track accessible time for maintenance is limited. We formulate all relevant costs in the component's life-cycle, including maintenance cost, fixed track-closure (possession) cost, social-economic cost related to the effects of maintenance time on the train operation, and service-life shortening cost due to the shifting of activities. Generally, it is beneficial to cluster and maintain several components in a single possession as this helps reduce the cost by occupying the track only once. However, the decision must depend on the available possession time. A sensitivity analysis is performed to highlight the effects of available possession time on the number of required possessions as well as the total cost incurred.

## INTRODUCTION

Maintenance of infrastructure is crucial for a safe and well-functioning railway system. The process of deciding what, when, and how infrastructure maintenance should best be performed is

25 a vital part in ensuring efficient operation. However, this is a complex planning problem since  
26 these decisions need to take several factors into consideration, such as track degradation, traffic  
27 condition, available resources, and other tangible and intangible issues (Uzarski and McNeil 1994).  
28 In large and extensively used railway networks, such as those in the US, UK, and continental  
29 Europe, maintenance planning is more challenging since a great amount of railway infrastructure is  
30 a mix of old and recently built assets that are often associated with a high demand for maintenance  
31 and are under pressure to increase operation time.

32 Railway infrastructure maintenance activities can be classified into two types: routine or  
33 ordinary maintenance, such as regular inspections and minor repairs, and major maintenance and  
34 renewal, such as rail grinding, ballast tamping, and renewal. These maintenance activities are  
35 performed in a track closure period called possession (Cheung et al. 1999; Higgins 1998; Budai  
36 2009; Lidén 2015). Ordinary maintenance activities require a relatively short time to execute and are  
37 often scheduled in a minor possession. Major maintenance and renewal activities require a longer  
38 time to execute and are thus scheduled in a major possession. Planning of a minor possession  
39 is not difficult since minor inspections and repairs can be done at night or in a period between  
40 two consecutive trains, which usually does not affect the train operation (Lidén 2015). However,  
41 planning a major possession is more complex since it affects the train operation and involves several  
42 parties, including the rail infrastructure manager, the train operating company, traffic control, and  
43 maintenance contractors. Thus, multiple and long possessions may have severe impacts on regular  
44 train timetables, and major maintenance and renewal jobs are often combined or clustered to reduce  
45 the total costs. This paper mainly focuses on the scheduling of major maintenance and renewal  
46 activities of components in a rail track system.

47 In this paper, the railway track is regarded as a system consisting of several components such  
48 as rail, ballast, sleepers, and switches, and our maintenance planning model considers all of these  
49 components simultaneously. Maintenance includes preventive maintenance (PM) and renewal.  
50 The former includes activities that restore the track components to a better condition such as  
51 rail grinding, ballast cleaning, and tamping, etc., and the latter is regarded as the replacement

52 of components when maintaining them is no longer practical and economical (Levy 2012). A  
53 PM activity can help restore the component to a better condition, and it is often cheaper than a  
54 renewal of that component. Very frequently, a PM activity can be done several times on the same  
55 component prior to a renewal to utilize the cost advantage of PM. An example of PM is "rail  
56 surfacing", which can help to improve the rail surface condition at a specific maintenance cost,  
57 but after a certain number of surfacing activities, the rail thickness can no longer sustain another  
58 surfacing, and a renewal at a much higher cost is needed. Generally, components can have different  
59 intervals of performing PM activities, and renewal is needed when a fixed number of PM activities  
60 have been performed. In this paper, we assume that the component has a worn-out stock which can  
61 be represented by the number of PM activities applied on it. After a PM, the component is in a  
62 good condition but not "as good as new", and the stock for the number of remaining PM activities  
63 to the next renewal decreases. After a renewal, the component is assumed to be "as good as new"  
64 and the number of remaining PM activities is reset to the maximum number, i.e. the same life-cycle  
65 is repeated.

66 In a highly utilized transportation system, the task of assigning a busy track for maintenance  
67 and for train operation is a critical issue (Lidén and Joborn 2016). The infrastructure accessible  
68 time for maintenance is often tightened up to a specified window. For example, in Hong Kong  
69 and Singapore, the maintenance window is 3 to 5 hours at night; in the Netherlands, ordinary  
70 maintenance activities are performed at night within a maintenance window of 4 to 5 hours. Major  
71 possessions are often limited to one or two days depending on how busy the track location is, and  
72 they are scheduled at a time when the impact on the train operation is less severe, e.g. weekends  
73 and holidays.

74 In this paper, we present a maintenance scheduling optimization model for multiple components  
75 in a railway track and formulate it as a binary integer programming (IP) model. The novelty of the  
76 proposed model is that the time needed for maintenance is incorporated in the railway maintenance  
77 problem and the limitation of possession time is modeled for the first time. This is, nowadays,  
78 very critical for busy railway systems, where the demand for train operation is very high and where

79 there is growing pressure to increase operation time, thus reducing the infrastructure accessible  
80 time for maintenance. In addition, two new ways of modeling the cost related to the impact of  
81 possession on operation that values the customers and the service-life shortening resulting from  
82 clustering maintenance activities are proposed. The model can help maintenance managers in  
83 determining the most cost-effective maintenance and renewal schedule for several components in  
84 the same railway track.

