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Citation: Mykhayliv D and Zauner KG (2015) The Q Theory of Investment with Private Benefits of Control, Soft Budget Constraints and Financial Constraints. Working Paper. The Authors.

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The Q Theory of Investment with Private Benefits of Control, Soft Budget Constraints and Financial Constraints

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December 2015

PRELIMINARY AND INCOMPLETE.

Abstract

In this paper, we extend Tobin's Q model under financial frictions (Hennesy, Levy, and Whited, Journal of Financial Economics (2007)), using a discrete-time version of their model, to include private benefits of control of managers and other stakeholders and soft budget constraints in the form of money injections into the firm. Managers are not viewed to maximise shareholder value, but to maximise the value of their shareholding plus their private benefits of control. Private benefits of control introduce elements of asset stripping into the model. We characterize the optimal investment policy, analyse comparative statics and discuss applications to firms in transitional economies.

Keywords: Corporate Governance, Financial Constraints, Investment, Private Benefits of Control, Soft Budget Constraints

1. Private Benefits of Control and Optimal Investment

In this paper, an investment model that allows for agency and control problems is presented. Mykhayliv and Zauner (2013) present a special case of this model and an empirical specification motivated by this model to study corporate investment in Ukraine.

The financial constraints include convex cost of external equity, debt overhang and collateral constraints on new borrowing as in Hennessy, Levy and Whited (2007). However, the managers of the firm do not maximize shareholders' wealth as in Hennessy, Levy and Whited (2007), but the wealth of managers which consists of the value of management shareholdings plus management's private benefits or a linear combination of the firm's value and the private benefits of managers in form of a fraction on the operating profits. This extension is motivated by Myers and Majluf (1984) and Wu and Wang (2005). In line with Wu and Wang (2005), the private benefits of managers have to be financed out of profits of the firm and exert a negative externality on outside shareholders.

The financial constraints are modelled as in Hennessy, Levy and Whited (2007). Investors are risk-neutral. The (risk-free) discount rate is β. The exogenous state variable ε_t models innovations in output prices, variable factor prices and productivity. The capital stock changes according to $K_t = (1-\delta)K_{t-1} + I_t$, where K_t is the capital stock at the beginning of period *t*, I_t is investment in period *t* and δ is depreciation. Gross profits are given by $F(K_t, \varepsilon_t)$ and capital adjustment costs by $G(I_t, K_t)$. Net profits are therefore $F(K_t, \varepsilon_t) - G(I_t, K_t)$. We make the standard assumption that F and G are twice continuously differentiable, F is strictly increasing and homogeneous of degree one in capital and G is strictly convex and homogeneous of degree one in *(I,K)*.

The monetary value of external equity financing is X_t, with $X_t < 0$ if the firm pays a dividend and shareholders receive X_t; and $X_t > 0$ if the firm raises external equity. There is a convex cost of external equity, $X_t > 0$, to model the impact of informational asymmetries (Myers and Majluf (1984)). The cost of equity function $H(X_t, K_t)$ satisfies $H(X_t, K_t) = X_t$ for all $X_t \leq 0$, that is, negative values of X_t or dividend payments, are equivalent to shareholders facing a negative cost. To guarantee existence of an optimal solution the function *H* is twice continuously differentiable, strictly increasing and weakly convex in X_t; it is decreasing and convex in K_t, and homogeneous in (X_t, K_t).

The firm has access to a bank credit line modelled by the endogenous state variable B_t, the credit line balance. The bank demands collateral $B_t \leq L(\varepsilon_t)K_t$, where $L(\varepsilon_t)$ is the liquidation value of a unit of capital K_t and the bank has absolute priority in case of default. The interest on the bank credit line is *r*, and because of the collateral it is risk free. If the firm saves it earns an interest rate of $r_s < r$. Denote the effective interest rate ρ. The changes of the bank credit line are governed by the financial policy g_t, that is, $g_t = B_t - B_{t-1}$. The bank enforces the collateral constraint by imposing $g_t \leq \gamma[L(\varepsilon_t)K_t - B_t]$, that is, the bank allows the firm to close the gap between the bank credit line balance and the upper bound *LK* at the rate γ. Intuitively, $g_t > 0$ and $B_t > 0$ means that the firm is drawing on the credit line, $g_t >$ 0 and $B_t < 0$ means that the firm is reducing its stock of cash; $g_t < 0$ and $B_t > 0$ means that the firm pays bank debt; $g_t < 0$ and $B_t < 0$ means that the firm increases its cash stock. Another financial constraint incorporated in the model is debt-overhang (Myers (1977)). The firm has pre-existing, non-negotiable public debt, denoted *D*, with a perpetual coupon payment of *b>0*.

