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RMS Capacity Utilisation: Product Family and Supply Chain

Abstract

The paper contributes to development of RMS through linkage with external stakeholders such as customers and suppliers of parts/raw materials to handle demand fluctuations that necessitate information sharing across the supply chain tiers. RMS is developed as an integrated supply chain hub for adjusting production capacity using a hybrid methodology of decision trees and Markov analysis. The proposed Markov Chain model contributes to evaluate and monitor system reconfigurations required due to changes of product families with consideration of the products life cycles.

The simulation findings indicate that system productivity and financial performance in terms of the profit contribution of product-process allocation will vary over configuration stages. The capacity of an RMS with limited product families and/or limited model variants becomes gradually inoperative whilst approaching upcoming configuration stages due to the end of product life cycles. As a result, reconfiguration preparation is suggested quite before ending life cycle of an existing product in process, for switching from a product family to a new/another product family in the production range, subject to its present demand. The proposed model is illustrated through a simplified case study with given product families and transition probabilities.

Keywords: Reconfigurable Manufacturing System, Capacity Utilisation, Product Life Cycle, Product-Process Configuration

1. Introduction

Reconfigurable manufacturing system (RMS) is a relatively new paradigm and has been developed and considered as the factory of the future or the new manufacturing paradigm called ‘industry 4.0’ through manufacturing responsiveness to demand changes. RMSs are designed to be capable of quickly adapting to variable volumes and types of products (flexible in capacity and functionality) for a given part family. RMSs are characterised by their rapid and cost-effective response to market changes, and therefore are frequently being built by global enterprises (Koren, 2013). The RMS components with reconfigurable machines, which are all connected into a system, will enable changes in the system structure to accommodate production of new product types with their desired volumes. Accordingly, the system is open-ended to produce a new product on an existing system (Mehrabi et.al. 2000).

RMS is designed at the outset for rapid changes in hardware and software components in order to quickly adjust to production capacity and functionality within a part family in response to sudden changes in the market or in regulatory requirements (Koren et al.1999).
The RMS key characteristics include modularity, integrability, convertibility, diagnosability, and customisation (Mehrabi et al. 2000). The key characteristics of changeable functionality and scalable capacity (Koren 2005), which are the focus of the paper, will reduce the system reconfiguration effort and the ramp-up time (Mehrabi et al. 2001). The responsive manufacturing system has, firstly, an adjustable production capacity to cope with demand fluctuations (Koren and Shpitalni, 2011). RMS performance can be assessed based on resources and lead-time (Erik et. Al 2016). Scalability can be applied for scaled performance and functionality, or scaling up/down a manufacturing characteristic (e.g. capacity) (Putnik, et al. , 2013).

The current status of research on RMS mainly focuses on manufacturing system design, product family formation, product-process configuration, and fast reconfigurable layout, machinery and robots to enhance rapid responses to market demand fluctuations. Further research on RMS changeability in terms of capacity and functionality is necessary and must concentrate on development of functional models with a generic structure along with adaptable and scalable methods (Wiendahl et al. 2007). For RMS performance evaluation, operational costs, reconfiguration costs and effective utilization of machines while minimizing the system complexity and maximizing its responsiveness need to be taken into account (Hassan et al. 2014).

Most studies in the field of optimal capacity allocation have been concerned with a manufacturing environment with advanced demand information, which could assist reaching production policies for efficient capacity usage (Ozer and Wei, 2004). The assignment of capacity to customers’ demand is complicated by demand changes and allocation of capacity before demand is fully known (Shumsky and Zhang, 2009). Nevertheless, a little attention has been paid to efficient usage of capacity and the economical impact of capacity allocation. Deif and ElMaraghy (2006) proposed a cost model consolidating the physical capacity cost based on capacity size and costs associated with the reconfiguration path comprised of both penalty and effort cost related to scalability. Dolgui (2010) proposed a dynamic programming model for capacity-extension scheduling. Wang and Koren (2012) proposed a methodology for scalability planning to determine the most economical way to add machines to an existing system. Capacity can be adjusted based on hormonising throughput time (Scholz-Reitera et al., 2016). Koren et al. (2016) developed a method for capacity planning using generic algorithm to evaluate throughputs of alternative configurations with capacity expansion in an RMS. Sharing capacity in a mix production environment under uncertainty has been
investigated by some researchers. Ceryan and Koren (2009) proposed an optimisation problem for finding optimal investment on flexible capacity in a firm producing two products with uncertain demand in the planning horizon.

The main interest of the proposed methodology for industries could be the internal-external linkage of the manufacturing system. The external linkage with the supply chain will help industries to update their information, from the market that includes product demands and their life cycles in time, and from their suppliers that includes the parts and raw materials during the life cycles before ending demands. External-Internal linkage will facilitate reconfiguring their manufacturing systems exactly when needed to meet the requirements infused by market and/or suppliers and/or manufacturing demands.

There is almost no published work addressing an RMS linked to its supply chain tiers, and particularly focusing on how an RMS can deal with the demand/supply changes, uncertainties and risk caused by the connected demand/supply layers. Although many researchers noticed a dynamic nature of demand, to the best knowledge of the authors, no research work, which addresses the impact of product family life cycle in evaluation of RMS capacity usage over configuration, has been published to date.

This paper proposes a methodology for evaluating RMS capacity and alternative configurations allocated to products families in an uncertain condition using Markov analysis considering end of product life cycle. The expected value consisting of revenue and changeover cost will be taken into account for product-process (re)configuration and optimum capacity utilisation over configuration stages in the planning horizon.

The paper contains a number of novel aspects as follows: 1) the indication of RMS distinguishing characteristic of scalability for capacity adjustment in a supply chain, 2) the investigation on the impact of product family life cycle on the corresponding life cycle production with three stages of set-up configuration, on-configuration and off-configuration in an RMS environment, 3) the demonstration of the proposed hybrid methodology of decision trees coupled with Markov analysis with consideration of the end of a product life cycle as an absorbing state through numerical examples.

In the next section, we review the relevant literature on RMS chain and its associated RMS configuration design via the reconfiguration link. We then describe our focus on capacity adjustment during a product-family life cycle, Capacity adjustment for production of a product family and propose a model for probabilistic reconfigurations using a decision tree diagram for two product families. We then extend the model to incorporate Markov analysis
for probabilistic RMS configurations. Finally, we develop a simulation of the proposed Markov model through an example and provide a discussion of findings and conclusion.

2. Reconfigurable Manufacturing Supply Chain

The criticality of simultaneously addressing three domains of product, manufacturing process, and supply chain for system design decisions is evident; and the lack of coordination of the three domains should outperform those decisions (Farza, et. al., 2005). A systematic review of the literature in reverse supply chains showed that partnership and collaboration, product design, service concepts, and IT solutions have been indicated as the main drivers for value creation across supply chain tiers (Schenkel e.al., 2015). Many problems such as parts shortages, delivery and quality problems, and cost increases, are rooted in the lack of effective internal and external supply chain integration (Rosenzweig et. al, 2003). Supplier integration can be defined as a state of syncretism among the supplier, purchasing and manufacturing (Das, et.al. 2006). The suppliers cannot individually lead to improve time-to-market as they are also dependent on the other tiers such as manufacturers who seek resources for their internal exploration activities (Perols, et.al, 2013). There is a dynamic strategic interaction between a manufacturer and retailers’ (customers’) demands in a multi-period dynamic supply chain with a trade-off between immediate and future sales and profits (Gutierrez and He, 2011). Many researchers considered direct linkages between supplier and manufacturers and evaluated this linkage by examining manufacturing flexibility (Malhotra and Mackelprang, 2012).

2.1 RMS chain and product-process configuration

To fulfil the gap between dynamic market demands and capacity and functionality of manufacturing systems, a reconfiguration link is necessary as to group products into families before manufacturing based on process similarities (Abdi and Labib, 2003). The product-process reconfiguration link integrates product and process (re)assignments according to permanent changes in market and capacity conditions through determination of products in the production range, grouping products into families, and allocating products families to RMS manufacturing system configurations. Any new product will then be allocated to a product family with a suitable configuration designated for the product family production. The key task designated to the reconfiguration link is to support decisions over selection of product families and production scheduling with the corresponding configurations through integration of the data derived from the suppliers and the market. Therefore, the appropriate
product families will be formed by grouping similar-product demands and can productively be manufactured over configuration stages. Similarly, in a reconfigurable assembly system, product family formation through a clustering method based on a product-similarity matrix and an assembly sequence can be applied to enhance product-assembly productivity and capacity utilisation (Kashkoush and ElMaraghy, 2014).

