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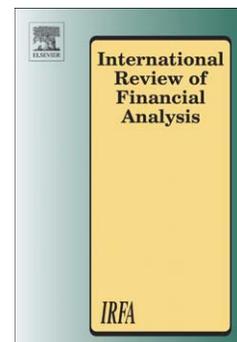
Negative bubbles and shocks in cryptocurrency markets

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**Negative bubbles and shocks in cryptocurrency markets****John Fry<sup>1</sup> and Eng-Tuck Cheah<sup>2</sup>****January 2016****Abstract**

In this paper we draw upon the close relationship between statistical physics and mathematical finance to develop a suite of models for financial bubbles and crashes. The derived models allow for a probabilistic and statistical formulation of econophysics models closely linked to mainstream financial models. Applications include monitoring the stability of financial systems and the subsequent policy implications. We emphasise the timeliness of our contribution with an application to the two largest cryptocurrency markets: Bitcoin and Ripple. Results shed new light on emerging debates over the nature of cryptocurrency markets and competition between rival digital currencies.

**Keywords:** Bitcoin; Ripple; Cryptocurrencies; Bubbles; Negative bubbles; Econophysics

**JEL Classification:** C1 E4 G1

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# Negative bubbles and shocks in cryptocurrency markets

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## Abstract

In this paper we draw upon the close relationship between statistical physics and mathematical finance to develop a suite of models for financial bubbles and crashes. The derived models allow for a probabilistic and statistical formulation of econophysics models closely linked to mainstream financial models. Applications include monitoring the stability of financial systems and the subsequent policy implications. We emphasise the timeliness of our contribution with an application to the two largest cryptocurrency markets: Bitcoin and Ripple. Results shed new light on emerging debates over the nature of cryptocurrency markets and competition between rival digital currencies.

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## 1 Introduction

Econophysics is an interdisciplinary subject that applies tools and techniques from theoretical and statistical physics to model financial and economic systems (Chen and Li, 2012; Mantegna and Stanley, 1999). For an introduction to econophysics and a comparison between finance and physics see Sornette (2014). As the econophysics movement has gained momentum our paper thus contributes to wider debates such as probabilistic and statistical approaches to econophysics (Bree and Joseph, 2013; Lin et al., 2014), the development of links between econophysics and mainstream financial models (Johansen et al., 2000) and the creation of tools to monitor the stability of financial systems and the subsequent policy implications that work in econophysics holds (Sornette, 2003; Fry, 2015).

Econophysics has been used to tackle a wide range of practical problems in finance and economics. This includes applications to options pricing and risk management (Bouchaud and Potters, 2003), the statistical characterisation of heavy-tailed asset returns distributions (Cont, 2001), empirical power-laws (Gabaix et al., 2003; Plerou et al. 2004), agent-based modelling (see e.g. Hommes, 2006; Le Baron, 2006), income-tax evasion (Pickhardt and Seibold, 2014), speculative bubbles (Johansen et al., 2000; Sornette, 2003) and the impact of high-frequency trading upon the stability of global financial markets (Filimonov and Sornette, 2012). One practical problem that econophysics has recently begun to address is the issue of Bitcoin and cryptocurrency markets.

Amid huge media and public interest Bitcoin and cryptocurrency markets present enormous legal (Grinberg, 2012; Plasaras, 2013), regulatory (European Central Bank, 2012; Ali et al., 2014; Gandal and Halaburda, 2014) and ethical (Angel and McCabe, 2015) challenges. Cryptocurrency markets have also been extremely volatile with the market share and market capitalisations of several cryptocurrencies fluctuating wildly (White, 2014). Though initially dominated

by other disciplines the financial and economic literature on Bitcoin and cryptocurrencies has recently started to emerge (see e.g. Dowd, 2014; Dwyer, 2015; Weber, 2014a). Cryptocurrencies have also been the focus of several recent econophysics papers (see e.g. Kristoufek, 2013; Garcia et al., 2014; Cheah and Fry, 2015). Their dependence on self-fulfilling expectations and the lack of a centralised governance body mean that, without wishing to exaggerate the importance of econophysics, cryptocurrency markets may prove especially amenable to econophysics approaches.

The objective of this paper is to showcase the use of tools and techniques from econophysics via a novel application to the two largest cryptocurrency markets. The importance of our contribution is fivefold. Firstly, we develop an econophysics model for bubbles and crashes that can be fitted to empirical financial data using standard statistical techniques such as maximum likelihood estimation. Whilst applicable to general financial markets our model may thus have specific relevance to cryptocurrency markets. Secondly, we contribute to a fledgling academic literature on Bitcoin and cryptocurrency markets. Thirdly, we find empirical evidence of negative bubbles in cryptocurrency markets – complementing earlier documented evidence of speculative bubbles in the literature. Fourthly, we address the issue of competition between rival cryptocurrencies. This is significant as the issue of contagion and co-dependence is particularly pertinent for cryptocurrency markets. One notable example of this is that the market share of the two largest cryptocurrencies Bitcoin and Ripple has fluctuated quite dramatically in recent months. Here, we find evidence of a spillover from Ripple to Bitcoin. Fifthly, we develop a model to independently verify the impact of putative market shocks (identified by academics and practitioners alike) upon Bitcoin. Results suggest that cryptocurrency markets are inherently complex and are often misunderstood by academics and practitioners alike.

The layout of this paper is as follows. Section 2 gives a brief overview of econophysics and reviews the academic literature on Bitcoin and cryptocurrencies. Section 3 develops the basic bubble/antibubble model and its extension to higher dimensions. Section 4 discusses a model for unpredictable market shocks. The model is later used to track the effect of events such as the closure of the illegal Silk Road website and the collapse of the Tokyo-based Bitcoin exchange Mt. Gox upon Bitcoin prices. Empirical applications are discussed in Section 5. Section 6 concludes.

## 2 Literature Review

### 2.1 Econophysics and the analysis of financial crashes

Econophysics is perhaps best defined as the use of paradigms and tools from theoretical and statistical physics to model financial and economic systems (Mantegna and Stanley, 1999). The movement has a long history. For a historical overview see e.g. Jovanovic and Schinckus (2013) and Chen and Li (2012). The term econophysics was first coined in Stanley et al. (1996) and the modern arm of the movement can trace its origins to several key developments that occurred in the 1990s. These include technical developments in the mathematics of Lévy processes and the development of increased computer processing power together with the ready availability of large electronic financial databases. Allied to the above as time has progressed several economics and finance journals have also become more receptive to ideas from econophysics (Jovanovic and Schinckus, 2013).

Speculation and financial crises have been endemic throughout human history (Reinhart and Rogoff, 2009). Bubbles typically occur when the price of an asset grows rapidly and does so in a manner far removed from realistic assessments of the asset's intrinsic value (Phillips and Yu,

2011). The implication is that such a dramatic price rise sets up asset prices for a subsequent fall. Kindelberger and Aliber (2005) describe bubbles as a sharp rise in asset prices – with the initial rise generating expectations of further rises and attracting new buyers via a process commonly labelled irrational exuberance (Shiller, 2005). However, beyond these definitions, and real economic suffering, both the theoretical existence of bubbles, and issues related to their empirical detection, remain hotly debated (Gurkaynak, 2008; Vogel and Werner, 2015).

From a statistical physics perspective stock market crashes represent a rupture event in a complex system (Feigenbaum, 2003). This analogy with exactly soluble models in statistical mechanics led to the development of phenomenological log-periodic power-law models for bubbles where the price exhibits unsustainably high super-exponential growth (Feigenbaum and Freund 1996; Sornette et al., 1996). A rational expectations version of the original model was then formulated in Johansen et al. (2000) thereby forming a potential bridge with the mainstream economics and finance literature. Despite some initial controversy (see e.g. Feigenbaum 2001a-b) econophysics modelling of financial bubbles has continued to grow from strength to strength – in part as a response to the increasingly volatile nature of global financial markets.

