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Effectiveness Factor of Self-Compacting Concrete in Compression for Limit Analysis of Continuous Deep Beams

Mahmoud Khatab and Ashraf Ashour

Abstract

The current design codes, such as ACI 318-14, EC2 and CSA23.3-04, in addition to previous research investigations suggested different expressions for concrete effectiveness factor for use in limit state design of concrete structures. All these equations are based on different design parameters and proposed for normal concrete deep beams. This research evaluates the use of different effectiveness factor equations in the upper and lower bond analyses of continuously-supported self-compacting concrete (SCC) deep beams. Moreover, a new effectiveness factor expression is suggested to be used for upper and lower bound solutions with the aim of improving predictions of the load capacity of continuously-supported SCC deep beams. For the range of deep beams considered, the strut-and-tie method with the proposed effectiveness factor formula achieved accurate predictions, with a mean of 1.01, a standard deviation of 6.7% and a coefficient of variation of 6.8%. For the upper-bound analysis, the predictions of the proposed effectiveness factor equation were more accurate than those of the formulas suggested by previous investigations. Overall, although the proposed effectiveness factor achieved very accurate predictions, further validation for the proposed formula is needed since the only data available on continuous SCC deep beams are those collected form the current study.

1 Introduction

Reinforced concrete deep beams are used in construction as load distribution members that receive a relatively high number of small loads, which are transferred to a very limited number of reaction points. They can be found in different civil engineering applications such as stores, hotels, offshore structures, theatres and many others. Although continuously-supported deep beams are commonly used in construction rather than simply-supported ones, all previous investigations have been conducted on simply-supported self-compacting concrete (SCC) beams [1-6]. In contrast, there are no research investigations on continuous reinforced SCC deep beams. This area of research is of special interest due to the high depth of deep beams and congested steel reinforcement, making it difficult for normal concrete (NC) to be properly placed and vibrated. SCC requires no vibration as it can easily flow and be placed under its self-weight with excellent surface finishes and homogenous distribution of concrete within the formwork, to the advantage of durability. However, the lower amount and smaller size of coarse aggregate used in SCC lead to more brittle behaviour and lower shear resistance as cracks can propagate further through the paste or mortar phase before stopped or diverted by a coarse aggregate particle, i.e. less contribution from aggregate interlock[1-6].

The current design codes, namely the ACI Building Code (ACI 318-14)[7], Euro Code 2 (EC2)[8] and Canadian Standard for the Design of Concrete Structures (CSA23.3-04)[9] classify deep beams as a discontinuity region in which the strain distribution is nonlinear. In this case, the classical theory of elasticity is only valid to describe the behaviour of deep beams before cracking. After cracking, however, major redistribution of stresses takes place and the elasticity theory becomes inapplicable[10, 11]. Therefore, the current design codes suggest that deep beams should be designed either by nonlinear analysis in which the nonlinear strain and stresses distributions are accounted for or by limit analysis, for example the strut-and-tie model (STM) and mechanism analysis. On the other hand, a number of researchers[12-14] developed a mechanism analysis based on the upper-bound theorem of the plasticity theory to predict the shear strength of deep beams.

The plasticity theory for rigid plastic structures mainly comprises three fundamental theorems, namely the lower-bound, upper-bound and uniqueness theorems. The lower-bound theorem can be developed by considering a safe and statically admissible stress distribution on or within the yield criteria[10, 11]. The load obtained by considering equilibrium of internal and external forces of such stress distribution, satisfying the static boundary conditions is lower than or at most equal to the collapse load. On the other hand, the upper bound theorem can be derived by considering a kinematically admissible failure mechanism and the load calculated from the energy principle is higher than the collapse load[10, 11]. The lower-bound analysis requires finding a load path to transfer forces from the load point to supports[11, 15]. However, for complicated loading conditions, it is easier to develop an upper-bound analysis as it just requires a geometrically admissible failure mechanism [13, 15]. The uniqueness theorem can be obtained by satisfying the two aforementioned theorems at the same time [10].

Applying the plasticity theory to reinforced concrete structures requires modifying the compressive strength of concrete by a reduction factor, called the effectiveness factor, v. This factor is introduced to overcome the shortcomings of applying the plasticity theory to concrete structures and account for the limited ductility of concrete[13]. It is also considered to account for the compressive strength reduction due to transverse tensile stresses or transverse reinforcement in tension. A number of studies [12, 13, 16] showed a good correlation between the plasticity analyses of reinforced concrete structures and experimental results when the compressive strength of concrete was reduced by an effectiveness factor.

The main aim of this paper is to evaluate the applicability of both the strut-and-tie model recommended by different design codes and the mechanism analysis proposed by Ashour and Morley [13] to continuously-supported SCC deep beams. The predictions from the two approaches are assessed using different effectiveness factor formulas available in the literature. Moreover, a new formula for the effectiveness factor is proposed for the lower and upper bound solutions in order to achieve more accurate predictions.

2 Experimental program overview

The experimental results of eight continuous SCC deep beams reported by the authors in a previous investigation[17] are used to examine the applicability of the design methods available for NC deep beams to predict the capacity of continuously-supported SCC deep beams. The overall geometrical dimensions along with the reinforcement details for all specimens are presented in Table 1, Figure 1 and Figure 2. The specimens were tested under a symmetrical two-point loading system, using a loading frame of a capacity of 2500 kN.

Table 1: Geometrical dimensions and reinforcement details of the beams tested by Khatab et al.[17]

Beam no.	*h* mm	*d* mm	*L* mm	Longitudinal reinforcement ratio (%)		Web reinforcement ratio (%)	
				Bottom	Top	Vertical	Horizontal
B1	600	560	2750	0.67	0.67	-	0.3
B2				0.67	0.67	0.3	-
B3				0.67	0.67	0.3	0.3
B4				0.67	0.67	0.3	0.6
B5				0.67	0.67	0.6	0.3
B6				1.10	1.10	0.3	0.3
B7	300	260		1.42	1.42	0.3	-
B8				2.37	2.37	0.3	-

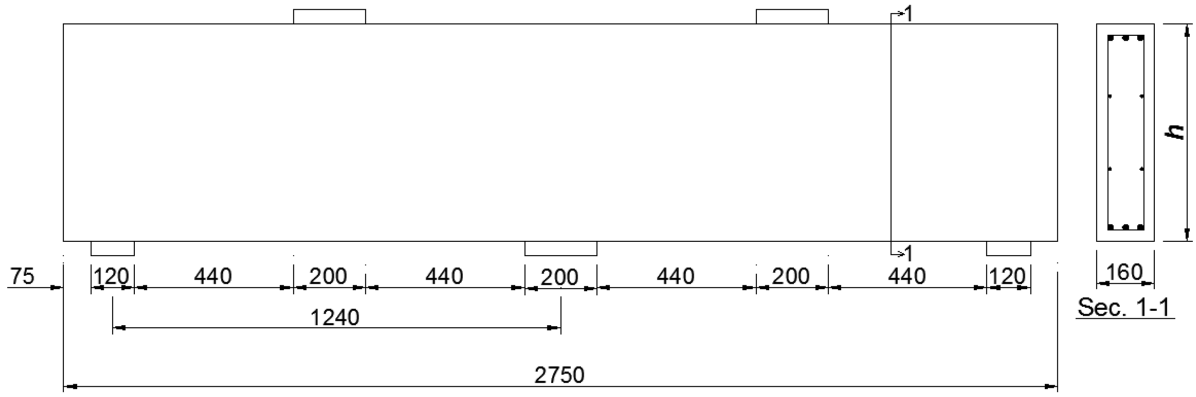

Fig. 1: Geometrical dimensions of the beams tested by Khatab et al.[17] (Dimensions in mm)

