Stochastic Equipment Capital Budgeting

with Technological Progress

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Abstract

We provide multi-factor real option models (and quasi-analytical solutions) for equipment capital budgeting under uncertainty, when there is either unexpected, or anticipated, or uncertain (volatile) technological progress. We calculate the threshold level of revenues and operating costs using the incumbent equipment that would justify replacement. Replacement is deferred for lower revenue thresholds. If progress is anticipated or highly uncertain, alert financial managers should wait longer before replacing equipment. Replacement deferral increases with decreases in the expected correlation between revenue and operating costs, and with increases in the revenue and/or operating cost volatility. Uncertain technological progress increases the real option value of waiting. The best approach for equipment suppliers is to reduce the expected revenue and/or cost volatility, and/or reduce the expected uncertainty of technological innovations, since then an incentive exists for the early replacement of old equipment when a technologically advanced version is launched.

EXECUTIVE SUMMARY

When should equipment be replaced, if both the incumbent's revenue and operating cost deteriorate over time (and usage) and are uncertain, and if there is technological progress in replacement equipment quality and efficiency? Is the best time for replacing the incumbent equipment altered if technological progress is highly uncertain? What is the value of the incumbent equipment including its replacement option?

We provide analytical solutions to these problems, which are easily executed in a spreadsheet format. The observable model values required for the solution are the current revenue and operating costs, their deterioration rates (and uncertainty), the expected new higher revenue and/or lower operating costs if equipment is replaced, including efficiencies due to any technological progress, and the investment cost (net of any salvage value). Many of these inputs are standard observables, while others, like expected revenue and cost volatility, technological progress uncertainty, correlation of revenue and costs, and technological advances, may require an element of managerial judgment. Given these prerequisites, equipment managers can determine, in a real time context, the current revenue and cost levels justifying equipment replacement. Further, the real value of existing equipment with a replacement option and the appropriate real depreciation rate can be evaluated as well.

In the presence of uncertainty and anticipated technological progress, we find that suppliers purely motivated by immediate sales may tend to seek out myopic financial managers using NPV, or at least those skeptical of technological progress or revenue or cost variability. The best approach for suppliers facing astute buyers with long horizons is to reduce the expected revenue and/or cost volatility, and/or reduce the expected uncertainty of technological innovations, since then an incentive exists for the early replacement of old equipment when a technologically advanced version is launched.

Stochastic Equipment Capital Budgeting with Technological Progress

1 Introduction

When a replaceable asset is installed, financial managers could assess its anticipated lifetime from a standard net present value (NPV) analysis for an infinite replacement chain, as in Lutz and Lutz (1951). This solution, though, is only strictly applicable for like-for-like replacements, but there are many assets with embedded technological progress that violate this assumption, including vehicles and aircraft with higher future fuel efficiency, robotic machine tools with greater functionality, mobile phones and computer-based products with faster and novel facilities. The presence of technological progress means that the evaluated ex-ante lifetime may not coincide with its ex-post value.

Thus, the real economic lifetime for capital equipment depends not only on its physical deterioration rate, but also on the technological progress embedded in the succeeding equipment as the incumbent suffers implied obsolescence. Since the ex-post lifetime is likely to be variable in the presence of technological progress, an evaluation using the traditional NPV method for multiple (infinite) replacements may be misleading due to its in-built assumption of an equal cycle time. Consequently, in this paper, we adopt a dynamic programming formulation for determining the optimal conditions signaling equipment replacement because that avoids a cycle time framework. This approach is applied to replaceable equipment that is subject to both revenue and operating cost deterioration and uncertainty, with technological progress that is unexpected, or anticipated or uncertain.

In the absence of uncertainty, the effect of unforeseen technological progress on the replacement policy is originally analyzed by Caplan (1940), who focuses on "premature abandonment" of old machines that "become old-fashioned" due to obsolescence, rather than just physical decline. Building on the economic lifetime models of Hotelling (1925) and Preinreich (1939), Caplan shows analytically that the consequence of unanticipated technological progress is to shorten the active life of the incumbent. When there is an unforeseen performance improvement in the succeeding asset or a more technologically advanced asset becomes available sooner than expected, the incumbent becomes prematurely obsolete. If the increase in profit potential from replacing the incumbent more than compensates the loss in recovering the original investment, then the incumbent is replaced before its ex-ante lifetime has expired. Caplan (1940) notes that real depreciation should be the first derivative of the equipment real value with respect to time as in Hotelling (1925), but that uncertainty and the degree of competition should also be considered.

Eilon, King and Hutchinson (1966) is an early example of capital equipment replacement in continuous time, with a closed-form solution for an optimal policy. Stapleton, Hemmings and Scholefield (1972) apply numerical simulation to show that if technological progress is foreseen, the optimal time between successive replacements is lengthened. Although these authors adopt a dynamic programming formulation to avoid the equal life assumption, Elton and Gruber (1976) show that an equal life policy is optimum for assets with technological improvements. However, these analyses focus on either anticipated or unanticipated technological progress, and do not provide simple operational rules for deciding the optimal conditions for replacing the incumbent.

Several authors have studied the adoption of technological innovations in a real options context, sometimes in a duopoly. Huisman and Kort (2003) assume that a new technology has a greater "efficiency" than the existing technology, and firms determine outcomes in a strategic context. Huisman and Kort (2004) use a similar approach, except that the new technology becomes available for adoption at some unknown time in the future. Tsekrekos, Shackleton and Wojakowski (2010) provide capital budgeting rules for multi-factor models of commodity prices. Armada, Kryzanowski and Pereira (2011) show the implications for investment when there may be hidden rivals. Adkins and Paxson (2011) formulate a two-factor, real-option replacement model for an asset that is subject to uncertainty in the magnitude of the input and output decay but do not allow for technological progress. Adkins and Paxson (2012) allow for technological progress, but not for revenue, cost or technological progress uncertainty.

We provide a format for some of these types of technological innovation, considering stochastic technological progress and also the real replacement value implied by various models, in contrast to Adkins and Paxson (2011, 2012). We allow technological advances to involve different successor initial operating cost levels compared to like-for-like equipment replacements; consider the possible volatility and time drifts of technological progress; and consider the correlation of technological progress with both the evolution of revenues produced by the incumbent technology, and the current operating costs of the existing technology.

We compare four equipment replacement models: deterministic NPV without technological progress, and then unexpected, or (alternatively) anticipated, or uncertain technological progress. For each case we adopt or derive an operational rule for replacement, and for the last three models examine analytically the differential impact of technological progress on the replacement policy. The resulting solutions provide the basis for the real replacement option value, whether the technological progress is unexpected, or anticipated, or uncertain.

In Section 2, we develop quasi-analytical solutions to the timing boundary for replacement with unexpected, anticipated and uncertain technological progress under revenue and operating cost uncertainty. These solutions are based on the premise that an optimal replacement occurs when the incremental value rendered by the replacement exceeds the re-investment cost. Numerical analysis is used to illustrate model behavior in Section 3. We show that while an anticipated technological progress prolongs the active life of the incumbent, expected revenue and cost uncertainty, and uncertainty regarding that progress, may result in an even longer life. The final section is a conclusion.

2 Equipment Replacement Models

We consider durable productive capital equipment, where both revenues attributable to the equipment and operating costs decay, and efficiency diminishes over time. The revenue produced by the equipment, denoted by P, changes at a risk-adjusted (see Dixit and Pindyck, 1994) continuous rate θ_P, assumed to be negative, while its operating cost, denoted by C, changes at the risk-adjusted continuous rate θ_C, assumed to be positive. When the incumbent attains a to be determined threshold, it is replaced by new equipment at a constant re-investment cost of K.

Technological progress is interpreted as an improvement in the initial attribute levels for the succeeding equipment relative to the incumbent. Unexpected technological progress is represented as a jump[1] in a favorable direction of one or more of the initial attribute levels for the succeeding equipment relative to the incumbent. An unexpected fall in the initial operating cost level for the succeeding equipment relative to the incumbent is indicative of an unforeseen technological improvement in the equipment performance. In contrast, a deterministic decline in the initial operating cost level for the succeeding asset is predictable, and because the improvement is foreseen, it is interpreted as anticipated technological progress.