Even before the 2008 crisis one dramatic unintended consequence of the increased computerisation of financial markets described above was increased speculation and volatility (Barber and Odean, 2001). This in turn has lent weight to the empirically oriented approach favoured by econophysics as financial markets evolved. Statistical physics, and its reincarnation into econophysics, is chiefly concerned with providing the best possible explanation of empirically observable phenomena. This contrasts sharply with the rigid theoretical frameworks often associated with both financial economics and mathematical finance (Jovanovic and Schinckus, 2013). The modelling of financial bubbles and crashes has thus emerged as a key part of the wider econophysics movement (Feigenbaum, 2003). This has gained added impetus since the 2008 crisis with several econophysics papers modelling bubbles and crashes appearing in well-respected finance and economics journals in recent years (see e.g. Kurz-Kim, 2012; Bree and Joseph, 2013; Lin and Sornette, 2013; Geraskin and Fantazzini, 2013; Lin et al., 2014).

A range of empirical findings reported in the literature suggest that econophysics can indeed generate useful insights into real markets. For example, econophysics can be used to shed new light into the monetary roots of bubbles and crashes (Corsi and Sornette, 2014). In line with this practically-minded approach the functioning of Bitcoin and other cryptocurrency markets represents an interesting problem in its own right and one that also presents new challenges to mainstream economics and finance (see Section 2.2).

## 2.2 The cryptocurrency market

The use of cryptocurrencies has gained traction in response to the perceived failures of government and central banks during the 2008 crash (Weber, 2014a). Bitcoin and other cryptocurrencies may also offer cheaper alternatives to existing debit and credit card systems (Angel and McCabe, 2015), in part reflecting recent technological innovations in regular monetary systems (European Central Bank, 2012; Böhme et al., 2015). Amid huge media and public interest the academic literature on Bitcoin is only recently starting to emerge (see e.g. Frisby, 2014; Vigna and Casey, 2015). Initially the debate surrounding the use of cryptocurrencies appeared to be dominated both by Bitcoin and by other disciplines such as computer science (see e.g. Sadeghi, 2013; Böhme et al, 2014) and law (see e.g. Grinberg, 2012; Plasaras, 2013). However, as discussed in this subsection, a fledgling economics and finance literature is starting to emerge. In particular, previous literature on Bitcoin raises several interesting questions such as the exact nature of Bitcoin, the long-term sustainability of Bitcoin, competition in the market for digital

currencies and an on-going ethical debate surrounding Bitcoin and cryptocurrencies. Our paper also contributes to these discussions and adds to a burgeoning econophysics literature on Bitcoin and cryptocurrencies.

The cryptocurrency market warrants close scrutiny given its increasingly high profile. The sums of money involved are substantial. The total cryptocurrency market capitalisation is estimated to be \$7.1 billion ([www.coinmarketcap.com](http://www.coinmarketcap.com), January 2016). Amongst over 600 traded cryptocurrencies Bitcoin and Ripple are the two most popular cryptocurrencies with around 91 percent and 2.8 percent of the entire cryptocurrency market capitalization respectively though as recently as May 2015 the market share of Ripple was as high as 6 percent. Ripple and Bitcoin share important characteristics such as their ultimate dependence upon the trust of their users and a unique network currency (Bitcoin for Bitcoin and XRP in Ripple respectively). However, there are also important differences. Ripple is primarily designed to serve as a medium of exchange and as a distributed payment system as opposed to an alternative currency per se. Ripple is thus arguably more like a new improved form of PayPal or Mastercard than a “digital dollar”. This ready convertibility means the Ripple network itself actually accepts Bitcoin and, inter alia, allows users to trade precious metals like gold and silver, cryptocurrencies, conventional currencies like the US Dollar and the GB Pound and even assets like air miles with lower transaction costs using its own native digital currency – the XRP. This added flexibility makes Ripple a strong potential rival to Bitcoin. Despite its obvious relevance and importance research into Ripple appears to be almost non-existent due to its lack of widespread usage prior to 2013.

Though it is a highly unconventional and fast-moving area it is important to note that there is a wealth of available statistical information about Bitcoin and other digital currencies that renders academic study of the area entirely possible. One key source of information (rather than an exchange site or place for investment advice is [www.coinmarketcap.com](http://www.coinmarketcap.com). Coinmarketcap.com has been a key data source used in previous academic studies (see e.g. Dowd, 2014; White, 2014; Cheah and Fry, 2015). Further, the site’s links with a diverse range of Bitcoin wikis and forums reinforces both its importance to practitioners and its wider usage. Coinmarketcap.com provides data about listed coins such as price, available supply, trade volume and market capitalisation (defined as the price multiplied by the available supply). Prices are calculated by averaging prices quoted at major exchanges weighted by volume. Every effort is made to provide timely information. Statistics are updated every five minutes and coins with stale datapoints (more than six hours old) are shown at the bottom of the listings accompanied by question marks and de-listed after seven days. Coinmarketcap.com also has a dedicated forum in [Bitcointalk.org](http://Bitcointalk.org) where suggestions for new listed currencies can be made.

However, in addition to providing timely information coins listed on [coinmarketcap.com](http://coinmarketcap.com) must also satisfy rigorous assessment criteria. Firstly, based on information such as the total number in circulation coins must be deemed to constitute a genuine cryptocurrency. Secondly, coins must be traded on a public exchange that is more than thirty days old and with an active Application Programming Interface (API) available. Essentially this means that all listed cryptocurrencies must be genuinely tradable. Thirdly, listed coins must satisfy a transparency requirement and have a public URL that displays the total supply (total coins used so far).

In the literature it remains unclear as to whether or not Bitcoin and cryptocurrencies should be seen as an alternative currency or as a speculative asset. Bitcoin’s own digital mining processes are intended to replicate the production costs associated with commodities like precious metals. Bitcoin’s convertibility and low transaction costs also share elements of currencies (Frisby, 2014). Bitcoin prices appear to be particularly susceptible to market sentiments and

dramatic boom-bust episodes (Weber, 2014a; Cheung et al., 2015; Cheah and Fry, 2015) undermining the role that Bitcoin might play as a store of value. The unpredictability and volatility in Bitcoin prices also requires that merchants accepting Bitcoin incorporate a spread over the price in the original currency. This undermines the role Bitcoin plays as a unit of account. Selgin (2015), Yermack (2013) and Baeck and Elbeck (2015) argue that Bitcoin should be seen as a speculative commodity rather than a currency. Yermack (2013) lists several features that could lead to Bitcoin failing as a currency such as cybersecurity risks, lack of a central governance structure, Bitcoin's comparatively small level of adoption, the diversity of Bitcoin prices across different exchanges and problems with the interpretability of Bitcoin prices due to the relatively high cost in Bitcoins for ordinary products and services.

Alongside these fears the long-term sustainability of Bitcoin has also come under scrutiny. Bitcoin was originally conceived as a decentralised network beyond the control of national governments (Weber, 2014a). However, some authors also see the lack of a centralised governance body as an essential weakness (Weber, 2014a). Others see money as inherently linked to the definition of the state (Van Alstyne, 2014; Dequech, 2013) – even if some elements of credit creation and money supply in modern economies lie beyond state control (Dequech, 2013). Ultimate limits upon the supply of Bitcoin have also raised fears about the potential for a debt-deflation spiral (Weber, 2014b; Böhme et al., 2015) in a similar manner to the problems that dogged gold-backed currencies in the last century due to the decoupling of credit creation and money supply. Allied to questions of long-term sustainability several studies also examine the issue of competition between Bitcoin and other alternative cryptocurrencies – sometimes labelled altcoins (Dowd, 2014; Gandal and Halaburda, 2014; Rogajanu and Badea, 2014). Concerns are raised in Dowd (2014) that its design flaws may ultimately make Bitcoin vulnerable to competing altcoins in the long-term. However, it remains unclear as to the extent to which the two largest cryptocurrencies Bitcoin and Ripple are in direct competition with each other (Coinsetter, 2013).