## 85 LITERATURE REVIEW

86 The existing literature on railway infrastructure maintenance scheduling can be classified into  
87 three categories. The first category concentrates on railway component degradation modeling  
88 and maintenance interval determination for a single component or a single type of maintenance  
89 activity. Ballast tamping is referred to as a preventive maintenance activity in a number of studies  
90 (Zhao et al. 2006; Andrade and Teixeira 2011; Caetano and Teixeira 2016; Vale et al. 2012). A  
91 life-cycle cost approach for ballast tamping and renewal is presented in Zhao et al. (2006). Vale  
92 et al. (2012) and Caetano and Teixeira (2016) use the track geometry degradation and tamping  
93 recovery quality modeling to determine the tamping schedule. Andrade and Caetano (2011) study  
94 the same problem but with a multi-objective optimization approach of minimizing both the costs of  
95 maintenance and train delay. The overhead contact line is analyzed to identify the best inspection  
96 frequency with a consideration of the criticality of components and area as well as the availability  
97 of inspection teams (Zorita A. L. et al. 2010). Although the models in this category can provide  
98 useful information and maintenance plans for a single type of component, there is a critique that  
99 these models have considered neither the track system as a whole nor the advantages of maintaining  
100 multiple components together.

101 In the second category, maintenance scheduling is considered as a “resource allocation” or  
102 “resource assignment” problem where maintenance jobs and resources, such as maintenance man-  
103 powers and available time slots, are the inputs, and maintenance schedulers need to assign the jobs  
104 to available resources as much as possible. Higgins (1998) considers maintenance scheduling as  
105 a job-allocation problem in which the maintenance activities should be assigned to a given set of

106 crews. Meanwhile, Cheung et al. (1999) regard railway maintenance as a time-allocation problem;  
107 that is, they aim to assign as many maintenance jobs as possible to the available time slots. Focusing  
108 on maintenance crew scheduling, Gorman and Kanet (2010) formulate it as a job-shop scheduling  
109 problem. In a more recent paper, Peng and Ouyang (2014) present a special approach for modeling  
110 the railway maintenance planning problem. It is modeled as a vehicle routing problem, where the  
111 maintenance crews are considered as “vehicles”, and there is a set of projects with several jobs in  
112 each project being regarded as “routes”. A vehicle, i.e. maintenance crew, needs to travel to several  
113 routes, i.e. jobs in the projects, in such a way that all the routes are covered and the total traveling  
114 cost is minimized. A drawback of the models in this category is that the number of maintenance  
115 tasks, crews, and the available time slots in the allocation problem are fixed. Thus, these models  
116 are difficult to be extended to a long-term planning problem where a maintenance task can be  
117 performed at any time in the planning horizon.

118 In the last category, the aim of railway maintenance scheduling is to find the time periods to  
119 perform maintenance activities for components with different maintenance intervals. In general,  
120 maintenance requires several set-up activities, and it is more economical to maintain several  
121 components at the same time to reduce the number of possessions and spend the set-up cost  
122 only once. The problem of maintenance planning for rail-track components belongs to a general  
123 maintenance class of multi-component systems with economic dependence (Dekker et al. 1997;  
124 Nicolai and Dekker 2008; Dao et al. 2014). Budai et al. (2006) formulate a preventive maintenance  
125 scheduling problem for repetitive routine works and renewal projects. A mathematical programming  
126 model is presented to minimize the total maintenance and possession costs in a finite planning  
127 horizon. Pouryousef et al. (2010) extend Budai’s model by considering the planning of multiple  
128 segments at the same time. Zhao et al. (2009) and Pargar et al. (2017) study the maintenance  
129 scheduling problem and consider multiple types of cost savings due to joint maintenance and  
130 renewal activities at multiple segments. The types of saving depend on the number of adjacent  
131 segments and the share of special machineries for maintenance. Focusing on the renewal of  
132 track components, Caetano and Teixeira (2013) optimize the components’ life-cycle cost and the

133 unavailability of track when it is available for maintenance.

134 In this paper, we study the last category of railway maintenance scheduling problem and address  
135 a situation where the possession time is limited. To our best knowledge, no paper in this category  
136 considers the limitation of possession time and its effects on the maintenance scheduling problem.  
137 By investigating the limitation of possession time, this paper can provide a solution for infrastructure  
138 managers of busy railway networks, where there is an increasing demand of using the track for train-  
139 path operation and where a shrinking time window is available for infrastructure maintenance. In  
140 addition, the existing maintenance planning models assume that a fixed possession cost is incurred  
141 when at least one activity is performed in a time period regardless of the number of activities to be  
142 clustered in that period. In fact, the possession cost varies depending on the possession time as well  
143 as the “social-economic” impacts, i.e. the expected number of customers and the cost per customer  
144 per hour, of the track location. Besides, when different maintenance activities are clustered in the  
145 same time period, it is often seen that a component is maintained or renewed in a period that is  
146 earlier than its recommended time. In this case, the service-life of the component is shortened  
147 compared to the service-life when its recommended maintenance interval is used. Thus, we also  
148 consider a service-life shortening cost due to early maintenance of components.

149 The maintenance scheduling problem is considered in a finite planning horizon using a rolling-  
150 horizon approach (Wildeman et al. 1997). In the rolling-horizon approach, the decision is made  
151 at the horizon starting time. At the end of the current horizon, the same planning horizon can be  
152 repeated, or if there is an information update, the scheduling model generates a new horizon using  
153 updated input, and so on. This approach is practical when a fixed-term planning is required and  
154 the information may be updated, or when the necessity of another planning term needs to be taken  
155 into consideration.

## 156 **PROBLEM FORMULATIONS**

157 In this section, we first provide a general problem statement of the railway maintenance schedul-  
158 ing problem under limited possession time. Then, we formulate four cost factors including main-  
159 tenance and renewal cost, fixed possession cost, social-economic cost, and service-life shortening

160 cost. Several constraints that a maintenance scheduler needs to consider are also discussed.