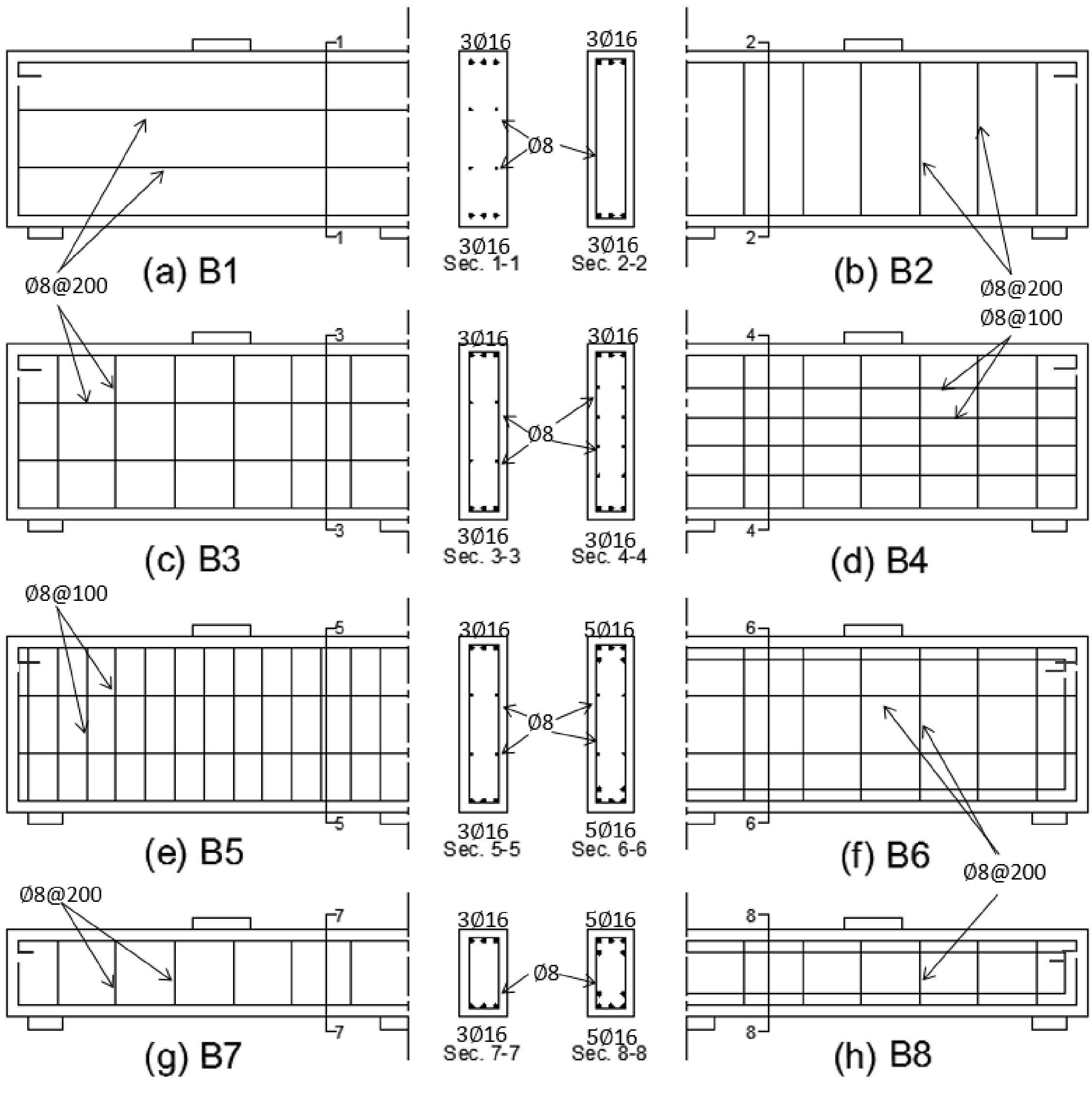

Fig. 2: Reinforcement details of beams tested by Khatab et al.[17]

The test specimens were made of SCC concrete having a cylinder compressive strength ranged between 31.1 MPa and 50.4 MPa. All the tested beams failed in shear due to a major diagonal crack in the internal shear span started at the mid-depth of the beam and extended along the distance between the edges of the load and intermediate support plates. The significant diagonal crack separated the beam into two concrete blocks: one rotated about the exterior support while the other was fixed over the other two supports. This failure mode was similar to that reported for NC continuously-supported deep beams [13, 14, 15]. The tested beams achieved different load capacities depending on their geometrical dimensions, reinforcement arrangement and concrete compressive strength. The results of the cylinder compressive strength, the maximum shear force and the total failure load for each beam are presented in Table 2.

Table 2: Results of cylinder compressive strength, maximum shear force and total failure load

Beam no.	f_c MPa	V kN	P_t kN
B1	31.1	352.3	1295
B2	42.5	464.3	1674
B3	36.0	377.2	1358
B4	46.0	539	1861
B5	47.8	588.5	1988
B6	50.4	543.5	1940
B7	32.0	181.8	579
B8	38.6	197.5	676

Note: f_c=compressive strength of concrete, V= maximum shear force, P_t = total failure load.

3 Effectiveness factor of concrete

The effectiveness factor of concrete, v, is introduced to overcome the shortcomings of applying the plasticity theorem to reinforced concrete, mainly to account for the limited ductility of concrete [10, 16]. The effectiveness factor proposed in the literature mainly depends on concrete properties, geometrical dimensions and reinforcement details [10-16]. There is disagreement among different codes of practice on the value of the effectiveness factor, as shown in Table 3. The ACI 318-14 [7] bases the value of the effectiveness factor on the amount of vertical and horizontal web reinforcement. This means that if the amount of web reinforcement satisfies the requirements presented in Table 3, the value of v is 0.64, otherwise v equals to 0.51. However, the value of the effectiveness factor suggested by the EC2 [8] depends on only concrete compressive strength. On the other hand, the Canadian Standard (CSA23.3-04)[9] recommends a value for the effectiveness factor based on the principal tensile strain of steel reinforcement (ε_1) and the angle between the tie and strut (θ). The value of the principal tensile strain can be approximated as ($\varepsilon_1 = \varepsilon_s + (\varepsilon_s + 0.002)/(\tan\theta)^2$), where ε_s is the tensile strain in the ties. For the purpose of design, ε_s can be considered as the yield strain of the steel reinforcement which was obtained by conducting a tensile test on the steel bars used in test specimens. On the other hand, the angle between the strut and tie depends on the a/d ratio ($\tan\theta = d/a$). In the current study, all the beams tested had the same type of reinforcement which means that the value of the tensile strain is constant for all beams while the value of the a/d ratio is different. As a result, the value of the effectiveness factor according to the Canadian Standard can be calculated based on the value of the a/d ratio as shown in Table 3.