Bitcoin and cryptocurrencies also raise several important ethical issues. These include concerns that the anonymity endowed by cryptocurrencies may encourage illegal activities, cybersecurity worries and fears over the continued ability of governments to raise taxes. Inter alia a large legal literature (see e.g. Kaplanov, 2012; Dion, 2013; Bryans, 2014; Doguet, 2012; Varriale, 2013; Yang, 2013; Twomey, 2013) thus reflects both a variety of ethical concerns raised by Bitcoin and a diverse international experience (Van Alstyne, 2014; Pilkington, 2014). In view of these concerns it is interesting to examine both the nature of the booms and crashes that occur on cryptocurrency markets and the extent to which government and law enforcement measures affect these markets.

Our paper also contributes to a burgeoning econophysics literature – directly motivated by the empirical study of cryptocurrency markets. An overview of the application of complex systems theory applied to Bitcoin is given by Pilkington (2014). Using an econophysics model Cheah and Fry (2015) find evidence of a bubble in Bitcoin. Soumya et al. (2015) examine the stylised empirical facts of cryptocurrency markets. Kandor et al. (2014) and Ober et al. (2013) apply network theory to study the empirical properties of the Bitcoin transaction graph. Kristoufek (2013) uses a bi-directional relationship between internet searches (google and wikipedia) and Bitcoin prices to quantify the recent bubble. An expanded version of this study is contained in Garcia et al. (2014) who consider both information on new Bitcoin users and word-of-mouth information on Social Media (Twitter) in addition to information on Internet searches (google trends) to explain Bitcoin price changes. Allied to wider questions of Bitcoin's financial definition and long-term sustainability a number of studies have looked into more detailed economic aspects of Bitcoin and other cryptocurrency markets. Owens and Lavich

(2013) study the implications of cryptocurrencies for online gambling with the comment made in Dwyer (2015) that online gambling stimulates a large amount of activity in Bitcoin. Yelowitz and Wilson (2015) use Google Trends search data to examine the determinants of interest in Bitcoin and find that interest in Bitcoin seems to be primarily driven by a mixture of computer-programming enthusiasts and illegal activity. Econometric evidence of bubbles in cryptocurrency markets is found in Cheung et al. (2015) and Cheah and Fry (2015) – thus hinting at the potential significance of econophysics for cryptocurrency markets.

In Sections 3-4 we derive the econophysics models needed for the analysis of empirical data in Section 5.

### 3 Bubbles and negative bubbles

#### 3.1 Univariate bubbles and negative bubbles

Let  $P_t$  denote the price of an asset at time  $t$  and let  $X_t = \log P_t$ . Following Johansen et al. (2000) our starting point is the equation

$$P(t) = P_1(t)(1 - \kappa)^{j(t)}, \quad (1)$$

where  $P_1(t)$  satisfies

$$dP_1(t) = [\mu(t) + \sigma^2(t)/2] P_1(t)dt + \sigma(t)P_1(t)dW_t, \quad (2)$$

where  $W_t$  is a Wiener process and  $j(t)$  is a jump process satisfying

$$j(t) = \begin{cases} 0 & \text{before the crash} \\ 1 & \text{after the crash.} \end{cases} \quad (3)$$

When a crash occurs  $\kappa\%$  is automatically wiped off the value of the asset. Prior to a crash  $P(t) = P_1(t)$  and it follows from Itô's formula that  $X_t = \log(P(t))$  satisfies

$$dX_t = \mu(t)dt + \sigma(t)dW_t - v dj(t), \quad (4)$$

where  $v = -\ln[1 - \kappa] > 0$ . Equation (4) shows us how the bubble will impact upon observed prices. Suppose that a crash has not occurred by time  $t$ . In this case we have that

$$E[j(t + \Delta) - j(t)] = \Delta h(t) + o(\Delta), \quad (5)$$

$$\text{Var}[j(t + \Delta) - j(t)] = \Delta h(t) + o(\Delta), \quad (6)$$

where  $h(t)$  is the hazard rate.

**Assumption 1 (Intrinsic Rate of Return)** *The intrinsic rate of return is assumed constant and equal to  $\mu$ :*

$$E[X_{t+\Delta} - X_t | X_t] = \mu\Delta + o(\Delta). \quad (7)$$

*First-order condition.* From Assumption 1 equations (4-5) and (7) give

$$\mu(t) - vh(t) = \mu; \quad \mu(t) = \mu + vh(t). \quad (8)$$

Equation (8) shows the rate of return must increase in order to compensate a representative investor for the risk of a crash. However, it can be shown that bubbles also impact upon the volatility (see below).

**Assumption 2 (Intrinsic Level of Risk)** *The intrinsic level of risk is assumed constant and equal to  $\sigma^2$ :*

$$\text{Var}[X_{t+\Delta} - X_t | X_t] = \sigma^2 \Delta + o(\Delta). \quad (9)$$

*Second-order condition.* For a bubble to develop a rapid growth in prices alone is not enough. The perceived price risk must also diminish. Similarly, from Assumption 2 equations (4), (6) and (9) give

$$\sigma^2(t) + v^2 h(t) = \sigma^2; \quad \sigma^2(t) = \sigma^2 - v^2 h(t). \quad (10)$$

The model thus states that it is the interplay between risk and return that fundamentally governs the behaviour of financial markets. Assumptions 1-2 show that bubbles can be identified via anomalous behaviour in the drift and volatility in equation (4). During a bubble a representative investor is compensated for the crash risk by an increased rate of return with  $\mu(t) > \mu$  the long-term rate of return. This is accompanied by a decrease in the volatility function  $\sigma^2(t)$  – a result which though counter-intuitive actually represents market over-confidence (Fry, 2012; 2014). Specification of the hazard rate thus completes the model. Here, we follow Fry (2012, 2014) in using

$$h(t) = \frac{\beta t^{\beta-1}}{\alpha^\beta + t^\beta}. \quad (11)$$

Equations (8) and (10) above mean that we can test for the existence of a speculative bubble by testing the one-sided hypothesis

$$H_0 : v = 0, \quad H_1 : v > 0. \quad (12)$$

Formulating the model in this way allows us to account for the fact that financial and economic time series often exhibit approximately exponential behaviour over long time horizons (Campbell et al., 1997). Further, this approach also allows us to account for the fact that prices may undergo substantial periods of growth even in the absence of a bubble.

A negative bubble represents the mirror image of a speculative bubble (Yan et al., 2012). Just as speculative bubbles result in dramatic price rises negative bubbles result in a dramatic price falls. Negative bubbles can be modelled by replacing  $v$  with  $-v$  in the above (Yan et al., 2012). In particular, we can test for the presence of a negative bubble by testing the one-sided hypothesis

$$H_0 : v = 0, \quad H_1 : v < 0. \quad (13)$$

This elegant formulation thus builds on the previous definition of negative bubbles in the econophysics literature (Yan et al., 2012). This complements an important body of work in the econophysics literature on antibubbles or time-reversed bubbles where  $t$  is replaced by  $-t$  in the usual definition of a speculative bubble. For more on the definition and empirical testing of antibubbles see e.g. Johansen and Sornette (1999), (2001) and Zhou and Sornette (2004a-b), (2005).