### 161 **Problem statement**

162 In the railway maintenance scheduling problem, we have to schedule PM and renewal activities  
163 for a track system consisting of  $N$  components in a discrete planning horizon  $T$  with a time index  
164  $t = 1, 2, \dots, T$ . Each component  $i, i = 1, 2, \dots, N$ , in the system has its recommended PM interval,  
165 i.e. the maximum number of time periods between two consecutive PMs, denoted by  $\tau_{p,i}$ . A  
166 renewal is required on the component after a maximum number of PMs, denoted by  $N_{p,i}$ , have  
167 been done. In the railway maintenance scheduling problem, both  $\tau_{p,i}$  and  $N_{p,i}$  are assumed to be  
168 known. Other input data include the time and cost of each maintenance activity, the cost of shifting  
169 activities to an earlier period (service-life shortening cost), the number of time periods elapsed  
170 from the last PM, the number of PM activities from the last renewal, and the available possession  
171 time. We need to determine the time periods to perform activities so that the total cost incurred  
172 in the planning horizon is minimized. A detailed summary of input and output of the railway  
173 maintenance scheduling problem in this paper is presented in Table 1.

174 *Table 1. Input and output of the railway maintenance scheduling problem*

175 The problem is formulated as a binary IP model. The main reason that binary IP is selected  
176 for modeling the problem is that it is appropriate for the type of railway maintenance scheduling  
177 problem studied in the paper. In this study, the objective is to find the time period to perform a  
178 preventive maintenance or renewal activity. By defining binary decision variables based on whether  
179 to do an activity at each time period or not, a solution that contains binary values for all periods  
180 in the planning horizon can be easily transferred to a maintenance schedule as required for the  
181 problem. The formulation of the railway maintenance scheduling problem is focused mainly on  
182 different cost elements and constraints in the planning horizon which are further explained in the  
183 remaining parts of this section.

## Maintenance and renewal cost and time

In the planning horizon  $T$ , two binary variables are defined as follows:

$$x_{i,t}^m = \begin{cases} 1 & \text{if component } i \text{ is maintained in period } t \\ 0 & \text{otherwise} \end{cases}, \quad \forall i = 1, 2, \dots, N \quad (1)$$

$$x_{i,t}^r = \begin{cases} 1 & \text{if component } i \text{ is renewed in period } t \\ 0 & \text{otherwise} \end{cases}, \quad \forall i = 1, 2, \dots, N \quad (2)$$

It is seen that the maintenance schedule for the track system with  $N$  components can be realized if the set  $\{x_{i,t}^m, x_{i,t}^r\}$  is explicitly determined with  $\forall i = 1, 2, \dots, N$ . In a time index  $t$ , a component cannot be maintained or renewed more than one time. Let  $c_{i,t}^m$  and  $c_{i,t}^r$  be the maintenance and renewal costs of component  $i$  in period  $t$  respectively. The total costs of maintenance and renewal for  $N$  components in  $T$  time periods are simply the summation of all individual component maintenance costs in each period. They are shown in (Eqs. 3 and 4).

$$C_M = \sum_{i=1}^N \sum_{t=1}^T c_{i,t}^m \times x_{i,t}^m \quad (3)$$

$$C_R = \sum_{i=1}^N \sum_{t=1}^T c_{i,t}^r \times x_{i,t}^r \quad (4)$$

Similarly, denote  $t_{i,t}^m$  and  $t_{i,t}^r$  as the maintenance and renewal times of component  $i$  in period  $t$ . It is assumed that the maintenance and renewal activities are performed sequentially in a possession. Then, the possession time for maintenance of all components in period  $t$  is calculated as in (Eq. 5).

$$T_t = \sum_{i=1}^N (t_{i,t}^m \times x_{i,t}^m + t_{i,t}^r \times x_{i,t}^r) \quad (5)$$

## Possession and social-economic costs

In period  $t$ , if a maintenance or a renewal is performed on at least one component, a track possession is needed to ensure that the track is available for maintenance and no trains can enter

205 the maintenance area. We define the following binary variable.

$$206 \quad x_t^p = \begin{cases} 1 & \text{if there is a possession in period } t \\ 0 & \text{otherwise} \end{cases}, \forall i = 1, 2, \dots, N \quad (6)$$

207 We have:

$$208 \quad x_t^p = \max_{\forall i} \{x_{i,t}^m, x_{i,t}^r\} \quad \text{or} \quad \{x_t^p \geq x_{i,t}^m \text{ and } x_t^p \geq x_{i,t}^r\}, \forall i = 1, 2, \dots, N \quad (7)$$

209 In a period, if the track is needed for maintenance, i.e. at least one maintenance or renewal  
 210 activity is performed, a fixed possession cost,  $c_p^0$ , is incurred (in this notation,  $p$  stands for posses-  
 211 sion).  $c_p^0$  represents the cost of having a track section for maintenance, which is thus not available  
 212 for train service. This concept is similar to the definition of possession cost in (Budai et al. 2006),  
 213 and it includes the cost of isolating the track section such as possession booking, re-timetabling,  
 214 and all relevant set-up costs that are needed before the maintenance crew can work on the track. In  
 215 addition, there is a social-economic impact that depends on the duration of possession, which can be  
 216 modeled as a social-economic cost, i.e. a variable cost in the possession. Let the expected number  
 217 of customers in period  $t$  and the cost per customer per unit time be ( $N_t^C$ ) and ( $c_p^e$ ) respectively.  
 218 The cost per customer per unit time is independent from component maintenance cost and it is a  
 219 new concept in this paper. This cost takes into account the effect of the train service disruption on  
 220 customers, i.e. they have to re-route and need extra hours to travel; it may also include "indirect"  
 221 costs such as the reputation lost, decreased customer satisfaction, and losses of future customers  
 222 due to disruptions. The total fixed possession cost -  $C_p^f$  and social-economic cost -  $C_p^e$  in the entire  
 223 planning horizon are calculated as in Eqs. (8) and (9) respectively.

