

The Time-Symmetric Gold Universe Reconsidered

Abstract

The present article proposes to re-examine the parity-of-reasoning or double-standard fallacy argument, which favours a time-symmetric Gold universe model over a cosmological arrow of time. There are two reasons for this re-examination. One is empirical: 1) the recent discovery of an expanding and accelerating universe questions the symmetry assumption of the Gold universe on empirical grounds; 2) the other is theoretical: the argument from t -symmetry fails to take into account some important aspects of the topology of phase space and recently developed typicality arguments. If the parity-of-reasoning argument, which depends on the t -symmetry of probability, is reconsidered in terms of the topology of phase space and typicality arguments, the double-standard fallacy argument loses much of its appeal. The Gold universe model itself suffers from unexplained dynamic asymmetries. The upshot of this paper is that the Gold universe model is implausible or far less plausible than asymmetric models.

I. Introduction

One of the most challenging problems in the physics and philosophy of time remains the question of the cosmological arrow of time. One familiar account holds that the arrow of time does not derive from, say, the validity of the Second law of thermodynamics alone but requires particular low-entropy boundary conditions at the birth of the universe to bestow an arrow of time on its cosmological evolution. If such an arrow exists, then initial and final conditions of the universe are very different, for instance in terms of their entropic states.

This approach, however, faces a fundamental objection, namely that it ‘favours’ initial over final conditions. This problem is best known as the ‘double standard fallacy’. The fallacy concerns the assumption of a cosmological arrow of time, which factors in ‘initial’ conditions but neglects ‘final’ conditions and assumes that the universe will evolve from a state of lower to a state of higher entropy. This account of the cosmological arrow of time falls foul of what has also been dubbed the parity-of-reasoning argument. (Sklar 1993; Price 1996; Schulman 1997) Hence symmetry between initial and final conditions, rather than asymmetry should be the default position. It should not be assumed, without further argument, that cosmological evolution is asymmetric. Rather, the default position should be a Gold universe model, which treats initial and final conditions of the universe as symmetric, i.e. the entropy is assumed to be low (or high respectively) at *both* ends of the lifeline of the universe.

There are two reasons for reviewing the argument for a Gold universe from temporal symmetry: 1) A Gold universe model seems to be excluded on empirical grounds. Cosmology has made significant empirical discoveries about the evolution of the universe since the 1990s: the evolution of the universe is no longer expected to end in a Big Crunch – the time-reverse of the Big Bang – but in a Big Chill. (Cf. Krauss/Scherrer 2008; North 2002) 2) But there is also a theoretical argument. The argument from t -symmetry does not take into account some important aspects of the topology of phase space and recently developed typicality arguments. When this is done, as the present article aims to show, the argument from double standards loses much of its appeal.

II. The Argument from Symmetry

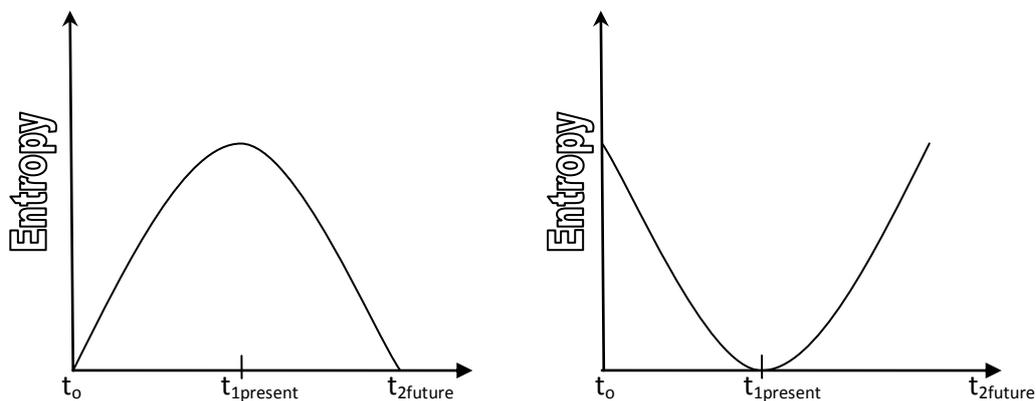
The insistence on parity-of-reasoning, coupled with the desire to avoid ‘temporal chauvinism’ (Carroll 2010, 288), is usually justified by appeal to the temporal neutrality of the notion of probability. Price (1996), for instance, argues for a view from Nowhere – an atemporal or Archimedean standpoint – which stipulates symmetry between the initial and final conditions of the universe. It is claimed that the ‘atemporal view’ avoids ‘double standards’, which means that it does not rely on statistical arguments in one time direction only. The view from Nowhere expects ‘entropy to increase in both directions’ unless there is a future constraint.