### 3.2 The multivariate model

In this subsection we discuss multivariate models for bubbles. These describe the price of more-than-one asset simultaneously and are significant for empirical applications (Fry, 2014; Sornette

and Malevergne, 2006). Let  $P_t$  denote the prices  $(P_t^1, \dots, P_t^p)$  of a basket of  $p$  assets at time  $t$ . Define  $X_t = (X_t^1, \dots, X_t^p)$  where  $X_t^i = \log P_t^i$ . For the multivariate model Assumptions 1-2 are replaced by their vector/matrix analogues.

**Assumption 1: [Intrinsic Rate of Return]** The intrinsic rate of return is assumed constant and equal to  $\mu$ :

$$E[X_{t+\Delta} - X_t | X_t] = \mu\Delta + o(\Delta). \quad (14)$$

**Assumption 2: [Intrinsic Level of Risk]** The intrinsic level of risk is assumed constant and equal to  $\Sigma$ :

$$\text{Var}[X_{t+\Delta} - X_t | X_t] = \Sigma\Delta + o(\Delta). \quad (15)$$

Co-ordinatewise our starting equation (1) becomes

$$p^i(t) = p_1^i(t)^{(1-\kappa^i)j^i(t)} \quad (16)$$

and before the crash  $X_t$  satisfies the vector-valued equation

$$dX_t = \mu(t)dt + \sqrt{\Sigma(t)}dW_t - v dj(t), \quad (17)$$

where  $v$  is the diagonal matrix satisfying  $v_{ii} = -\ln(1 - \kappa^i) = v_i$ . As in Section 3.1 replacing  $v_i$  with  $-v_i$  in the above yields a model for a multivariate negative bubble.

Assumption 1 above yields a vector-valued re-statement of equation (8):

$$\mu(t) - v h(t) = \mu; \quad \mu(t) = \mu + v h(t). \quad (18)$$

Similarly, Assumption 2 shows that the second-order condition now becomes

$$\Sigma(t) + v \Sigma_j v^T h(t) = \Sigma; \quad \Sigma(t) = \Sigma - v \Sigma_j v^T h(t). \quad (19)$$

where  $\Sigma_j$  denotes the correlation matrix of  $j(t)$ . Equation (19) shows how correlation in the bubble process is transferred to prices prior to the crash. Genuinely high-dimensional and multivariate models are possible though it seems that these may lose some interpretability. Bivariate models seem to be by far the most convenient and natural to use in applications – see Section 3.3.

### 3.3 A bivariate bubble model

In a bivariate extension of the preceding univariate and multivariate models equation (17) becomes

$$dX_t = \mu(t)dt + \sqrt{\Sigma(t)}dW_t - v dj(t), \quad (20)$$

where  $X_t = (X_1(t), X_2(t))^T$  denotes the log-price of Assets 1 and 2 at time  $t$ ,  $\Sigma(t)$  is the instantaneous covariance and  $W_t$  is standard bivariate Brownian motion. Assumption 1 gives

$$\mu_1(t) = \mu_1 + v_1 h(t); \quad \mu_2(t) = \mu_2 + v_2 h(t). \quad (21)$$

Assumption 2 gives

$$\begin{aligned}\Sigma(t) &= \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix} - \begin{pmatrix} v_1 & 0 \\ 0 & v_2 \end{pmatrix} \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \begin{pmatrix} v_1 & 0 \\ 0 & v_2 \end{pmatrix} h(t), \\ &= \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix} - \begin{pmatrix} v_1^2 & \rho v_1 v_2 \\ \rho v_1 v_2 & v_2^2 \end{pmatrix} h(t).\end{aligned}\tag{22}$$

In addition to equation (10) the phase-transition condition also gives

$$\min_t |\Sigma(t)| = 0; \quad \min_t \sigma_{12} - \rho v_1 v_2 h(t) = 0.\tag{23}$$

Equation (23) thus clarifies that the bubble constitutes a phase transition between random and deterministic behaviour in prices (see e.g. Fry, 2012).

*Historical Estimation Bias.* The above also serve to highlight possible dangers regarding historical estimation bias and coincides with fears over illusory diversification raised in Vogel and Werner (2015). During a bubble regime prices may be rising at artificially high rates with comparatively little volatility compared to the underlying long-term values. Equation (22) is also useful in highlighting that using historical prices in a bubble regime may lead to under-diversified portfolios as a consequence of under-estimating long-term correlation levels in returns series. If a crash occurs at time  $t_0$ , in addition to an increase in marginal volatility, the covariance of  $\Delta X_1(t_0)$  and  $\Delta X_2(t_0)$  increases by a factor of  $\rho v_1 v_2 h(t_0)$  (from  $\sigma_{12} - \rho v_1 v_2 h(t_0)$  to its equilibrium value of  $\sigma_{12}$ ).

### 3.3.1 Spillovers and Contagion

In this section we develop a method to test for the presence or absence of contagion during bubbles and negative bubbles. We therefore present a mathematical treatment of a serious practical problem. Many papers discuss the modelling of spillovers on conventional markets (see e.g. Blasco et al., 2012; Hsieh, 2013). However, doing a comparable analysis on cryptocurrency markets is more challenging and requires a more mathematical approach. For cryptocurrency markets detailed information on direct transactions (e.g. paired occurrences of short positions in one currency and long positions in another) or indeed direct transaction volumes between two different cryptocurrencies do not typically exist.

The above discussion leads naturally to an empirical test for contagious effects that arise with bubbles and negative bubbles. As discussed below this involves testing the hypothesis shown in equation (27). Suppose we have two assets whose prices are given by  $e^{X(t)}$  and  $e^{Y(t)}$ . Let  $\Delta X_t = X_{t+1} - X_t$ . Under the model (20) knowledge of  $Y(t)$  reduces uncertainty in  $X(t)$  by

$$\begin{aligned}\text{Var}[\Delta X(t)] - \text{Var}[\Delta X(t)|\Delta Y(t)] &= \text{Var}[\Delta X_t] - (1 - \text{Cor}^2(\Delta X_t, \Delta Y_t))\text{Var}[\Delta X_t] \\ &= \text{Cor}^2(\Delta X_t, \Delta Y_t)\text{Var}[\Delta X_t].\end{aligned}\tag{24}$$

Similarly, knowledge of  $X(t)$  reduces uncertainty in  $Y(t)$  by the amount

$$\text{Var}[\Delta Y(t)] - \text{Var}[\Delta Y(t)|\Delta X(t)] = \text{Cor}^2(\Delta X_t, \Delta Y_t)\text{Var}[\Delta Y_t].\tag{25}$$

The constraints  $\sigma_X^2(t) \geq 0$  and  $\sigma_Y^2(t) \geq 0$  imply that

$$\sigma_X^2 = \frac{v_X^2(\beta - 1)^{1-\frac{1}{\beta}}}{\alpha}; \quad \sigma_Y^2 = \frac{v_Y^2(\beta - 1)^{1-\frac{1}{\beta}}}{\alpha}.\tag{26}$$

Contagion from  $Y(t)$  to  $X(t)$  occurs if  $Y(t)$  is more informative about  $X(t)$  than  $X(t)$  is about  $Y(t)$ . From equations (24-26) contagion from  $Y(t)$  to  $X(t)$  occurs if

$$\begin{aligned} \text{Cor}^2(\Delta X_t, \Delta Y_t) \text{Var}[\Delta X_t] &< \text{Cor}^2(\Delta X_t, \Delta Y_t) \text{Var}[\Delta Y_t] \\ \text{Var}[\Delta X_t] &< \text{Var}[\Delta Y_t] \\ v_X^2 \left[ \frac{(\beta - 1)^{1-\frac{1}{\beta}}}{\alpha} - \ln \left( \frac{\alpha^\beta + (t+1)^\beta}{\alpha^\beta + t^\beta} \right) \right] &< v_Y^2 \left[ \frac{(\beta - 1)^{1-\frac{1}{\beta}}}{\alpha} - \ln \left( \frac{\alpha^\beta + (t+1)^\beta}{\alpha^\beta + t^\beta} \right) \right] \\ v_X^2 &< v_Y^2. \end{aligned} \quad (27)$$

Equation (27) is significant as it shows that contagion occurs as the overall bubble process becomes dominated by price rises and speculation in Asset  $Y$ . Similarly in a negative bubble contagion from  $Y(t)$  to  $X(t)$  occurs as speculation that drives down the price of  $Y(t)$  becomes the dominant effect.