$$224 \quad C_p^f = \sum_{t=1}^T c_p^0 \times x_t^p \quad (8)$$

$$225 \quad C_p^e = \sum_{t=1}^T c_p^e \times N_t^C \times \sum_{i=1}^N T_i = \sum_{t=1}^T c_p^e \times N_t^C \times \sum_{i=1}^N (t_{i,t}^m \times x_{i,t}^m + t_{i,t}^r \times x_{i,t}^r) \quad (9)$$

227 It is also noted that not all costs are easy to gather. However, a great deal of money is involved  
 228 in rail infrastructure maintenance (Lidén 2015), and it is worthwhile to spend resources and acquire  
 229 the cost data in practice. Regarding the possession cost, it can be obtained by general accounting  
 230 data sharing between railway infrastructure manager, traffic planning, and railway train operators.  
 231 On the other hand, the cost per customer per unit time greatly depends on how the railway companies  
 232 value their customers and the infrastructure manager has to discuss this with railway train services  
 233 in order to quantify this value.

### 234 **Service-life shortening cost**

235 Each component  $i$  in the system has its recommended PM interval, i.e. the maximum number  
 236 of time periods between two consecutive PMs,  $\tau_{p,i}$ , and the maximum number of PM activities  
 237 before its renewal,  $N_{p,i}$ . If the component is considered individually, its PM and renewal activities  
 238 should be performed exactly in the due time period. However, when components are considered  
 239 simultaneously, it is more economical to cluster and jointly maintain several components in the  
 240 same time period. As a result, PM and renewal activities of a component may be performed earlier  
 241 than its individual due time, and this will reduce the service-life of the component compared to its  
 242 service-life in the individual schedule.

243 In this paper, we define an original schedule as the schedule obtained by directly using the  
 244 individual PM intervals and the maximum number of PM activities to schedule maintenance and  
 245 renewal of each component. Since  $\tau_{p,i}$  and  $N_{p,i}$  are given, the set of  $\{x_{i,t}^{m0}, x_{i,t}^{r0}\}$ , which corresponds  
 246 to the decision to do PM and renewal activities in the original schedule, can be explicitly determined  
 247 for all  $i = 1, 2, \dots, N$  and  $t = 1, 2, \dots, T$ .

248 Denote  $c_i^s$  as the service-life shortening cost per time period of component  $i$ , i.e. a cost of  $c_i^s$   
 249 is realized if a maintenance activity is performed a period earlier than the recommended period.  
 250 This cost takes into account of utilizing the track until its end of service life as much as possible  
 251 and preventing too early maintenance and renewal of rail-track components. Considering all  
 252 components in the system, the total service-life shortening cost of a current schedule with  $\{x_{i,t}^m, x_{i,t}^r\}$

253 is presented in Eq. (10).

$$254 \quad C_S = \sum_{i=1}^N c_i^s \left\{ \tau_{p,i} \sum_{t=1}^T (x_{i,t}^m - x_{i,t}^{m0}) + \sum_{t=T-\tau_{p,i}+1}^T (T-t)(x_{i,t}^m - x_{i,t}^{m0}) + \sum_{t=T-\tau_{p,i}+1}^T (T-t)(x_{i,t}^r - x_{i,t}^{r0}) \right\} \quad (10)$$

255 Eq. (10) implies that the total service-life shortening cost of each component is equal to a product  
 256 of the service-life shortening cost per time period and the total number of periods shortened due to  
 257 the maintenance and renewal of that component. The first term in the curly bracket represents the  
 258 number of periods shortened if the number of PM actions on component  $i$  in the current schedule  
 259 is greater than that in the original schedule. This newly introduced term is critical in reducing the  
 260 possibilities of having additional PM activities when the planning horizon is shorter than the asset's  
 261 life-cycle. The second and third terms take into consideration the number of periods in the last PM  
 262 cycle, i.e. from  $T - \tau_{p,i} + 1$  to  $T$ , and offset the difference between the times to do the last activity  
 263 in the two schedules. These two terms can play a similar role as in Budai et al. (Budai et al., 2006)  
 264 to value the last PM interval and to schedule the last PM and renewal activity as late as possible.

### 265 **Total cost in the planning horizon**

266 In summary, the total cost in the whole planning horizon, i.e. the objective function to be  
 267 minimized, is presented as in Eq. (11).

$$268 \quad C = C_M + C_R + C_P^f + C_P^e + C_S \quad (11)$$

### 269 **Sets of constraints in the railway maintenance scheduling problem**

270 The following constraints are considered in this paper.

- 271 • Latest possible time to do preventive maintenance activities: This set of constraints ensures  
 272 that the PM activities are performed within the specified PM interval.
- 273 • Latest possible time to do renewal activities: This set of constraints ensures that the a  
 274 renewal activity is performed once the number of PMs reaches its limit.
- 275 • Maximum available time in each possession: This set of constraints guarantees that the

276 activities are performed within the available time of blocking a railway section for mainte-  
 277 nance, and

- 278 • Variables constraints.