Parity-of-reasoning requires that inferences into the past and future be treated as symmetrical. (Sklar 1995) Statistical reasoning about the evolution of thermodynamic systems of the universe contains no in-built asymmetry. If entropy is expected to increase in the future – barring some future constraint (Price 1996, 20) – statistical reasoning alone requires that we should expect it to increase into the past. In order to avoid double standards the statistical argument must not be restricted to one direction of time (usually the future direction). Without the ‘favouring’ of initial conditions there is no reason to assume that entropy will rise towards the future. However, if initial conditions are factored in, as St. Hawking complains, ‘the second law is really a tautology.’ Entropy increases with time because ‘we define the direction of time to be that in which entropy increases.’ (Hawking 1994, 348)

Statistical reasoning thus leads to the postulation of two-time boundary conditions of the universe. Models with two-time boundary conditions are known as Gold universe models, according to which initial and final conditions are approximately identical. In the usual Gold model scenario the universe starts in a low-entropy past and ends in a low-entropy future, with the possibility that a future collapse could influence cosmic events today, through some form of retro-causality. (Schulman 1997) The reason for a consideration of two-time boundary conditions is that symmetry is a fundamental motivation in scientific reasoning, which avoids temporal bias. For instance, the fundamental equations of physics are said to be time-reversal invariant, which means that the equations allow both time-forward and time-reversed states as possible solutions. (See Lebowitz 1994; Earman 2002) It is usually assumed that a Gold universe undergoes a cycle from a Big Bang to a Big Crunch (Price 1996, 81-2; Carroll 2010, 347-8; see Figure 1a), although there seems to be no strong evidence in

Gold's writings for such an interpretation. Gold regards the universe, due to its expansion, as a radiation heat sink, which leads to asymmetric boundary conditions. In Gold's model the entropic arrow tracks the cosmic arrow, with the consequence that the arrow of time can be reversed. (See Gold 1962; 1966; 1974) Many cosmological models assume a *de facto* low-entropy past, but far fewer assume a low-entropy future. But from the symmetric point of view low entropy boundary conditions need explaining whilst high entropy conditions both at the beginning and end of the universe's history are expected. At any rate, if the elegance of symmetry is valued and double standards are to be avoided, it is reasonable to envisage an alternative Gold universe, one that starts and ends in a high entropy state. (Figure 1b)

Figure 1 a, b: The Gold universe and its inverse



The symmetry view satisfies the theoretical demand for symmetry, at least for the symmetry of the boundary conditions, but there is little factual support for the Gold universe. Recent Nobel Prize winning discoveries have shown that the expansion of the universe is in fact accelerating (Krauss/Scherrer 2008; Schmidt 2005) and that there is no evidence of a future low-entropy state. Furthermore, a number of cosmological models are now available, which no longer stipulate low entropy initial conditions but derive them from pre-Big Bang scenarios. (Carroll 2010; Penrose 2010)

The lack of evidence for any future constraint, the discovery of an accelerated expansion of the universe, hurling towards a Big Chill (rather than a Big Crunch) and the *de facto* existence of a low-entropy Big Bang pose problems for two-time boundary scenarios. In terms of the ‘parity-of-reasoning’ objection, however, the logical problem remains, namely to investigate the cosmological properties of the universe without favouring initial conditions.

T -symmetric models rely on the temporal symmetry of statistical reasoning. But even if the empirical fact of an expanding universe is ignored it turns out that a t -symmetric model suffers from unexplained dynamic asymmetries, which make it implausible. They emerge if we consider arguments from the topology of phase space and typicality arguments, contrary to the parity-of-reasoning demand, they arrive at an arrow of time. This discrepancy gives rise to the question whether an unjustified favouring of initial conditions can be avoided, whilst accounting for the observable, *de facto*, entropy increase in the universe.

III. The Argument from Asymmetry

The argument from asymmetry usually starts from the postulation of low-entropy initial conditions but this postulation is subject to the double-standard fallacy argument. Hence an argument from asymmetry should attempt to show that asymmetry can be obtained without favouring initial conditions. Attempts in these directions have concentrated either on the topology of phase space (Section III) or on the notion of typicality (Section IV; Earman’s ‘time direction heresy’ will be briefly considered in this section). The question is whether a consideration of the structure and volume of phase space can avoid the parity-of-reasoning argument over and above the empirical fact that the universe is most likely to end in a Big Chill rather than a Big Crunch.