Equation (27) leads to the following hypothesis test to determine the presence of contagion during both bubbles and negative bubbles

$$H_0 : |v_X| = |v_Y|; \quad H_1 : |v_X| \neq |v_Y|. \quad (28)$$

The direction of departure from the null hypothesis in equation (28) indicates the direction of contagion. If  $|v_X| < |v_Y|$  we have contagion from  $Y$  to  $X$ . In contrast if  $|v_X| > |v_Y|$  we have contagion from  $X$  to  $Y$ .

## 4 Unpredictable market shocks

Suppose that the market is exposed to an unpredictable shock. The timing of the shock is assumed to be completely unpredictable. If the shock is exogenous in nature then its affect is merely transitory (Sornette and Helmstetter, 2003). In contrast, the after-effects of an endogenous shock are potentially much longer lasting.

The shock occurs at time 0 and results in an initial decrease in drift by the amount  $\mu_0$  and an initial increase in volatility by the amount  $\sigma_0^2$ . As an arbitrage opportunity has to be eliminated, the market recovers at the random time  $t_0$  – the drift increases by  $\mu_0$  and volatility decreases by  $\sigma_0^2$ . The time  $t_0$  of the market recovery is a random variable with hazard function  $h(t)$ . Since the effect of an exogenous shock is transitory it follows that in this case  $h'(t) > 0$ , since as time progresses a market rebound becomes increasingly likely. Also, since the shock is assumed to happen at  $t = 0$  it follows that we must also have  $h(0) = 0$ :

$$h'(t) > 0; \quad h(0) = 0. \quad (29)$$

The price dynamics prior to the market recovery are described by the following equation

$$dX_t = \mu(t)dt + \sigma(t)dW_t + dj(t). \quad (30)$$

The form of  $j(t)$  is explicitly chosen to represent the fact that when a recovery happens the effect is an increase in the drift (representing better returns post-recovery) and a decrease in variance (representing a reduction in the level of risk post-recovery and necessitating use of the complex number below). As such we follow Fry (2012) in assuming  $j(t)$  satisfies

$$dj(t) = \mu_0 \delta(t - t_0)dt + i\sigma_0 \delta(t - t_0)dW_t, \quad (31)$$

where  $i = \sqrt{-1}$  and  $\delta(\cdot)$  denotes Dirac's delta function. Prior to the recovery we have that

$$E[X_{t+\Delta} - X_t | X_t] = (\mu(t) + \mu_0 h(t))\Delta + o(\Delta). \quad (32)$$

Thus, from Assumption 1 (equation 7) it follows that

$$\mu(t) = \mu - \mu_0 h(t). \quad (33)$$

Equation (33) shows that the shock reduces the level of return. The risk (variance) associated with equation (30) is

$$\text{Var}(X_{t+\Delta} - X_t | X_t) = \text{Var}[\sigma(t)(W_{t+\Delta} - W_t)] + \text{Var}[j(t+\Delta)|j(t) = 0] \quad (34)$$

This gives

$$\text{Var}(X_{t+\Delta} - X_t | X_t) = \sigma^2(t)\Delta + \text{Var}[E(j(t+\Delta)|j(t) = 0)] \quad (35)$$

$$+ E[\text{Var}(j(t+\Delta)|j(t) = 0)] + o(\Delta); \quad (36)$$

$$\text{Var}(X_{t+\Delta} - X_t | X_t) = (\sigma^2(t) + (\mu_0^2 - \sigma_0^2) h(t)) \Delta + o(\Delta). \quad (37)$$

Similarly, it follows from Assumption 2 (equation 9) that

$$\sigma^2(t) + (\mu_0^2 - \sigma_0^2) h(t) = \sigma^2; \quad \sigma^2(t) = \sigma^2 + (\sigma_0^2 - \mu_0^2) h(t). \quad (38)$$

If  $\sigma_0^2 \geq \mu_0^2$  the shock affects volatility more than it does the drift. The shock thus results in an increase in market volatility alongside a decrease in drift. If  $\sigma_0^2 \leq \mu_0^2$  the shock actually results in a reduction in volatility. However, irrespective of the effect upon market volatility the shock decreases the rate of return so is still likely to remain bad news for investors. If  $\sigma_0^2 = \mu_0^2$  market volatility remains unaffected.

In empirical work we choose

$$h(t) = \lambda[1 - (1+t)^{-\alpha}]. \quad (39)$$

Not only does  $h(t)$  in (39) satisfy (29) but the special case  $\alpha = 0.5$  in (39) recreates both the empirical power-law reported in Sornette et al. (2003) and related phenomenology in Sornette and Helmstetter (2003). Equation (39) also provides a natural empirical test for the presence of an exogenous/endogenous shock (see below).

From (39) it follows that

$$\sigma^2(t) = \sigma^2 + \beta[1 - (1+t)^{-\alpha}], \quad (40)$$

where  $\beta = \lambda(\sigma_0^2 - \mu_0^2)$ . The case  $\alpha = 0$  corresponds to the case of an efficient market where price changes are completely unpredictable and we are left with the classical random walk or Black-Scholes model:

$$dX_t = \mu dt + \sigma dW_t. \quad (41)$$

In empirical work we test the hypothesis

$$H_0 : \alpha = 0; \quad H_1 : \alpha \neq 0. \quad (42)$$

The link with statistical physics means that we can interpret rejection of the null hypothesis in (42) as follows:

*Endogenous shock.* If  $\alpha < 0$  (and  $\beta < 0$ ) then  $\sigma^2(t)$  increases without bound. This represents the fundamental uncertainty related to an endogenous shock (Fry, 2012).

*Exogenous shock.* If  $\alpha > 0$  (and  $\beta > 0$ ) the market recovery becomes the inevitable phase transition between random and deterministic behaviour with

$$\lim_{t \rightarrow \infty} \sigma^2(t) = 0. \quad (43)$$

This suggests that

$$\sigma^2 + \beta = 0; \quad \sigma^2 = -\beta. \quad (44)$$

## 5 Empirical applications

### 5.1 Bubbles and negative bubbles

The global cryptocurrency market is still evolving and, as such, represents a fascinating field of study - especially for econophysics. Information on market capitalisation and market share are readily available from the authoritative website [coinmarketcap.com](http://coinmarketcap.com). In particular, as discussed in White (2014) the cryptocurrency market shows some fluidity with both market share and market capitalisations fluctuating wildly in recent years. This is especially true for Bitcoin whose market share has decreased from around 91% from as recently as November 2014 to as little as 84% in February 2015 back to around 91% again in January 2016 during the latest revision of this paper. These fluctuations in market share reflect wider concerns raised about Bitcoin's long-term sustainability (Dowd, 2014). Against this backdrop Ripple has recently emerged as the largest alternative cryptocurrency (or altcoin) to Bitcoin. At the time of the first draft of this paper (May, 2015) the market share of Ripple was as high as 6% though this has recently dropped to around 2.8% in January 2016 during the revisions for this paper.