279 In the first and second sets of constraints, the first preventive maintenance and renewal activities  
 280 need to be performed earlier than the regular interval if the asset is not new at the start of the  
 281 planning horizon, i.e. time  $t = 0$ . Let  $\tau_{p,i}^0$  and  $N_{p,i}^0$  be the number of periods and number of PM  
 282 activities elapsed from the last PM/renewal of component  $i$ . The first maximum interval of doing  
 283 PM and renewal for component  $i$  is presented as in Eqs. (12) and (13) respectively.

$$284 \tau_{p,i}^1 = \tau_{p,i} - \tau_{p,i}^0 \quad (12)$$

$$285 \tau_{r,i}^1 = \tau_{p,i} \times (N_{p,i} - N_{p,i}^0) - \tau_{p,i}^0 \quad (13)$$

287 The available possession time constraint is critical for a busy railway system where maintenance  
 288 planners have to coordinate with train operators to reserve the track for maintenance. Since the  
 289 demand for operation is very high, too long possessions are not desirable, and the train operation  
 290 usually limits the time for possession on the railway track, for example one day a week or two days  
 291 a month.

292 The mathematical formulations of each constraint is presented in the following section.

### 293 **The railway maintenance scheduling optimization model**

294 The railway maintenance scheduling problem under a limitation of available possession time is  
 295 formulated as a IP model. Details are as follows:

$$296 \begin{aligned} \text{Min } C = & \sum_{i=1}^N \sum_{t=1}^T c_{i,t}^m \times x_{i,t}^m + \sum_{i=1}^N \sum_{t=1}^T c_{i,t}^r \times x_{i,t}^r + \sum_{t=1}^T c_P^0 \times x_t^p + \sum_{t=1}^T c_P^e \times N_t^C \times \sum_{i=1}^N (t_{i,t}^m \times x_{i,t}^m + t_{i,t}^r \times x_{i,t}^r) \\ & + \sum_{i=1}^N c_i^s \left\{ \tau_{p,i} \sum_{t=1}^T (x_{i,t}^m - x_{i,t}^{m0}) + \sum_{t=T-\tau_{p,i}+1}^T (T-t)(x_{i,t}^m - x_{i,t}^{m0}) + \sum_{t=T-\tau_{p,i}+1}^T (T-t)(x_{i,t}^r - x_{i,t}^{r0}) \right\} \end{aligned} \quad (14)$$

297 Subject to:

$$298 \sum_{t=1}^{\tau_{p,i}^1} x_{i,t}^m \geq 1, \forall i = 1, 2, \dots, N \quad (15)$$

$$299 \sum_{t=1+k}^{\tau_{p,i}+k} x_{i,t}^m \geq 1, \forall i = 1, 2, \dots, N; k = 1, 2, \dots, T - \tau_{p,i} \quad (16)$$

$$301 \sum_{t=1}^{\tau_{r,i}^1} x_{i,t}^r \geq 1, \forall i = 1, 2, \dots, N \quad (17)$$

$$302 \sum_{t=1+k}^{\tau_{r,i}+k} x_{i,t}^r \geq 1, \forall i = 1, 2, \dots, N; k = 1, 2, \dots, T - \tau_{r,i}; \tau_{r,i} = \tau_{p,i} \times N_{p,i} \quad (18)$$

$$303 \sum_{i=1}^N (t_{i,t}^m \times x_{i,t}^m + t_{i,t}^r \times x_{i,t}^r) \leq T_t^0, \forall t = 1, 2, \dots, T \quad (19)$$

$$304 x_t^p \geq x_{i,t}^m, \forall i = 1, 2, \dots, N; t = 1, 2, \dots, T \quad (20)$$

$$305 x_t^p \geq x_{i,t}^r, \forall i = 1, 2, \dots, N; t = 1, 2, \dots, T \quad (21)$$

$$306 x_t^p, x_{i,t}^m, x_{i,t}^r \in \{0, 1\}, \forall i = 1, 2, \dots, N; t = 1, 2, \dots, T \quad (22)$$

313 The objective of the railway maintenance scheduling problem is to minimize the total cost,  $C$ , that  
 314 is incurred in the planning horizon. Each element in the objective function has been previously  
 315 described in this section.

316 Constraints (15) and (16) ensure that the PM activities are performed within the allowed interval,  
 317  $\tau_{p,i}$ . Constraints (17) and (18) ensure that the renewal activities are performed on component  $i$  when  
 318 the number of PM activities reach  $N_{p,i}$ . In Constraints (16) and (18), index  $k$  starts from 1 instead of  
 319 0 because when  $k = 0$  these constraints are very similar to (15) and (17), except the upper limit of  
 320 the sum on the left-hand side. Following the reasoning from (12), the upper limit in (15) is smaller  
 321 than that in (16). Thus, Constraint (16) is automatically satisfied for  $k = 0$ . In another words,  
 322 Constraint (15) already covers (16) for the case  $k = 0$  and thus the index in (16) starts from 1. It is  
 323 noted that Constraints (15) and (17) have taken the current ages of components into consideration

324 for the first maintenance and renewal. Constraint (19) presents the effect of possession time, which  
 325 guarantees that the total maintenance times of all components in a possession must be within the  
 326 available possession time in each period,  $T_i^0$ . The available possession time,  $T_i^0$ , is given and the  
 327 evaluation of possession time in each period is shown in Eq.(5). Constraints (20) and (21) ensure  
 328 that a possession is required whenever a maintenance or renewal activity takes place which is  
 329 equivalent to Eq. (7). Constraint (22) sets the binary conditions of decision variables.

### 330 ILLUSTRATIVE EXAMPLE

331 In this section, we present an example for a railway track system consisting of  $N = 5$  components  
 332 to illustrate the maintenance scheduling problem in a finite planning horizon with  $T = 12$  time  
 333 periods. The input data on PM intervals, the maximum number of PMs in each life-cycle, the cost  
 334 and time to do a PM/renewal, the unit service-life shortening cost, and the time elapsed from the last  
 335 maintenance are given in Table 2. Other input data on the fixed possession cost, the social-economic  
 336 cost, and the available possession time are provided in Table 3.

