

# Analytical solution of shallow water equations for ideal dam-break flood along a wet bed slope

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**Abstract:** The existing analytical solutions of dam-break flow do not consider simultaneously the effects of wet downstream bottom and bed slope on the dam-break wave propagation. In this study, a new analytical solution for the shallow-water equations (SWE) is developed to remove this limitation to simulate the wave caused by an instantaneous dam-break. The approach adopts the method of characteristics and has been applied to simulate the dam-break flows with different downstream water depths and slopes. The analytical solutions have been compared with predictions by the lattice Boltzmann method and the agreement is good. Although the proposed analytical solution treats an idealized case, it is nonetheless suitable for assessing the robustness and accuracy of numerical models based on the SWE without the frictional slope.

**Keywords:** Analytical solution; Dam-break; Rarefaction wave; Shock wave; Slope; Wet bed.

## Introduction

Analytical studies of dam-break flows date back to over 120 years ago. Well-known analytical solutions for the dam-break flood waves in a horizontal channel include Ritter (1892) and Stoker (1957). Ritter (1892) derived an analytical solution for instantaneous dam-break flows on a

27 frictionless and dry bed. For wet bed conditions downstream of the dam, a shock wave develops in  
28 the downstream region. To investigate the effect of the initial downstream water depth on the  
29 dam-break wave, Stoker (1957) proposed a theoretical solution which included three interrelated  
30 equations and three unknown variables. For the dam-break problem on a sloping bed, some exact and  
31 approximate analytical solutions exist (Dressler 1958; Hunt 1983; Fernandez-Feria 2006; Ancy et al.  
32 2008; Chanson 2009). However, none of these considers simultaneously the effects of both the wet  
33 bed condition and the bottom slope on the dam-break flow.

34 In the present study, a new analytical solution of the shallow-water equations is proposed for an  
35 infinite volume of an ideal (frictionless) fluid released instantaneously from upstream of a dam with  
36 initial wet horizontal and sloping channel. The omission of the friction is based on the following  
37 considerations: (1) the frictional slope is a nonlinear term that hinders one from solving the  
38 Saint-Venant equations (SVE) analytically (Chanson 2009); (2) a frictionless fluid is often  
39 considered in the development of the analytical solutions for dam-break problems (Ritter 1892;  
40 Stoker 1957; Wu et al. 1999; Fernandez-Feria 2006; Ancy et al. 2008; Chen et al. 2011; Wang et al.  
41 2017; Cozzolino et al. 2017); (3) dam-break flow can be considered as frictionless flow for relatively  
42 high flow velocity and little flow separation from solid boundaries (Batchelor 2000; Guo et al. 1998;  
43 Guo 2005), and (4) although a truly frictionless flow does not occur in nature, the frictionless case  
44 constitutes an unambiguous end-member as well as a clear target case for testing numerical models  
45 (Ancy et al. 2008). A typical example is that Zoppou and Roberts (2003) conducted an examination  
46 of the performance of twenty explicit numerical schemes used to solve the shallow water wave  
47 equations for simulating the dam-break problem by comparing the results from these schemes with  
48 analytical solutions and expected more analytical solutions to be developed for testing the numerical  
49 schemes. The proposed analytical solution is developed using the method of characteristics. Its  
50 performance will be examined by comparing with the numerical simulations based on the Lattice  
51 Boltzmann Method (LBM).

52 During the past two decades, the LBM has become a successful alternative numerical method  
 53 for computational fluid dynamics. As a microscopic method, the LBM has been developed and used  
 54 to successfully solve the shallow water equations (Zhou 2004; Peng et al. 2011a, b).

## 55 **General models for dam-break wave**

56 Figure 1 sketches the problem investigated in which  $x_M$  and  $x_N$ , respectively, denote the positions of  
 57 the front and tail for the rarefaction wave; and  $x_R$  represents the location of the shock wave-front. The  
 58 dam releases immediately the water at both sides initially stationary (i.e.,  $u_u = u_d = 0$ ) with two  
 59 different water depths (i.e.,  $h_u > h_d > 0$ ) (Fig.1). After dam collapses, the flow can be divided into  
 60 four zones: Zone 1 is the undisturbed far upstream; Zone 2 is a rarefaction wave; Zone 3 is a constant  
 61 state where water is not at rest; and Zone 4 is the quiet downstream that is terminated on the  
 62 upstream side by the shock wave. The water depths and flow velocities in Zones 2 and 3 are denoted  
 63 as  $h$  and  $u$  as well as  $h_c$  and  $u_c$  respectively.  $\zeta$  is the shock wave-front celerity.