2.1 Model I: No Technological Progress

As a benchmark, we propose a deterministic NPV model, where no technological process is anticipated. The revenue and operating cost levels for the incumbent at installation and replacement are denoted by P_I and C_I, respectively, and r is the discount rate. Following Lutz and Lutz (1951), and as derived in Adkins and Paxson (2011), the optimal cycle time $\hat{T}$ is the solution for:

$$\frac{P_I\left(1-\mathrm{e}^{\theta_P\hat{T}}\right)}{r-\theta_P}-\frac{C_I\left(1-\mathrm{e}^{\theta_C\hat{T}}\right)}{r-\theta_C}=K+\left[\frac{-\theta_P P_I\mathrm{e}^{\theta_P\hat{T}}}{\left(r-\theta_P\right)}+\frac{\theta_C C_I\mathrm{e}^{\theta_C\hat{T}}}{\left(r-\theta_C\right)}\right]\frac{\left(1-\mathrm{e}^{-r\hat{T}}\right)}{r}. \tag{1}$$

This states that the optimal cycle time occurs when the net incremental value rendered by the replacement, represented by the left hand side of (1), exactly balances the re-investment cost plus a positive amount. This positive amount is the weighted sum of the values for the revenue and operating cost for an incumbent at the optimal lifetime, adjusted by an annuity factor with an optimal lifetime horizon. For a replacement to be optimal, the rendered net incremental value has

[1] Indeed, somewhat more complex models might incorporate a jump process.

to exceed the re-investment cost. Given the initial revenue and operating cost levels for the incumbent, finding the optimal replacement time involves equating the LHS and RHS numerically by varying the cycle time. For the deterministic case without technological progress, Model I, the optimal thresholds for revenue $\hat{P}_D$ and operating cost $\hat{C}_D$ are given by $\hat{P}_D = P_I \,\mathrm{e}^{\theta_P \hat{T}}$ and $\hat{C}_D = C_I \,\mathrm{e}^{\theta_C \hat{T}}$, respectively.

Our primary contribution is developing three new stochastic models, II, III and IV, to illustrate the distinctions among unexpected, anticipated and uncertain technological progress, respectively. For each model, we derive an operational rule for deciding whether the incumbent should or should not be replaced. Replacement is characterized by a timing boundary, which is created from a quasi-analytical solution to the corresponding real replacement option problem.

2.2 Model II: Unexpected Technological Progress

We seek to find the threshold signaling the optimal replacement of the incumbent when technological progress is feasible, but not expected. This threshold is represented by a function of the trigger levels for the revenue and operating cost, denoted by $\hat{P}_U$ and $\hat{C}_U$, respectively, which divides the decision space into two mutually exclusive exhaustive regions of continuance and replacement. When plotted on this decision space, if the prevailing levels of the revenue and operating cost lie within the continuance region, then the optimum strategy is to continue with the incumbent, or if the prevailing levels belong to the replacement region, then replacing the incumbent is the optimal decision.

We assume that the two variables follow distinct geometric Brownian motion processes with drift. For $X \in \{P, C\}$:

$$\mathrm{d}X = \alpha_X X \,\mathrm{d}t + \sigma_X X \,\mathrm{d}z_X , \tag{2}$$

where α_X is the instantaneous drift rate, σ_X is the instantaneous volatility rate, and $\mathrm{d}z_X$ is the increment of a standard Wiener process. Dependence between the two uncertain variables is described by the instantaneous covariance term $\rho_{P,C}\sigma_P\sigma_C$. The function F is defined as the value of the incumbent equipment including its embedded replacement option. All replacement decisions are treated as being made in isolation to any other enacted policies, so there are no scale or other flexibilities and no competition. The value of F depends on the prevailing revenue and operating cost levels so $F = F(P, C)$, assuming risk neutrality, which is expressed as a two-dimensional partial differential equation ("PDE"). This valuation relationship is described by:

$$\begin{aligned} &\tfrac{1}{2}\sigma_P^2 P^2 \frac{\partial^2 F}{\partial P^2} + \tfrac{1}{2}\sigma_C^2 C^2 \frac{\partial^2 F}{\partial C^2} + \rho_{P,C}\sigma_P\sigma_C PC \frac{\partial^2 F}{\partial P \partial C} \\ &\qquad + \theta_P P \frac{\partial F}{\partial P} + \theta_C C \frac{\partial F}{\partial C} + (P - C) - rF = 0. \end{aligned} \tag{3}$$

where r is the discount rate, and θ_P and θ_C are the risk-adjusted drift rates respectively for revenues and operating costs. We assume a world without taxes, and $r - \theta_X > 0$ for $X \in \{P, C\}$.

The value of the existing equipment and its replacement option is:

$$F_{II}(P, C) = A_{II} P^{\beta_{II}} C^{\eta_{II}} + \frac{P}{r - \theta_P} - \frac{C}{r - \theta_C}, \tag{4}$$

with coefficient A_{II} and parameters β_{II} and η_{II}. The function F_{II} is composed of two elements. The term $A_{II} P^{\beta_{II}} C^{\eta_{II}}$ is interpreted as the replacement option value, which being positive means that $A_{II} > 0$. Since the incentive to replace the incumbent grows as the revenue decreases, but as the operating cost increases, we conjecture $\beta_{II} < 0$ and $\eta_{II} > 0$. The second element:

$$\frac{P}{r-\theta_P} - \frac{C}{r-\theta_C}$$

denotes the equipment value to the owner in the absence of any optionality and at any P and C levels.

Originally, when the incumbent is installed, its initial revenue and operating cost levels are specified by P_I and C_I, respectively, but when the incumbent is replaced, the initial revenue and operating cost levels for its successor are specified by P_S and C_S, respectively. This specification implies that the attribute levels of the succeeding equipment may be allowed to differ from those of the incumbent that it replaces, so there is no underlying presumption that the incumbent is replaced by a replica with identical attribute levels. Consequently, we can model the presence of an unexpected technological progress as the unexpected move in the initial attributes from C_I to C_S where $C_I > C_S$, or from P_I to P_S where $P_I < P_S$. In our formulation, technological progress is characterized as an improvement in the equipment initial operating cost level, so $C_I > C_S$ while $P_I = P_S$. Even though a technological advance leads to improved initial levels for the succeeding asset, we assume that the respective drift rates remain unchanged.

Value conservation at replacement requires that the incumbent value has to be exactly balanced by the net value for the succeeding equipment. The incumbent value at replacement $F_{II}\left(\hat{P}_U, \hat{C}_U\right)$ is determined from the valuation function (4), defined at the respective threshold levels, $\hat{P}_U$ and

$\hat{C}_U$. These thresholds represent the values for the respective attributes that economically justify replacing the incumbent equipment. As the value for the succeeding asset at installation is $F_{II}\left(P_S, C_S\right)$, its net value is $F_{II}\left(P_S, C_S\right) - K$, so the value-matching relationship can be expressed as:

$$A_{II}\hat{P}_U^{\beta_{II}}\hat{C}_U^{\eta_{II}} + \frac{\hat{P}_U}{r-\theta_P} - \frac{\hat{C}_U}{r-\theta_C} = A_{II}P_S^{\beta_{II}}C_S^{\eta_{II}} + \frac{P_S}{r-\theta_P} - \frac{C_S}{r-\theta_C} - K\,. \tag{5}$$

Although value conservation is enforced by the value-matching relationship, the requirement governing optimal replacement is specified by the two smooth pasting conditions, one for each of the two factors. These can be expressed as:

$$\beta_{II}A_{II}\hat{P}_U^{\beta_{II}-1}\hat{C}_U^{\eta_{II}} + \frac{1}{r-\theta_P} = 0\,, \tag{6}$$

$$\eta_{II}A_{II}\hat{P}_U^{\beta_{II}}\hat{C}_U^{\eta_{II}-1} - \frac{1}{r-\theta_C} = 0\,, \tag{7}$$

which affirms our conjecture that $\beta_{II} < 0$ and $\eta_{II} > 0$. The characteristic root equation (8), which defines the requisite relationship amongst the parameters for (4) to be a viable solution to (3), is:

$$\frac{1}{2}\sigma_P^2\beta_{II}(\beta_{II}-1) + \frac{1}{2}\sigma_C^2\eta_{II}(\eta_{II}-1) + \rho_{P,C}\sigma_P\sigma_C\beta_{II}\eta_{II} + \theta_P\beta_{II} + \theta_C\eta_{II} - r = 0\,. \tag{8}$$

The four equations, (5), (6), (7) and (8), constitute the model solution from which the optimal timing boundary can be found. The real replacement option value at the current output price P and input cost C, ROV_U, is obtained by solving (6) for A_{II}, and substituting in (4):

$$ROV_U = \frac{\hat{P}_U}{-\beta_{II}(r-\theta_P)}\left[\frac{P^{\beta_{II}}}{\hat{P}_U^{\beta_{II}}}\frac{C^{\eta_{II}}}{\hat{C}_U^{\eta_{II}}}\right] \tag{9}$$

Under similar conditions with $C_S=C_I$ and $P_S=P_I$, and assuming nil revenue and cost volatility, the solutions to Model I and Model II are identical.

2.3 Model III: Anticipated Technological Progress

The threat of new technology is likely to motivate equipment suppliers to continuously improve product performance. If these improvements are realized through continuous changes in the initial attributes, then over a period of time, we would observe falls in either the initial operating cost level or re-investment cost for the succeeding equipment, or increases in its initial revenue level. The dynamic programming framework allows the initial attribute to change with time, so Model III treats anticipated technological progress as being expressed through a time dependent successor operating cost level.