A classical system in mechanics is standardly described in terms of a $6N$ -dimensional phase space, Γ , in which each individual particle has three position coordinates (x, y, z) and three momentum coordinates (since ‘momentum = mass x velocity’ is a vector quantity). Single particle systems or many particle systems are represented by a single point X , its micro-state, which moves around in phase space according to the deterministic laws of Hamiltonian mechanics. Phase space is an expression of the number of ways in which a macro-state, like temperature, can be realized by a configuration of micro-states. The phase

space usually comes endowed with a Lebesgue measure, μ , which roughly is a volume measure of the phase space, available to the systems. For Hamiltonian systems the Lebesgue measure is invariant under the dynamics; this statement is equivalent to the Liouville theorem, which asserts that the dynamics preserves the phase-space volumes.

To any particular macro-state, M_i ($i = 1 \dots m$), of which there is assumed to be a finite number, there corresponds a number of micro-states, which make up the macro-state.¹ A macro-region, Γ_{M_i} ($1 \leq i \leq m$), characterizes the area for which the region is in macro-state M_i , and which contains all the macro-states, which correspond to M_i . The different sizes of M , and hence the difference in the volume values of $|\Gamma_M|$, depend on the number of states, which make up the macro-regions, and the amount of spreading. As is well-known from Boltzmann's entropy equation, almost all the phase points, which are initially in Γ_{M_i} will evolve (if the dynamics is undisturbed) to newly available regions of phase space, such that the system will eventually reach equilibrium state, M_{eq} .

Liouville's theorem in classical mechanics states that a volume element along a flowline preserves the classical distribution function $f(r, v) dr dv$:

$$f(t+dt, r+dr, v+dv) = f(t, r, v)$$

(Kittel/Kroemer 1980, 408; Albert 2000, 73f)

Liouville's theorem says that the dynamic evolution of a classical system preserves the *volume* of the initial phase-space region but not its *shape*. The shape of the phase-space region can become very unstructured, disordered, a physical state for the description of which sometimes the term 'fibrillation' is used. (Sklar 1993, 55-6; Penrose 2010, 25-34) A rather uniform (smooth) region in the initial stage of dynamic evolution can become very fibrillated since the shape of phase space regions is not invariant. The division of phase space into different cells – with different topology – is known as 'coarse-graining'. That is, the volume of the available phase space regions is invariant over time even though the

¹ The converse does not hold: to one particular micro-state there corresponds exactly one macro-state, which can be regarded as a snapshot of the micro-states, which make up a particular macro-state at a moment in time. As a macro-state, like temperature T , is defined as mean molecular kinetic energy, many particular configurations of the micro-states can constitute the particular macro-state, T .

motion of point X along an evolution curve within this volume can start from smooth initial states or end up in fibrillated final configurations.

When speaking of the space volume of a particular macro-system, like a gas, some care must be taken as to the precise meaning of this term. When Maxwell's Demon sits in a sealed container, with a small opening in a partition wall, which allows the Demon to separate the slow from the fast molecules, he is confined to a three-dimensional Euclidean space. The phase space of such a gas is fixed and does not change with time. It describes every possible state, which a system may occupy. (Cf. Carroll 2010, 336, 340-2; Penrose 2005, 701) However, an n-particle system in $6N$ -dimensional phase-space can move along different evolution curves, according to deterministic laws. The phase space remains invariant but the configurations within that state – the shape of the coarse-grained cells – will change and evolve. Although a number of evolution curves are available to the system, it may not actually occupy all these curves.