### 64 ***Governing equation for rarefaction wave***

65 The propagation of a dam-break wave is governed by the SVE (Chow 1959). For an infinitely long  
 66 prismatic channel of mild constant slope, the SVE can be written if the effect of wall friction is  
 67 neglected as follows:

$$68 \quad \frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} + \frac{A}{B} \frac{\partial u}{\partial x} = 0 \quad (1a)$$

$$69 \quad \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + g \frac{\partial h}{\partial x} = gS_o \quad (1b)$$

70 Here  $h$  = the flow depth,  $x$  = the distance along flow direction,  $t$  = the time,  $g$  = gravity acceleration,  
 71  $u$  = the average flow velocity,  $B$  = the water surface width,  $A$  = the area of cross-section, and  $S_o$  = the  
 72 bottom slope.

73 Applying the characteristic method to the system of partial differential equations (1a)–(1b), a  
 74 characteristic system of equations results as:

$$75 \quad \frac{d}{dt} \left( u + \int_0^h \sqrt{g \frac{B}{A}} dh \right) = gS_o \quad \text{along} \quad \frac{dx}{dt} = u + \sqrt{g \frac{A}{B}} \quad (2a)$$

$$76 \quad \frac{d}{dt} \left( u - \int_0^h \sqrt{g \frac{B}{A}} dh \right) = gS_o \quad \text{along} \quad \frac{dx}{dt} = u - \sqrt{g \frac{A}{B}} \quad (2b)$$

77 Along the forward characteristics:

$$78 \quad \left( u + \int_0^h \sqrt{g \frac{B}{A}} dh \right) - \left( u_u + \int_0^{h_u} \sqrt{g \frac{B}{A}} dh \right) = gS_o t \quad (3)$$

79 where subscript  $u$  = the undisturbed upstream reach, i.e., Zone 1.  $h_u$  and  $u_u$  = the initial water depth  
80 and flow velocity respectively in the undisturbed upstream reservoir.

81 As the upstream reach is undisturbed,  $u_u = 0$ . Substituting it into Eq. (3) yields:

$$82 \quad u = \int_0^{h_u} \sqrt{g \frac{B}{A}} dh - \int_0^h \sqrt{g \frac{B}{A}} dh + gS_o t \quad (4)$$

83 Substituting Eq. (4) into Eq. (2b) yields:

$$84 \quad \int_0^h \sqrt{\frac{B}{A}} dh + \sqrt{\frac{A}{B}} = \int_0^{h_u} \sqrt{\frac{B}{A}} dh - \left( \frac{1}{\sqrt{g}} \frac{dx}{dt} - \sqrt{g} S_o t \right) \quad (5)$$

85 Eq. (5) is the transformed SVE and applies for the prismatic channel of arbitrary shape.

### 86 ***Governing equation for shock wave***

87 The motion of the shock wave is described by the conservation equations for mass and momentum  
88 as:

$$89 \quad A_c (u_c - \xi) = A_d (u_d - \xi) \quad (6)$$

$$90 \quad A_d u_d (u_d - \xi) - A_c u_c (u_c - \xi) = g (A_c \bar{h}_c - A_d \bar{h}_d) \quad (7)$$

91 where subscripts  $c$  and  $d$  = the reaches upstream and downstream of the shock respectively;  $\bar{h}$  = the  
92 centroid water depth for the cross section; and  $\xi$  is defined as:

$$93 \quad \xi = \frac{dx_R}{dt} \quad (8)$$

94 In this study, the downstream flow is at rest, i.e.,  $u_d = 0$ . Therefore Eqs. (6)–(7) simplify to:

95 
$$\xi = \frac{A_c u_c}{A_c - A_d} \quad (9)$$

96 
$$u_c = \left[ g \frac{(A_c \bar{h}_c - A_d \bar{h}_d)(A_c - A_d)}{A_c A_d} \right]^{1/2} \quad (10)$$