We start by assuming that for the succeeding equipment, the new initial operating cost level, which is denoted by C_N, can be adequately expressed by a growth function with a continuous constant rate θ_N, that is $\mathrm{d}C_N = \theta_N C_N \mathrm{d}t$. This growth parameter is expected to be negative, since performance improvements are presumed to be embedded in the succeeding equipment with C_N declining over time. The presence of a new initial operating cost level in the model means that the value function, which is denoted by F_{III}, depends on three factors, the new initial operating cost level as well as the prevailing levels for the revenues and operating cost. In the two-factor model (4), the replacement option value is expressed as a product power function of the two factors, revenues and operating costs. For the three-factor model under consideration, we similarly adopt a product power function but now of three factors, revenues, operating costs and new successor initial operating cost level, to represent the replacement option value. So, the valuation function becomes:

$$F_{III}\left(P,C,C_N\right)=A_{III}P^{\beta_{III}}C^{\eta_{III}}C_N^{\gamma_{III}}+\frac{P}{r-\theta_P}-\frac{C}{r-\theta_C}, \qquad (10)$$

where $A_{III}P^{\beta_{III}}C^{\eta_{III}}C_N^{\gamma_{III}}$, with $A_{III}>0$, represents the option value with power parameters β_{III}, η_{III} and γ_{III}. Again, the term $P/\left(r-\theta_P\right)-C/\left(r-\theta_C\right)$ denotes the value of the equipment to the owner in the absence of any optionality. As before, we conjecture that β_{III} is negative, and η_{III} positive. We now consider the sign of γ_{III}. Since a stronger economic incentive exists for replacing the incumbent when the successor initial operating cost level is low rather than high, we would expect the replacement option to increase in value as C_N decreases, so we conjecture that the value of γ_{III} should be negative. The replacement event is signaled when the three factor levels, P, C and C_N, simultaneously attain their respective optimal threshold levels, $\hat{P}_A$, $\hat{C}_A$ and $\hat{C}_{NA}$. Collectively, these three optimal thresholds form the timing boundary, which is determined from the model solution, made up of the economic conditions signaling an optimal replacement, that is the value-matching relationship and the smooth pasting conditions, plus the characteristic root equation.

Because value is conserved at replacement, the incumbent value $F_{III}\left(\hat{P}_A,\hat{C}_A,\hat{C}_{NA}\right)$ has to exactly balance the succeeding asset value $F_{III}\left(P_S,\hat{C}_{NS},\hat{C}_{NS}\right)$, less the re-investment cost K. By using (10), the value-matching relationship can be expressed as:

$$A_{III}\hat{P}_A^{\beta_{III}}\hat{C}_A^{\eta_{III}}\hat{C}_{NA}^{\gamma_{III}}+\frac{\hat{P}_A}{r-\theta_P}-\frac{\hat{C}_A}{r-\theta_C}=A_{III}P_S^{\beta_{III}}\hat{C}_{NS}^{\eta_{III}+\gamma_{III}}+\frac{P_S}{r-\theta_P}-\frac{\hat{C}_{NS}}{r-\theta_C}-K. \qquad (11)$$

Replacement is optimal whenever the smooth pasting conditions are obtained. Associated with (11), there are three smooth pasting conditions, for P, C and C_N, respectively, which can be expressed as:

$$\beta_{III} A_{III} \hat{P}_A^{\beta_{III}-1} \hat{C}_A^{\eta_{III}} \hat{C}_{NA}^{\gamma_{III}} + \frac{1}{r-\theta_P} = 0, \tag{12}$$

$$\eta_{III} A_{III} \hat{P}_A^{\beta_{III}} \hat{C}_A^{\eta_{III}-1} \hat{C}_{NA}^{\gamma_{III}} - \frac{1}{r-\theta_C} = 0, \tag{13}$$

$$\gamma_{III} A_{III} \hat{P}_A^{\beta_{III}} \hat{C}_A^{\eta_{III}} \hat{C}_{NA}^{\gamma_{III}-1} - \left(\eta_{III} + \gamma_{III}\right) A_{III} P_S^{\beta_{III}} \hat{C}_{NS}^{\eta_{III}+\gamma_{III}-1} + \frac{1}{r-\theta_C} = 0. \tag{14}$$

We observe from (12) and (13) that $\beta_{III} < 0$ and $\eta_{III} > 0$. Also, since $P_S^{\beta_{III}} \hat{C}_{NS}^{\eta_{III}} < \hat{P}_A^{\beta_{III}} \hat{C}_A^{\eta_{III}}$ because $\hat{P}_A \leq P_S$ and $\hat{C}_A > \hat{C}_{NA}$, then from (14) $\gamma_{III} < 0$. The three smooth pasting conditions affirm our conjecture on the signs of the power parameters.

The final component of the model is the characteristic root equation:

$$\frac{1}{2}\sigma_P^2 \beta_{III}(\beta_{III}-1) + \frac{1}{2}\sigma_C^2 \eta_{III}(\eta_{III}-1) + \rho_{P,C}\sigma_P\sigma_C\beta_{III}\eta_{III} + \theta_P\beta_{III} + \theta_C\eta_{III} + \theta_N\gamma_{III} - r = 0. \tag{15}$$

There are five equations for Model III. These are (i) the value-matching equation, (11), (ii) three smooth pasting equations (12), (13) and (14) and (iii) the characteristic root equation, (15). Eliminating β_{III}, η_{III}, γ_{III} and A_{III} enables the timing boundary to be derived as a single relationship linking $\hat{P}_A$, $\hat{C}_A$ and $\hat{C}_{NA}$.

The real replacement option value at current P, C and C_N, ROV_A, is obtained by solving (12) for A_{III}, and substituting in (10):

$$ROV_A = \frac{\hat{P}_A}{-\beta_{III}(r-\theta_P)}\left[\frac{P^{\beta_{III}}}{\hat{P}_A^{\beta_{III}}}\frac{C_N^{\gamma_{III}}}{\hat{C}_{NA}^{\gamma_{III}}}\frac{C^{\eta_{III}}}{\hat{C}_A^{\eta_{III}}}\right] \tag{16}$$

2.5 Model IV: Uncertain Technological Progress

Finally, we consider the case where technological progress follows a geometric Brownian motion process with drift (2), where θ_N is the risk-adjusted drift rate for technology specific to the particular type of equipment, σ_N is the instantaneous volatility rate for C_N. There may be dependence among three uncertain (volatile) variables, where N denotes C_N, described by the instantaneous covariance terms $\rho_{P,C}\sigma_P\sigma_C$, $\rho_{P,N}\sigma_P\sigma_N$, and $\rho_{C,N}\sigma_C\sigma_N$.

Now the value of F depends on the prevailing revenue and operating costs and the technological progress, expressed as the following PDE:

$$\begin{aligned}
&\tfrac{1}{2}\sigma_P^2 P^2 \frac{\partial^2 F_{IV}}{\partial P^2} + \tfrac{1}{2}\sigma_C^2 C^2 \frac{\partial^2 F_{IV}}{\partial C^2} + \tfrac{1}{2}\sigma_N^2 C_N^2 \frac{\partial^2 F_{IV}}{\partial C_N^2} \\
&+\rho_{P,C}\sigma_P\sigma_C PC \frac{\partial^2 F_{IV}}{\partial P \partial C} + \rho_{P,N}\sigma_P\sigma_N PC_N \frac{\partial^2 F_{IV}}{\partial P \partial C_N} + \rho_{C,N}\sigma_C\sigma_N CC_N \frac{\partial^2 F_{IV}}{\partial C \partial C_N} \\
&+\theta_P P \frac{\partial F_{IV}}{\partial P} + \theta_C C \frac{\partial F_{IV}}{\partial C} + \theta_N C_N \frac{\partial F_{IV}}{\partial C_N} + (P-C) - rF_{IV} = 0.
\end{aligned} \tag{17}$$

Since the solution to the homogenous element is a product power function, the valuation function satisfying (17) is:

$$F_{IV} = A_{IV} P^{\beta_{IV}} C^{\eta_{IV}} C_N^{\gamma_{IV}} + \frac{P}{r-\theta_P} - \frac{C}{r-\theta_C}. \tag{18}$$

The associated characteristic root equation is:

$$\begin{aligned}&\tfrac{1}{2}\sigma_P^2\beta_{IV}\left(\beta_{IV}-1\right)+\tfrac{1}{2}\sigma_C^2\eta_{IV}\left(\eta_{IV}-1\right)+\tfrac{1}{2}\sigma_N^2\gamma_{IV}\left(\gamma_{IV}-1\right)\\&+\rho_{P,C}\sigma_P\sigma_C\beta_{IV}\eta_{IV}+\rho_{P,N}\sigma_P\sigma_N\beta_{IV}\gamma_{IV}+\rho_{C,N}\sigma_C\sigma_N\eta_{IV}\gamma_{IV}\\&+\theta_P\beta_{IV}+\theta_C\eta_{IV}+\theta_N\gamma_{IV}-r=0.\end{aligned} \tag{19}$$

In (18), the equipment value is composed of the replacement option value $A_{IV}P^{\beta_{IV}}C^{\eta_{IV}}C_N^{\gamma_{IV}}$ and the equipment value to the owner in absence of any optionality $P/\left(r-\theta_P\right)-C/\left(r-\theta_C\right)$. Since the option value increases as the incumbent factors, P and C, deteriorate, then β_{IV} must be negative and η_{IV} positive. Further, there is a greater likelihood of replacing the incumbent for low values of C_N, so γ_{IV} is negative.