337 *Table 2. Input data of components*

338 *Table 3. Possession cost and available possession time*

339 In Table 2, the number of periods elapsed from the last PM/renewal activities is the duration  
 340 from the last PM/Renewal to the starting time of the planning horizon. It is defined to accommodate  
 341 the fact that components are generally in use for certain periods of time at the start of the planning  
 342 horizon, or some maintenance activities have been performed in the past.

343 Generally, the service-life shortening cost per unit of time (the sixth column in Table 2 -  $c_i^s$ ),  
 344 depends on the component's renewal and maintenance costs as well as its recommended service-  
 345 life. The later term can be determined based on component's PM interval,  $\tau_i^p$  and the maximum  
 346 number of PMs in a renewal cycle  $N_{p,i}$ . We present the following equation to calculate  $c_i^s$ .

$$347 \quad c_i^s = \frac{c_i^r + N_{p,i}c_i^m}{\tau_{p,i}(N_{p,i} + 1)} \quad (23)$$

348 The nominator of the right hand side in Equation ( 23) is the total maintenance and renewal

349 costs between two consecutive renewals of component  $i$ , and the denominator is the component's  
350 recommended service-life. The above formula implies that the service-life shortening cost of a  
351 component is an average of maintenance and renewal costs per period of use in its renewal cycle.

## 352 **RESULTS AND DISCUSSIONS**

353 The results of maintenance scheduling models with and without the limitation on available  
354 possession time are investigated in terms of maintenance activities clustering, and the analysis is  
355 based on the cost and time breakdown as in the objective function ( 14) and constraint ( 19). In  
356 addition, the sensitivity analysis of the total cost and number of possessions over the available  
357 possession time is also performed.

358 In this paper, the binary linear IP model is solved using IBM CPLEX optimizer. The reason of  
359 using CPLEX is that it is able to find a true optimal solution for the linear IP model in this paper.  
360 CPLEX is a commercial software, but available for researchers to use, and it has been successfully  
361 applied to solve several similar linear programming models in the railway maintenance context  
362 (Budai et al. 2006; Vale et al. 2012; Vale and Ribeiro 2014; Vansteenwegen et al. 2016).

### 363 **Maintenance schedules with and without limitation of possession time**

364 First, we solve the railway maintenance scheduling problem and analyze two maintenance  
365 schedules: A – when there is no limitation on the available possession time, i.e. without the  
366 limitation on possession time, and B – when there is a possession time limitation of  $T^0 = 24$  hours  
367 (1 day). Schedule B can be obtained with the given optimization model in the previous section. The  
368 optimization model to obtain Schedule A is obtained by removing constraint (19) out of the model.  
369 These two schedules are compared with the original schedule which is generated by assigning the  
370 latest possible time to do a maintenance/renewal activity as the planned time to do it. The three  
371 schedules are presented in Figs 1, 2, and 3 respectively.

372 *Fig. 1. The original maintenance schedule*

373 *Fig. 2. Maintenance schedule A - without limitation of possession time*

374 *Fig. 3. Maintenance schedule B - with 24 hrs limitation of possession time*

375 From Fig. 1, we can see that maintenance and renewal activities in the original schedule are  
376 spread out in the planning horizon. This is because the components have different preventive  
377 maintenance and renewal intervals. There are in total ten possessions, and activities are rarely  
378 performed together in the same period. On the other hand, many maintenance and renewal activities  
379 are clustered in Schedule A, i.e. when there is no limitation on the possession time (Fig. 2). There  
380 are in total five possessions in the planning horizon in periods 1, 3, 6, 7, and 11. Noticeably, three  
381 maintenance activities of components 1, 3, and 4 are clustered in period 3, and the renewal activity  
382 of component 4 is also clustered with two other maintenance activities of components 1 and 5 in  
383 period 11. When considering the limitation of available possession time, maintenance and renewal  
384 activities in Schedule B are also combined but not as much as in Schedule A. The four maintenance  
385 activities of components 1, 2, 4, and 5 cannot be clustered as in Schedule A since it takes 28 hours,  
386 i.e. more than  $T^0 = 24$  hours to perform these activities in the same time period. Also, each  
387 renewal of the two components 1 and 4 can only be combined with one more maintenance activity.  
388 There are, in total, six possessions with either two maintenance activities or a maintenance and a  
389 renewal activity to be clustered in Schedule B.

390 The clustering of maintenance activities in a schedule can be measured by two factors: the  
391 average number of activities in a possession and the total number of possessions in the schedule.  
392 The clustering of activities is high when the former factor is big and the later one is small. These  
393 factors are calculated and tabulated in a Table 4. We compare Schedule B with the original  
394 schedule and observe that the average number of activities in a possession clearly increases, by 1.7  
395 times, and the number of possessions reduces from 10 (in the original schedule) to 6 possessions  
396 (in Schedule B).

397 *Table 4. Clustering of different maintenance schedules*

### 398 **Cost and time analysis**

399 A cost analysis is performed to compare three different schedules: the original schedule,  
400 Schedule A (without limitation of possession time), and Schedule B (with limitation of possession  
401 time) to identify the similarities and differences. Four cost elements in the objective function,

402 i.e. maintenance, renewal, possession, and service-life shortening costs, are calculated for each  
403 schedule and presented in Table 5 and Fig. 4.