There are two kinds of (re)configuration in an RMS: 1) products reconfiguration that means indication of product families, which are feasible in terms of economy and process requirements, and 2) RMS (re)configurations that corresponds to manufacturing facilities arrangement for production of each product family. The RMS reconfiguration might be complicated with various rearrangements of machines and tools and fixtures, material handling redirection, process rerouting, layout differentiation, and labour reassignment. Due to uncertain demand and vague data reflecting configuration criteria, alternative configurations for each product family can be evaluated through multi criteria decision (Abdi, 2009). The flexibility of the manufacturing system is embodied by the degrees of freedom in configuration and described by the number of possible configurations of an RMS (Unglert et al., 2015). Capacity utilisations can be optimised by alternative routing and rearrangement of machines cells in an RMS during production cycles (Eguia, et al. 2016).

Figure 1 highlights the RMS chain, as the supply chain hub, containing a reconfiguration link between market demand and a set of RMS configurations. In the RMS chain, product data analysis, new product introduction, product grouping and product family formation are performed through the reconfiguration link. This is followed by allocation of each product family to the corresponding manufacturing configuration at each configuration stage.

The customers of produced product families A, B, C, .. will deliver the products and their behaviour changes e.g. failing interest in a product family would affect the market demand and the range of product types entering to the reconfiguration link in the upcoming stages.

From the integrated supply chain shown in Figure 1, all the information regarding the product types e.g. A, B, and C demanded in the market will be derived over time. In addition, the product life cycle of each type is estimated based on information updated from the market and the suppliers. Product types may move out with ending their life cycles due to market requirements or the suppliers’ circumstances. The four stages of a product life cycle that includes introduction, demand growth, demand maturity, and demand decline will be derived from the reconfiguration link. In addition, internal integration of the link will facilitate gathering information regarding the capacity and (re)configuration conditions over time from the manufacturing system. Therefore, all the input data to the proposed mathematical models
for a real application in an RMS firm will be supplied by the data-based reconfiguration link as the centre of the RMS chain. The assumptions and the input data required for modelling the problem are identified in Section 3.1.

Figure 1 RMS chain with a product-process reconfiguration link

From the integrated supply chain shown in Figure 1, all the information regarding the product types e.g. A, B, and C demanded in the market will be derived over time. In addition, the product life cycle of each type is estimated based on information updated from the market and the suppliers. Product types may move out with ending their life cycles due to market requirements or the suppliers’ circumstances. The four stages of a product life cycle that includes introduction, demand growth, demand maturity, and demand decline will be derived from the reconfiguration link. In addition, internal integration of the link will facilitate gathering information regarding the capacity and (re)configuration conditions over time from the manufacturing system. Therefore, all the input data to the proposed mathematical models for a real application in an RMS firm will be supplied by the data-based reconfiguration link as the centre of the RMS chain. The assumptions and the input data required for modelling the problem are identified in Section 3.1.

2.2 RMS configuration design via a reconfiguration link
In an RMS design, the main question of ‘what is the optimal configuration of both products and manufacturing facilities?’ must be answered. Therefore, manufacturing facilities for producing the selected products, which are grouped into families and set in the production range, are designated in conjunction with their specific configurations. Economic and operational feasibility of the existing product families in the market are considered for possible production through a reconfiguration link, which also facilitate grouping identical products with operational similarities into families over configuration stages (Abdi and Labib, 2004). A product family with common operations can also be formed based on their commonality of alternative machines considering machine usage. Grouping methods such as the average linkage clustering method proposed by Navaei and ElMaraghy (2014) can derive a products-machines usage matrix through linking a products-operations similarity matrix and an operations-machines probability matrix. A hybrid methodology based on networked sequence of operations and operational similarity is used to group parts/product variants/models from a large product family in order to reduce changeover time and ease reconfiguration (Navaei and ElMaraghy, 2016). Accordingly, a suitable and identical configuration must be designed and allocated for manufacturing each product family with its variants/models in the planning horizon.

A continuous reconfiguration process is necessary to allocate suitable configurations to product families. Figure 2 illustrates a product-process loop for an RMS design, which must be reconfigurable to cope with various circumstances imposed by market demand and available manufacturing capacity. Considering market requirements, availability of supply of parts and raw materials and on-hand capacity, preliminary designs of potential configurations with determination of manufacturing facilities for existing product families are provided through economic and operational feasibility.

Having allocated a product-process configuration, RMS performance will be evaluated by measuring system throughput, capacity utilisation, changeover time, changeover cost. For evaluating RMS performance, maximum numbers of orders to the product families can be reassigned through formulating a semi-Markov process (Xiaobo et.al, 2001). A Markovian in-house production capacity with independent random demand levels in different time points can facilitate a production policy for capacity outsourcing when required (Yang, et. al., 2005). On the other hand, critical analysis of RMS performance can be performed via analytical methods such as holonic architecture linked to analytical network process (ANP) while considering both operations and economical aspects (Abdi and Labib, 2011). Dev et.al (2016) developed a real time decision support system using decision tree and holonic
structure to evaluate supply chain performance with respect to inventory levels in mobile industry under uncertain environment. The supply chain key performance indicators such as information sharing, lead time, inventory policy and product demand with the life cycle stages were evaluated by means of discrete event simulation linked to a decision tree classifier algorithm. The results showed that short life cycles of products increased variability in lead time that affected the level of inventory required to meet the customer service level.

Figure 2 RMS product-process reconfiguration loop
For RMS performance evaluation, the gap between the desired capacity usage and the actual capacity used is analysed over subsequent configuration stages while considering manufacturing criteria with updated (re)configuration requirements. Reusability of capacity and products contributes to green manufacturing and a reverse supply chain, which connects tiers of suppliers and the customers through a reconfiguration link required for an environment friendly RMS design.

3. Capacity adjustment during a product-family life cycle

Manufacturers can predict their product demands via online marketplace for planning their production (Chong et.al, 2015). Product families are selected based on their market demands and available capacity and then arranged according to their operational similarities or operation sequences (Goyal et. al., 2013). Production order will be the key inputs to (re)arrangement of RMS configurations. The similarity-based arrangement of product-process configuration will result in increasing reusability of manufacturing capacity over configuration stages while considering the selected product families life cycles. A probability distribution function can be used to reflect demand forecast and/or a capacity range with the commonality that represents a production rate, as a function of available production time and throughput (ElMaraghy et.al., 2012). The decision about capacity policy in term of initial level and rate of change needs to be made carefully and early enough to avoid unexpected production shortage resulting customers’ disappointment or overproduction, and find the minimum production capacity to achieve unimpeded diffusion of new products (Balakrishman and Pathak, 2014).

Wang and Koren (2012) defined system scalability as the complementary percentage value of proportion of the smallest possible increment to the existing capacity. For example, if 1% of an existing capacity can be added to the existing system, its scalability will be 99%. In contrast, dedicated manufacturing (serial configuration) has zero scalability as the smallest increment would be gained by an additional full production line with the same capacity [0% = 100 (1-1/1)%]. Consequently, a manufacturing system design yielding a lower capacity increment promotes scalability with fine tuning capacity.