Competition between different cryptocurrencies is an interesting issue (see e.g. White, 2014; Gandal and Halaburda, 2014; Rogojanu and Badea, 2014; Dowd, 2014). However, one further complication is that it is far from clear that in practical terms Ripple and Bitcoin are in direct competition with each other. Some practitioners have expressed the view that the growth of the Ripple network may actually make Bitcoins substantially easier to buy and sell. Further, one prediction made was that XRPs (the native currency of the Ripple network) may be used to transfer money across financial institutions whilst Bitcoins may be primarily held by individual investors (Coinsetter, 2013).

When analysing Bitcoin and Ripple it is important to recognise key differences in the construction of both currencies. Bitcoin was conceived as a decentralised currency supposedly immune from successive episodes of devaluation that have dogged centrally backed national currencies (Dowd, 2014). Bitcoin thus had an initial monetary supply of zero and its own digital mining processes are intended to mimic a key tenet of currencies backed by precious metals i.e. scarcity reinforced by limited supply and free from artificial manipulation (Frisby, 2014). In contrast, Ripple had an initial monetary supply of 100 billion XRP and remains vulnerable to accusations of artificial price manipulation and hoarding by its originators. To a greater or lesser extent similar comments apply to any currency, especially national currencies, without Bitcoin's decentralised mining processes that rigorously enforce the scarcity of supply (Dowd, 2014). These concerns notwithstanding Ripple has achieved significant market penetration. At the time of writing (January 2016) data on [coinmarketcap.com](http://coinmarketcap.com) lists Ripple as having an

available supply of 33.5 billion XRP out of a total supply of 100 billion XRP. The market capitalisation (calculated as price  $\times$  available supply) ranks Ripple significantly ahead of the third largest cryptocurrency Litecoin (\$202.8m versus \$153.8m).

As discussed above differences in construction mean that roughly 2/3 of Ripple’s total supply is retained by its originators. However, this is akin to the estimated 73% of Bitcoins held in dormant accounts (Weber, 2014a) and linked to speculative behaviour in Yermack (2013). Since speculation plays such an important role in both markets it is interesting to examine the potential for spillovers from Bitcoin to Ripple and vice versa. Further, since the amount of currency in active circulation appears to be comparable across both markets we argue that results are unlikely to be unduly influenced by artificial hoarding of Ripple by its originators.

The data consist of closing values of the Bitcoin Coindesk Index (downloaded from the website [Coindesk.com](http://Coindesk.com)) and weekly closing prices of Ripple XRP (downloaded from the website [Ripplecharts.com](http://Ripplecharts.com)). Both sources represent the industry standard and are, for instance, regularly referenced on [Coinmarketcap.com](http://Coinmarketcap.com). A plot of weekly Ripple (XRP) and Bitcoin prices in US Dollars over time is shown below in Figure 1. Both series show a dramatic spike upwards throughout 2013 before falling precipitously throughout 2014. Empirical evidence of bubbles in Bitcoin and cryptocurrency markets has now been well-documented (see e.g. Cheah and Fry, 2015; Garcia et al., 2014; Cheung et al., 2015). Hence, a more interesting question would appear to be to test for the presence of a negative bubble from 2014 onwards.

The results of the statistical tests are shown below in Table 1. Results from both the univariate and bivariate tests give strong evidence of a negative bubble. We also obtain a positive result for the test for contagion. This suggests that during the negative bubble there is a spillover from Ripple which then exacerbates the subsequent falls in Bitcoin. This interpretation mirrors findings in Gandal and Halaburda (2014) where the values of several altcoins are documented to have risen relative to Bitcoin over a similar time period. A plot of Ripple (XRP) measured in units of Bitcoin is shown below in Figure 2 and shows a general increase over the period in question. Thus, with its added flexibility, Ripple was thought by some to represent a serious potential rival to Bitcoin. However, following helpful comments from an anonymous referee we fitted the univariate bubble model in Section 3.1 to Ripple prices measured in units of Bitcoin. This gives significant evidence ( $p = 0.000$ ) of a bubble in Ripple. The implication would appear to be that Ripple appears over-valued relative to Bitcoin. This appears to set the stage for subsequent falls in the value Ripple relative to Bitcoin in the period between initial submission and revision of this paper. Over the same period Bitcoin has also regained significant market share from Ripple ([coinmarketcap.com](http://coinmarketcap.com)).

## 5.2 Endogenous or exogenous market shocks

As an empirical application we examine the impact of various market shocks that are popularly believed to have significantly impacted upon bitcoin prices. Following established methodology (Sornette et al., 2003; Johansen and Sornette, 2010; Fry, 2012) we test for the presence of an exogenous shock in the first 100 trading days following a putative shock. In particular, we test the null hypothesis shown in equation (42). Events examined include a technical glitch in the core Bitcoin software on March 11th 2013 (Dourado and Brito, 2014), the seizure by the Department of Homeland Security of assets belonging to the Mt Gox bitcoin exchange (May 15th, 2013), the FBI’s closure of the Silk Road website (October 2nd, 2013), the People’s Bank of China’s prohibition of Chinese financial institutions from using bitcoins (December 5th, 2013), the suspension of trading on the Bitcoin exchange Mt Gox due to technical issues on February 7th 2014 (Dourado and Brito, 2014) and the cyber attack on the Canada-based bitcoin bank