97 The hydraulic characteristics, i.e., flow depth and velocity, are unique in the plane  $N-N$   
 98 connecting Zones 2 and 3. Applying Eq. (4) to this boundary condition yields the flow velocity in  
 99 Zone 3:

100 
$$u_c = \int_0^{h_u} \sqrt{g \frac{B}{A}} dh - \int_0^{h_c} \sqrt{g \frac{B}{A}} dh + g S_o t \quad (11)$$

101 Substituting Eq. (10) into Eq. (11) yields:

102 
$$\int_0^{h_u} \sqrt{\frac{B}{A}} dh - \int_0^{h_c} \sqrt{\frac{B}{A}} dh + \sqrt{g} S_o t = \left[ \frac{(A_c \bar{h}_c - A_d \bar{h}_d)(A_c - A_d)}{A_c A_d} \right]^{1/2} \quad (12)$$

103 Eq. (12) is the integral form of the momentum equation for the shock wave and applies for a  
 104 channel of arbitrary shape.

## 105 **Dam-break wave in wet rectangular channel**

### 106 *Analytical solution for rarefaction wave*

107 For a rectangular channel, the cross-sectional area is the product of the flow depth and the water  
 108 surface width. Applying this condition for the rarefaction wave, Eq. (5) simplifies to:

109 
$$\int_0^h \frac{1}{\sqrt{h}} dh + \sqrt{h} = \int_0^{h_u} \frac{1}{\sqrt{h}} dh - \left( \frac{1}{\sqrt{g}} \frac{dx}{dt} - \sqrt{g} S_o t \right) \quad (13)$$

110 Integrating Eq. (13) yields the dimensionless flow depth:

111 
$$h^* = \frac{h}{h_u} = \frac{1}{9} \left[ 2 - \frac{1}{\sqrt{g h_u}} \left( \frac{dx}{dt} - g S_o t \right) \right]^2 \quad (14)$$

112 Let

$$113 \quad X = \frac{1}{\sqrt{gh_u}} \left( \frac{dx}{dt} - gS_o t \right) \quad (15)$$

$$114 \quad X_1 = \frac{1}{\sqrt{gh_u}} \frac{dx}{dt} \quad (16)$$

$$115 \quad X_2 = \sqrt{\frac{g}{h_u}} S_o t \quad (17)$$

116 where  $X_1$  = a dimensionless distance and  $X_2$  = a dimensionless variable to account for the effect of  
 117 bed slope. Eq. (15) can then be rewritten as:

$$118 \quad X = X_1 - X_2 \quad (18)$$

119 From Eqs. (14) and (15), one gets:

$$120 \quad h^* = \frac{1}{9} (2 - X)^2 \quad (19)$$

121 Substituting Eq. (19) into Eq. (2b) yields the dimensionless velocity:

$$122 \quad u^* = \frac{u}{\sqrt{gh_u}} = \frac{1}{3} [2(1 + X_1) + X_2] \quad (20)$$

123 For consistency, the flow discharge is normalized as:

$$124 \quad \frac{Q}{A_u \sqrt{gh_u}} = \frac{Bhu}{B_u h_u \sqrt{gh_u}} \quad (21)$$

125 The top widths in Zones 1 and 2 are equal, i.e.,  $B = B_u$ , thus the dimensionless discharge is:

$$126 \quad Q^* = \frac{Q}{A_u \sqrt{gh_u}} = \frac{h}{h_u} \cdot \frac{u}{\sqrt{gh_u}} = h^* \cdot u^* = \frac{1}{27} (2 - X)^2 [2(1 + X_1) + X_2] \quad (22)$$

### 127 *Analytical solution for shock wave*

128 For a rectangular channel, the centroid depth for the cross section is half of the flow depth.

129 Applying this condition for the shock wave, Eq. (12) simplifies to:

$$130 \quad \int_0^{h_u} \frac{1}{\sqrt{h}} dh - \int_0^{h_c} \frac{1}{\sqrt{h}} dh + \sqrt{g} S_o t = \left[ \frac{(h_c^2 - h_d^2)(h_c - h_d)}{2h_c h_d} \right]^{1/2} \quad (23)$$

