The Valuation of No-Negative Equity Guarantees and Equity Release Mortgages

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Abstract

We outline the valuation process for a No-Negative Equity Guarantee in an Equity Release Mortgage loan and for an Equity Release Mortgage that has such a guarantee. Illustrative valuations are provided based on the Black '76 put pricing formula and mortality projections based on the M5, M6 and M7 mortality versions of the Cairns-Blake-Dowd (CBD) family of mortality models. Results indicate that the valuations of No-Negative Equity Guarantees are high relative to loan amounts and subject to considerable model risk but that the valuations of Equity Release Mortgage loans are robust to the choice of mortality model. Results have significant ramifications for industry practice and prudential regulation.

Keywords: Actuarial Science, Black '76 model, CBD mortality models, Equity Release, No Negative Equity Guarantee, Prudential Regulation

JEL Classification: G2, G3.

1 Introduction

No-Negative Equity Guarantees (NNEGs) are a standard feature of UK Equity Release Mortgage (ERM) loans. An ERM is a loan made to a property-owning borrower late in life that is collateralised by the value of their property. The amount of the loan compounds over time at the loan interest rate and the loan is repaid when the borrower leaves their property by dying or going into care. The NNEG is a guarantee that stipulates that the amount of the loan due for repayment is capped by the property value at the time the loan is repaid, i.e., the borrower owes no more than the minimum of the rolled-up loan amount and the property value at the time of repayment. This obligation to repay the minimum of two future values implies that the NNEG involves put options granted by the lender to the borrower.

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In the UK context, the main earlier NNEG literature appear to consist of Hosty et al. (2008), Li et al. (2010), Dowd (2018), Buckner and Dowd (2019) and a series of regulatory documents set out by the UK Prudential Regulatory Authority (see e.g., PRA, 2016, 2018). From the methodological perspective, the key ingredients are (a) a mortality model, (b) a house price model and (c) a valuation approach. A good sense of the approaches used can be obtained from, e.g., Li et al. (2010); Chen et al. (2010); Lee et al. (2012); Shao et al. (2015); Kim and Li (2017); Kogure et al. (2014) and Lee et al. (2018). The mortality model might be, e.g., Lee and Carter (1992) or Cairns et al. (2006, 2009), known as CBD. The house price model might be ARMA-GARCH, ARMA-GARCH, DCC-GARCH, VARX etc. The pricing approach might be an Esscher Transform, Wang Transform, minimum entropy or some hedonic approach.

We use a much simpler approach based on the Black '76 put option formula (Black, 1976) We argue that Black '76 is a natural model for NNEG valuation. It is simple, intuitive and straightforward to implement. We would also argue that less parsimonious models such as ARMA-EGARCH are overparameterised (see Buckner and Dowd, 2019, pp. 148-150). Moreover, despite views to the contrary, we suggest that Black '76 can be applied even when house prices are autocorrelated (Cornalba et al., 2002).

This paper outlines a simple approach to value both NNEGs and ERMs based on a combination of the Black '76 put pricing model and the CBD mortality model. The paper then provides some illustrative NNEG and ERM valuation results and a sensitivity analysis. Leaving aside earlier unrefereed work by two of the current authors, this article is the first to: (a) give Black '76 NNEG valuation results based on a new expected volatility approach proposed by Buckner and Dowd (2019); (b) give NNEG valuation results based on different versions of the CBD mortality models; (c) give *any* ERM valuations; and (d) establish and explain the finding that ERM valuations are more robust to mortality model risk than NNEG valuations.

The layout of this paper is as follows. Section 2 obtains future house-exit probabilities using projections from three different mortality models. Section 3 explains the principles of NNEG and ERM valuation based on these exit probabilities. Section 4 provides some illustrative valuations. Section 5 presents the results of some sensitivity analysis. Section 6 discusses the significance of the results. Section 7 concludes.

2 House-exit probabilities

An ERM contract specifies that, excepting cases of early exercise, the loan is repaid when the borrower leaves the house. Assuming away morbidity or ill-health, e.g., a prolonged stay in a hospital or nursing home ante mortem, then the borrower exits the house on death. Under this simplifying assumption the exit probability for any future year t is the probability of death in year t conditional on surviving to year t. The exit probability for year t is therefore equal to

$$\text{exit prob}_t = q_t \times S_t, \quad (1)$$

where q_t is the mortality rate for year t and S_t is the probability that an individual alive now will survive to year t. Note that $S_0 = 1$ and $S_t = (1-q_{t-1}) \times S_{t-1}$ for all $t > 0$.