In a more mathematical sense a $6N$ -dimensional abstract phase space, Γ , describes all the particle states, which belong to the system. If energy conservation is taken into account, 'the motion of the system is confined to a $(6N-1)$ -dimensional energy hypersurface Γ_E' which describes the phase space *available* to the evolution curves (or phase flow ϕ_t). (See Werndl 2013, 471) When phase space (or the space of states) is conceived as the phase space of all the possible states, which a system could hypothetically occupy, the reversibility of the fundamental equations requires that this available phase space remain invariant. (Cf. Carroll 2010, 336, 340-2; Penrose 2005, 701) The phase space must remain invariant if the trajectories are allowed, in theory, to return to their initial conditions. According to the Liouville theorem, μ is a volume-preserving measure. But asymmetry between the initial and final states or the coarse-grained regions implies some evolution: what changes is not the number of *accessible* regions of phase space but the number of *actual* configurations, which occupy phase space at any given time before reaching equilibrium. That is, the volume of phase space, which the configurations actually occupy before reaching equilibrium, is smaller than the volume of phase space available to them. Due to the difference between phase space accessed and phase space accessible, the system evolves, such that the equilibrium macro-state is larger than any other state. In terms of initial and final conditions this evolution means that, although the volumes remain invariant under

Liouville's theorem topologically they differ (they are not shape-preserving under Liouville's theorem and *de facto* a continuous undoing of final conditions requires highly atypical fine-tuning). This measure of fibrillation or spreading refers to both senses of the phase space. For both the phase space in the sense of all hypothetical possible states and the phase space of the occupied space see their volumes but not their topology preserved.

As a system, like the universe, undergoes expansion, it begins to occupy different volumes, due to the different configurations it can occupy – for instance $|\Gamma_{M_{equilibrium}}| \gg |\Gamma_{M_{initial}}|$ – and these different volumes allow the construction of volume ratios – $|\Gamma_{M_i}| / |\Gamma_{M_{eq}}|$ – which become important for asymmetry arguments. Eddington regarded these volume differences as another criterion – apart from entropy – for a cosmological arrow of time. (Eddington 1935, 67-8) But an immediate consequence of this theorem is that even though the *volume* is preserved the *shape* of this macro-region is not preserved and this implies a dynamic evolution of the states along evolution curves within the accessible phase space region. For two coarse-grained regions cannot differ from each other without an evolution of the states from occupied to accessible space. It also implies that a reversed evolution of the trajectories will preserve the volume but not necessarily the shape, unless very special conditions apply. Hence a bundle of reversed trajectories need not be invariant with respect to the shape of the phase space region.

But does this language of fibrillation – of the non-invariance of the shape of some region at time t , $T_t \Gamma_{Ma}$ – not surreptitiously flout the 'parity-of-reasoning' argument? The appeal to Liouville's theorem assumes that the evolution of the system develops from an ordered to a less ordered state, in accordance with the Second law. In order to fully respect the force of the parity-of-reasoning argument and not to assume that a system is in an initial low-entropy condition, it is best not to postulate a particular state of entropy, as in standard universe models.

Although notions like 'fibrillation' and 'spreading' have an intuitive physical appeal, in the statistical-mechanical literature spreading usually refers to the realisability of the macro-state with respect to the available combinations of micro-states. Realisability describes the number of micro-states, which are compatible with or make up a given macro-state, as

reflected in Boltzmann's definition of entropy: $S = k_B \log V$ (where V is the volume of the coarse grained region, which contains X). It is tempting to argue that the realisability of different configurations 'extracts a direction of time even though the molecular collisions which give rise to the diffusion of the gas are each time-reversible.' (Landsberg 1982b, 75; cf. Kupervasser *et al.* 2012) For if a system has a greater degree of realisability available to its macro-states – a greater amount of spreading into the available phase space –, this evolution could serve as a criterion for an arrow of time.

According to the parity-of-reasoning argument probabilities are t -symmetric and hence do not allow to extract an arrow of time. The double-standard fallacy is committed when an unjustified favouring of initial over final conditions occurs, which seems to make the Second law a tautology. The realisability argument has the advantage that it does not need to invoke initial conditions. It appeals to 'molecular collisions' to explain the evolution towards equilibrium. The realisability of more macro-states from fewer macro-states is more likely than the realisability of fewer macro-states from more macro-states because of the shape difference between accessed and accessible phase space regions. As mentioned above, Eddington treated the volume ratios $|\Gamma_{M_i}| / |\Gamma_{M_{eq}}|$ as a separate criterion for the arrow of time. The increasing arrow of time can therefore be understood as a function of the ratio of occupied to available phase space. These occupied regions of phase space are regarded as statistically irreversible, whilst the equations of motion, which govern the trajectories, remain time-reversal invariant. According to some recent estimates a return of all the particles in a two-chamber system to just one chamber has a probability of $10^{-6 \times 10^{22}}$ and the mean time $\langle T \rangle$ for such an occurrence is of the order of $10^{6 \times 10^{22}}$ s, the estimated age of the earth. (See D'Abramo 2012)

But such an appeal to realisability tacitly assumes that there is no future constraint, which acts on the current state. (Price 1996, 47) Or, in metaphorical terms, that there is no *deus ex machina* – Loschmidt's Demon (see Weinert 2016, Part III) – who is able to achieve the fine-tuning that would be required to ensure a reversal of the trajectories (phase flows) to their previous low-entropy configuration. Although on empirical grounds such a future constraint can probably be excluded, it enjoys equal probability on logical grounds. But then the realisability argument seems to commit the double standard fallacy: realisability is

greater towards the future than towards the past but from a purely statistical point of view, realisability should be equally likely in both directions.