131 Integrating Eq. (23) yields:

$$132 \quad (h_c^*)^3 - 9h_d^*(h_c^*)^2 + 8 \left( 2 + \sqrt{\frac{g}{h_u}} S_o t \right) (h_d^*) (h_c^*)^{\frac{3}{2}} - (h_d^*) \left( h_d^* + 8 + 8 \sqrt{\frac{g}{h_u}} S_o t + \frac{2}{h_u} g S_o^2 t^2 \right) (h_c^*) + (h_d^*)^3 = 0$$

133 (24)

134 where  $h_c^* = h_c / h_u$  and  $h_d^* = h_d / h_u$  = the dimensionless flow depths upstream and downstream of  
 135 the shock wave respectively. Combining Eq. (17) with Eq. (24) yields the following equation:

$$136 \quad (h_c^*)^3 - 9h_d^*(h_c^*)^2 + 8(2 + X_2)(h_d^*)(h_c^*)^{\frac{3}{2}} - (h_d^*)(h_d^* + 8 + 8X_2 + 2X_2^2)(h_c^*) + (h_d^*)^3 = 0 \quad (25)$$

137 The hydraulic characteristics in Zone 2 are identical as those in Zone 3 at the junction, i.e.,  $N-N$ .

138 Therefore, applying Eq. (20) for the flow in junction yields the dimensionless velocity:

$$139 \quad u_c^* = \frac{u_c}{\sqrt{gh_u}} = \frac{1}{3} [2(1 + X_{1N}) + X_{2N}] \quad (26)$$

140 where  $X_{1N}$  and  $X_{2N}$  = dimensionless variables referring to the junction  $N-N$ , defined as:

$$141 \quad X_{1N} = \frac{1}{\sqrt{gh_u}} \frac{dx_N}{dt} \quad (27)$$

$$142 \quad X_{2N} = \sqrt{\frac{g}{h_u}} S_o t \quad (28)$$

143 Applying Eq. (19) for the flow at the junction yields

$$144 \quad h_c^* = \frac{1}{9} (2 - X_N)^2 \quad (29)$$

145 where  $X_N$  = a dimensionless variable referring to the junction  $N-N$  and can be expressed as:

$$146 \quad X_N = X_{1N} - X_{2N} \quad (30)$$

147 Substituting Eqs. (29)-(30) into Eq. (26) yields:

$$148 \quad u_c^* = X_{2N} + 2 \left( 1 - \sqrt{h_c^*} \right) \quad (31)$$

149 Equation (26) or (31) can be used to calculate the dimensionless velocity. Similarly, the  
 150 dimensionless discharge is obtained by combining Eqs. (29) and (31):

151 
$$Q_c^* = \frac{Q_c}{A_u \sqrt{gh_u}} = \frac{h_c}{h_u} \cdot \frac{u_c}{\sqrt{gh_u}} = h_c^* \left[ X_{2N} + 2 \left( 1 - \sqrt{h_c^*} \right) \right] \quad (32)$$

152 Substituting Eq. (31) into Eq. (9) yields the dimensionless wave-front celerity:

153 
$$\xi^* = \frac{\xi}{\sqrt{gh_u}} = h_c^* \cdot \frac{X_{2N} + 2 \left( 1 - \sqrt{h_c^*} \right)}{h_c^* - h_d^*} \quad (33)$$

## 154 **Dam-break wave in slope and frictionless channel**

155 To verify the accuracy of the proposed analytical solution, numerical simulations by the LBM are  
156 carried out correspondingly.

### 157 *Lattice Boltzmann model for shallow water equations*

158 In this study, the most common Lattice Boltzmann Model (D2Q9) is adopted and the lattice  
159 Boltzmann equation for 2D shallow water equations reads (Zhou 2004):

160 
$$f_\alpha(\mathbf{x} + e_\alpha \Delta t, t + \Delta t) - f_\alpha(\mathbf{x}, t) = -\frac{1}{\tau} \left[ f_\alpha(\mathbf{x}, t) - f_\alpha^{eq}(\mathbf{x}, t) \right] + \Delta t F_\alpha \quad (34)$$

161 where  $f_\alpha$  and  $f_\alpha^{eq}$  = the distribution functions,  $\tau$  = the single relaxation time,  $e_\alpha$  = the particle  
162 velocity,  $F_\alpha$  = force term as defined by Peng et al. (2011a, b).