Table 3: Effectiveness factor v according to different design codes

Reference	Effectiveness factor	Notes
ACI 318-14 [7]	$v = 0.85\beta_s$	$\beta_s = 0.75$ if: $\sum \frac{A_{si}}{bs_i} \sin \propto_i \geq 0.003$ $\beta_s = 0.6$ Otherwise
EC2 [8]	$v = 0.6\left(1 - \frac{f_c}{250}\right)$	
CSA23.3-04 [9]	$v = \frac{1}{1.20 + 0.74(a/d)^2}$	≤ 0.85

Note: β_s is a factor to account for the effect of cracking and confining reinforcement on the effective compressive strength of concrete in a strut, A_{si} is the area of surface reinforcement crossing the strut, s_i is the spacing between the surface reinforcement bars crossing the strut, b is the beam web width, $\propto_i$ is the angle between the axis of strut and the surface reinforcing bars crossing the strut, a is the shear span and d is the effective depth of the beam.

On the other hand, a large number of research investigations suggested different formulas for v. As shown in Table 4, three equations for v were selected to be used in the analysis presented in this paper. The selection of these formulas was based on the accuracy of the predictions compared to the experimental results in previous investigations. As can be clearly seen from Table 4, the three selected formulas were based on different material and geometrical properties. Neilsen [16] proposed a formula for v based on the value of f_c. The value of v resulting from this formula ranges from 0.3 to 0.8 for a concrete strength up 100 MPa. However, Vecchio and Collins [18] considered v as a function of concrete strength and principal tensile and compressive strains in steel reinforcement. This formula was modified by Yang and Ashour [15] to reflect the size effect as shown in Table 4. It should be noted that this formula was proposed for the upper-bound analysis and it results in low effectiveness factor values. Another formula was proposed by Warwick and Foster [19], which considers the effect of a/d ratio in addition to the concrete strength with an upper limit of 0.85.

Table 4: The value of the effectiveness factor v according to previous studies

Reference	**Effectiveness factor**	**Notes**
Nielsen [16]	$v = 0.8 - \frac{f_c}{200}$	
Yang and Ashour [15]	$v = \frac{\xi}{1.0 + k_c k_f}$	$\xi = \frac{1}{\sqrt{1 + \frac{d}{25d_a}}}$ $k_c = \frac{1 + sin\alpha}{1 - sin\alpha} \geq 1.0$ $k_f = 0.1825\sqrt{f_c'} \geq 1.0$
Warwick and Foster [19]	$v = 1.25 - \frac{f_c}{500} - 0.72\left(\frac{a}{d}\right) + 0.18\left(\frac{a}{d}\right)^2$	≤ 0.85

Note: f_c is the cylinder compressive strength of concrete, f_y is the yield strength of steel reinforcement, ξ is the size effect factor, d is the beam effective depth, d_a is the maximum size of aggregate, a is the shear span, α is the angle between the relative displacement δ_c and the yield line chord as shown in Figure 4.3.

Figures 3 and 4 show the effect of concrete compressive and shear span-to-depth ratio on the value of the effectiveness factor using the expressions considered in the current investigation.

It can be seen that the value of the effectiveness factor can significantly change with changing these parameters. This is can be attributed to the fact that most of the effectiveness factor equations are based on rational background and relies on one parameter only. For example, adopting the empirical value proposed by Nielsen [16] has several shortcomings. Initially, it suggests that the only design parameter affecting the behaviour and strength of the compression strut is the concrete compressive strength. Moreover, for high strength concrete (more than 90 MPa), the effectiveness factor value would drop to 0.3 which leads to unreasonably conservative results. On the other hand, the ACI 318-14 suggested two constant values for the effectiveness factor depending on the shear reinforcement ratio. In this case, the value of the effectiveness factor is assumed to be independent from other parameters such as compressive strength and a/d ratio. However, the effectiveness factor was shown to be dependent on a number of parameters such as a/d ratio and compressive strength [20]. Moreover, the current expressions for the effectiveness factor that relies on the compressive strength are unreasonably conservative when applied to high strength concrete. Similar observation can be noted for the equations that are based on the a/d ratio. It can be concluded that the effectiveness factor must be a function of more than one parameter to represent the effect of the different design parameters on the strength of the concrete strut.