There is a trade-off between system scalability and investment cost for selecting optimal configuration (Koren et al., 2016). In general, parallel facility/machine configuration increases scalability as each parallel manufacturing route can accommodate its contribution to capacity considering line balancing. In automotive industry, product family-based
platforms are scalable through stretching, shrinking, or reconfiguring operations on the platform to satisfy market demand in terms of product variant and volume. Figure 3 illustrates a product-family life cycle including four stages of product family, introduction, demand growth, demand maturity, and demand decline. It shows that a product family has a gradual typical demand growth following its introduction to the market. RMS capacity ought to be adjusted to demand variations of the active product family at each configuration stage, which happens at time \( t \) when identical/different models within the family are manufactured. Capacity volume at time \( t \) of configuration stage \( k \) is indicated by \( C(t_i) \), \( t_m \geq t_k \geq t_0 \), and \( m \geq k \geq 0 \), where \( m \) is equal to the total number of configuration stages during the whole product family life cycle including four stages as follows:

Stage 0 will be the introduction and/or development of the product family through functional testing of a prototype for at least for one of the product family models following substantial (online) market research undertaken via the reconfiguration link, in which the consumer requirements are fully updated. Demands for products are predicted and derived from various sources e.g. products’ online sales (Chong et.al, 2015). The existing/potential orders from existing customers or potential orders from potential customers in the online/offline market facilitate selection of products for production and grouping them into families. The selected product families in the production range could be transferred into the product design/development phase with the intention of being (re)designed based on their modular structures. Different combinations of individual modules used in the product design will accommodate production of different product families and models with using common resources. This also facilitates the modularity integration throughout the product-process design stage that will smooth the reconfiguration process with variant modular manufacturing elements. As a result, the modular structure increases the RMS adaptation to unpredictable changes in the product design and its processing needs through easily upgrading of hardware and software instead of the replacements of manufacturing facilities.

Production of a (new) product family is included in the master production plan with allocation of a preliminary configuration at time \( t_0 \) subject to operational and economical feasibility. The capacity allocated to a new product is typically low and shared with the other product family(ies), which have already been positioned in more advanced stages of their product life span. Having introduced a new product family, RMS will face risks due to a small number of customers, low profits, and unpredictable technological problems in the newly configured system for its production at time \( t_0 \).
In stage 1 (demand growth), the demand for at least one of the product family models starts to rise. Depending on full/partial acceptance of a product family by its consumers there might be a sudden/gradual growth in demand. In stage 2 (demand maturity), the product family is well established in the market with due to consumer satisfaction, the high and steady product demand with an unlikely growth. Manufacturing processes and their corresponding configurations are allocated to the product families. The capacity of each manufacturing facility/workstation/machine for the operations/tasks allocated is indicated. Set up for orders in process includes operations for preparation of machines, tools, and operators’ technical skills and their (re)assignment for new models of each product family. Therefore, the RMS configuration for the product family is constantly operative with predictable capacity usage during this stage. However, due to continuous demand and the pressure from the manufacturing competitors for increasing their market share, which is being limited by the RMS, the decisive managers need to update their product design strategy through (re)investment in design/development of new models under the same product-family umbrella. In this stage, specific functions must be carried out as follows:

- System balance for efficient capacity adjustment and avoiding bottleneck by adding machines and/or rearranging their connection.
- Material accuracy in terms of quality and volume with on time supply for each production cycle allocated to each product family while minimising inventory level and cost.
- Machine availability for operations required for allocated product families through continuous monitoring machine functionality and sustainability (energy consumption).
- Production control via data acquisition across the manufacturing executive system to monitor the whole manufacturing process from set up to the end of configuration due to the end of product cycle.

In stage 3 (demand decline), all the product family models cannot satisfy the consumer desires anymore and new product choices offered by the competitors seem to be more attractive. The extension of new models within the product family is an option to sustain its market demand, which leads to sudden/gradual decline depending on the failure/success of a product redesign with new model(s) introduction. The stage will eventually be terminated with the end of product-family life cycle for all of the models, and consequently through production control the corresponding manufacturing configuration will become inoperative and eventually disappear from the upcoming configurations.
There are failure possibilities and exceptions for maintaining the profile foreseen for the product life cycle. A product family/model may be put out of the market before maturity phase, particularly with launching a new product. This may also happen due to the product’s structure and complexity, degree of fitness to customer needs, and the presence of competitive substitutes. In such a situation, manufacturers need to foresee and update the life cycle profile through the reconfiguration link dynamically. Therefore, the proposed maturity level is skipped and a decline stage starts. The integrated supply chain presented in Figure 1 show how data based reconfiguration link can help an RMS deal with products becoming early leavers from market/manufacturing that leads to out-of-configuration. Having various product families in the production range will help manufacturing sustainability in terms of continuous capacity usage through substitution of product families/models over production cycles.

![Graph showing product family life cycle with capacity adjustment](image)

**Figure 3** Product family life cycle with capacity adjustment
The RMS capacity for a product family with demand $D(t)$ at time $t$ is the production rate $P(t)$ of the same product family with all the active on-configuration product models during the life cycle. Capacity reusability with facilities sharing is considered as the cost-effective use of active manufacturing capacity while reconfiguring RMS. The optimal allocation of a system configuration with the capacity required for production of each product family at a configuration stage must be obtained via reflection of changeover cost, changeover time and the reusability level of available hardware/software equipment.

3.1 Capacity utilisation and adjustment: assumptions and formulation

An identical manufacturing configuration can be (re)set for production of different models of a product-family, with minor reconfiguration, during its life cycle. A manufacturing schedule could occur in different configuration stages, which are different from phases within the product life cycle, so those periods are not necessarily harmonized. For example, an identical configuration for a product family is set/reset in configuration stages 1 and 3, which occur in the introduction phase (1) and the growth phase (2) respectively, whereas in configuration stage 2, the production capacity is allocated to another configuration processing on another product family. Therefore, the reconfiguration frequency during the planning horizon might vary across the life cycles. For instance, an identical configuration may be (re)set and used in five occasions in the life cycle during the production planning horizon (e.g. a year); one occasion during the introduction phase, another occasion during the growth phase, two occasions during the maturity phase, and one occasion during the decline phase. The five occasional configurations occur in a year i.e. planning horizon and the capacity in the rest of the year will be allocated to the other product families by setting their corresponding configurations.

Capacity planning can be undertaken through expanding capacity with adding new machines to match a new market demand (Wang and Koren, 2012). A scalability planning methodology was presented to determine the most economical way to add machines to an existing system. In comparison, this study focuses on maximum/efficient capacity utilisation with current facilities and without extra investment on new machines.

The main objective of the proposed model is to maximise overall capacity utilisation over production cycles. Assuming all the workstations have equal capacities will help prevent bottlenecks and hence increases throughput.

We hypothesize that RMS firms are concerned with high product variety and aimed at scale-efficient production for multiple product families having different life cycles whilst seeking capability of new product introduction to continuously meet the market requirements and the
customer preferences. The degree of automation for such RMS firms could vary from a medium level to a high level according to the volume of production and the range of product variety. This hypothesis implies rapid adjustment of the firms’ capacity and functionality in a coherent way to match product variety with the supply chain tiers’ requirement. For instance, automotive industries incorporating through an integrated supply network with employing advanced automated technologies such as product platform configurations are the potential end users of the proposed methodology.

The other assumptions for suitability of the manufacturing environment for implementation of the proposed include:

- All the workstations have equal capacities. This will help prevent bottlenecks and homogenous throughput of the workstations.
- Product variants in terms of changing product types occur frequently. In the integrated supply chain, the trend of product variety and product development are analysed, and the types of products with the number of variants for production are indicated through the reconfiguration link.
- Reconfiguration of manufacturing processes can occur by any kind of, or combination of, manufacturing facilities reformation in terms of process rerouting, layout reconfiguration e.g. machine relocation, departments expansion or shrinkage, conveyor redirections , and labour reassignment.
- The manufacturing processes with potential variability of system (hardware and software) to produce the current products in the market that are selected to place in the production range are defined.
- Standardisation of product and processes with their integration is needed for efficient reconfigurations over product variants.
- Time, effort and cost for system reconfigurations over product variant are identified and quantified.