It seems that as long as the argument is formulated in terms of probabilities, motivated by the desire to avoid the double standard fallacy, a commitment to a Gold-type or *t*-symmetric universe is unavoidable, despite its empirical implausibility. (Note that it is possible to justify probability measures as typicality measures, as argued in Volchan [2003] and Werndl [2013]). The next section will therefore address the question whether realisability can be saved if realisability in one direction can be said to be more *typical* than realisability in the opposite direction. If this strategy worked it would undermine the *t*-symmetry of statistical reasoning and question the logical viability of the Gold universe. The proposal here is to characterize the topology of phase space by appeal to volume ratios, without reference to the notion of entropy.

IV. Typicality

Why does time-asymmetric behaviour occur? There have been a number of tentative answers to this question. They appeal to the Past Hypothesis in connection with the Second law, interventionism and external environmental disturbances. (Cf. Sklar 1993; Kupervasser *et al.* 2012) But all these approaches assume asymmetry and fall foul, in one way or another, of the parity-of-reasoning fallacy. In this connection it is worth considering Earman's 'Time Direction Heresy' (Earman 1974), according to which the arrow of time becomes a geometric feature of space-time, making the appeal to entropy seemingly redundant. This geometric approach makes space-time time-orientable so that the arrow of time is built into the modelled fabric of space-time. Although this approach invests space-time with a conventional temporal direction, it does not invest it with a temporal arrow because a temporally orientable space-time is not the same as a temporally oriented space-time.

Temporal orientability is merely a necessary condition for defining the global arrow of time, but it does not provide a physical, nonarbitrary criterion for distinguishing between the two directions of time.' (Castagnino *et al.* 2003, 2496; cf. Aiello *et al.* 2008; Lehmkuhl 2012)

As it turns out a temporally oriented space-time still requires 'energy fluxes', typically associated with the Second law, to establish temporal asymmetry. It is therefore appropriate for an evaluation of the parity-of-reasoning objection, which is concerned with

the system's boundary conditions in a time-symmetric universe, to consider typicality approaches. In line with the above proposal these approaches characterize typicality as the ratio of overwhelmingly many to a small number of divergent cases. Typical cases follow the expected temporal evolution of the system towards equilibrium; atypical cases constitute rare entropy-decreasing evolutions. Do they escape the 'parity-of-reasoning' objection?

In order to evaluate the 'parity-of-reasoning' argument, the following formulation of the typicality view will be used (Goldstein and Lebowitz 2004): 'for any [micro-region] Γ_{Mi} the relative volume of the set of micro-states $[x]$ in Γ_{Mi} for which the Second law is violated (...) goes to zero rapidly (exponentially) in the number of atoms and molecules in the system.' (Quoted in Frigg 2011, 84) Such a notion of typicality resembles the notion of weak t -invariance (Landsberg 1982a, 8), since a weakly t -invariant process, such as diffusion and heat conduction, allows for its (improbable) time-reverse without violating the Second law of thermodynamics. But such processes are never observed in nature, hence their realisability is practically zero. These characterizations, which focus on volume ratios, seem to appeal to what will be dubbed here *volume-ratio* typicality, since they make an implicit appeal to the distinction between *occupied* and *accessible* phase space, their ratios and the non-invariance of the structural shape of initial and final conditions. It will be argued below that they hold the key to the avoidance of the double-standard fallacy without assuming a symmetric Gold universe.

As this paper is *not* concerned with the question of the evolution of thermodynamic systems towards equilibrium, volume-ratio typicality in the present sense simply refers to the volume ratios of accessed and accessible phase-space volumes. As all the notions reviewed so far failed to respect the parity-of-reasoning concern, the question imposes itself whether volume-ratio typicality arguments are not susceptible to the double fallacy argument? The temporal neutrality of statistical relations means that in the fullness of time both small and large fluctuations will occur, and hence thermal fluctuations may give rise to highly ordered, low-entropy complex systems, like Boltzmann brains, in an otherwise high-entropy universe.