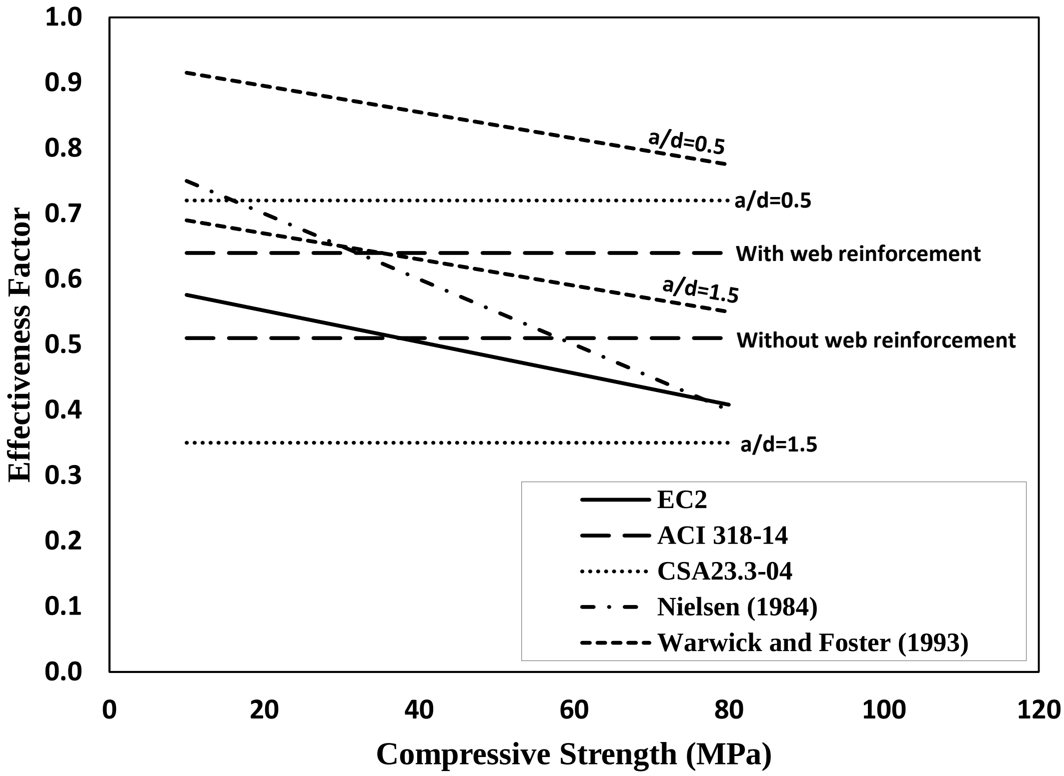

Fig. 3: Effect of compressive strength on the effectiveness factor

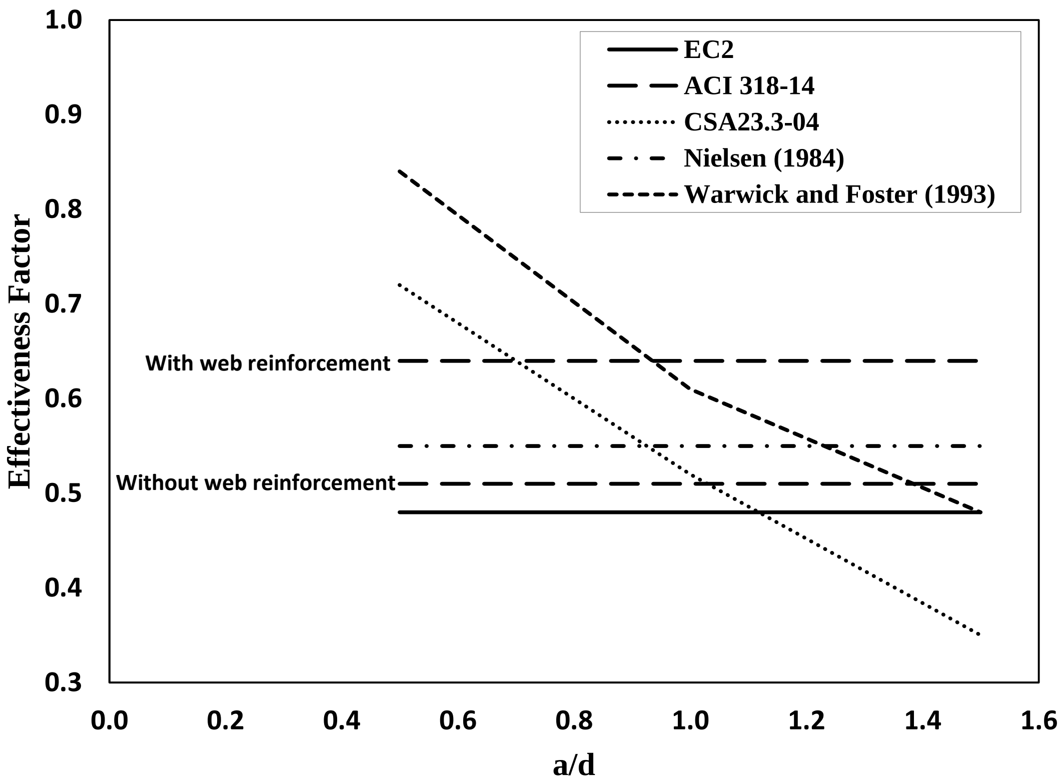

Fig. 4: Effect of shear span-to-depth ratio on the effectiveness factor

4 Strut-and-tie model in current design codes

The current design codes suggest that deep beams shall be designed using either nonlinear analysis or strut-and-tie model (STM). In this section, comparisons between the experimental results and the STM suggested by different design codes are carried out, namely the ACI Building Code (318-14)[7] , the Euro Code 2 (EC2)[8] and the Canadian Standard for the Design of Concrete Structures (CSA23.3-04)[9]. The main aim is to assess the validity of the STM, proposed for NC deep beams, for predicting the load capacity of SCC continuous deep beams. The STM will be used to predict the load capacity of continuous SCC deep beams using the values of the effectiveness factor presented in Tables 3 and 4.

The total applied load is estimated by using a set of equations based on a simple STM shown in Figure 5. For two spans continuous deep beams, the total load, P_t, due to the failure of concrete struts can be determined from equations (1) to (4) below:

$$P_t = 2 v f_c b \left[w_{ES} + w_{IS} \right] Sin\ (\theta) \qquad (1)$$

$$w_{ES} = w_t\ Cos\ (\theta) + \frac{[l_{EP} + 0.5 l_{LP}]}{2}\ Sin\ (\theta) \qquad (2)$$

$$w_{IS} = w_t\ Cos\ (\theta) + \frac{[\ l_{LP} + l_{IP}]}{4} Sin\ (\theta) \qquad (3)$$

$$\theta = tan^{-1} \frac{(h - c - c')}{a} \qquad (4)$$

where v is the effectiveness factor of concrete, f_c is the cylinder compressive strength of concrete, b is the beam width, w_{ES} is the average effective width of the exterior concrete strut, w_{IS} is the average effective width of the interior concrete strut, θ is the angle between the concrete strut and the longitudinal axis of the beam, l_{EP} is the width of the exterior bearing plate, l_{IP} is the width of the interior bearing plate, l_{LP} is the width of the load bearing plate, h is the total height of the beam, c and c' are the concrete covers of the bottom and top longitudinal reinforcement, respectively, a is the shear span and w_t is the effective tie width which equals twice the concrete cover ($w_t = 2c$).

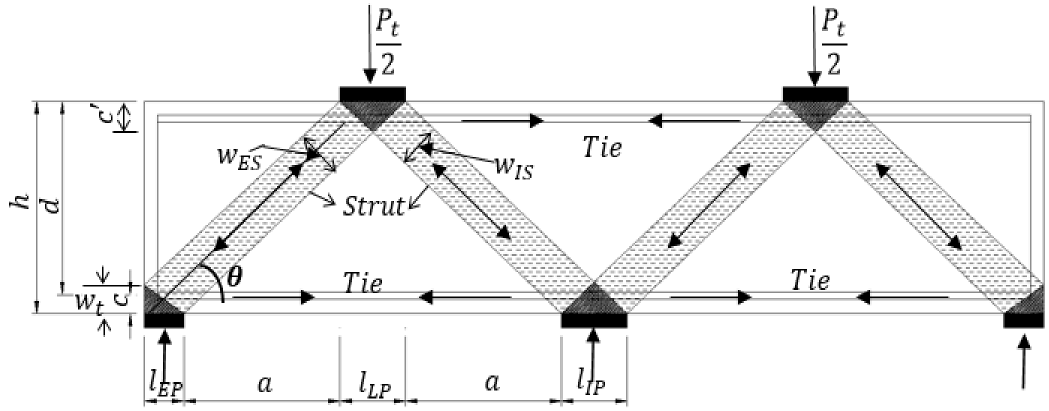

Fig. 5: Schematic STM for continuous deep beams

In the above equations, the effectiveness factor of concrete is the only difference among the three design codes considered in the current comparison. Each design code suggests a different formula for the effectiveness factor as presented in Table 3. Moreover, a number of researchers suggested different values for the effectiveness factor. Some of these values were selected to be used in this analysis as it was shown in Table 4.

Table 5 and Figure 6 show the comparisons between the experimental results and those predicted by the STM using effectiveness factor formulas suggested by different design codes whereas Table 6 and Figure 7 show similar comparisons but for effectiveness factor formulas proposed by different researchers. It should be noted that the effectiveness factor proposed by Yang and Ashour [15] was excluded from the comparisons as it was proposed for the mechanism analysis of the upper-bound theorem and it results in very low predictions compared to the experimental results. Overall, the comparisons showed that the ACI prediction was the closest to the experimental results with a mean of 1.15, a standard deviation of 4.1% and a coefficient of variation of 3.6%. The predictions of all the considered codes were conservative for all the SCC beams tested. Moreover, the predictions of the

Canadian Code underestimate the results of SCC beams, specifically those having high shear span-to-depth ratio. Furthermore, the effectiveness factor formulas proposed by Nielsen [16] and Warwick and Foster [18] also resulted in conservative predictions and the results were less accurate than those predicted by the ACI code. It can be concluded that the available formulas for the effectiveness factor resulted in highly conservative predictions. Therefore, a proposed effectiveness factor for SCC is needed with the aim of achieving more accurate predictions.