Input data:
- Various orders from the customers are classified into several product groups (product family i, i= 1,2,3...m) through the reconfiguration link in an RMS. Each order is referred to as a single product /model belonging to a product family and the number of orders fit into a product family i is denoted by Di
• Demand for a product family i follow a deterministic or known probabilistic pattern e.g. uniform passion distribution function.
• The production rate and available capacity for an order of product family i is assumed to be known as Pi and Ci.
• The maximum number of orders in a family i should not exceed than production rate Pi at the time of production cycle k (tk) or configuration stage k.
• The selection policy of processing orders is based on a priority given to a product family based on its delivery time to the customers and the volume of orders while matching up with the available capacity at the time of production cycle k (tk).
• Orders for product family i can be produced by the corresponding configuration denoted as Con\textsuperscript{tk}\textsubscript{i} at the time of production cycle k (tk).
• The system configuration Con\textsuperscript{tk}\textsubscript{i} for product family i can be changed to Con\textsuperscript{tk}\textsubscript{j} based on the selection policy for processing product family j at the time of production cycle k (tk).
• Each system configuration can have a production rate with different revenue at configuration stage tk Rev(Con\textsuperscript{tk}\textsubscript{i}).
• The production time for an order of product family i is assumed to follow an exponential distribution with the average production time (1/\mu_{i,k})
• Changeover cost for switching from a system configuration i at tk to configuration j at tk+1 is denoted by g\textsubscript{ij}(Con\textsuperscript{tk}\textsubscript{i}, Con\textsuperscript{tk+1}\textsubscript{j}). Changeover time can be taken into account in changeover cost depending on the efforts needed to switch from a configuration to another. More similar configurations need less time/cost and effort for being interchanged. For the models within a family, changeover time is neglected.

Recalling from Figure 3, manufacturing capacity (discrete horizontal lines) follows market demand fluctuations (continuous curve) for a product family during its life cycle. On the other hand, discrete P(t) at time t is equal to the used capacity and equal or lower than the nominated total capacity. While simultaneously producing a number of product models within a family, the accumulated production rates at any configuration stage must not exceed the maximum capacity. As a general rule, assuming that m product models are identified for a product family based on a make-to-order policy with their feasible production rates P\textsubscript{i}(t) at time t for product model i, i = 1,.., m, then the total production (used capacity) should not
exceed either the total capacity, or the minimum value between the total demand $D_i(t)$ and the available capacity $C(t)$ at time $t$ in the same configuration stage being as illustrated in equations (5) and (6). Similarly, $C(t)$ can be defined as 'the maximum production rate’ at time $t$ as illustrated in equations (7). In addition, the production rates of each product type $i$ denoted by $P_i$ should not exceed the individual demand of each product type $i$ denoted by $D_i(t)$ at time $t$ in the same configuration stage as presented in equations (4).

Capacity utilisation can be calculated by using equation (1):

$$CU(t) = 100\% - \left(\frac{C(t) - P(t)}{C(t)}\right)100\%$$  \hspace{1cm} (1)

Where $CU(t)$ is the capacity utilisation at time $t$. If $C(t) = P(t)$, $CU(t)$ will be $100\%$ that means the nominal capacity is fully utilised at time $t$. For example, with $P(t_k)= 40$ unit/day and $C(t_k) = 50$ unit/day, then $CU(t_k) = 80\%$ this means $20\%$ of the nominal capacity is unutilised at time $t_k$. The utilization of capacity has to be below $100\%$ to justify its feasibility; the closer to $100\%$ the system utilization makes an RMS more economically efficient.

The optimisation problem with the objective function of maximising total capacity utilisation or minimising unused capacity over the planning horizon $T$ or $(0, T)$ will be:

$$\text{Max } CU = \text{Min}\left(\int_0^T \left(\frac{C(t) - P(t)}{C(t)}\right) d(t)\right)$$  \hspace{1cm} (2)

We denote where $C$ as the matrix of available capacity for products/models $i = 1, 2, \ldots, m : C = (C_1, C_2, \ldots, C_m)$ and $P$ is the matrix of actual production (used capacity) for products $i = 1, 2, \ldots, m : P = (P_1, P_2, \ldots, P_m)$. By considering $m$ models for a product family and assuming that the sums of discrete capacity utilisation during $t_1$ to $t_m$ for production of the models in the production range the objective function will be:

$$\text{Max } CU(C, P) \approx \text{Min}\left(\sum_{t_1}^{t_m}C(t_i) - P(t_i) / C(t_i)\right)$$  \hspace{1cm} (3)

Subject to the model constraints with given the time variant demand of each product model:

$$P(t_i) \leq D(t_i) , \hspace{0.5cm} i = 1, \ldots, m$$ \hspace{1cm} (4)

$$\sum_{t_1}^{t_m}P(t_i) = P(t)$$ \hspace{1cm} (5)
\[
\sum_{i=1}^{m} D_i(t_i) = D(t) \quad (6)
\]
\[
P(t) \leq \min (C(t), D(t)) \quad (7)
\]

Other objectives related to cost and revenue for a configuration and switching two consecutive configurations can also be taken into account as follows:

\[
\text{Max } \left[ \sum_{i=1}^{m} \text{Rev} (\text{Con } t_i) - \sum_{i=1}^{m} \sum_{j=1}^{m} g_{ij} (\text{Con } t_i, \text{Con } t_j) - \sum_{i=1}^{m} h_i (C(t_i) - P(t_i)) \right] \quad (8)
\]

Where

\text{Rev} (\text{Con } t_i) \text{ represents the income from selling product } i \text{ with its corresponding configuration that can be calculated by multiplying the unit price by the demand/production value, and } g_{ij} \text{ reflects the changeover cost of changing configuration from } i \text{ to } j, \text{ and } g_{ij} = 0 \text{ if } i = j.

(Con t_i, Con t_j) \text{ for the corresponding product models } i \text{ and } j \text{ at } t_i \text{ to } t_j; \text{ } h_i \text{ is the unit cost of unused capacity for product } i. \text{ This could reflect the missed opportunity of sales of product family } i \text{ with price } r_i (\approx h_i) \text{ for the unused capacity.}

For example, assuming that two product models A and A’ within a product family with similar process requirements are selected to be simultaneously produced, their feasible production rates (P_A and P_A’) are limited by the available capacity (C) as given by equation (9). Similarly, for three products A, A’, A” we will have the constraint given in equation (10). The linear non-equations and their feasible areas are graphically represented for a RMS with two and three products respectively in Figure (4_a,b).

\[
P_A (t) + P_{A'} (t) \leq C (t) \quad (9)
\]
\[
P_A (t) + P_{A'} (t) + P_{A''} (t) \leq C(t) \quad (10)
\]
The modular structure of a RMS beginning from the product design stage integrated into the process design facilitates reconfigurability of system elements in terms of changing their capacity and functionality whilst changing product volume and/or product type. The policy of no more, no less in manufacturing flexibility whether in capacity or functionality should be sought, using optimisation techniques such as analytical and/or simulation methods.

Recalling from Figure (2), the expected demand for the reconfiguration period $k$ denoted as $E[D(t)]$ can be derived from the integral given in equation (11). This expected value can be used as an estimation for the required capacity to be fixed over the period $k$ (between two sequential reconfigurations) denoted as $C(t)$, given $t_{k-1} \leq t \leq t_k$. The volume of capacity changes for a product type at reconfiguration time $t_k$ is equal to $C(t_k) - C(t_{k-1})$.

$$E[D(t) \mid t_{k-1} \leq t \leq t_k] = \left( \int_{t_{k-1}}^{t_k} D(t) \, dt \right) / (t_k - t_{k-1}) \quad (11)$$

Despite the product demand being stochastic and not fully predictable; it can be estimated through fitting their uncertain parameters to those in known probability distribution functions such as normal, geometric, or Poisson process functions.
Figure 5 illustrates how an RMS could adapt to demand variations in terms of adjustability of capacity for each configuration, and functionality to switch from a configuration to another configuration while changing on-configuration (running) product families.

An RMS dealing with production of more product families is appreciated as a more reconfigurable system. Therefore, more configuration stages with more often sub-stages of set-up configurations and off-configuration are expected in the planning horizon.

![Diagram of RMS configurations and capacity usage changes](Diagram)

**Figure 5 Changes of capacity used in three configuration sub-stages**

### 3.2 Reconfiguration process with capacity usage changes

The capacity usage may vary over configuration stages and/or during changeover time in three following sub-stages for the active product families:

- **$t_{A0}^1$:** Set-up configuration, which is the time required to set up the configuration to launch product family A. The sub-stage the product models within a product family is disregarded due to the short changeover time.