To obtain these exit probabilities we need a model to project the borrower's future mortality rates q_t. To do so, we use three models from the CBD family of mortality models calibrated on England & Wales male death rates data and spanning the period 1971-2017 and ages 55-89. The data come from the Life & Longevity Markets Association. The CBD family of models is particularly suitable for old age projections and its goodness of fit and performance evaluation are assessed elsewhere (Cairns et al., 2011; Dowd et al., 2010).

The first of these models is model *M5* – a reparameterised version of the original CBD model (Cairns et al., 2006) – which posits that

$$logit(q_t(x)) = log\left(\frac{q_t(x)}{1-q_t(x)}\right) = \kappa_t^{(1)} + \kappa_t^{(2)}(x-\bar{x}), \quad (2)$$

where the mortality rate $q_t(x)$ explicitly depends on age x and period effects $\kappa_t^{(1)}$ and $\kappa_t^{(2)}$, and where $\bar{x}$ is the mean of the sample age range used to calibrate the model.

The second model, *M6*, posits that

$$logit(q_t(x)) = log\left(\frac{q_t(x)}{1-q_t(x)}\right) = \kappa_t^{(1)} + \kappa_t^{(2)}(x-\bar{x}) + \gamma_{t-x}, \quad (3)$$

where γ_{t-x} is a zero trend cohort effect dependent on the year of birth $t-x$.

The third model, *M7*, posits that

$$logit(q_t(x)) = log\left(\frac{q_t(x)}{1-q_t(x)}\right) = \kappa_t^{(1)} + \kappa_t^{(2)}(x-\bar{x}) + \kappa_t^{(3)}((x-\bar{x})^2 - \hat{\sigma}_t^2) + \gamma_{t-x}, \quad (4)$$

where $\kappa_t^{(3)}$ is a third period effect and $\hat{\sigma}_t^2$ is the variance of the ages in the sample range. See Cairns et al. (2009) for technical details of the model fitting and identifiability constraints.

Figure 1 shows the exit probabilities for a male who has just turned 70. The low horizon exit probabilities on the left hand side reflect low early mortality rates and high early survival probabilities. The mortality rates are initially dominant, so exit probabilities rise as the mortality rate increases. Eventually, however, the declining survival rates become the dominant factor, so the exit probabilities peak (in this case, in the late 80s age range) and thereafter decline towards zero. The three exit probability curves have similar humped shapes but have somewhat different peaks and old age tails. These differences reflect mortality model risk.

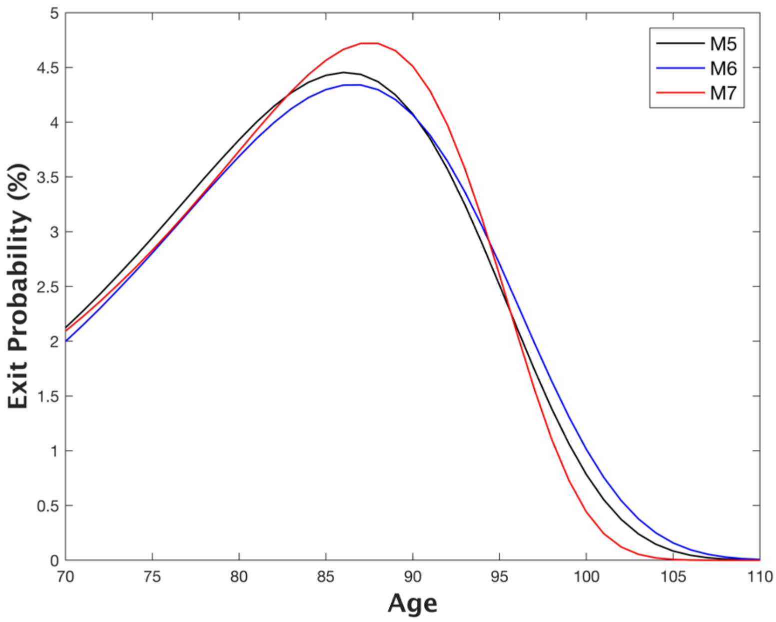

Figure 1: Exit probabilities for a UK male aged 70 obtained using models M5, M6 and M7 calibrated to England & Wales male deaths rate data over the years 1971-2017 and ages 55-89.