We now consider the boundary conditions that hold along the timing boundary $P=\hat{P}_V$, $C=\hat{C}_V$ and $C_N=\hat{C}_{NV}$. The value-matching relationship ensures value conservation by requiring the net values instantaneously before and after replacement to be equal, so $F_{IV}\left(\hat{P}_V,\hat{C}_V,\hat{C}_{NV}\right)=F_{IV}\left(P_S,\hat{C}_{NS},\hat{C}_{NS}\right)-K$, or:

$$A_{IV}\hat{P}_V^{\beta_{IV}}\hat{C}_V^{\eta_{IV}}\hat{C}_{NV}^{\gamma_{IV}}+\frac{\hat{P}_V}{r-\theta_P}-\frac{\hat{C}_V}{r-\theta_C}=A_{IV}P_S^{\beta_{IV}}\hat{C}_{NS}^{\eta_{IV}+\gamma_{IV}}+\frac{P_S}{r-\theta_P}-\frac{\hat{C}_{NS}}{r-\theta_C}-K\,. \tag{20}$$

Replacement is optimal whenever the smooth pasting conditions are obtained. Associated with (20), there are three smooth pasting conditions, for P, C and C_N, respectively, which can be expressed as:

$$\beta_{IV}A_{IV}\hat{P}_V^{\beta_{IV}-1}\hat{C}_V^{\eta_{IV}}\hat{C}_{NS}^{\gamma_{IV}}+\frac{1}{r-\theta_P}=0, \tag{21}$$

$$\eta_{IV}A_{IV}\hat{P}_V^{\beta_{IV}}\hat{C}_V^{\eta_{IV}-1}\hat{C}_{NS}^{\gamma_{IV}}-\frac{1}{r-\theta_C}=0, \tag{22}$$

$$\gamma_{IV} A_{IV} \hat{P}_V^{\beta_{IV}} \hat{C}_V^{\eta_{IV}} \hat{C}_{NS}^{\gamma_{IV}-1} - (\eta_{IV} + \gamma_{IV}) A_{IV} P_S^{\beta_{IV}} \hat{C}_{NS}^{\eta_{IV}+\gamma_{IV}-1} + \frac{1}{r-\theta_C} = 0. \qquad (23)$$

The optimal timing boundary for the stochastic version of the replacement model is obtained as a relationship linking $\hat{P}_V$, $\hat{C}_V$ and $\hat{C}_{NS}$, by using the five equations (19) –(23). Eliminating β_{IV}, η_{IV}, γ_{IV} and A_{IV} enables the timing boundary to be derived as a single relationship linking $\hat{P}_V$, $\hat{C}_V$ and $\hat{C}_{NS}$.

The real replacement option value at current P, C and C_N, ROV_V, is obtained by solving (21) for A_{IV}, and substituting in (18):

$$ROV_V = \frac{\hat{P}_V}{-\beta_{IV}(r-\theta_P)} \left[\frac{P^{\beta_{IV}}}{\hat{P}_V^{\beta_{IV}}} \frac{C_N^{\gamma_{IV}}}{\hat{C}_{NS}^{\gamma_{IV}}} \frac{C^{\eta_{IV}}}{\hat{C}_V^{\eta_{IV}}} \right] \qquad (24)$$

3 Numerical Illustrations

Since the solutions for Models II, III and IV involve solving sets of simultaneous equations, their behavior is investigated through numerical illustrations. Table 1 exhibits the base case data we use to illustrate the solution. For comparison, we also show Model IIa and Model IIIa in Table 2 assuming no technological progress, that is C_S=C_I, and also Model IIb and IIIb where C_S<C_I. The differences between their solutions (a and b) reflect the impact of technological progress on the replacement policy. Also, the successive initial operating cost level for Model IIIb is set to decline at a known rate. This suggests that the initial operating cost implied by technological progress inherent in the succeeding asset can be anticipated.

INSERT TABLE 1

The optimal timing boundary discriminates between the regions of continuance and replacement and is represented by the feasible set of threshold levels. Whenever the prevailing levels pertain to the continuance region, the optimal decision is to continue with the incumbent, while if they do not, then replacement becomes the optimal decision. In the case for the deterministic model, the set of threshold levels is defined by a single point, but if the number of factors in a stochastic model is two or more, the set of threshold levels maps out a locus. The timing boundary for Model II is represented by a locus defined in a two-dimensional plane and is calculated from equations 5-6-7-8 to solve for the revenue threshold for a pre-specified operating cost threshold, and then repeatedly by allowing the revenue threshold to vary. The optimal timing boundaries for Models III and IV involve three distinct thresholds $\hat{P}_A$, $\hat{C}_A$ and $\hat{C}_{NA}$, and $\hat{P}_V$, $\hat{C}_V$ and $\hat{C}_{NV}$, respectively. For comparison, we assume $\hat{C}_D = \hat{C}_U = \hat{C}_A = \hat{C}_V$ =31.039 and $\hat{C}_{NA} = \hat{C}_{NV}$ =15, and then show the corresponding $\hat{P}_D, \hat{P}_U, \hat{P}_A$ *and* $\hat{P}_V$. When illustrated in a two-dimensional plane the timing boundary can be represented by a nested set of boundaries each for a pre-specified value of $\hat{C}_{NA}$ or $\hat{C}_{NV}$ For any given $\hat{C}_{NA}$ or $\hat{C}_{NV}$, the timing boundary for Model III is determined from equations 11-12-13-14-15 and for Model IV from equations 19-20-21-22-23.

3.1 Replacement Policy Solutions

Using the base case data[2], the optimal cycle time for the deterministic Model I is calculated to be 10.99, and its derived revenue and operating cost threshold levels are presented in Table 2. This table also includes a single point solution on the timing boundaries for Models IIa, IIb, IIIa, IIIb

[2] While these parameter values are hypothetical, Knittel (2011) states that technological progress (measured as miles per gallon adjusted for the effects of changes in weight, horsepower, torque and acceleration) in light truck fuel efficiency is not deterministic. For instance, the average annual increase in fuel efficiency for US light trucks from 1981-2006 was 8.0%, σ_{N} = 9.3%, and a correlation with gasoline prices=.36. He also reports some discrete engine design movements such as new fuel injection systems. Knittel shows it is hard (but not impossible) to isolate one type of technological progress under *ceteris paribus* assumptions.

and IV. For that point, we select the thresholds $\hat{C}_D = \hat{C}_U = \hat{C}_A = \hat{C}_V$, in order for the various solutions to be comparable.

INSERT TABLE 2

The distance of the trajectory for the incumbent attributes from initial to threshold levels is a measure of its lifetime. We observe from Table 2 that the deterministic replacement policy without technological progress has the highest revenue threshold and so is the least deferred. Even without technological progress, the effect of introducing uncertainty, assuming all other parameters levels remain unchanged, is to make waiting valuable in case more favourable revenue and cost values are realised. A revenue threshold decrease indicates deferred replacement.

However, this deferral effect is partially mitigated when the attributes of the successor are improved. Table 2 reveals that the Model IIb revenue threshold increases from 54.850 to 57.771 when the initial operating cost level is improved upon replacement from 20 to 15. But, if the equipment owner is aware that this jump in the initial operating cost level is to be followed by a continuous improvement in the initial operating cost level at the rate of 5% (Model IIIb), then this awareness produces a slight revenue threshold decrease to 56.925 due to the advantages in waiting for an improved version. These findings extend the results produced by Caplan (1940) to the world of uncertainty. An announced jump in the attribute levels for the successor that is unexpected will hasten replacement, even to the extent of motivating an immediate replacement, while an anticipated change leads to a deferral. A long deferral is justified when technological progress is highly uncertain, but here the specific assumed parameter values of C_N volatility and correlation with P and C are critical. Note the significant increase in the ROV as technological progress and additional uncertainty in the variables are considered.

The stochastic equipment replacement models are naturally sensitive to changes in expected volatilities and correlations. We simulate (i) the sensitivity of thresholds and the option to replace to changes in the volatility of the operating costs, (ii) the sensitivity of thresholds to changes in the correlation of revenue and operating costs, (iii) the sensitivity of thresholds and replacement option values to changes in technological progress C_N, (iv) the sensitivity of thresholds and replacement option values to changes in the investment cost, and (v) for Model IV the sensitivity of thresholds and option values to changes in the expected volatility of technological progress.

3.2 Variations in the Cost Volatility

Increases in the underlying volatility are expected to defer the replacement decision. This characteristic should be reflected in falls for the revenue threshold, all else kept constant. Figure 1 illustrates this negative relationship between the revenue threshold and the operating cost volatility, holding the correlation constant at -0.5. The waiting time, as indicated by the revenue threshold, is least for the deterministic version of the models, having an operating cost volatility of zero (while there is a positive revenue volatility). Moreover, in line with the Caplan (1940) finding, the revenue threshold is higher for unexpected technological progress, even for stochastic revenue. The effect of anticipatory technological progress is to defer the replacement decision, since additional waiting is economically justified by the anticipated fall in the initial operating cost level for the successor.