- **$t_{A0}^2$:** On-configuration, which is the time that the production of product family A is continued with possible changes of the product models. Therefore, no major reconfiguration is required and therefore capacity can be steadily adjusted to each product model over the sub-stage. Nevertheless, the manufacturing capacity usage or production rate may vary across the product models.

- **$t_{A0}^3$:** Off-configuration, which is the time required to switch the manufacturing system from an existing product family (A) to the next product family (B) at configuration stage 0 with the production order determined in the production plan. Hence, RMS configuration for product
family A is switched off while the configuration of product family B is switched on. According to the process routes and the layout configurations, particular machines may be re-functioned or interchanged through retooling and/or relocation.

It is important to note that the off-configuration sub-stage could coincide with the set-up configuration sub-stage of the next product family in the following stage. Attaining a mean value for the demand of each active product facility could facilitate obtaining the optimal capacity utilisation while allocating suitable capacity for configuration installation at each stage/sub-stage. Having set the capacity according to the demand mean value of a product family, the manufacturing facilities may operate under-capacity during the production, and particularly during the set-up configuration sub-stage and the off-configuration sub-stage. The optimal configuration with the matching capacity at each configuration stage must be found considering the changeover cost, the changeover time and the level of reusability of available hardware and software equipment.

3.3 A proposed model for probability decision tree with notations

In this section, a probability decision tree is proposed to demonstrate probabilistic allocation of manufacturing configurations to product families in different production cycles with various outcomes including reproduction, reconfiguration between product families, and end of life cycles of product families. The model notations are as follows:

Product family \( i, i=1,2,3,...,m \), with up to \( m \) product families indicated in the RMS. There is no practical limitation for number of product families (\( m \)). The model is acceptable/adjustable with a finite/infinite value of \( m \) with the following inequality formula:

\[
P_i(t_k) \leq D_i(t_k); \quad i = 1,2,3,...,m; \quad k = 0,1,2,...,n, \tag{12}
\]

Where \( P_i(t_k) \) is production rate for product family \( i \) at production cycle \( t_k \); and \( D_i(t_k) \) is demand for \( i^{th} \) product family at production cycle \( t_k \). Although there is up to \( m \) production cycles indicated in the planning horizon, no real practical limitation for the number of production cycles (\( m \)) exists.

\[
C_i(t_k); \quad i = 1,2,3,...,m; \quad k = 0,1,2,...,n.
\]

Where \( C_i(t_k) \) is capacity for production of product family \( i \) at production cycle \( t_k \) and:

\[
P_b i(t_1); \quad i = 1,2,3,...,m; \quad k = 0,1,2,...,n
\]

Probability of production of product family \( i \) at initial production cycle \( t_k \) (\( k=1 \)) or \( t_1 \):

Probability of production switch from product family \( i \) to product family \( j \) at production cycle \( t_k \) and \( t_{k+1} \):

20
\[ P_{b_{ij}}(t_k); \ i, j= 1,2,3,..m; \ k = 0,1,2..n. \]

Probability of reproduction of product family i at production cycle \( t_k \) and \( t_{k+1} \):
\[ P_{b_{ij}}(t_k); \ i = j= 1,2,3,..m; \ k = 0,1,2..n. \]

Probability of ending production of product family i at production cycle \( t_k \) due to end of its life cycle: \( P_{b_{i-out}}(t_k) \).

In the following sections, in order to simplify the model illustration, product family i or j is shown in terms of characters such as \( i = A, B, C \), and \( j = A, B, C \); when \( i = j \) the same product family with the same configuration will be under process in two consecutive production cycles \( t_k \) and \( t_{k+1} \), \( k = 0,1,2..n \).

### 3.4 Probabilistic reconfigurations using a decision tree diagram for two product families

Assume there are two product families A and B with two switchable corresponding configurations A and B, as shown in Figure 6. At each stage, only one product family (with its models) can be manufactured on the corresponding manufacturing configuration and layout, which can be switched to another configuration required for another product family, or could remain on the same configuration, manufacturing the same product family. Therefore, a configuration stage does not necessitate reconfiguration, especially when no change of a product type/family happens.

![Decision Tree Diagram](image)

**Figure 6- Schematic diagram of RMS transition with two product families A and B**

Assuming the transition takes place following known/estimated probabilities, which are derived from the system/market statistics and the frequencies of changes in the past, a tree diagram can be built as shown in Figure 7. End of a product life cycle for a product family (A or B) may happen at any current/future stage that causes termination of branching of the ‘end or life cycle’ node. Market demands for product families are the key factors for selecting the appropriate product family (A/B) with the corresponding configurations at stage \( t_k \), as notified by \( t_k^{A/B} \). Zero demand or a low demand below the predetermined threshold could
cause the end of the product family, and thus its production. In contrast, new product models introduced in the market will be added to the existing product family.

![Figure 7- Probability tree diagram for an RMS with two product families](image)

Depending on the market demand at stage 0, an RMS configuration: configuration A for product A and configuration B for product B will be set. The market demand will continuously affect the configuration selection for configuration A/B in the following stages: stage 1, stage 2, .., etc. over the planning horizon.

4. Economic Evaluation of Probabilistic RMS Configurations Using Markov Analysis

In this section, a mathematical formulation is presented to evaluate the expected value (returnable benefit) of system reconfigurations over product families. The evaluation consist of the cost Markov analysis while considering revenue of each configuration allocated to each product family and changeover costs.

The cost Markov analysis deals with probabilities of future (known) finite events through using current known probabilities. The stochastic analysis is based on an assumption
reflecting an initial state that system can start operating towards limited forthcoming probabilistic events. Accordingly, a matrix of transition probabilities is established to present the likelihood of changing from one state to another state. In particular, the Markov process would enable a system to predict the future events/conditions.

In a Markovian process with n exhaustive, chronological and mutually exclusive states, the probabilities at a specific point at time $t_k$, $k=0, 1, 2, \ldots, m$ are adopted from (Taha, 2011) as shown in equation 13.

$$Pb_{ij} = Pb\{X_{t_{k+1}} = j \mid X_{t_k} = i\}, \quad i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, n, \quad k = 0, 1, 2, \ldots, m \quad (13)$$

This is known as the one-step transition probability of moving from state $i$ at time $t_k$ to state $j$ at time $t_{k+1}$. By definition, the sum of the transition probabilities from state $i$ to all the possible state $j$s will be equal to 1. Matrix $Pb$ (shown by equation (14)) presents a Markov-chain transition probabilities:

$$Pb = \begin{pmatrix}
Pb_{11} & Pb_{12} & \ldots & Pb_{1n} \\
Pb_{21} & Pb_{22} & \ldots & Pb_{2n} \\
\vdots & \vdots & & \vdots \\
Pb_{n1} & Pb_{n2} & \ldots & Pb_{nn}
\end{pmatrix} \quad (14)$$

In this section, the probability decision tree presented in section 3 is linked to a Markov chain for modelling product (family)-process (re)configuration and capacity adjustment. The model is proposed to demonstrate probabilistic analysis of optimum product family allocation to configurations with feasible production and capacity allocation.

Set of feasible configurations at discrete stage/production cycle $t_k$ for product family $i$: $Con_{i}^{t_k}$ where $i=1, 2, \ldots, m$. and $Con_{i}^{t_k} \in Con^{t_k}$, which represents total configurations available in an RMS in production cycle $t_k$.

Assuming the unit production cost $q_i$ of product family $i$ does not exceed than the product price $r_i$ the product revenue at configuration stage $t_k$ will be:

$$Rev(Con_{i}^{t_k}) = Con_{i}^{t_k}(r_i - q_i) \times P_i(t_k) \quad (15)$$

This value will be the revenue associated with producing an order belonging to product family $i$ using its configuration at stage/production cycle $t_k$ during its life cycle.