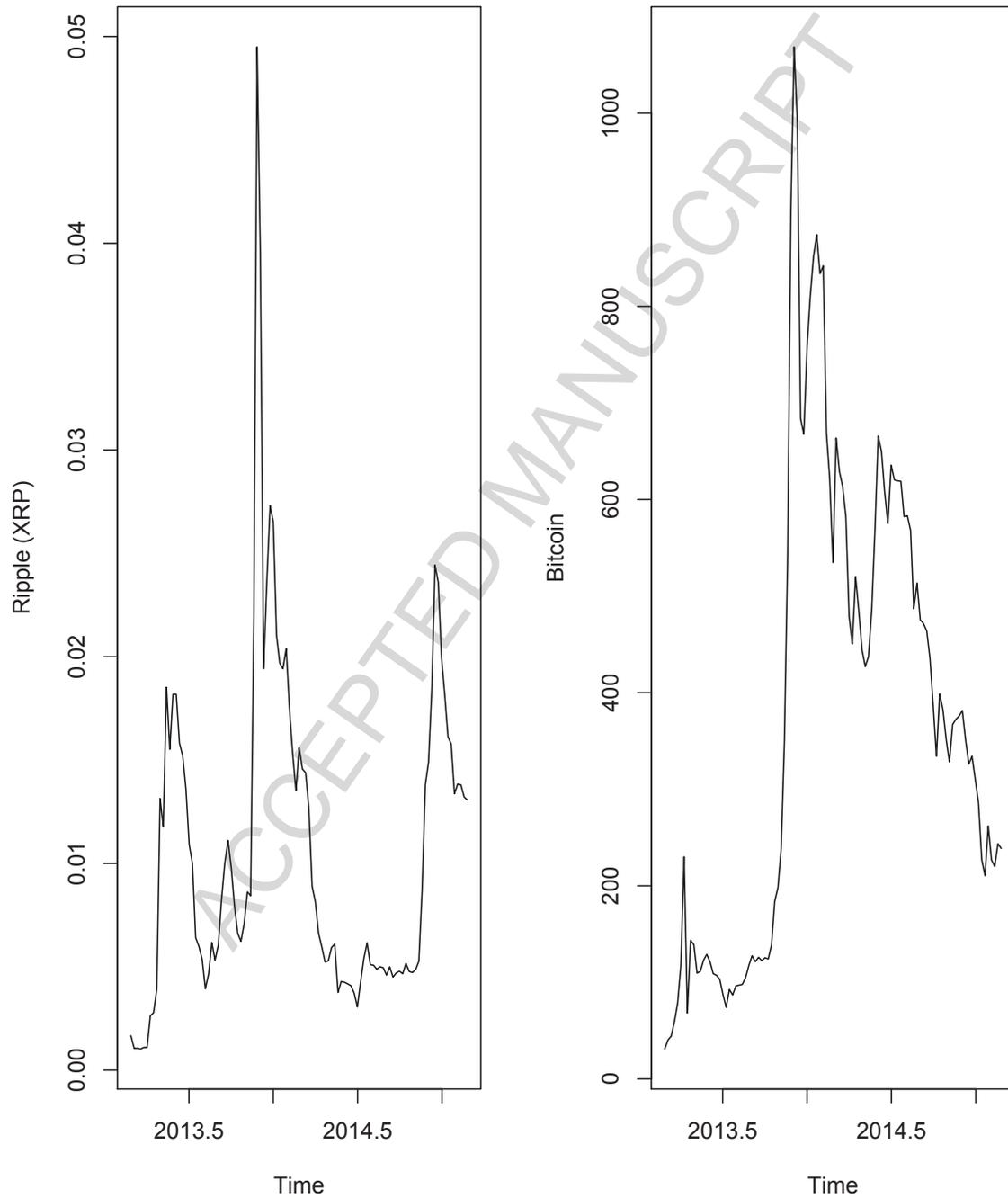


Figure 1: Plot of weekly Ripple (XRP) and Bitcoin prices in US Dollars over time from Feb 26th 2013 to Feb 24th 2015. Left panel: Ripple (XRP). Right panel: Bitcoin.

Univariate negative bubble test				
	$\hat{v}$	e.s.e $\hat{v}$	$t$ -value	$p$ -value
Ripple (XRP)	-0.168	0.007	23.254	0.000
Bitcoin	-0.058	0.001	40.705	0.000
Bivariate negative bubble test				
	$\hat{v}$	e.s.e $\hat{v}$	$t$ -value	$p$ -value
Ripple (XRP)	-0.285	0.025	11.417	0.000
Bitcoin	-0.174	0.015	11.417	0.000
Test for contagion			$t$ -value	$p$ -value
			3.128	0.003

Table 1: Results for the statistical tests for negative bubbles (equation 13) and contagion (equation 28).

Flexicoin (March 2nd 2014). We also look at market shocks identified in the academic literature such as the end of the 2011 bubble on October 18th 2011 (as identified in Garcia et al., 2014), the so-called Halving Day on November 28th 2012 which is the point at which returns from mining Bitcoins halved (Garcia et al., 2014) and a Bitcoin crash on April 13th 2013 identified in Rogojanu and Badea (2014). The results obtained are shown below in Table 2 and Figure 3.

Numerical evidence in Table 2 and graphical evidence in Figure 3 both suggest that these events have had a mixed impact upon the Bitcoin market. In particular, our model would appear to dispute the timings of Bitcoin crashes previously defined in the literature. Neither the events of October 18th 2011 (identified in Garcia et al., 2014) nor of April 13th 2013 (identified in Rogojanu and Badea, 2014) seem to have a lasting impact. Indeed not long after April 13th 2013 the Bitcoin bubble re-inflates.

There are also several instances when the market events have no detectable effect. These include the seizure of Mt. Gox assets by the Department of Homeland Security (May 15th 2013), the suspension of trading on Mt. Gox due to technical issues (February 7th 2014) and the cyber attack on Flexicoin (March 2nd 2014). In contrast, the prohibition of Chinese financial institutions from using Bitcoin does have a detectable effect upon prices. However, this event is classified by our model as an exogenous shock. There is an interesting link here to wider economic debates on the legitimacy and definition of Bitcoin and the relationship between nation states and their national currencies (see e.g. Van Alstyne, 2014; Weber, 2014a; Dequesch, 2013). However, the suggestion would appear to be that official sanctions may have only a temporary effect upon decentralised cryptocurrency networks. The relatively temporary nature of this China-related shock also appears consistent with the interpretation of Pilkington (2014).

In other cases it seems that the effects of potentially damaging market events are dwarfed by the sheer intensity of the Bitcoin bubble (Cheah and Fry, 2015). One example of this occurs in the aftermath of the Bitcoin halving day (November 28th 2012). Further, both the glitch in Bitcoin software (March 11th 2013) and the FBI closure of the Silk Road website (October 2nd 2013) also result in renewed bubble phases – possibly over fears of a limited future supply of Bitcoins.

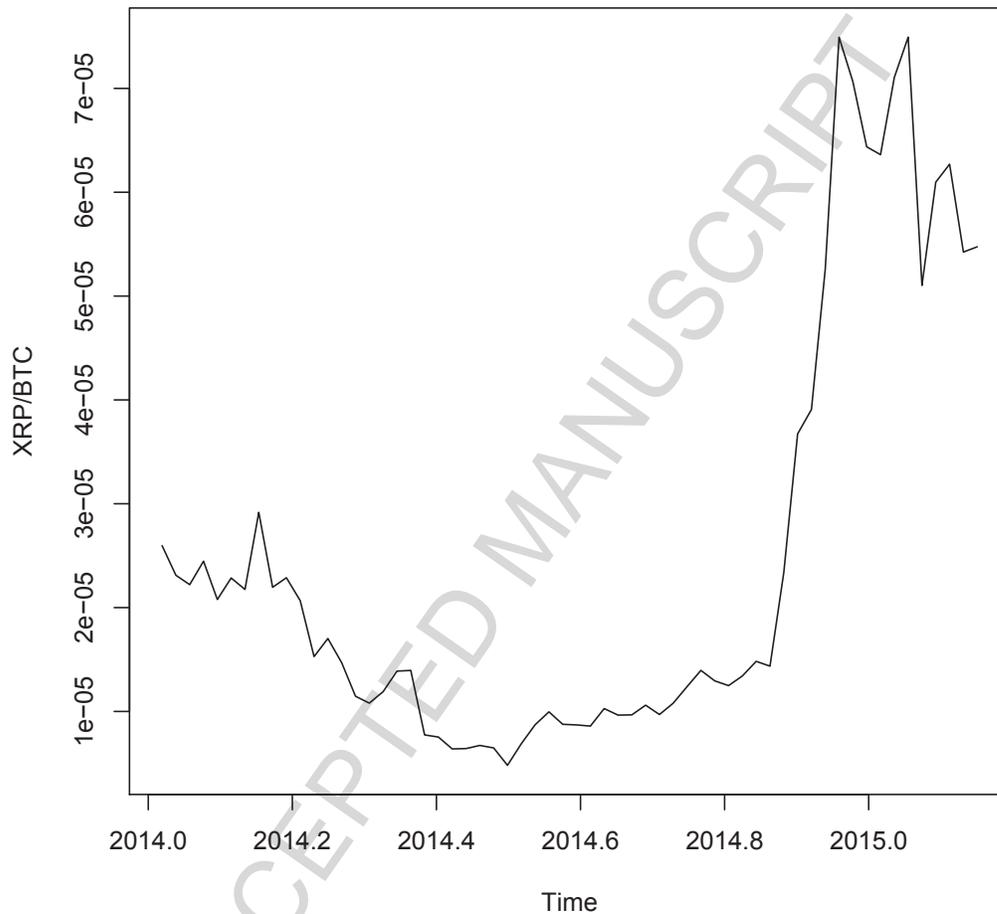


Figure 2: Plot of weekly Ripple (XRP) price measured in Bitcoins from Jan 7th 2013 to Feb 24th 2015.