The manager maximizes his wealth and does not maximise shareholder value. This wealth consists of the market value of the shareholdings of the managers, that is, the percentage of manager's shareholding of all outstanding shares times the present discounted value of the firm plus private benefits that are a fraction c of the present discounted value of operating profits or cash flow. More precisely, the objective function of the manager is

$$E_0\{-\sum_t \beta^t \alpha H(X_t, K_t) + \sum_t \beta^t c[F(K_t, \varepsilon_t) - G(I_t, K_t)]\},$$

where $\alpha \in [0,1]$ is the percentage of managers' shareholding and $c \in [0,1]$ is the percentage of the operating profits that the managers obtain. For simplicity it is assumed that the private benefit parameter *c* does not vary over time and is exogenously given.

The following budget constraint has to be satisfied for each t,

$$X_t + g_t + F(K_t, \varepsilon_t) = I_t + G(I_t, K_t) + b + \rho B_t + c[F(K_t, \varepsilon_t) - G(I_t, K_t)]$$

The left hand side displays the sources of funds, that is, external equity, new bank borrowing or reduction in the cash buffer stock and cash inflow from current operations. The right hand side displays the uses of funds, that is, investment cost, (indirect) capital adjustment cost, payments on public debt, interest on the bank credit line, and the costs of the private benefits. The budget constraint can be rewritten as

$$X_t = I_t + (1-c)G(I_t, K_t) + b + \rho B_t - g_t - (1-c)F(K_t, \varepsilon_t)$$

The outside shareholders play a passive role. However, being shareholders, the outside shareholders also bear the costs of the private benefits of managers because the budget constraint above contains the private benefits of managers as a cost, $c[F(K_t, \varepsilon_t) - G(I_t, K_t)]$.

The state of the system at each point in time *t* is described by $(K_t, B_t, \varepsilon_t)$. At each *t*, the manager selects a financial policy g_t and an investment policy I_t and an optimal time to default *T* to maximise the objective function of managers given above. The maximisation problem of the managers is as follows

$$\max_{\{g_t, I_t, T\}_{t=0}^{T}} E_0\{-\sum_t \beta^t \alpha H(X_t, K_t) + c\sum_t \beta^t [F(K_t, \varepsilon_t) - G(I_t, K_t)]\},$$

subject to $K_t = (1-\delta)K_{t-1} + I_t$,

$$g_t = B_t - B_{t-1},$$

$$g_t \le \gamma[L(\varepsilon_t)K_t - B_t], \text{ and}$$

$$X_t = I_t + (1-c)G(I_t, K_t) + b + \rho B_t - g_t - (1-c)F(K_t, \varepsilon_t).$$

Let $\lambda_{t,K}, \lambda_{t,B}, \lambda_t$ be the Lagrange multiplier of the first, second and third constraint, respectively. The Lagrange expression is given by

$$L = E_0\{\sum_t \beta^t [(-1)\alpha H(X_t, K_t) + c[F(K_t, \varepsilon_t) - G(I_t, K_t)] +$$
$$\lambda_{t,K}(I_t + (1-\delta)K_{t-1} - K_t) + \lambda_{t,B}(g_t - B_t + B_{t-1}) +$$
$$\lambda_t(\gamma[L(\varepsilon_t)K_t - B_t] - g_t)]\},$$

where it is implicitly assumed that the budget constraint has been substituted in place of X_t, that is, $I_t + (1-c)G(I_t, K_t) + b + \rho B_t - g_t - (1-c)F(K_t, \varepsilon_t)$ and that $B_{-1} = 0$ and $K_{-1} = 0$.