Table 5: Comparisons between test results and predictions of STM using effectiveness factors suggested by different design codes

Beam no.	P_{EXP}	P_{ACI}	P_{EC2}	P_{CSA}	P_{EXP}/P_{ACI}	P_{EXP}/P_{EC2}	P_{EXP}/P_{CSA}
B1	1295	1074	885	877	1.21	1.46	1.48
B2	1674	1466	1145	1197	1.14	1.46	1.40
B3	1358	1243	1001	1015	1.09	1.36	1.34
B4	1861	1587	1219	1296	1.17	1.53	1.44
B5	1988	1650	1256	1348	1.20	1.58	1.47
B6	1940	1739	1307	1420	1.12	1.48	1.37
B7	579	500	410	165	1.16	1.41	3.51
B8	676	602	480	198	1.12	1.41	3.41
Mean					1.15	1.46	1.93
Standard deviation (%)					4.1	7.2	95
Coefficient of variation (%)					3.6	4.9	49.2

Table 6: Comparisons between test results and predictions of STM using effectiveness factors suggested by different researchers

Beam no.	P_{EXP}	P_1	P_2	P_{EXP}/P_1	P_{EXP}/P_2
B1	1295	923	1049	1.40	1.23
B2	1674	1149	1380	1.46	1.21
B3	1358	1028	1196	1.32	1.14
B4	1861	1207	1477	1.54	1.26
B5	1988	1234	1526	1.61	1.30
B6	1940	1271	1594	1.53	1.22
B7	579	427	379	1.36	1.53
B8	676	488	444	1.39	1.52
Mean				1.45	1.30
Standard deviation (%)				10	15
Coefficient of variation (%)				7.0	11

Note: P_1 and P_2 are the total loads predicted by the STM using the effectiveness factor formula proposed by Nielsen [16] and Warwick and Foster [19], respectively.

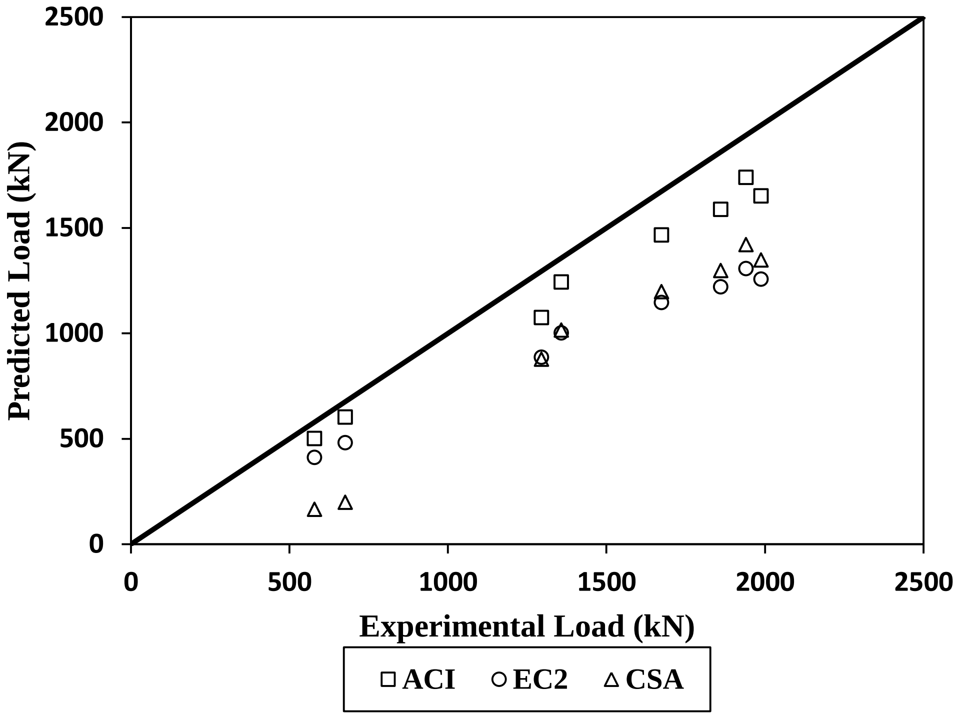

Fig. 6: Comparisons between experimental results of SCC continuous deep beams and predictions of STM using effectiveness factors suggested by different design codes

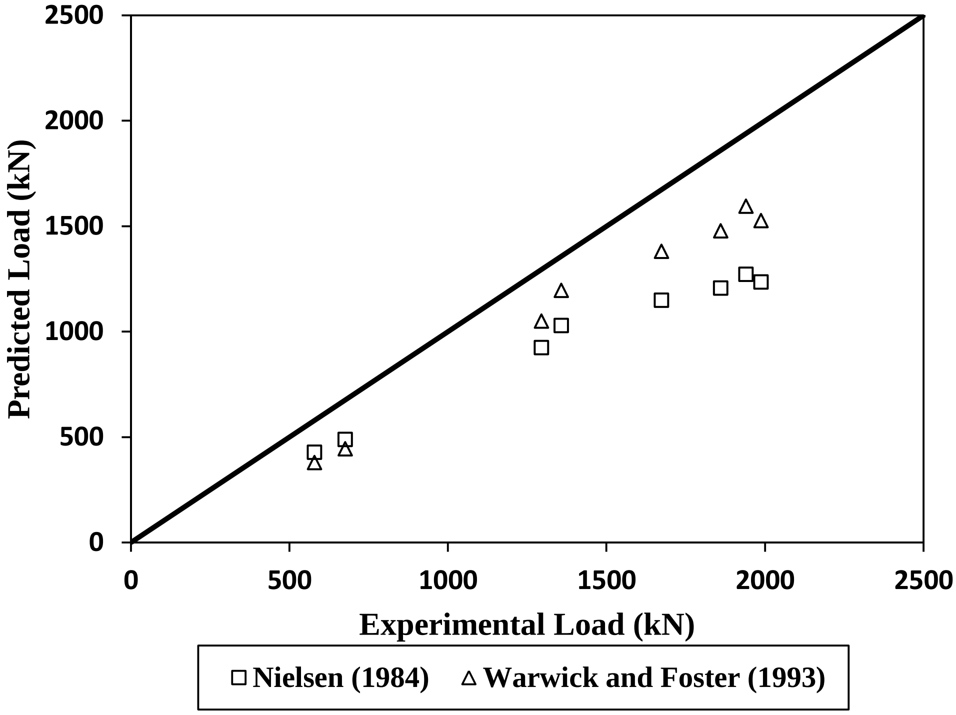

Fig. 7: Comparisons between experimental results of SCC continuous deep beams and predictions of STM using effectiveness factors suggested by different researchers

5 Upper-bound analysis

Previous experimental studies on NC continuous deep beams[13, 21-24] in addition to the experimental investigation on SCC continuous deep beams reported by the authors in a previous study [17] showed that the main cause of failure in continuous deep beams is a major diagonal crack formed between the applied mid-span load and the intermediate support separating the beam into two blocks: the first one rotated around the end support, leaving the rest of the beam fixed on the other two supports. In this section, the mechanism analysis proposed by Ashour and Morley[13] is used to predict the load capacity of continuously-supported SCC deep beams. In their study, the yield line is used to represent the diagonal crack for the mechanism of failure described above as shown in Figure 8 and the continuous deep beam is assumed to be in a state of plane stresses. The concrete is modelled as a rigid perfectly plastic material obeying the modified Coulomb failure criteria with zero tension cutoff while the steel reinforcement is assumed to be a rigid perfectly plastic material in tension and compression having a yield strength, f_y, and carrying only longitudinal tensile and compressive stresses. As the resistance of concrete is very weak in tension compared to compression and the ductility of concrete in tension is very limited, the tensile strength of concrete is not taken into account in this analysis [13].