Also, there is a shortening of the spread between the revenue thresholds for the unexpected and anticipated technological progress. This suggests that for higher operating cost volatility levels, the effect of anticipated technological progress on the solution becomes increasingly less important, so any gains from deferring replacement due to anticipated technological progress becomes increasingly less important relative to deferral arising from increases in volatility. Deferral is greatest for the volatile C_N, especially if the assumed correlation of C and C_N is -1.0. Similar results are obtained for revenue volatility.

INSERT FIGURE 1

The cited literature does not consider the sensitivity of the real replacement option at current revenues and costs to changes in volatility. Figure 1 also shows that both real replacement option values (at current P and C) ROV_U, ROV_A and ROV_V, at the base parameter values, increase with increases in cost volatility, although at a decreasing rate. Note that when there is no cost volatility (although there is a positive revenue volatility), $ROV_U < ROV_A < ROV_V$, the opposite of the revenue threshold order.

3.3 Variations in the P and C Correlation

The threshold levels depend on the extent of the correlation between revenue and operating cost through the various Q functions. Figure 2 presents the revenue threshold levels for variations in the correlation when the operating cost threshold is set equal to 31.039, its deterministic value. This reveals that an increase in the correlation creates a revenue threshold increase for the stochastic Models II, III and IV, which implies a hastening of the replacement decision. Also, the threshold spreads between unexpected, anticipated and volatile models increase with correlation.

Standard real-option theory tells us that the extent of the deferral varies positively with the underlying volatility. Now, the critical element affecting the replacement policy is the net revenue, that is the difference between revenue and operating cost, and since the volatility of net revenue varies inversely with correlation, the extent of the deferral is attenuated for correlation increases.

An alternative explanation is that revenue can be conceived as a partial hedge for movements in the operating cost whenever their correlation is positive, so an increase in correlation leads to a rise in the revenue threshold. Finally, we observe that the effect of changes in correlation affects the timing boundaries for Models II, III and IV in a very similar way.

INSERT FIGURE 2

The impact of correlation on the solution can also be viewed through the replacement option value. Figure 2 also illustrates the modifications in replacement option value due to correlation changes. It shows that the replacement option value declines for increases in the correlation and this feature is explained by exactly the same mechanism producing a threshold rise. This finding is consistent with other exchange-type real options.

3.4 Variations in the Successor Cost Level at Replacement

The value for the succeeding asset is expected to rise for decreases in its initial operating cost level upon replacement, and this desirable effect should be reflected in hastening the replacement of the incumbent. Figure 3 illustrates the variations in the revenue thresholds due to changes in the initial operating cost level for the successor. As expected, the revenue thresholds increase for decreases in the initial operating cost level, and consequently, the incumbent tends to be retired earlier. The figure also shows that the spread among the thresholds for unexpected, anticipated and uncertain technological progress declines as the successor operating cost level falls.

INSERT FIGURE 3

Figure 3 also illustrates the effect of variations in the successor operating cost level on the replacement option value. Both unexpected and anticipated real replacement option values decrease with increases in the successor cost level (and consistently with the lower revenue thresholds), indicating that it is the reversionary replacement operating cost levels that drive real replacement value. But ROV_V increases with increases in C_N, whether the sign of the correlation of C and C_N is negative or positive. Possibly this indicates that although the revenue threshold at which it is optimal to initiate equipment replacement increases with decreases in the assumed successor initial operating cost, the opportunity value of being able to do this repeatedly, even at relatively high successor operating costs, increases if technological progress is uncertain.

3.5 Variations in the Re-investment Cost

The re-investment cost K fundamentally affects the successor's net value and for any investment cost increase, the successor becomes less valuable. This should be reflected in postponing the incumbent's replacement and lowering the revenue threshold. Figure 4 shows that a decrease in the re-investment cost leads to revenue threshold increases for all models including Model I (NPV), justifying earlier replacement of the incumbent. The figure shows that for unexpected technological progress, the incumbent is always replaced earlier than for anticipated technological progress, and in turn earlier than for volatile C_N, but the spread between the four revenue thresholds widens for re-investment cost increases. This effect is explained by the benefits from waiting for the improvement becoming more attractive as the re-investment cost rises.

It might be imagined that the replacement investment cost, K, would be a critical factor in the equipment replacement decision. But Figure 4 shows that a 50% increase in K causes the revenue threshold to fall by 13-17%, and that at low K there is not much difference between the deterministic, unexpected, anticipated and volatile model revenue thresholds.

INSERT FIGURE 4 and 5

Figure 5 shows a real option value decline accompanies any increase in the re-investment cost for the successor, while $ROV_V > ROV_A > ROV_U$. Note that an increase in K of +10 naturally decreases NPV-K by the same amount, but at lower K, ROV_U and ROV_A decrease by more than 10, while at high K by less than 10. At very high K, the ROVs are hardly affected by changes in K.

A lesson for suppliers is that in designing new equipment, such as airplane engines or fuel efficient cars, to replace the existing stock, lower re-investment costs may not be as important as lower successor initial operating costs (such as fuel efficiency), for these parameter values.

3.6 Variations in Technological Progress Uncertainty

Increases in the expected uncertainty of technological progress (TP) are expected to lead to deferral of the replacement decision. Figure 6 illustrates this negative relationship between the revenue threshold and TP uncertainty, holding the $\rho_{C,N}$ correlation constant at -1. With nil volatility, the results are the same as for Model IIIb with the base parameter values. However, the ROV_V increases with increases in TP uncertainty, indicating more value for the purchaser of replacement equipment, waiting for large breakthroughs in operating cost improvements.

INSERT FIGURE 6

3.7 Focus of Participants in the Equipment Replacement Process

Suppose following Hotelling (1925) and Caplan (1940) that real depreciation should be the first derivative of equipment value with regards to time (or other factors), and this is the same for ROV_U, ROV_A and ROV_V. All ROVs decrease (and all revenue thresholds increase, indicating early replacement, shorter life) with increases in P and C correlation, and decreases in P and C volatility. A decrease in ROV and increase in revenue threshold should be accompanied by an increase in the real depreciation. However, ROV_U and ROV_A decrease (but revenue thresholds also increase, indicating later replacement, longer life) with increases in C_N, and increases in K. The net effect on real depreciation depends on the balance of the lower ROV to be depreciated over a longer life.

Real accountants (and alert practical managers), and investors looking at capital equipment intensive activities, such as airlines and ground transportation, will want to transform historical-data-based accounts into real-accounts to see if real profits are different from traditional accounting profits.

In a world of uncertainty, an alert Chief Options Manager ("COM") should adopt real-time decision making and attend to all the model parameters, especially P_I, C_I, C_S, C_N, the volatility of P, C and C_N, and correlations as well as current P and C. Further, the COM should periodically assess replacement policy as parameter values may change over time and usage. If not, then there is a strong likelihood of the COM destroying value. Replacement decisions will generally be in error when based on net present values, or deterministic discounted cash flow methods, treating revenue and operating cost as certain, and ignoring technological progress.

Equipment suppliers adopt distinct marketing policies dependent on whether their view is short- or long-term. In seeking immediate replacement equipment sales justified by the largest threshold spread, short-term suppliers may advise their clients to follow traditional capital budgeting rules. Shrewd suppliers would tend to emphasize those elements resulting in high P thresholds, such as high correlation between revenue and costs, low or no revenue or cost volatility, and obviously, lower successor initial operating cost levels upon replacement.

By adopting a longer-term view in the interests of their clients, reputable suppliers might focus on those elements increasing the real replacement option values, such as low or negative correlation between P and C, high P and C volatility, and obviously low successor initial operating costs upon replacement. A lower re-investment cost would increase the real replacement option value and also result in an earlier replacement decision, but, of course, it may result in lower supplier profits in the absence of lower equipment manufacturing costs. Perhaps some long-term horizon suppliers will focus on the uncertainty of technological progress, since at these base parameter values, since with high TP uncertainty, $ROV_V > ROV_A > ROV_U$.

4 Conclusion

When technological progress is interpreted as an improvement in one or more of the initial attribute levels for the succeeding asset, particularly in an uncertain initial operating cost level, we show that unexpected, anticipated and volatile technological progress can be considered in a dynamic programming formulation. The benefits of this approach are not only its avoidance of the shortcomings of a cycle time conceptualization, but its provision of the optimal replacement policy as a simple operational formula expressed in terms of thresholds. An unexpected improvement in the successor operating level has the potential to hasten the act of replacement, while the act of replacement is deferred whenever the technological progress can be fully anticipated.

The spreads between the unexpected case threshold $\hat{P}_U$, the anticipated case threshold $\hat{P}_A$ and the volatile case threshold $\hat{P}_V$ decrease somewhat as cost volatility increases. The effect of increased cost volatility on the real replacement option value is the opposite of that for the thresholds, consistent with real option theory that the lower the revenue justifying a replacement, the greater the real replacement value. The spreads between the unexpected ROV_U , the anticipated ROV_A and ROV_V decrease somewhat as cost volatility increases. The effect of increased correlation is to increase the spreads among $\hat{P}_U$, $\hat{P}_A$, and $\hat{P}_V$ and to increase the spreads among ROV_U, ROV_A and ROV_V.