## 6 Conclusions

Amid huge media interest cryptocurrencies have attracted significant amounts of popular attention (Vigna and Casey, 2015; Frisby, 2014). From an economic perspective the sums of money involved are substantial. Several cryptocurrencies, including Bitcoin, have experienced dramatic fluctuations in both market capitalisation and market share in recent years. Against this backdrop the academic literature on cryptocurrencies has begun to emerge with a burgeoning economics and finance literature (Böhme et al, 2015; Weber, 2014; Cheah and Fry, 2015) complementing earlier studies related to law (Grinberg, 2012; Plasaras, 2013) and computer science (Böhme et al, 2014; Sadeghi, 2013).

Econophysics thus has a crucial role to play as global cryptocurrency markets continue to evolve. Here, in addition to evidence of speculative bubbles (see e.g. Cheah and Fry, 2015; Garcia et al., 2014; Cheung et al., 2015) recently developed econophysics models for negative

Date	$\hat{\alpha}$	e.s.e	$t$ -value	$p$ -value	Tentative Conclusion
1. October 18th 2011	0.281	0.149	1.884	0.060	No evidence of an effect
2. November 28th 2012	-0.640	0.141	-4.526	0.000	Endogenous shock (bubble)
3. March 11th 2013	1.000	0.0002	4791.905	0.000	Exogenous shock (bubble)
4. April 13th 2013	0.628	0.131	4.780	0.000	Exogenous shock (bubble)
5. May 15th 2013	-0.220	0.203	1.082	0.279	No evidence of an effect
6. October 2nd 2013	-1.527	0.211	7.237	0.000	Endogenous shock (bubble)
7. December 5th 2013	0.821	0.160	5.12195	0.000	Exogenous shock (crash)
8. February 7th 2014	0.069	0.200	0.343	0.731	No evidence of an effect
9. March 2nd 2014	-0.110	0.265	0.416	0.678	No evidence of an effect

Table 2: Results for the test of endogenous vs. exogenous shocks (equation 42)

bubbles also provide a useful description of cryptocurrency markets. Evidence for a negative bubble is found from 2014 onwards in the two largest cryptocurrency markets Bitcoin and Ripple. Further, evidence suggests that there is a spillover from Ripple (XRP) to Bitcoin that exacerbates recent price falls in Bitcoin. This finding does reflect both concerns raised about the long-term sustainability of Bitcoin (Dowd, 2014) and increased competition between rival cryptocurrencies (Gandal and Halaburda, 2014) that are potentially more flexible (see Section 2). However, over the period in question the main conclusion appears to be that Ripple is over-priced relative to Bitcoin.

Allied to the above a model for endogenous/exogenous shocks developed in Fry (2012) also leads to new insights into cryptocurrency markets. We use our model to study a series of events that are popularly thought to have affected Bitcoin markets. It is interesting to note that the effect of these events upon Bitcoin markets is mixed. Inter alia the effect of some market shocks is simply dwarfed by the scale and extent of the speculative bubble in Bitcoin (Cheah and Fry, 2015). However, certain events do have a detectable impact upon the market. A technical glitch in the Bitcoin software is shown to temporarily raise prices during the Bitcoin bubble in March 2013. The FBI's closure of the illegal Silk Road website in October 2013 has a similar effect. Finally, The People's Bank of China's banning of Chinese financial institutions from using Bitcoin is classified as an exogenous shock. Thus, it appears that though the Bitcoin bubble fundamentally destabilises prices (Cheah and Fry, 2015) the bubble is actually brought to an end by an exogenous shock – a picture that seems qualitatively similar to the bursting of the internet stocks bubble in 2000 (Fry, 2012). This comparison between the Bitcoin and dot com bubbles is made in purely qualitative terms in Yermack (2013).

This paper has explored a number of themes of wider importance such as statistical and probabilistic approaches to econophysics, the relationship between econophysics and mainstream

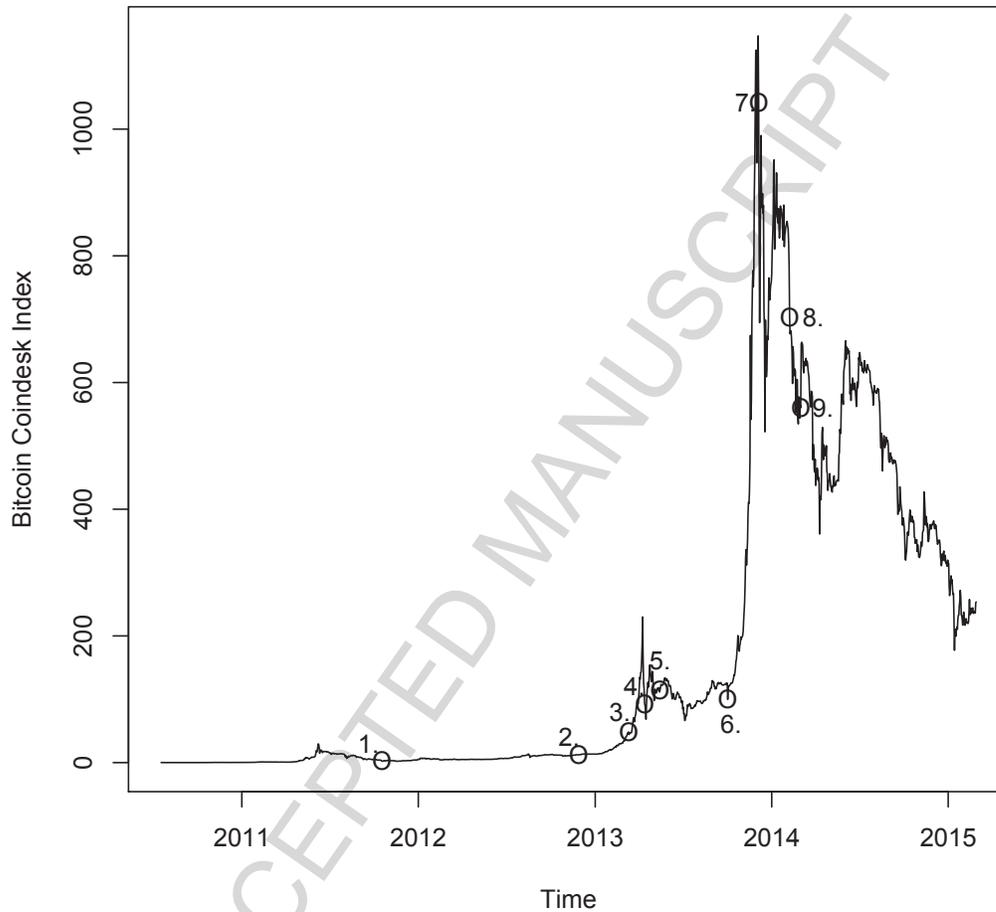


Figure 3: Plot of daily Bitcoin Coindex Index from July 18th 2010 to February 28th 2015 and the impact of putative market events (circled). The numbering of the market events is as shown in Table 2.

financial models and the creation of tools to monitor financial stability and to assist economic policy. Econophysics clearly has much to contribute. Future work will discuss the modelling of self-fulfilling and self-denying prophecies in complex socio-technical systems (Biggs, 2009; Heylighen and Joslyn, 2001). Potential financial applications include the development of new trading and hedging strategies. Future work incorporating more detailed information on transaction pairs of long and short positions would give a more comprehensive account of spillovers across different cryptocurrencies.

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**Highlights**

- To date Bitcoin and cryptocurrency markets have been under-explored
- Bitcoin and cryptocurrency markets contain a considerable speculative component and are extremely volatile
- We use econophysics models to examine shocks and crashes in cyptocurrency markets
- We examine competition between rival cryptocurrencies and find evidence of a spillover from Ripple to Bitcoin
- The extent to which law enforcement and government measures can affect Bitcoin markets appears mixed