In the mechanism analysis of the upper-bound theorem, the total load carrying capacity, P_t, of a two-span continuous deep beam can be calculated by equating the total internal energy to the external energy. The total internal energy can be calculated by adding the energy dissipated by concrete along the yield line to the energy in the steel reinforcing bars (longitudinal, vertical and horizontal) crossing the yield line as shown in Figures 8 and 9 while the total external energy is the work done by the total applied load. Therefore, the total load capacity, P_t, can be found from equation (5) below:

$$P_t = \frac{b}{a}\left[\frac{f_{ce} h r_c \,(1 - sin\alpha)}{sin\beta} + 2\sum_{i=1}^{n} (r_s)_i \,(A_s)_i (f_y)_i cos(\alpha_s)_i\right] \qquad (5)$$

where b is the beam width, a is the shear span of the beam measured from the centre of the support to the point of the applied load, f_{ce} is the effective compressive strength of concrete, h is the beam total depth, r_c is the distance from the instantaneous centre to the middle point of the yield line chord, α is the angle between the relative displacement δ_c and the yield line chord, β is the inclination of the yield line chord, n is the number of reinforcing bars crossing the yield line, $(r_s)_i$ is the distance between the instantaneous centre to the point where the reinforcing bar i crosses the yield line, $(A_s)_i$ and $(f_y)_i$ are the area and yield strength of the reinforcing bar i crossing the yield line, ω is the rotation of the rigid block i (See Figures 8 and 9) and $(\alpha_s)_i$ is the angle between the reinforcing bar i crossing the yield line and the relative displacement δ_s.

The effective compressive strength of concrete f_{ce} used in the prediction of the load capacity is calculated from equation (6) below:

$$f_{ce} = v f_c \tag{6}$$

where v is the effectiveness factor of concrete presented earlier and f_c is the cylinder compressive strength of concrete.

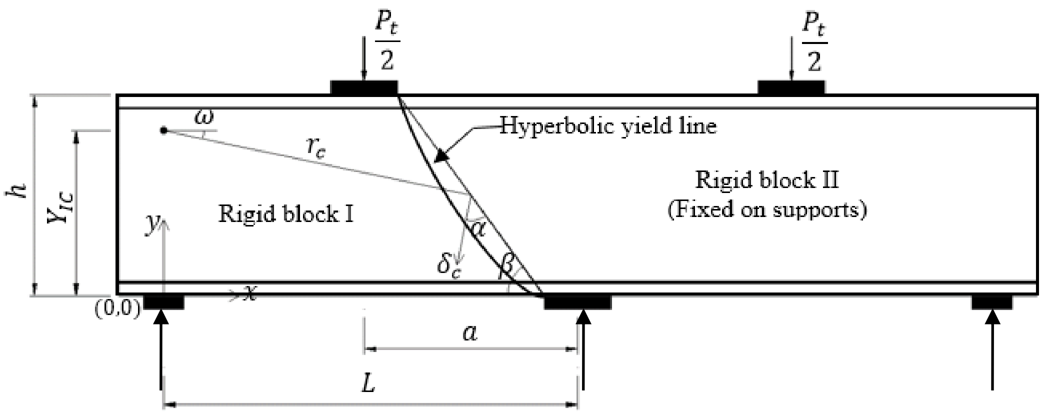

Fig. 8: Failure mechanism of two-span continuous deep beams [13]

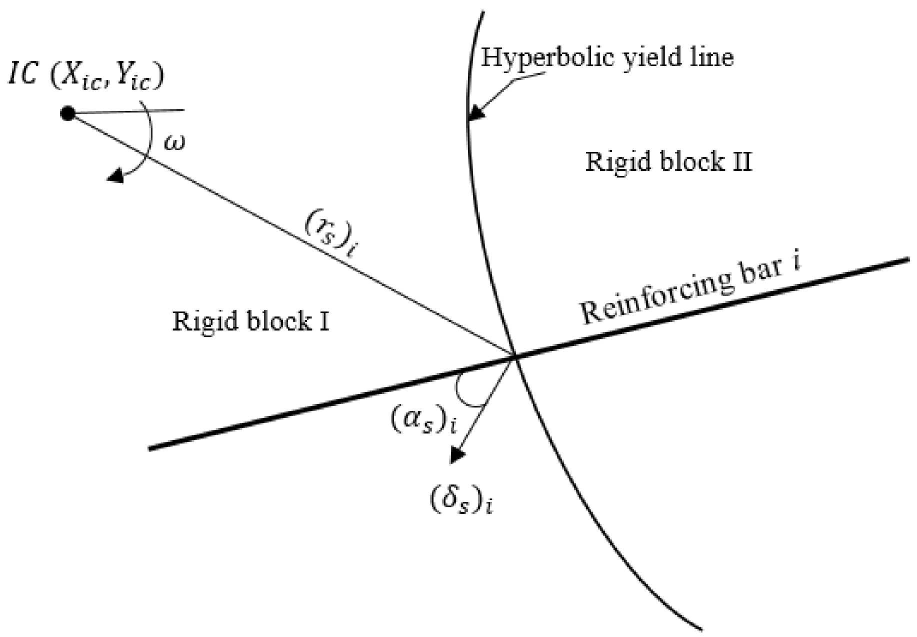

Fig. 9: Reinforcement crossing the yield line [13]

As mentioned earlier, equation (5), proposed by Ashour and Morley [13] for NC continuous deep beams, is applied in the present study to predict the total load capacity of SCC continuous deep beams using different values for the effectiveness factor of concrete. The results obtained from this analysis are presented below.

Tables 7 and 8 and Figures 10 and 11 show the comparisons between the experimental results of SCC continuous deep beams and those predicted by the upper-bound analysis for different effectiveness factor formulas collected in the current study. It can be clearly noticed that all the considered codes showed reasonable predictions for beams having web orthogonal web reinforcement only. The most accurate results were obtained by using the effectiveness factor recommended by ACI 318-14 [7] with an average of 1.03, standard deviation of 20% and a coefficient of variation of 20%. The predictions clearly underestimate the load capacity of beams having web reinforcement in one direction only (B1 and B2). Moreover, the accuracy of the load capacity predicted by the upper-bound analysis considerably decreased for beams having high shear span-to-depth ratio (B7 and B8). It can be concluded that, although the mean value for the prediction was reasonable, the variations of the results around the mean were very scattered as indicated by the large values of standard deviation and coefficient of variation. On the other hand. The results obtained using the effectiveness factor proposed by Warwick and Foster [19] were the most accurate with an average of 1.02, a standard deviation of 18% and a coefficient of variation of 17%. The predictions were more accurate for beams having orthogonal web reinforcement (B3 to B6). However, using the effectiveness factor proposed by Yang and Ashour [15] led to more accurate predictions for beams having high shear span-to-depth ratio (B7 and B8).