404 *Table 5. Cost of alternative maintenance schedules*

405 *Fig. 4. Possession cost vs. service-life shortening cost*

406 From Table 5, we see that the original schedule has a significantly higher total cost in comparison  
407 with the other two schedules. Schedule A gives the least total cost of 84.05 cost units, while the  
408 total cost of Schedule B is higher at 86.48 cost units. The maintenance and renewal costs in the  
409 three schedules are similar since the number of maintenance and renewal activities are identical.  
410 The major differences are in the possession and service-life shortening costs. The possession  
411 cost significantly reduces when considering the clustering of maintenance activities. The original  
412 schedule requires the highest possession cost of 29.1 cost units. Schedules A and B have lower  
413 possession cost with approximately 34.5% and 27.5% less than the original schedule, respectively.  
414 This result can be explained by the reduced number of possessions in Schedules A (5 possessions)  
415 and B (6 possessions). Regarding the service-life shortening cost, Schedule B, with a limitation  
416 of possession time, results in slightly higher cost compared to Schedule A, which implies that the  
417 activities in Schedule A are shifted to earlier periods more often in the planning horizon.

418 In addition, it is noticed that a too long possession may not be possible due to the limitation  
419 of the available possession time. The possession times in the three schedules are calculated and  
420 compared to the available possession time as shown in Table 6 and Fig. 5.

421 *Table 6. Possession time comparisons*

422 *Fig. 5. Unused and overused possession time*

423 Although Schedule A requires the least number of possessions, it ignores the limitation of  
424 available possession time and is the only schedule that violates the limitation of possession time.  
425 Two out of five possessions in Schedule A exceed the time limitation,  $T^0$ , with total excessive  
426 time of 7 hours. Thus, Schedule A may not be applicable, or there will be a huge penalty when  
427 implementing it. Schedule A also has a short possession of 3 hours which is not desirable. The  
428 original schedule requires many possessions with short durations, e.g. 3 or 6 hours, which indicate

429 that there are opportunities for clustering and reducing the number of possessions. Meanwhile, the  
430 possession time in Schedule B is more equally distributed compared to the other two schedules.

431 The number of possessions and possession times results in a great disparity in average unused  
432 and overused possession times of the three schedules. On average, the original schedule does not  
433 use 14.9 hours out of 24 hours (62%), which indicates that this schedule has a very low maintenance  
434 time utilization, and to improve this schedule, activities should be clustered. Schedule A (without  
435 limitation of possession time) shows less unused possession time of 7.2 hours (30%). However, it  
436 is the only schedule that overuses the time available for maintenance with 1.4 hours per possession.  
437 The schedule with a consideration of the limitation of possession time is the best schedule in terms  
438 of available time utilization with only 4.8 hours unused per possession and zero hours of overused  
439 available possession time.

#### 440 **Sensitivity analysis of the available possession time**

441 We further investigate the effects of the available possession time –  $T^0$  on the maintenance  
442 scheduling problem. The two distinct factors characterizing a maintenance schedule, which are  
443 the total cost and the number of required possessions, are calculated when the available possession  
444 changes. In this experiment,  $T^0$  is set to vary from 18 to 28 hours, and the results are presented  
445 in Fig. 6. Here, it is not possible to get a feasible schedule when the available possession time is  
446 less than 18 hours; when the available possession time is greater than 28 hours, the solution does  
447 not change compared to the one with  $T^0$  of 28 hours. Generally, the corresponding total cost and  
448 number of possessions are negatively proportional with the available time for maintenance.

#### 449 *Fig. 6. Sensitivity analysis of the available possession time*

450 From Fig. 6, it can be seen that both the total cost and the number of possessions tend to decrease  
451 as the available possession time,  $T^0$ , increases. However, the total cost gradually decreases while  
452 the number of possessions sometimes remains constant when  $T^0$  increases, e.g. between  $T^0 = 20$   
453 and  $T^0 = 22$ ; between  $T^0 = 24$  and  $T^0 = 26$ . The reduction of the total cost indicates that a different  
454 maintenance schedule is obtained when the available possession time varies. The results of this  
455 analysis can help the maintenance decision makers in two ways:

- 456 • Select the best maintenance schedule with a specific available possession time: This is highly  
457 applicable for busy railway sections with limited flexibility on the available possession time,  
458 and
- 459 • Decide whether to modify the threshold on the available possession time: this is more  
460 practical for the case that there is some flexibility of available possession time. For example,  
461 it may be of interest to decrease the available possession time for the studied section from  
462 24 hrs to 22 hrs but not from 22 hrs to 20 hrs because the additional expense in the first  
463 change (0.19 cost units) is notably less than that of the second change (1.53 cost units).

464 In this paper, due to the sensitivity of cost data, the cost data in our example are presented in  
465 terms of cost units, and we have tried to select the relative cost as close to practical data of rail  
466 track components (rails, ballast, switch, etc.) as possible. However, it is also noted that the cost  
467 for each type of component can vary depending on several factors, e.g. the sleeper's material, the  
468 switch's size and type of turn-out. The proposed model in this paper is a conceptual model that can  
469 be flexibly applied for different rail-track systems.

## 470 **CONCLUSIONS**

471 A maintenance scheduling model for several rail infrastructure components considering a  
472 limitation of possession time is presented in this paper. This model is practical for busy railway  
473 networks associated with a high demand of train operation. Several costs factors related to railway  
474 maintenance and operation have been formulated and optimized. We have analyzed the effects of  
475 the available possession time on the maintenance decisions and the total cost in a finite planning  
476 horizon, and our main finding is that the limitation of available possession time can alter the  
477 resulting maintenance schedule and the total cost. Furthermore, it may be worthwhile to change  
478 the limitation of possession time for the studied rail-track section. However, it is also noted that the  
479 detailed course of actions vary depending on the actual input costs as well as the type of railway  
480 section under investigation.

481 The proposed new maintenance scheduling model in this paper is for different components in a

482 single track link. Based on the model, several research directions can be identified as follows.