The first order condition of the Lagrange expression with respect to g_t, the financing policy, gives $-\lambda_{t,B} = \alpha H_{X_t}(\cdot) - \lambda_t$. The complementary slackness condition yields the following optimal financing rule. If $\lambda_t = 0$, we have $-\lambda_{t,B} = \alpha H_{X_t}(\cdot)$. If $\lambda_t > 0$, then we have $-\lambda_{t,B} < \alpha H_{X_t}(\cdot)$ and $g_t = \gamma[L(\varepsilon_t)K_t - B_t]$.

The shadow cost of bank debt is given by λ_{tB} and H_{X_t} represents the marginal cost of equity finance and, therefore, αH_{X_t} represents the marginal cost of equity finance managers are facing. When the collateral constraint is not binding, then the optimal finance policy set the marginal cost equal to the shadow cost of bank debt. When the collateral constraint is binding, the marginal cost of equity is larger than the marginal cost of bank debt because of credit rationing.

The first order condition with respect to investment I_t is

$$q \equiv \lambda_{tK} = \alpha H_{X_t}(\cdot)(1+(1-c)G_{I_t}) + cG_{I_t} = (\lambda_t - \lambda_{tB})(1+(1-c)G_{I_t}) + cG_{I_t}$$

where q is defined as the shadow value of a unit of (installed) capital. This optimality condition states that the shadow value of a unit of capital is equal to the marginal cost of investment taking into account the cost of the funds and the private benefits of the managers.

Optimal investment under financial constraints and firm value maximisation (Hennessy, Levy and Whited (2007)) is a special case of this model when there are no private benefits of managers and managers maximize the market value of the firm. In this case $\alpha = 1$ and $c = 0$. The optimality condition, $q \equiv \lambda_{tK} = H_{X_t}(1+G_{I_t}) = (\lambda_t - \lambda_{tB})(1+G_{I_t})$, or,

$G_{I_t}(I_t^*, K_t) = \dfrac{q}{H_{X_t}} - 1$ coincides with the one in Hennessy, Levy and Whited (2007). The optimality condition shows that the standard implication that the rate of investment is higher for firms with high marginal q. However, in contrast to neoclassical models of investment, conditional on marginal q, firms that issue equity invest less (Hennessy, Levy and Whited (2007), p. 699). Under perfect capital markets, the model further specializes. In this case, firms do not default, that is, $T = \infty$, there are no collateral constraints that are binding and the cost of external equity is one. This implies that $\lambda_t = 0$ and $H_{X_t} = 1$. It follows that marginal q is equal to average q (Hayashi (1982)).

The model can be adapted to also include a soft budget constraint and private benefits of other controllers or stakeholders. To model soft budget constraints, assume that the government or governmental organization injects a sum of money into the firm in each period, denoted S_t. Each controller or ownership category *i*, enjoying private benefits, has a share $c_i, 0 \le c_i \le 1, i = 1,2,...$, of operating profits of the firm. Let c_1 denote the private benefits of the management, that is, the share of the managers on the operating profit. The budget constraint of the firm is now given by

$$X_t + g_t + F(K_t, \varepsilon_t) + S_t = I_t + G(I_t, K_t) + b + \rho B_t + \sum_i c_i [F(K_t, \varepsilon_t) - G(I_t, K_t)]$$

,where the left hand side displays the sources of funds and the right hand side displays the uses of the funds. The left hand side now includes the money injections of the government or other organizations and the right hand side the costs of the private benefits of stakeholders and the managers. Note that the private benefits of controllers other than the managers acts like a cost to the firms, but does not influence the objective function. This is consistent with the view that these controllers do not interfere with running the firm. If this is the case, the model can be suitably amended.

The first order condition with respect to investment I_t is

$$q \equiv \lambda_{tK} = \alpha H_{X_t}(\cdot)(1 + (1 - \sum_i c_i) G_{I_t}) + c_1 G_{I_t} = (\lambda_t - \lambda_{tB})(1 + (1 - \sum_i c_i) G_{I_t}) + c_1 G_{I_t}$$

where $H_{X_t}(\cdot)$ is evaluated at $I_t + (1 - \sum_i c_i) G(I_t, K_t) + b + \rho B_t - g_t - (1 - \sum_i c_i) F(K_t, \varepsilon_t) - S_t$

and *q* is defined as the shadow value of a unit of (installed) capital.

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