The contrasting impact on the replacement policy of unexpected, anticipated and uncertain technological progress has distinct implications for owners and suppliers of replaceable equipment. Owners are predisposed to capturing as much of the full value embedded in the incumbent as is economically viable, so they are motivated to search the historical records for patterns of anticipated change because of its effect on deferring replacement. However, any indications that prolong the act of replacement are unlikely to be in the suppliers' short-term interest. Although at the base parameter values, perhaps short-term horizon suppliers might focus on $\hat{P}_D > \hat{P}_U > \hat{P}_A > \hat{P}_V$ and so de-emphasize technological progress uncertainty, long-term horizon suppliers and COMs might focus on $ROV_V > ROV_A > ROV_U$, and so act accordingly in a world of high uncertainty.

Extensions of this paper include more general models to allow for taxes and for uncertainty in salvage values and reinvestment costs for each succeeding asset, and, viewing these replacement models in a competitive environment as Caplan (1940) suggested more than a seven decades ago.

Table 1

Base Case Data

Description	Parameter	Value
Re-investment Cost	K	100
Initial Revenue Level for Incumbent	P_I	80
Initial Operating Cost Level for Incumbent	C_I	20
Initial Revenue Level for Succeeding Asset	P_S	80
Initial Operating Cost Level for Succeeding Asset	C_S	20 (IIa, IIIa) 15 (IIb, IIIb)
Succeeding Operating Cost Tech Progress	C_N	15
Revenue Growth Rate	θ_P	-2%
Operating Cost Growth Rate	θ_C	4%
Initial Operating Cost Growth Rate	θ_N	-5%
Discount Rate	r	12%
Revenue Volatility	σ_P	20%
Operating Cost Volatility	σ_C	20%
Revenue and Cost Correlation	$\rho_{P,C}$	-.5
Revenue and C_N Correlation	$\rho_{P,N}$	-.1
OpCost and C_N Correlation	$\rho_{C,N}$	-1.0
C_N Volatility	σ_N	50%

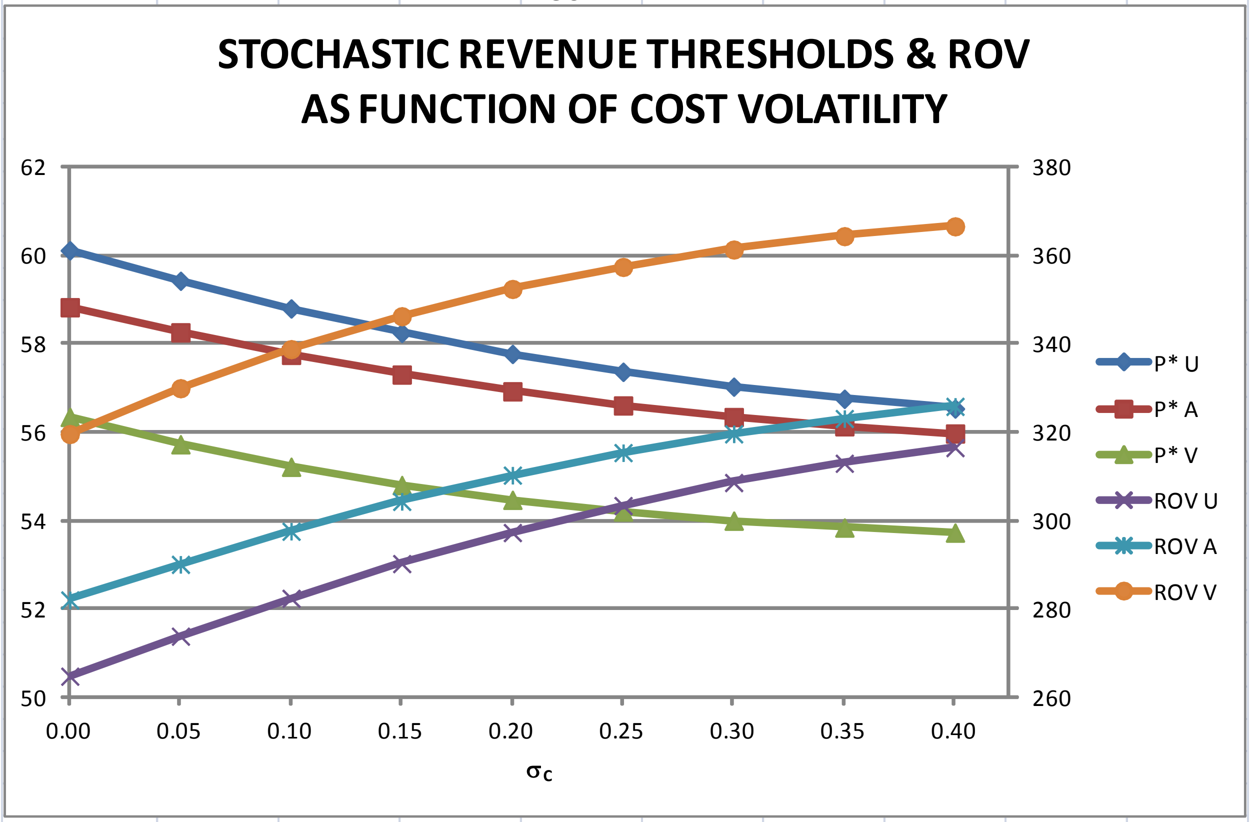
Table 2

COMPARING RESULTS

	NO TECHNOLOGICAL PROGRESS			TECHNOLOGICAL PROGRESS		
				UNEXPECTED	ANTICIPATED	UNCERTAIN
	MODEL I	MODEL IIa	MODEL IIIa	MODEL IIb	MODEL IIIb	MODEL IV
P*	64.217	54.850	54.325	57.771	56.925	54.472
C*	31.039	31.039	31.039	31.039	31.039	31.039
ROV		275.419	299.575	297.317	310.306	352.411

This table depicts the set (P*,C*) that indicates when P=<P* and
C=>C*, equipment replacement is justified, equating the RHS and LHS of Eq 1 for Model I,
setting Eqs 5,6,7,8=0 for Model II, setting Eqs 11,12,13, 14,15 =0 for Model III, and Eqs 19, 20, 21, 22, 23=0 for Model IV.
Assumes Table 1 values and zero volatility for Model I.
No tech progress C_S=20, tech progress C_S=C_N=15.
ROV is determined from Eqs 9 (M IIa,b), 16 (M IIIa,b) and 24 (M IV).

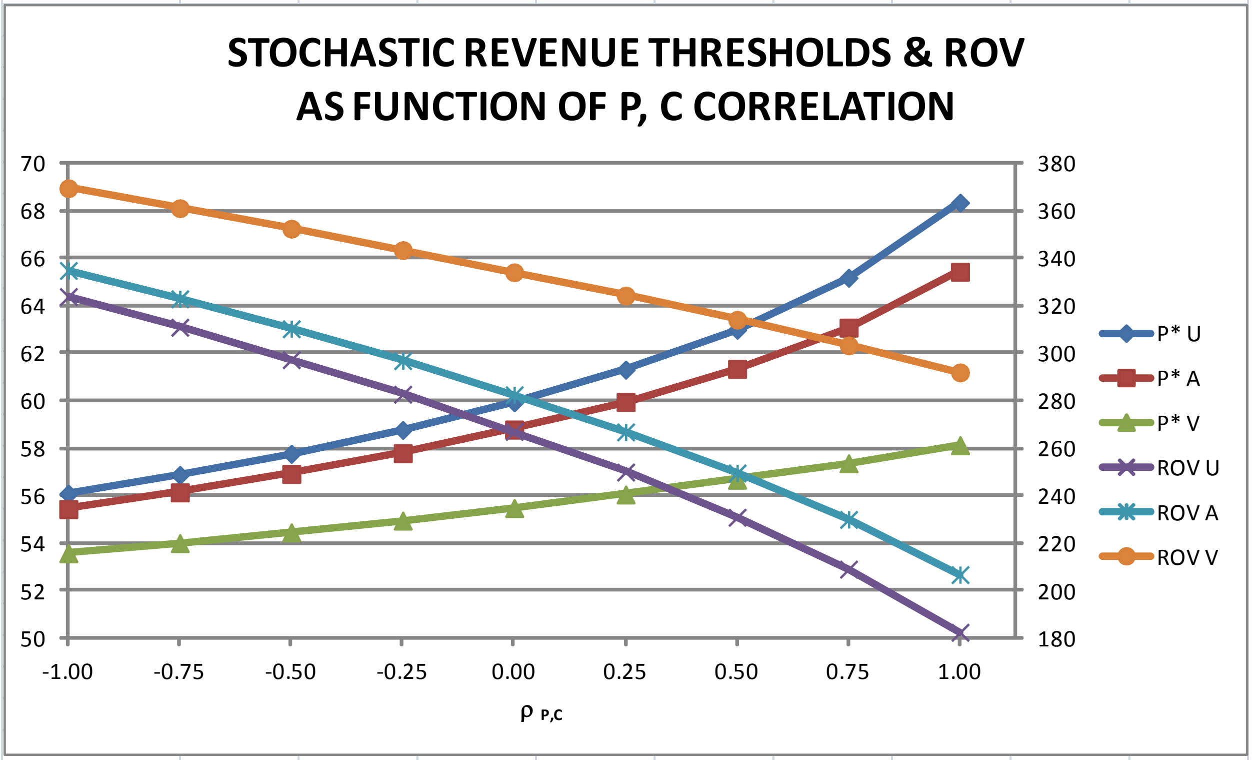
FIGURE 1



P* U	60.114	59.425	58.800	58.248	57.771	57.367	57.031	56.756	56.535
P* A	58.827	58.260	57.751	57.306	56.925	56.606	56.345	56.135	55.971
P* V	56.346	55.728	55.220	54.806	54.472	54.208	54.002	53.846	53.732
ROV U	264.842	273.891	282.453	290.307	297.317	303.426	308.634	312.982	316.536
ROV A	282.069	290.129	297.616	304.371	310.306	315.390	319.640	323.104	325.848
ROV V	319.617	329.977	338.826	346.264	352.411	357.391	361.333	364.360	366.593
σ_C	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40