- 483 • Study the maintenance scheduling problem in a railway network context: when considering  
484 multiple components in a railway network, it is necessary to model the relationships between  
485 different track links to guarantee a smooth operation of the network. For example, when a  
486 track link is possessed for maintenance, it is not possible to possess another link since the  
487 block of both tracks will cause severe disruptions to the network operation.
- 488 • Study the methods to obtain the solution: The linear integer optimization model is currently  
489 solved by CPLEX, a commercial software for solving linear optimization problems. With  
490 this, the computational time increases exponentially when the number of components and  
491 the number of time periods increase. Heuristics and evolutionary computing methods are  
492 suggested for large-size problems and are recommended for future study.

**APPENDIX I. NOTATION**

*The following symbols are used in this paper:*

$C$  = Total cost;

$c_{i,t}^m$  = Cost of maintaining component  $i$  in period  $t$ ;

$c_{i,t}^r$  = Cost of renewing component  $i$  in period  $t$ ;

$c_i^s$  = Service-life shortening cost per time period of component  $i$  ;

$c_P^0$  = Fixed set-up cost occurred once per possession;

$c_P^e$  = Social-economic cost per customer per unit time;

$C_M$  = Maintenance cost;

$C_P^e$  = Variable social-economic cost;

$C_P^f$  = Fixed possession cost;

$C_R$  = Renewal cost;

$C_S$  = Service-life shortening cost;

$i$  = Component index;

$N$  = Number of components;

$N_{p,i}$  = Number of PM activities between two renewals of component  $i$ ;

$N_{p,i}^0$  = Number of PM activities elapsed from the last renewal of component  $i$ ;

$N_t^C$  = Expected number of customers per unit time in period  $t$ ;

$t$  = Time index;

$t_{i,t}^m$  = Time of maintaining component  $i$  in period  $t$ ;

$t_{i,t}^r$  = Time of renewing component  $i$  in period  $t$ ;

$T$  = Maximum time in the planning horizon;

$T_t$  = Total maintenance and renewal time for all components in period  $t$ ;

$T_t^0$  = Available possession time in period  $t$ ;

$\tau_{p,i}$  = Maximum number of time periods between two PMs of component  $i$ ; and

$\tau_{p,i}^0$  = Number of time periods elapsed from the last PM of component  $i$ .

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**TABLE 1.** Input and output of the railway maintenance scheduling problem

| Input                                      | Output                                  |
|--|---|
| Component PM interval and elapsed periods  | Time of doing each maintenance activity |
| Number of PMs between two renewals         | Time of doing each renewal activity     |
| PM cost and time per component             | Total maintenance cost                  |
| Renewal cost and time per component        | Total renewal cost                      |
| Service-life shortening cost per component | Total service-life shortening cost      |
| Cost per possession                        | Total possession cost                   |
| Available possession time                  | Total number of possessions             |

**TABLE 2.** Input data of components

| Component ( $i$ ) | PM interval (periods) | Number of PMs ( $N_{p,i}$ ) | Main. cost (cost units) | Ren. cost (cost units) | Ser. short. cost | $t_i^m$ (hrs) | $t_i^r$ (hrs) | $\tau_{p,i}^0$ (periods) | $N_{p,i}^0$ |
|-------------------|-----------------------|-----------------------------|-------------------------|------------------------|------------------|---------------|---------------|--------------------------|-------------|
| 1                 | 4                     | 9                           | 2                       | 6                      | 0.6              | 9             | 18            | 1                        | 3           |
| 2                 | 6                     | 8                           | 6                       | 15                     | 1.17             | 6             | 16            | 4                        | 3           |
| 3                 | 9                     | 7                           | 5.5                     | 20                     | 0.81             | 8             | 23            | 5                        | 2           |
| 4                 | 8                     | 10                          | 4                       | 18                     | 0.66             | 10            | 16            | 4                        | 5           |
| 5                 | 5                     | 6                           | 4                       | 25                     | 1.4              | 3             | 12            | 4                        | 2           |

**TABLE 3.** Possession cost and available possession time

| Input parameter                    | Value |
|------------------------------------|-------|
| Fixed possession cost (cost units) | 2     |
| Cost per customer (cost units)     | 0.001 |
| Number of customers                | 100   |
| Avail. possession time (hrs)       | 24    |

**TABLE 4.** Clustering of different maintenance schedules

| Schedule | Number of possessions | Ave. number of activities per possession |
|----------|-----------------------|--|
| Original | 10                    | 1.10                                     |
| A        | 5                     | 2.20                                     |
| B        | 6                     | 1.83                                     |

**TABLE 5.** Cost of alternative maintenance schedules (cost units)

| Schedule | Maintenance cost | Renewal cost | Possession cost | Service-life shortening cost | Total cost |
|----------|------------------|--------------|-----------------|------------------------------|------------|
| Original | 37.5             | 24           | 29.1            | -                            | 90.6       |
| A        | 37.5             | 24           | 19.1            | 3.45                         | 84.05      |
| B        | 37.5             | 24           | 21.1            | 3.88                         | 86.48      |

**TABLE 6.** Possession time comparisons (in hours)

| Schedule | Possession number |    |    |    |    |   |    |   |    |    | Excess $T^0$ ? |
|----------|-------------------|----|----|----|----|---|----|---|----|----|----------------|
|          | 1                 | 2  | 3  | 4  | 5  | 6 | 7  | 8 | 9  | 10 |                |
| Original | 3                 | 6  | 9  | 10 | 8  | 3 | 18 | 6 | 12 | 16 | No             |
| A        | 9                 | 27 | 3  | 24 | 28 | - | -  | - | -  | -  | <b>Yes</b>     |
| B        | 9                 | 19 | 11 | 24 | 19 | 9 | -  | - | -  | -  | No             |

"-": not applicable

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