Table 7: Comparisons between experimental results and predictions of upper-bound analysis for different v values recommended by design codes

Beam no.	P_{EXP}	P_{ACI}	P_{EC2}	P_{CSA}	P_{EXP}/P_{ACI}	P_{EXP}/P_{EC2}	P_{EXP}/P_{CSA}

B1	1295	968	977	957	1.34	1.33	1.35
B2	1674	1347	1337	1333	1.24	1.25	1.26
B3	1358	1446	1354	1338	0.94	1.00	1.01
B4	1861	1646	1506	1509	1.13	1.24	1.23
B5	1988	1932	1782	1789	1.03	1.12	1.11
B6	1940	2016	1851	1865	0.96	1.05	1.04
B7	579	710	714	630	0.82	0.81	0.92
B8	676	903	902	806	0.75	0.75	0.84
Mean					1.03	1.07	1.10
Standard deviation (%)					20	21	18
Coefficient of variation (%)					20	19	16

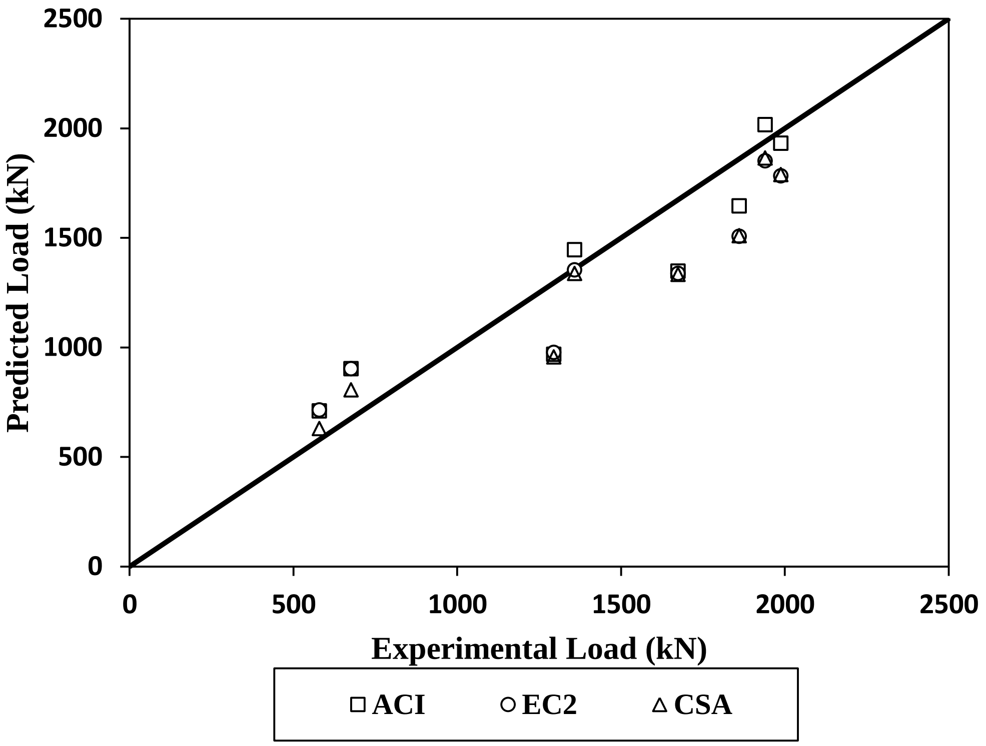

Fig. 10: Comparisons between experimental results and predictions of upper-bound analysis for different v values recommended by design codes

Table 8: Comparisons between test results and predictions of upper-bound analysis for different v values recommended by previous studies

Beam no.	P_{EXP}	P_1	P_2	P_3	P_{EXP}/P_1	P_{EXP}/P_2	P_{EXP}/P_3
B1	1295	1053	772	1027	1.23	1.68	1.26

B2	1674	1415	1063	1408	1.18	1.57	1.19
B3	1358	1431	1117	1412	0.95	1.22	0.96
B4	1861	1581	1212	1584	1.18	1.54	1.17
B5	1988	1855	1479	1864	1.07	1.34	1.07
B6	1940	1922	1535	1939	1.01	1.26	1.00
B7	579	743	625	707	0.78	0.93	0.82
B8	676	932	797	895	0.73	0.85	0.76
Mean					1.02	1.3	1.02
Standard deviation (%)					19	30	18
Coefficient of variation (%)					19	23	17

Note: P_1, P_2 and P_3 are the total loads predicted by the upper-bound analysis using the effectiveness factor formula proposed by Nielsen , Yang and Ashour [15] and Warwick and Foster (1993), respectively.

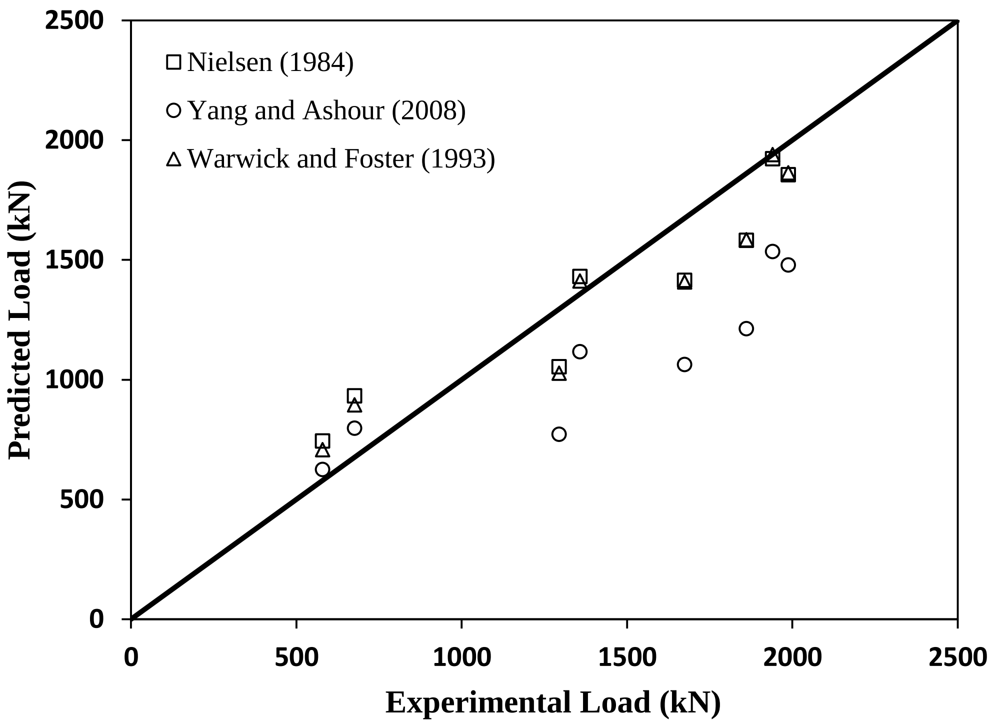

Fig. 11: Comparisons between experimental results and predictions of upper-bound analysis for different v values collected from previous studies

6 Proposed effectiveness factor for SCC

As discussed earlier, all of the effectiveness factor formulas considered in the current analysis for continuously supported SCC deep beams resulted in unreasonable predictions for all of the beams considered. Therefore, a modified value for the effectiveness factor for SCC is needed with the aim of achieving better predictions.