This figure depicts revenue thresholds given the set (P*, C*=31.039) that indicates when P=<P* and C=>C*, replacement is justified, setting Eqs 5, 6,7,8 =0 for Model II, Eqs 11,12,13, 14, 15=0 for Model III, and setting Eqs 19, 20, 21, 22, 23=0 for Model IV. Assumes σ_C as shown, otherwise Table 1 values P_I=80, C_S=C_N=15, K=100, σ_P=.20, ρ=-.50, r=.12, θ_P=-.02, θ_C=.04, θ_N=-.05. This figure also depicts replacement option value from Eqs 9, 16 and 24 when P=75, C=30.

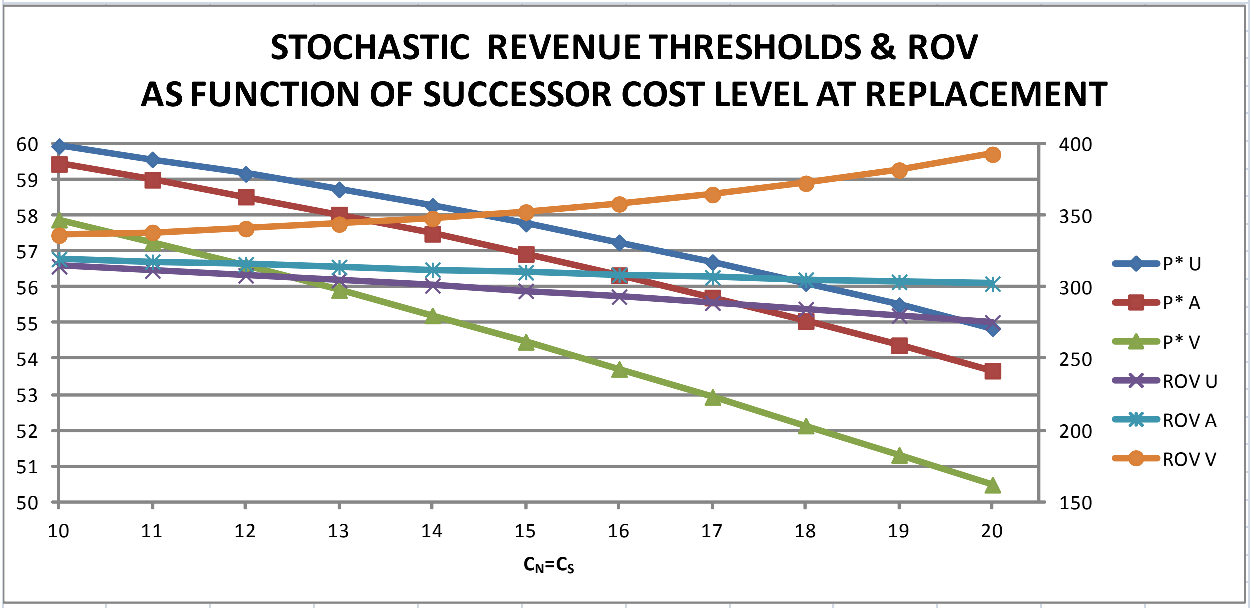
FIGURE 2



P* U	56.090	56.883	57.771	58.777	59.938	61.312	62.994	65.175	68.339
P* A	55.447	56.147	56.925	57.798	58.794	59.954	61.342	63.079	65.428
P* V	53.588	54.016	54.472	54.962	55.490	56.061	56.685	57.371	58.134
ROV_U	323.854	310.961	297.317	282.772	267.115	250.035	231.035	209.225	182.644
ROV_A	334.835	322.906	310.306	296.905	282.528	266.916	249.674	230.123	206.891
ROV_V	369.444	361.072	352.411	343.429	334.087	324.338	314.120	303.360	291.959
$\rho_{P,C}$	-1.00	-0.75	-0.50	-0.25	0.00	0.25	0.50	0.75	1.00

This figure depicts revenue thresholds given the set (P*, C*=31.039) that indicates when P=<P* and C=>C*, replacement is justified, setting Eqs 5, 6,7,8 =0 for Model II, Eqs 11,12,13, 14, 15=0 for Model III, setting Eqs 19, 20, 21, 22, 23=0 for Model IV. Assumes $\rho_{P,C}$ as shown, otherwise Table 1 values P_I=80, C_S=C_N=15, K=100, σ_P=.20, σ_C=.20, r=.12, θ_P=-.02, θ_C=.04, θ_N=-.05. This figure also depicts replacement option value from Eqs 9, 16 and 24 when P=75, C=30.

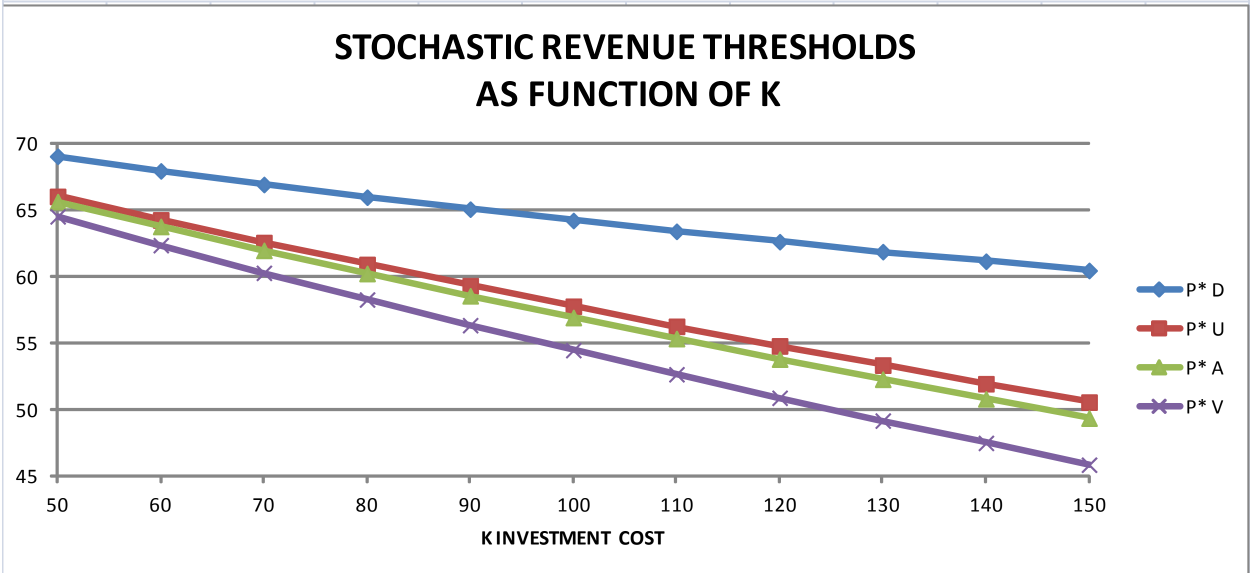
FIGURE 3



P* U	59.933	59.561	59.159	58.727	58.264	57.771	57.247	56.693	56.108	55.494	54.850
P* A	59.434	58.994	58.523	58.021	57.488	56.925	56.331	55.708	55.056	54.375	53.668
P* V	57.880	57.254	56.600	55.917	55.207	54.472	53.714	52.934	52.134	51.317	50.485
ROV U	314.49	311.48	308.25	304.81	301.16	297.32	293.28	289.06	284.67	280.12	275.42
ROV A	319.56	317.75	315.90	314.03	312.16	310.31	308.50	306.77	305.15	303.66	302.35
ROV V	336.17	338.21	340.78	343.93	347.78	352.41	357.96	364.57	372.41	381.69	392.65
$C_N=C_S$	10	11	12	13	14	15	16	17	18	19	20

This figure depicts revenue thresholds given the set (P^*, $C^*=31.039$) that indicates when $P=<P^*$ and $C=>C^*$, replacement is justified, setting Eqs 5, 6,7,8 =0 for Model II, Eqs 11,12,13, 14, 15=0 for Model III, setting Eqs 19, 20, 21, 22, 23=0 for Model IV. Assumes C_N and C_S as shown, otherwise Table 1 values $P_I=80$, $K=100$, $\sigma_P=.20$, $\sigma_C=.20$, $\rho_{P,C}=.50$, $\rho_{P,CN}=-.1$, $\rho_{C,N}=-1$, $r=.12$, $\theta_P=-.02$, $\theta_C=.04$, $\theta_N=-.05$.
This figure also depicts replacement option value from Eqs 9, 16 and 24 when $P=75$, $C=30$.