3 Principles of NNEG and ERM valuation

The present value *ERM* of the Equity Release loan can be considered to be the present value L of a risk-free loan, i.e., a loan which is guaranteed to be repaid in full, minus the present value *NNEG* of the NNEG guarantee:

$$ERM = L - NNEG. \tag{5}$$

The loan value grows at the loan rate l from its current amount until the time when the loan ends. In equation (5) L can be calculated as

$$L = \sum_t exit\ prob_t \times current\ loan\ amount \times e^{(l-r)t}, \tag{6}$$

where exit prob_t is the probability of exiting the house in period t given by equation (1) and r is the risk-free rate of interest. Note the implicit distinction here between the loan amount (the original loan amount or rolled up loan amount), on the one hand, and L, the (economic) value of the loan, on the other. The former is the amount loaned plus the interest accumulated since the inception of the loan, whereas the latter is the value of the loan to the lender, including the expected profit to be made on the loan.

In equation (2) $NNEG$ is the sum of the products of the exit probabilities for each future time t and the present value of the NNEG guarantee for each future time t:

$$NNEG = \sum_t exit\ prob_t \times NNEG_t, \tag{7}$$

where $NNEG_t$ is the present value of the NNEG guarantee for period t. The question is then how to value each of these individual $NNEG_t$ (or NNEGlet) terms and thence the NNEG guarantee.

The right to repay the minimum of two future values (one of which, the future house price, is uncertain) at some given future time implies a European put option granted by the lender to the borrower. Since the time of exercise is uncertain we can think of the NNEG as involving a portfolio of such put options. We thus need an option pricing model that allows for an underlying with a continuous rental benefit. The simplest such model is the so-called Black '76 model (Black, 1976), which gives the following formula for the price p_t of a European put option with maturity t on a forward contract bearing a continuous yield q:

$$p_t = e^{-rt}[K_t\Phi(-d_2) - F_t\Phi(-d_1)], \tag{8}$$

where K_t is the strike price or exercise price for period t, F_t is the forward house price for period t, $\Phi(\cdot)$ is the standard normal cumulative distribution function, σ is the volatility of the forward house price and d_1 and d_2 are given by

$$d_1 = \frac{\ln(F_t/K_t) + \sigma^2 t/2}{\sigma\sqrt{t}};\ d_2 = d_1 - \sigma\sqrt{t}. \tag{9}$$

The strike price K_t is then the rolled up or accumulated loan amount by period t:

$$K_t = \text{current loan amount} \times e^{lt}. \tag{10}$$

The forward price F_t, the price agreed now to be paid on possession in period t, is given by

$$F_t = \text{current house price} \times e^{(r-q)t}, \tag{11}$$

where the continuous yield q is the deferment rate which is also equal to the house net rental rate i.e., the rental yield net of dilapidation, insurance costs, management costs and void. This put option model is the same as that used by the PRA to value NNEGs (PRA, 2018; Section 3.20).

The use of Black '76 in this context is sometimes criticised on the grounds that Black '76 assumes geometric Brownian motion, but abundant empirical evidence (e.g., Chen et al., 2010; Li, et al., 2010; Tunaru and Quaye, 2019) suggests that house prices are autocorrelated. We agree that house prices are autocorrelated. However, house price autocorrelation does not imply that Black '76 is inapplicable, but rather that care needs to be taken with the volatility calibration.

Cornalba et al. (2002) provide a fairly general analysis of the impact of temporal correlation on option pricing and their conclusions are clear (Section 1): "In the Gaussian case, we find that the effect of [auto-] correlations can be compensated by a change in the hedging strategy and therefore options should be priced using the standard uncorrelated Black-Scholes [or here, Black '76] model." Thus, any required change can be implemented by an adjustment to the volatility calibration. A fuller discussion of this issue is provided in Buckner and Dowd (2019, pp. 49-66).

4 Illustrative valuations

We now build an ERM and NNEG valuation model based on plausible input parameter values. Following Buckner and Dowd (2019, pp. 21-22) we work with the following baseline parameter values:

- The current age is 70, a typical age for ERM borrowers when they take out ERM loans.
- The Loan to Value ratio (LTV) = 40% p.a. This LTV is consistent with an "age minus 30" rule of thumb, i.e., LTV in percent = borrower age minus 30, which approximately describes the LTVs used in the UK equity release industry.
- The risk-free rate r = 1.5% p.a.
- The ERM loan rate l = 5.25%, which is in line with recent typical empirical loan rates.
- The deferment rate q = 4.2%.

The determination of the volatility σ is a little more involved. Unlike previous approaches by other authors, we do not use a single 'off the shelf' volatility rate that would be applied in all cases. Instead, we use an expected volatility given by

$$\sigma = \sum_t exit\ prob_t \times \sigma_t, \tag{12}$$

where σ_t is a volatility term structure calibrated by Buckner and Dowd (2019, pp. 49-62, 176).