163 The fluid kinematic viscosity  $\nu$  is defined as:

164 
$$\nu = \frac{e^2 \Delta t}{6} (2\tau - 1) \quad (35)$$

165 The local equilibrium distribution function  $f_\alpha^{eq}$  in Eq. (34) is defined as:

166 
$$f_\alpha^{eq} = \begin{cases} h - \frac{5gh^2}{6e^2} - \frac{2h}{3e^2} u_i u_i, & \alpha = 0 \\ \frac{gh^2}{6e^2} + \frac{h}{3e^2} e_{\alpha i} u_i + \frac{h}{2e^4} e_{\alpha i} e_{\alpha j} u_i u_j - \frac{h}{6e^2} u_i u_i, & \alpha = 1, 3, 5, 7 \\ \frac{gh^2}{24e^2} + \frac{h}{12e^2} e_{\alpha i} u_i + \frac{h}{8e^4} e_{\alpha i} e_{\alpha j} u_i u_j - \frac{h}{24e^2} u_i u_i, & \alpha = 2, 4, 6, 8 \end{cases} \quad (36)$$

167 The macroscopic water depth and velocity can be obtained from the following equations:

168 
$$h = \sum_{\alpha} f_{\alpha} \quad (37)$$



169

$$u_i = \frac{1}{h} \sum_{\alpha} e_{\alpha i} f_{\alpha} \quad (38)$$

170 More details on LBM for shallow water equations can be found in Zhou (2004) and Peng et al.  
171 (2011a, b).

### 172 ***Comparison with simulations by LBM***

173 Three smooth, rectangular flumes are 60 m long, 1m wide, and 1.2 m high with bottom slopes of 0.1,  
174 0.3, and 0.5% respectively are used in experiments. A virtual dam located 30 m away from the  
175 downstream flume end is adopted to simulate an instantaneous dam failure. Three wet-bed conditions  
176 are tested:  $h_d = 0.12, 0.24$  and  $0.36$  m, corresponding to  $h_d^* = 0.2, 0.4$  and  $0.6$ . Due to length  
177 limitation, only the simulated results for the case with  $h_d^* = 0.2$  are shown in Figs. 2 as the results of  
178 other two cases are similar.

179 Figure 2 compares the water surface profiles predicted by the proposed analytical method and  
180 LBM for  $t = 0.5, 1.5$  and  $3.0$  s. Generally the water surface profile by the proposed analytical method  
181 agrees well with the prediction by LBM. In Zone 2, the water depth simulated by LBM is slightly  
182 smaller than the analytical solution. Both the front and the tail of the rarefaction wave captured from  
183 LBM propagate faster than those from the analytical model, resulting in an extension of rarefaction  
184 fans. In Zone 3, the water surface profiles predicted by LBM are under the analytical results.  
185 Compared with the numerical results, the evolution position of the shock wave in the analytical  
186 solution lags slightly and the difference between the analytical and LBM solution is smaller than  
187 10% for all of three slopes during the first ten seconds. This difference may be due to the finite  
188 difference method adopted by the LBM model.

189 The discharges along the flume predicted by the proposed analytical method versus LBM are  
190 also shown in Fig. 2. It can be seen that the discharge increases as bottom slope of the flume  
191 increases. With the increase of downstream water depth, the flow discharge in Zone 3 generally  
192 decreases. The analytical solution captures the motion characteristics well. Based on the comparisons

193 between analytical solutions and numerical results, it is clear that the analytical and simulated results  
194 agree well in both Zone 1 and Zone 2, while the numerical results are slightly higher than the  
195 analytical solutions in both Zone 3 and Zone 4, and the difference between them tends to increase  
196 with time. Especially in Zone 4, the analytical model assumes that the water body is not affected by  
197 the shock wave. Therefore, the water is stationary, and the corresponding discharge is zero. When the  
198 downstream water depth is small (e.g.  $h_d^* = 0.2$ ), the analytical solution of discharge in Zone 3 agrees  
199 well with the numerical results. That demonstrates that the proposed analytical method can be  
200 applied to accurately predict the flood wave propagation generated by dam-break along a sloping and  
201 initially wet downstream bed.