Recalling from equations (8) and (15), we will then gain the total expected value (EV) for the planning horizon with the probability of production swap between product families $i$ and $j$, $i$ and $j = 1, 2, \ldots, m,$ and $i \neq j$ with $Pb_{ij}(t_k)$ at stage/production cycle $t_k$ as follows:
\[ EV = \sum_{k=0}^{n} \sum_{i=1}^{m} \beta^k [Rev(Con_i^{t_k}) - h_i(C(t_i) - P_i(t_k) P_{bi}(t_k))] - \]
\[ \sum_{k=0}^{n} \sum_{i=1}^{m} \sum_{j=1}^{m} \beta^k [g_{ij} (Con_i^{t_k}, Con_j^{t_k+1}) P_{bij}(t_k)] \]  

Where \( \beta^k \) is a discounted rate for calculating the present value connected to the configuration stage \( k \) in the planning horizon, and \( g_{ij} \) is the cost associated with changeover while switching production of product family \( i \) with configuration at stage/production cycle \( t_k \) to product family \( j \) with configuration at stage/production cycle \( t_{k+1} \); and \( i=j=1,2,..,m; k=0,1,2,..,n; \) \( g_{ij} = 0 \) if \( i=j \) where reproduction of an identical product family \( (i \ or \ j) \) occurs with the same configuration in two consecutive production cycles \( t_k \) and \( t_{k+1} \). This includes the cost of operator (re)assignments, setup and layout rearrangement and could be measured by the reconfiguration time.

**Assumptions:**

State probabilities of production remain the same with production rate \( P_i(t_k) \) at production cycles \( t_k \), which depends on the state probabilities of mixed production of \( n \) product families, where \( i=1,2,..,m; k=0,1,2,..,n \).

State probabilities of production swap between product families \( i \) and \( j \): \( P_{bij}(t_k) \) remain the same at all stages \( t_k \), \( k=0,1,2,..,n \).

The EV of a single (re)configuration is simply the probability of that configuration occurrence multiplied by the monetary value of that outcome i.e. revenue minus changeover cost. The total EV will be the sum of the expected values of production cycles through various (re)configurations in the planning horizon in all stages \( t_k \), \( k=0,1,2,..,m \).

The proposed model will be coupled with a Markov-chain model for further analysis of various states with the corresponding capacity usages. In the following sections, in order to simplify the model illustration, product family \( i \) or \( j \) is shown in terms of characters such as \( i \) and \( j = A, B, C \).

**4.1 Illustration of the Markov analysis for product families via an example**

In this section, a time-invariant and memoryless Markov chain is proposed with a discrete-time process reflecting independent configuration stages over time. Assume there are three product families A, B and C without considering the end of their life cycles and with the given transition/reconfiguration probabilities as shown in Figure 8. As explained in section...
3.4, the unique configuration arranged for production of each product family including its all models can remain the same or be transformed to another configuration arranged for another product family over two consecutive configuration stages.

Figure 8 Tree diagram of transition/reconfiguration probabilities for three product families

For example, Pb_{AA} = 0.8 is the probability of remaining in configuration A in two consecutive configuration stages. The transition matrix Pb or Π (0) (transition probability matrix at stage 0) is shown below:

\[
P_b = \begin{pmatrix}
0.8 & 0.1 & 0.1 \\
0.1 & 0.7 & 0.2 \\
0.2 & 0.2 & 0.6
\end{pmatrix}
\] (17)

The Makov chain is assumed to be time homogenous that means transition probability is independent from time shifting. The transition matrix shown in Equation (17) is a one-step reversible transition matrix and the probability remains unchanged while variables such as product demand changes during different stages of a product life cycle. The fixed transition probabilities are used to simplify the proposed finite Markov Chain model with limited existing product families. In addition, in the condition where the probability distribution functions of products demands and their life cycles are known within a finite planning horizon the changes of the transition probabilities could be slight and ignored.

Assume that the market shares (or the corresponding capacity allocation) for products A, B, and C at stage 0 is a vector indicated as:

\[\Pi (0) = (0.40, 0.30, 0.30).\] Therefore, the market share/capacity allocation for the next stage (stage 1) will be (0.41, 0.31, 0.28) as shown below:
Having considered the RMS reconfiguration being as a memoryless Markov chain with independent configuration stages, we have:

\[
\Pi^{(2)} = \Pi^{(1)} \cdot P_b^{1}, \quad \Pi^{(n+1)} = \Pi^{(n)} \cdot P_b^n, \quad \text{and} \quad \Pi^{(n)} = \Pi^{(0)} \cdot P_b^n \quad (19)
\]

Therefore, the market share of the products for any future stages can be predicted using the equation above. By reaching a steady state in which:

\[
\Pi^{(n+1)} = \Pi^{(n)}, \quad (20)
\]

The equilibrium market share of the three products can be computed using the equation below:

\[
\Pi^{(n)} = \Pi^{(n)} \cdot P_b \quad (21)
\]

Where the future market share will be equal to the current market share multiplied by the transition probability matrix. Three variables \(X_A, X_B, \) and \(X_C\) stand for market share of products \(A, B,\) and \(C\) respectively. We have:

\[
(X_A, X_B, X_C) = (X_A, X_B, X_C) \times \begin{pmatrix}
0.8 & 0.1 & 0.1 \\
0.1 & 0.7 & 0.2 \\
0.2 & 0.2 & 0.6
\end{pmatrix} \quad (22)
\]

By solving the simultaneous equations above, as described by (Render, et.al. 2009), the values of the three variables can be found as follows:

\[X_A = 0.42\]
\[X_B = 0.32\]
\[X_C = 0.26\]

Considering the memoryless Markov chain, the random variables \(X_A, X_B, X_C\) in equation (22) are not independent. The variables at time \(t\) only depend on stage \((t-1)\) whereas the whole history earlier than time \((t-1)\) is forgotten. As a result, the market share will be very stable in the future with a very little increase in demands for products \(A\) and \(B\), and a very little demand reduction for product \(C\) considering an equilibrium condition and/or an unchanged configuration. Equilibrium condition is a condition in which the state probabilities of a future configuration remain the same as the state probabilities in the previous configuration. In addition, an unchanged configuration happens when reconfiguration is not
required over two consecutive configuration stages even though product models within the product family could be swapping.

4.2 Markov analysis for two products with the ‘end of life cycle’ event

The open-ended tree structure shown in Figure 7 can be simplified to transition nodes through Markov analysis in order to avoid repeating presentation of the same branch/event i.e. a RMS configuration. Accordingly, the RMS configurations for two product families A and B, the transition probabilities are noted on the arrows as follows:

- Probability of (re)configuration between the two products, form product A to product B and vice versa: $P_{BA}, P_{AB}$
- Probability of returning on the identical configuration at two consecutive configuration stages : $P_{BB}, P_{AA}$
- For running out of the product life cycle when the RMS configuration for product A or B will be out of order respectively: $P_{B_{out}}, P_{A_{out}}$

If a product family (model) is run out from the RMS production due to the end of its life cycle, the unused capacity will be allocated to another product model including a newly designed model within the product family, or to another existing product family, or newly defined product family with a different configuration. However, in the case of more than one option for the capacity replacement the product family/model with less reconfiguration time/cost will be selected for replacement (Abdi, 2012)

There are two kinds of an ending life cycle: 1) the end of a product-model’s life cycle within a product family, and 2) the end a product-family’s life cycle that means all the models within the product family are declined in the market. Ending product life cycles begins from a post maturity stage to a decline stage of the product life cycle, in which there will be no more demand for the product model/family in the current market. Accordingly, some of the product models within a family are declined whereas the rest are still appealing in the market, so their RMS configurations remain operational. Ending life cycle might follow different trends such as smooth, steady, sudden, or sharp decline depending to the current/future market and its (un)certain conditions.

Assuming two product families A and B existing in a RMS production plan, if one of the product (A or B) ends its life cycle due to the lack of demand at the configuration stage there will be no need of more production by the corresponding configuration, and the
production will move towards the other product which is still demanded by the market. The Markov process structure remains the same over stages as shown in Figure 9.