We assume an illustrative house price of £100 which, combined with the assumed loan to value ratio of 40%, implies a loan amount of £40.[2] Our baseline NNEG valuation results are shown below in Table 1. We see that the L and NNEG valuations vary considerably across the mortality

[2] As an alternative, one might wish to use, say, a national house price average. According to the Nationwide House price index https://www.nationwide.co.uk/about/house-price-index/download-data#xtab:uk-series the average UK house price is currently £215,910 (2019 Quarter 2). In that case, one would replace the stated house price (£100) with £215,910 and the all the valuations subsequently given would be multiplied by a factor of 21,591.

projections depending on the mortality model used, but that their impacts on ERM valuations largely offset each other, and this offset produces a more robust ERM valuation.

Mortality Projection	***L***	***NNEG***	***ERM***
CBD M5	£74.8	£32.2	£42.7
CBD M6	£76.5	£34.3	£42.2
CBD M7	£74.3	£31.5	£42.8

Table 1: Valuation of NNEGs and ERMs under alternative mortality projections. Notes: L is the present value component of the Equity Release Mortgage, *NNEG* is the present value of the NNEG guarantee, *ERM* is the present value of the Equity Release Mortgage. Results based on the baseline assumptions: male aged 70, LTV = 40%, current house price = £100, r = 1.5%, l = 5.25%, q = 4.2% and volatilities (σ) = 14.8% for M5, 15% for M6 and 14.7% for M7. Mortality projections are based on England &Wales male mortality rate data spanning the years 1971-2017 and ages 55-89.

5 Sensitivity analysis

Table 3 shows the sensitivities of L, $NNEG$ and ERM to changes in key parameter inputs. These are expressed in elasticity form, i.e., where the elasticity of the relevant output with respect to a change in an input is the % change in the output divided by the % change in the input.

Elasticity wrt	***L***	***NNEG***	***ERM***
Model M5			
r	-27	-54	-7
l	94	189	23
q	0	57	-43
σ	0	25	-19
LTV	100	174	44
Model M6			
r	-28	-54	-6
l	98	190	23
q	0	54	-44
σ	0	23	-19
LTV	100	170	43
Model M7			
r	-26	-53	-7
l	92	187	23
q	0	58	-43
σ	0	26	-19
LTV	100	176	43

Table 2: Sensitivities of Valuations in Elasticity Form. Notes: As Per Table 1.

We see, for example, that: a rise in r leads to falls in L and $NNEG$, but a very small fall in ERM; a rise in LTV leads to large rises in L and $NNEG$ and a smaller rise in ERM; and a rise in volatility leads to no change in L, small rise in $NNEG$ and a smaller rise in ERM. The greater robustness of ERM valuations relative to NNNEG valuations reflects the largely offsetting effects of mortality factors on L and $NNEG$. Results also indicate that the elasticities are highly robust to mortality risk.

6 Implications

The key methodological message is the importance of distinguishing between a forward house price as defined in the standard option pricing literature and in (8) above, which is also the underlying variable in the put price equation (5) vs the projected house price at future period t (as recommended in Hosty et al., 2008). The projected future house price has no role in the put pricing formula. In reporting the results of a survey of UK practitioners, the Prudential Regulation Authority reports that a number of respondents conflated the projected future house price with the forward house price (PRA, 2016). The use of the incorrect underlying in the put pricing equation is a quantitatively significant error because it produces NNEG valuations that are in the region of an order of magnitude too low (Buckner and Dowd, 2019). This NNEG undervaluation implies that ERMs are seriously overvalued and raises concerns about the profitability of the ERM sector and about the financial health of some firms in it (Buckner and Dowd, 2019). These concerns in turn raise questions about the sustainability of the sector and about the adequacy of its current system of prudential regulation.

7 Conclusions

This paper sets out a simple approach to the valuation of NNEGs and ERMs. Results based on the Black '76 put option pricing formula and CBD family mortality models calibrated to England & Wales mortality data suggest that NNEG valuations are considerably higher and ERM valuations considerably lower than is commonly believed. NNEG valuations are also subject to considerable mortality model risk, but ERM valuations are much less so. Our results have some significance given documented evidence of widespread of practitioner mis-pricing (PRA, 2018) and raise questions about the financial condition of the UK equity release sector and the adequacy of current UK prudential regulation of the sector (Dowd, 2018).

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