## 202 **Conclusions**

203 In the present study, a new analytical solution, based on the method of characteristics, has been  
204 developed for the shallow-water equations (SWE) which can be used to validate numerical models  
205 based on the one-dimensional SWE as long as the term of the frictional slope is neglected. The  
206 analytical solution accurately predicts the effects of bottom slope and initial downstream water depth  
207 on the propagation of a flood wave generated by the dam-break, which is difficult to be achieved by  
208 previously analytical solutions. The propagation of the dam-break flood waves on sloping and wet  
209 beds predicted by the analytical model was compared with the numerical simulations based on the  
210 LBM. A satisfactory agreement between the analytical and numerical solutions is found in both  
211 Zones 1 and 2, while mild distinction exists in both Zones 3 and 4.

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## 219 **Notation**

220 *The following symbols are used in this paper:*

221  $A$  = cross-sectional area;

222  $B$  = water surface width;

223  $e$  = particle velocity;

224  $f$  = distribution function of particle;

225  $F$  = force term;

226  $g$  = gravity acceleration;

227  $h$  = flow depth;

228  $\bar{h}$  = centroid water depth;

229  $Q$  = flow discharge;

230  $S_o$  = bottom slope;

231  $t$  = time

232  $\Delta t$  = time space

233  $\tau$  = single relaxation time

234  $u$  = average flow velocity;

235  $\nu$  = fluid kinematic viscosity

236  $X = X_1 - X_2$ ;

237  $X_1$  = dimensionless distance;

238  $X_2$  = dimensionless variable accounting for bed slope effect;

239  $x$  = distance along flow direction originated from dam site;

240  $\xi$  = shock wave-front celerity.

## 241 **Superscripts**

242  $eq$  = equilibrium ;

243 \* = dimensionless quantity.

## 244 **Subscripts**

245  $\alpha$  = direction of lattice;

246  $c$  = reach upstream of the shock;

247  $d$  = reach downstream of the shock;

248  $i$  = direction of velocity or force;

249  $j$  = direction of velocity or force;

250  $M$  = position of rarefaction wave front;

251  $N$  = position of rarefaction wave tail;

252  $R$  = location of shock wave-front; and

253  $u$  = undisturbed upstream reach.

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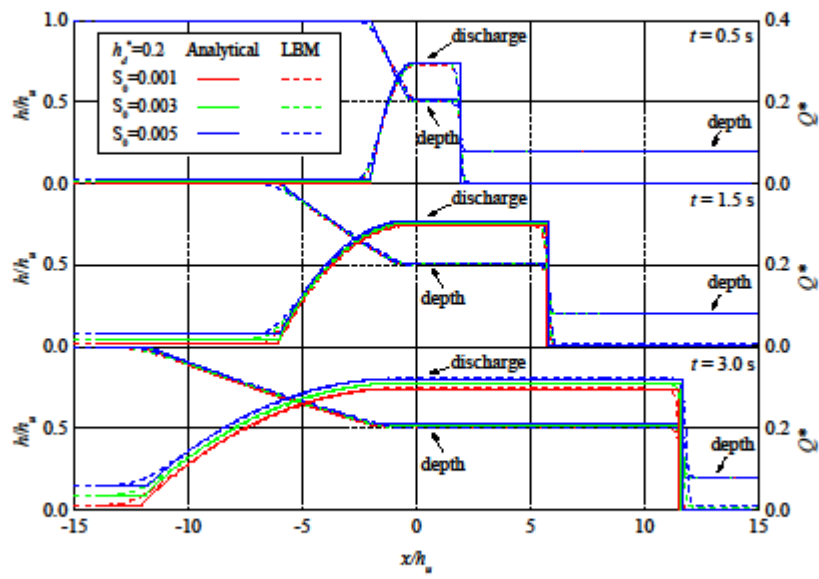


Figure 2. Profiles of dimensionless depth and discharge for  $h_j^* = 0.2$ .

298

299