Based on a regression analysis of the experimental results of the beams considered in the current study, a new formula for the effectiveness factor of SCC is suggested to achieve more accurate predictions from the STM and mechanism analysis used for NC continuous deep beams. As mentioned earlier, the value of the effectiveness factor mainly depends on the material properties and size effect. Therefore, the proposed equation has been expressed in terms of concrete strength, effective depth and maximum size of coarse aggregate as shown in equation (7) below:

$$v = 0.43 + 0.6\left(1 - \frac{f_c}{250}\right)\frac{1}{\sqrt{1 + \frac{d}{25d_a}}} \tag{7}$$

As can be seen from equation (7), the proposed effectiveness factor formula was based on the effectiveness factor equation suggested by EC2. The ratio between the beam depth, d, and maximum size of aggregate, d_a, is included in the proposed equation to reflect the influence of size effect. In the plasticity theory, the size effect could not be considered because of the fact that the nominal stress at failure must be independent of size [25]. Therefore, in order to take the size effect into account, the only way is to consider it in the effectiveness factor. The maximum size of aggregate presents one of the main differences between SCC and NC as smaller size of coarse aggregate is required for SCC. In addition, it is well known that the shear strength decreases as the beam depth increases. It was proved that[25] the nominal shear stress is inversely proportional to the term $\left[\sqrt{1 + \frac{d}{d_a}}\right]$. In the proposed equation, the higher the beam depth, d, the lower is the value of the effectiveness factor, leading to lower shear strength.

The comparisons between the experimental load capacity of continuously-supported SCC deep beams and the predictions of the lower-bound and upper-bound analysis using the proposed concrete effectiveness factor equation are presented in Table 9 and Figure 12. It can be observed that the proposed concrete effectiveness formula achieved more accurate predictions for the load capacity of the beams tested than these using the previous formulas. The STM predictions were slightly more accurate than that of the mechanism analysis, achieving a mean of 1.01, a standard deviation of 6.8% and a coefficient of variation of 6.7%. However, the load capacities obtained from STM are lower than those from experiments as it is a lower bound analysis and the web reinforcement is not considered. Moreover, the predictions of the mechanism analysis highly overestimate the results of SCC beams specifically those having high shear span-to-depth ratio. However, the predictions of the mechanism analysis using the proposed equation were slightly more accurate than the predictions resulted from other effectiveness factor equations proposed in the literature. Overall, the proposed effectiveness factor achieved very accurate predictions for the range of beams considered. However, more validation for the proposed formula is needed as the current experimental data available on continuous SCC deep beams is limited.

Table 9: Comparisons between experimental results and predictions of effectiveness factor formula suggested in current study

Notation	P_{EXP} (kN)	Lower-Bound		Upper-Bound	
		P_{PRE}	P_{EXP}/P_{PRE}	P_{PRE}	P_{EXP}/P_{PRE}
B1	1295	1216	1.06	1101	1.18
B2	1674	1625	1.03	1517	1.10
B3	1358	1395	0.97	1501	0.90
B4	1861	1748	1.06	1704	1.09
B5	1988	1811	1.10	1989	1.00
B6	1940	1899	1.02	2073	0.94
B7	579	625	0.93	733	0.79
B8	676	742	0.91	930	0.73
Mean			1.01		0.97
Standard deviation (%)			6.8		16

Coefficient of variation (%)	6.7		16

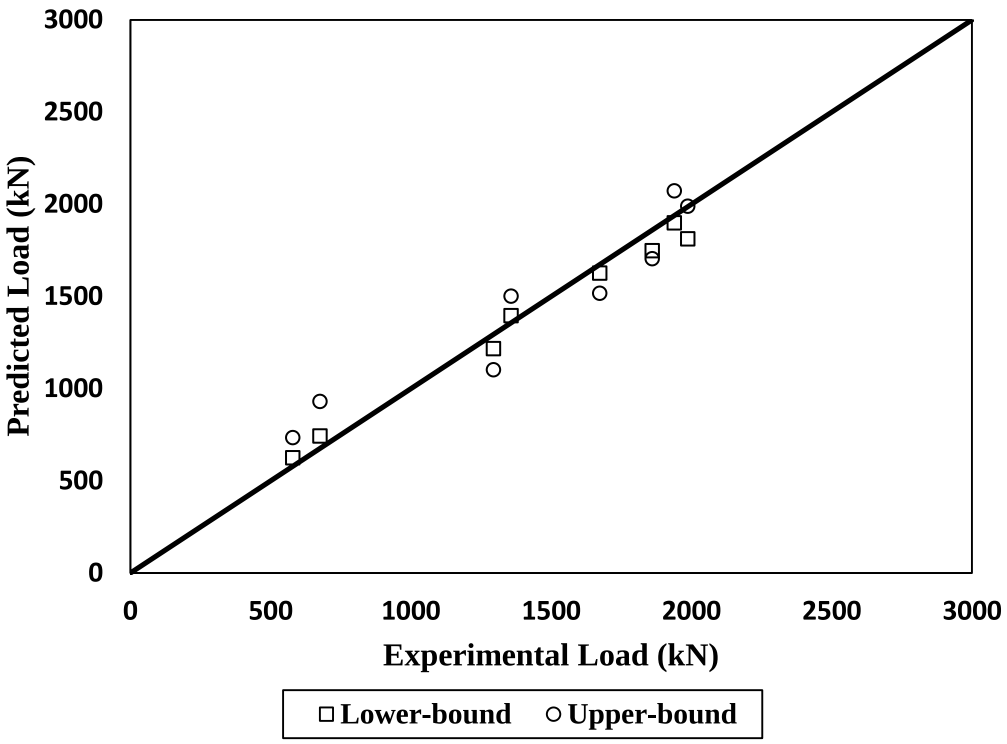

Fig. 12: Comparisons between experimental results of SCC continuous deep beams and predictions using effectiveness factor suggested in the current study

7 Conclusions

This paper presented a critical examination of different effectiveness factor equations suggested by a number of design codes of practice and previous research investigations for the prediction of the load capacity of SCC continuously-supported deep beams. The investigation was based on the use of the suggested effectiveness factor equations in the strut-and-tie model and mechanism analysis of the upper-bound theory developed earlier for NC continuously-supported deep beams. This study also suggests a new effectiveness factor equation to be used in both design methods considered. Based on analysis presented in this paper, the following conclusions are drawn:

- There is a clear disagreement among different design codes and researchers about selecting the appropriate effectiveness factor of concrete. Most of the available effectiveness factor equations are based on one parameter. However, the effectiveness factor of concrete strut seems to be significantly influenced by a combination of more than one parameter such as compressive strength, shear span-to-depth ratio and shear reinforcement ratio.

- Lower and upper bound analyses are efficient, simplified tools for predicting the load capacity of continuously-supported SCC deep beams. However, the lower-bound analysis provided less scatter predictions than the upper-bound analysis for the beams considered in the current investigation.
- For the range of beams considered, the proposed effectiveness factor formula achieved accurate predictions, especially when applied for the lower-bound analysis. For the upper-bound analysis, the predictions of the proposed effectiveness factor equation were more accurate than those suggested by previous investigations. However, the upper-bound analysis clearly underestimated the load capacity of beams having high shear span-to-depth ratio and those having shear reinforcement in one direction only.

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