Figure 9 Tree diagram for two product families’ configurations with ending product life cycles

4.3 End of product life cycles as absorbing states

An absorbing state is a state which is not returnable. In other words, if a system is in an absorbing state, it cannot move to another state between any two periods. Assuming two product families A and B can be produced in an RMS given both product families are favourably demanded in the current market condition with no preference. However, each product family may face ‘end of life-cycle’ as a non-returnable state and cannot be transformed to other states as shown by transition diagram (Figure 10) with the probabilities shown in Table 1. The system configuration initially starts at configuration A, which is a transitional node and can remain or move to end of life or configuration B. The probabilities that a non-absorbing state (a product on configuration) end with an absorbing state (end of a product life cycle) are presented through a numerical example.
Figure 10 Transition diagram for configurations of two product families (Con A, Con B) with ending product life cycles (End of A, End of B)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>0.1</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>0.3</td>
<td>0.6</td>
<td>0</td>
<td>0.1</td>
</tr>
<tr>
<td>End of Product Family A life cycle</td>
<td></td>
<td>0</td>
<td>0.1</td>
</tr>
<tr>
<td>End of Product Family B life cycle</td>
<td></td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 1 Transition probabilities for two product families A and B with end of life-cycle

The transition matrix P consists of four quarters of matrix B, matrix A, an identity matrix I (1s on the diagonal and 0s elsewhere) and zero matrix 0 (with all values 0s), as described by Render et.al. (2009) will be:

\[
P = \begin{pmatrix}
B & A \\
0.7 & 0.2 & 0.1 & 0 \\
0.3 & 0.6 & 0 & 0.1 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 1 & & \\
\end{pmatrix}
\] (23)

Therefore, the fundamental matrix (F), as described by Render et.al. (2009), will be:
The output of multiplication of matrices $F$ and $A$ will be the probability of moving from non-absorbing state or returnable configuration to an absorbing/nonreturnable state i.e. ‘out of configuration’ as shown in equation 25.

$$
F \times A = \begin{bmatrix}
6.67 & 3.33 \\
5 & 5
\end{bmatrix} \times \begin{bmatrix}
0.1 & 0 \\
0 & 0.1
\end{bmatrix} = \begin{bmatrix}
0.67 & 0.33 \\
0.5 & 0.5
\end{bmatrix}
$$

The $F \times A$ output values reflect the probabilities of the configuration for product family A terminates with the ‘end of life cycle’ of products family A or B are 0.67 and 0.33 respectively. In comparison, the probability of the configuration for product family B terminates with the ‘end of life cycle’ of products family A or B is equal (0.5). On the other hand, all the configurations for existing product families will be eventually terminated by the ‘end of life cycle’. Assuming the total number of RMS configurations set for all the models of each product family A/ B is the same and equal to 100 installations during their entire product life cycles, the number of phases that the configuration terminates with ‘end of life cycle’ of the models of product family A and B will be 117, 83 respectively as shown below:

$$(100, 100) \times \begin{bmatrix}
0.67 & 0.33 \\
0.5 & 0.5
\end{bmatrix} = (117, 83) \quad (26)$$

Therefore, the probability of product A being on an RMS configuration and ending up to an absorbing state of being out of product family A production (Product A, not-on-configuration) is 67%, whereas the probability of product family A ending up with finishing product B life cycle (Product B, not-on-configuration) is 33%. In contrast, the probability of product family B (on-configuration) ends with ‘end of life cycle of either product family A or B is the same. Therefore, in overall, regardless of the operative configuration, loosing demand and inoperativeness of product family B seems to be more likely than product family A. On the other hand, assuming the (re)configuration cost for each product family is equal to £1000, the expected total cost of reconfiguration of active product families of A and B becomes inoperative during its life cycle will be almost £117000 (117*£1000), £83000 (83*£1000) respectively.

The more frequent demand of a product family causes the less risk of not-on-configuration and capacity change because of end of life cycle. The product models with less steady
demand over consecutive stages will be replaced with more frequently demanded products with proper reconfiguration. Considering the same example and assuming that each product family A/B has an equal production capacity of 1000 unit with the same market demand per configuration stage, the total RMS capacity used for all the models of both product families will be \((100+100) \times 1000 = 200,000\) units, from which the total capacity share of product family A and B will be 117000, 83000 respectively. However, the total capacity of 100 units per product family per configuration stage will ultimately be halted due to the end of life cycles of all the models belonged to both product families.

5. Simulation of the proposed Markov model through a simplified industrial case study

Automotive industry has been leading mass customization with system reconfigurability to offer a range of products in the market for satisfying the particular requirements of different kinds of customers without massive increase in the operational cost. Product platforms used in automotive industry are allocated to product families, which are sets of products that share a number of common components and functions with each product having its unique specifications to meet demands of certain customers' (Pirmoradi et. al., 2014). Zapico et. Al. (2015) developed application of product platform design as a design space of possible configurations for materials handling vehicles industry. A configuration subset could be used for a specific products supplemented with specific components.

In this paper, the proposed method is shown with a simplified case study undertaken in an automotive firm (Company A, UK) with make-to-order production. The assembly line produces various ranges of trucks from 8 to 44 tonnes with production rate over 14,000 trucks per year in total. The daily production of 8hrs per shift is in range of \((10,100)\) with an average of around 55 trucks. Achieving efficient production according to truck specifications with variants options, from assembly and material flow perspective has been a challenging objective for the company. The productivity has recently improved about 10% through implementing advanced lean manufacturing principles in terms of work standardisation, visual team processes and lean material flow.

The truck production line is designed based upon product platform architecture; with alternative configurations consisting of various workstations i.e. mostly automatic for the purpose of mass assembly of various product families whilst minimising reconfiguration time and effort. By using product platform architecture, similarities of process configurations increase; hence by switching product families, the change over time and cost are sharply
reduced. In order to focus on capacity utilisation, we assume that all the workstations on product platform have equal capacities and no bottleneck is created.

The production capacity for various trucks varies according to the truck specifications such as axle configuration (in proportion to truck capacity), chassis type, cap, and engine power. The demands for the models within product families vary over time. A product platform has a direct effect on the performance and operations cost and product development time. Most products will face changes and redesigns during their life cycle. The production time and cost for product development via new models depend basically on how the original platform is designed and adaptable to the changes. A sustainable product platform must contain appropriate margins for future change and will lower the redesign costs in the long term and will delay the necessity for a completely new design (Eckert et al. 2012).

Table 2 shows two product families A and B with their axle configurations according to engine power, chassis type, execution, and cap type. Considering engine power (hp) and axle configuration that are placed in the same classification with 6 different engines (excluding overlaps), there are 80 (=5*16) possible different models for each product family, which can be produced on the same product platform (configuration). By adding new futures such as color, types of brake system, tyre sizes and compounds, seat and trim materials, lights, seat suspensions, and heating systems, the total number of possible configurations can exponentially rise into thousands. However, in accordance to the market, some models/ configurations might have no demand; and some others might have a short life span. Those products reaching at the end of their life cycles will be eliminated from the production plan. By changing production of models within a product family the product platform remains unchanged with slight changes in operations. On the other hand, the product platforms and their corresponding hardware/software configurations and operations are rearranged when product families are switched.

<table>
<thead>
<tr>
<th>Product family</th>
<th>Product platform axle configuration (tones)</th>
<th>Engine Power (hp)</th>
<th>Chassis type (for A and B)</th>
<th>Execution (for A and B)</th>
<th>Cap type (for A and B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(8-12 tones)</td>
<td>150, 180, 210, 220, 250</td>
<td>Tractor , Rigid</td>
<td>Standard, Construction</td>
<td>Sleeper, Day cab, Space cap, Super space cab</td>
</tr>
<tr>
<td>B</td>
<td>(14-16 tones)</td>
<td>180, 210, 220, 250, 280</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2 Product families A and B with various possible models/configurations
Figure 11 depicts the initial stage of the simulation through using the DecisionPro software (2002). By starting production of the models within product family A with its corresponding platform configuration, the production capacity used during (re)configurations of the two product families A and B with certain models can be obtained by the Markov chain simulation considering the product-models’ end of life cycles. The tree structure with the same Marcovian states noted in the previous section shows that the system starts with a full capacity (100%) with parts/raw materials ready for production of product family A’s models with 100 units in state 0 followed by the three upcoming events of the production of product family A or B or the end of product A’s life cycle with the given fixed probabilities. The simulation start with production of 100 units reflecting 100% of the capacity of a model of product family A, and zero production of product family B with no units left at the end of life cycle of product A/B. The transition probabilities for three different outcome are: 1) configuration A remains the same (A to A) when models within product family A are interchanged in two consecutive production stages, or 2) configuration changes to configuration B, for production of a model of product family B, or 3) terminates with end of the model’s life cycle, and assumed to be 80%, 15% and 5% respectively. In practice, the accurate probabilities are derived from the latest frequencies of the product families’ swap and the proportion of the products life cycles in the planning horizon, and the period of each configuration stage that can vary over production stages.