FIGURE 4

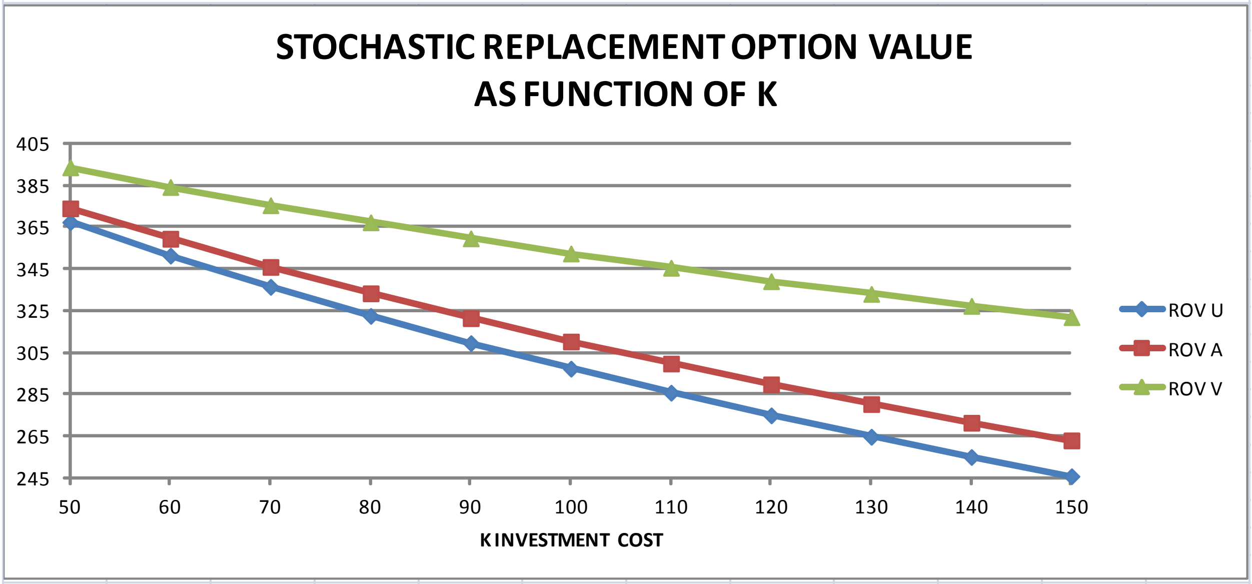


P* D	69.05	67.94	66.92	65.97	65.07	64.22	63.40	62.62	61.88	61.15	60.46
P* U	66.03	64.26	62.56	60.91	59.32	57.77	56.26	54.79	53.35	51.94	50.56
P* A	65.64	63.77	61.97	60.23	58.56	56.92	55.34	53.79	52.28	50.81	49.36
P* V	64.53	62.36	60.28	58.28	56.34	54.47	52.66	50.89	49.17	47.49	45.86
K	50	60	70	80	90	100	110	120	130	140	150

This figure depicts revenue thresholds given the set (P*, C*=31.039) that indicates when P=<P* and C=>C* equipment replacement is justified, setting Eqs 5,6,7,8 =0 for Model II, Eqs 11,12,13,14,15=0 for Model III and setting Eqs 19, 20, 21, 22, 23=0 for Model IV. Assumes K as shown, otherwise Table 1 values.

P_I=80, C_I=20, C_S=C_N=15, σ_P=.20, σ_P=.20, ρ=-.50, r=.12, θ_P=-.02, θ_C=.04, θ_N=-.05.

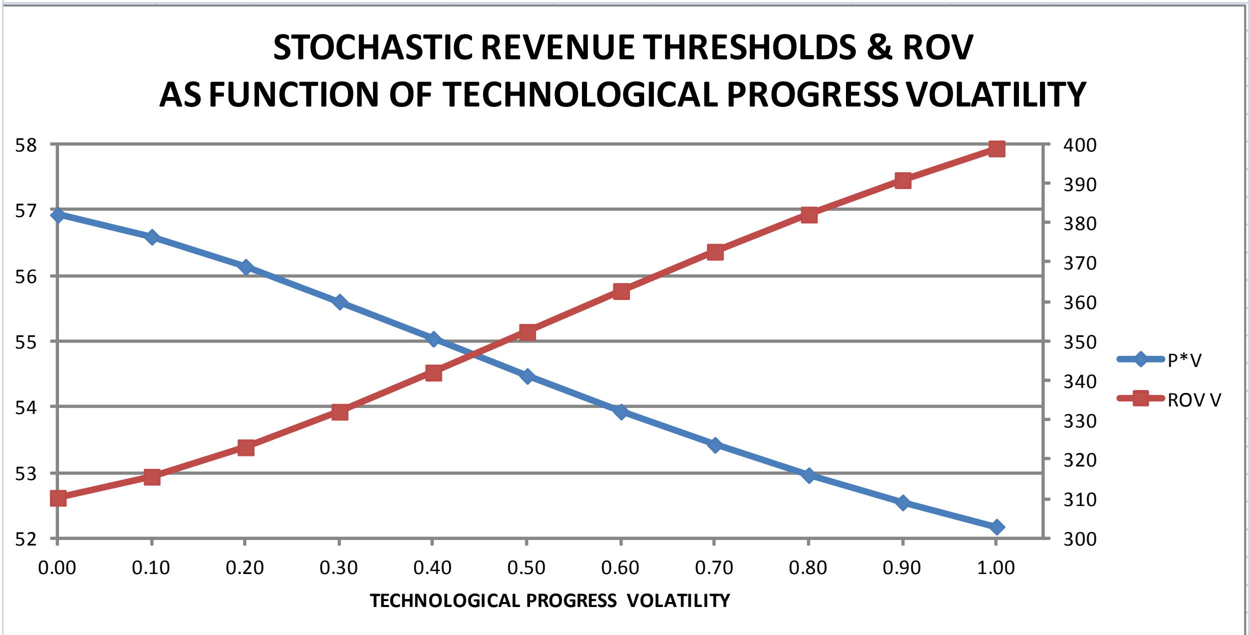
FIGURE 5



NPV-K	110.71	100.71	90.71	80.71	70.71	60.71	50.71	40.71	30.71	20.71	10.71
ROV U	367.44	351.38	336.47	322.55	309.54	297.32	285.82	274.98	264.74	255.06	245.88
ROV A	374.01	359.51	346.00	333.35	321.48	310.31	299.77	289.81	280.39	271.46	262.98
ROV V	393.59	384.23	375.50	367.32	359.64	352.41	345.60	339.17	333.10	327.36	321.94
Δ (NPV-K)		-10.00	-10.00	-10.00	-10.00	-10.00	-10.00	-10.00	-10.00	-10.00	-10.00
Δ ROV U		-16.06	-14.92	-13.91	-13.02	-12.22	-11.50	-10.84	-10.24	-9.69	-9.18
Δ ROV A		-14.49	-13.51	-12.65	-11.87	-11.17	-10.54	-9.96	-9.42	-8.93	-8.47
Δ ROV V		-9.35	-8.73	-8.18	-7.68	-7.23	-6.81	-6.43	-6.07	-5.74	-5.42
K	50	60	70	80	90	100	110	120	130	140	150

This figure depicts replacement option value from Eqs 9, 16 and 24 when P=75, C=30,
Assumes K as shown, otherwise Table 1 values.
P_I=80, C_I=20, C_S=C_N=15, σ_P=.20, σ_P=.20, ρ=-.50, r=.12, θ_P=-.02, θ_C=.04, θ_N=-.05.
NPV is based on the last two RHS terms of Equation 4.

FIGURE 6



P* V	56.925	56.590	56.135	55.603	55.038	54.472	53.930	53.424	52.963	52.548	52.179
ROV V	310.306	315.644	323.114	332.120	342.063	352.411	362.732	372.710	382.135	390.887	398.912
σ_N	**0.00**	**0.10**	**0.20**	**0.30**	**0.40**	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**1.00**

The LHS of this figure depicts the boundary set (P*,C*=31.039) that indicates when P=<P* and C=>C*, equipment replacement is justified, setting Eqs 19, 20, 21, 22, 23=0 for Model IV.
Assumes technological progress volatility as shown, otherwise Table 1 values.

P_S=80, C_S=15, K=100, σ_P=.20, σ_C=.5, $\rho_{P,C}$=-.50, $\rho_{P,N}$=-.1, $\rho_{C,N}$=-1, r=.12, θ_P=-.02, θ_C=.04, θ_N=-.05.

The RHS of this figure shows the replacement option value from Eq 24 when P=75, C=30.

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