![Figure 11- Example of a tree structure with a Markov chain for configurations of two product families A and B](image-url)
Figure 12 depicts the proportion of used/unused capacity for on-configuration product family A/B and off-configuration of their corresponding models due to the end of their life cycles, which might take place at each configuration stage. It can be seen that the replacement of system configuration A (for product family A) by system configuration B (for product family B) occurs more frequently in the beginning of production horizon than later as the life cycles of the product B’s models are supposed to be shorter than the product A’s models. However, by ending product life cycle of both product families over the time, neither system (re)configuration nor production of either product is required.

![Graph of used/unused capacity](image)

**Figure 12** The proportion of used/unused capacity for production of two product families with end of life cycles

Similarly, Figure 13 represents a synopsis of the used/unused capacity over configuration stages for the same problem with the same input values. It can be seen if the product families and/or the product models within each family are limited, the ‘used capacity’ will gradually be converted into ‘unused capacity’ over configuration stages in the planning horizon. A configuration stage may vary from a short period (a week) to a long period (a month) depending on the demand and the configuration requirements. Introducing random demands with different distribution patterns along with transition probabilities of product families according to their product life cycle makes the Markov analysis considerably complex. However, to check sustainability level of the results, the Markov simulation was also carried out by using random demands generated from various distribution functions such as normal, passion and uniform distributions. Despite differences in the used/unused capacity patterns, the results reflected more or less similar perspective of the used/unused capacity over configuration stages. For example, by using random demand uniform distribution (10, 100)
for product families A and B while keeping the other parameters unchanged, the capacity usage is increased sharply, after a delay in the beginning of production, and then reduced smoothly over configuration stages till near the end of production cycles with zero utilized capacity that means similar sustainable results.

Figure 13 Synopsis of used/unused capacity for production of two product families with end of life cycles a) with fixed transition probabilities, and b) with uniform distributions of product family demands

Figure 14 highlights the fact that at configuration stage 5 (after 5 consecutive (re)configurations), the plant uses only 50% of the manufacturing capacity for production of product families A/B, and the rest of the capacity (50%) will become idle if it is not allocated to any other product family or newly introduced models within the exiting active product families. As a result, if the production plan is limited to two product families with the transition probabilities given remained the same, the capacity will unwillingly be inoperative.
by the following 15 upcoming configuration stages. Therefore, the plan needs to get prepared for switching from a product family (A or B) to another product family (e.g. product families C or D) or introduce new models (if demanded) within the exiting active product families (A and B) in order to maximise its capacity utilisation.

The company continuously develops its products with new engine design for ultra-clean and fuel consumption efficiency, greater load space, three-axle trailer, economical aerobody through new product families, which necessitate different components and product platforms (configurations). The predecessor families/models remains in the production range, but in the maturity or decline stage in Europe whereas former product families are still produced in the factories outside Europe such as the former trucks suitable for transportation of agricultural products in Africa. The simulation is extended to accommodate production of new product families (heavier trucks) introduced to the plant as shown in Table 3.

<table>
<thead>
<tr>
<th>Product family</th>
<th>Product platform axle configuration (tones)</th>
<th>Engine (horse power)</th>
<th>Chassis type</th>
<th>Execution</th>
<th>Cap type</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>(19-30 tones)</td>
<td>220, 250, 280, 290, 310</td>
<td>Tractor, Rigid</td>
<td>Standard, Construction</td>
<td>Sleeper, Day cab, Space cap, Super space cab</td>
</tr>
<tr>
<td>D</td>
<td>(30-44 tones)</td>
<td>330, 370, 400, 410, 440, 460, 510</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3 Product families C and D introduced to the company in the planning horizon

Figure 14 Used/unused capacity trends over configuration stages
By introducing new product families or models within each existing product family the capacity reusability can be improved. As shown in Figure 15, by ending life cycles of existing product families A and B, new product families C and D can be introduced with corresponding transition probabilities $p_{A\rightarrow C}$ and $p_{B\rightarrow D}$, and again with ending life cycles of new product families C and D.

**Figure 15** Tree diagram of probabilistic configurations by introduction two new product families followed by ending life cycles of two product family (models)

Assuming 80% chance of producing a new product family C/D at the end of life cycle of product family A/B, the capacity usage can be updated over configuration stages. Figures 16 and 17 illustrate the Markov model without/with introduction of new product C/D with the expected value (EV) as presented in equation (16) calculated by simulation. Expected value (net present value) considering a fixed interest rate (0.01) per each stage through over 100 configuration stages is calculated when profit contribution of product models of families A, B, C, D are assumed to be alike with $5000$, $4000$, $5000$, $4000$ respectively. Since, the company income statement does not reflect the profit for delivery of each product family/type, the profit contribution is approximately calculated according to sales, revenue and cost in the previous seasons divide by the number of products (or families) delivered in
Europe. The EV net present value with/without possibility of new product introduction i.e. product families C and D is $2,656,275/$2,230,086. Although existence of change over cost is evident there is no record or calculation of platform interchanges for swapping product families in the company. Hence, it is assumed that the changeover cost for switching production between the products families equally remains as $10000. It is shown that by adding new product family, EV can be significantly increased depending on the changeover cost.

Figure 16 A tree diagram of the Markov model with EV without new product introduction
Figure 17  A tree diagram of the Markov model with EV with new product introduction (C and D)
Figure 18 (a,b) Synopsis of the used/unused capacity for production of two product families with end of life cycles

6. Conclusions
The paper explores the radical idea of necessity of a continuous linkage between market, suppliers and manufacturing to optimise the capacity usage in RMS. The significance of product life cycle for allocating product families to the corresponding configurations is demonstrated.

The paper contributes to the indication of RMS distinguishing characteristic of scalability for capacity adjustment in a supply chain with the impact of product family life cycle on the
corresponding life cycle production with three stages of set-up configuration, on-configuration and off-configuration. The paper develops a stochastic model consisting of a decision diagram coupled with Markov analysis to determine the gap between used and unused capacity over discrete configuration stages. In the proposed model, ‘end of product life cycle’ of product families (models) is considered as an absorbing state, in which the product (model) will be out of configuration and inoperative. Numerical examples with given transition probabilities highlight how an RMS firm with available capacity can operationally cope with demands of various product families with Markovian flow of their life cycles. The Markov process of product families’ life cycles and the capacity usage is found to be statistically monotone.

As a result of simulation performed for the industrial case study, the production slowdown of an RMS with very limited product families including limited model variants featuring short life cycles indicates that the major capacity will be gradually inoperative through upcoming configuration stages. In order to maximise the capacity reusability, the RMS needs to get prepared for switching from existing product families to new product families before ending the life cycles, or alternatively introduce new models (if demanded) within the exiting operative product families. In addition, it is found that the monetary present value is sharply increased by adding new product families/models for the product families with a short life cycle while considering their profit contributions and changeover costs over (re)configurations.

We assumed that all the workstations have equal capacities to help prevent bottlenecks and hence increases throughput. Balancing workload for workstations with different capacities that create bottlenecks can be considered as future research. In addition, for future research, time varying transition probabilities can be obtained by dynamically updating data using simulation with data generation process incorporating data from the previous stages for finding most likely transition probabilities over infinite upcoming configuration stages. Due to our focus on the capacity utilisation and missed opportunity of unused capacity as a result of ending product life cycle, other decision variables and criteria such as inventory levels and bottlenecks, which are not included in the model, can be considered for optimal allocation of production resources as our future research. The proposed model can also be developed by fitting distribution functions for generating transition probabilities, demand, maintenance and changeover time and cost. In addition to capacity utilisation, configurations revenue and changeover costs indicated in the paper, reconfiguration time can also be
considered as another key performance indicator. The results can be extended for the infinite-horizon version of the problem through a real case study.

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