One way out of this dilemma is to postulate low-entropy initial conditions, from which the Second law can then explain the evolution towards increasing equilibrium. But in order to protect this argument from the accusation of double standards, the low-entropy initial

condition of the universe cannot simply be 'put in by hand'. However, typicality arguments assume that there are differences in the volumes of accessed phase spaces, for otherwise no ratio could be established. Does this not mean that typicality arguments also fall foul of the parity-of-reasoning objection because they assume that a low-entropy initial condition exists? How can such an initial measure be justified? On empirical grounds the assumption of a low-entropy initial condition is justified because all cosmological models (to the author's knowledge) assume an initial low-entropy condition of our universe (even if it is part of the multiverse). But there are also plausibility arguments, which favour typicality approaches over the Gold universe. Typicality arguments do not assume, as symmetry considerations do, that what is theoretically possible is also equally probable. It is theoretically possible for a Loschmidt Demon to reverse all the trajectories in an evolved phase space but in an expanding universe there is practically no guarantee that the trajectories will exactly return to their initial conditions. Liouville's theorem only guarantees an invariance of volume, not of shape. In the evolved phase space the smooth initial conditions have turned into final fibrillated conditions. The final conditions are not the time-reverse of the initial conditions, which introduces a temporal asymmetry.

One advantage of volume-ratio typicality arguments is that they are not based on probabilities; hence the problem of the t -invariance of probabilities does not immediately arise. These approaches do not start from the assumption of the temporal neutrality between initial and final conditions. They do not assume that all evolutions are equally probable. Earlier it was suggested that the topology of phase space should be characterized in terms of Liouville's theorem. Usually Liouville's theorem expresses the fact that the dynamics of the state of a system is volume-preserving but that the phase space is not shape-invariant. Hence the topology of the system changes over its thermodynamic evolution. According to typicality approaches the evolution to a fibrillated state is typical, whilst a return to a smooth, ordered state is atypical. Hence typicality approaches do not give rise to a demand for parity-of-reasoning: a quasi-identity of initial and final conditions is highly atypical, although it is not excluded. The number of degrees of freedom in a fibrillated state is much greater than in a smooth state, and hence their return to a more ordered state requires 'perfect aiming', which, given the physical constraints on a system, make it extremely atypical. A proponent of a t -symmetric model will ask by what right we

can assume that evolution towards the future will be typical. On the empirical level, a proponent of typicality can point to the cosmological fact that the universe is expanding at an accelerated pace and will – in the fullness of time – reach what the 19th century dubbed the ‘heat death’. On an epistemological level the proponent of typicality will, as will now be argued, appeal to plausibility arguments.

A further reflection on a time-symmetric universe reveals the dynamical problems it faces, under the assumption of identical boundary conditions - where these boundary conditions can either be in a state of high or low entropy. It shows that typicality does not need to favour initial conditions and that temporal asymmetry is more plausible than temporal symmetry in a Gold universe.

V. Typicality in a Gold universe

The symmetry approach regards it as an advantage that there is no need for the stipulation of specific initial conditions. If we take the parity-of-reasoning approach seriously, two scenarios need to be considered:

- Either the current state (of a Gold universe) evolved from a low-entropy state in the past and will again collapse to a lower entropy state in the future (Figure 1a).
- Or the current state is an entropy-decreased fluctuation from a higher entropy state in the past such that it will again be higher in the future (Figure 1b)

In a Gold universe, of either type, the current state of the system has reached a certain level of entropy but the two scenarios invite different questions.

Question 1: How likely is it that the system evolves from a higher state of entropy, at $t_{1\text{present}}$, to a lower state of entropy at $t_{2\text{final}}$? (Figure 1a) This part of the evolution curve displays anti-thermodynamic behaviour and cannot be explained by appeal to the Second law. The Second law can only explain the thermodynamic behaviour from t_0 to t_1 . Hence whilst the Gold universe respects the demand for parity of the boundary conditions, it requires a violation of dynamic symmetry for different parts of the evolution curve, because it implicitly assumes an asymmetry between anti-thermodynamic and thermodynamic behaviour.

Question 2: This question is a mirror image of the first question. How likely is it that the system will return to a state of lower entropy, from t_0 to $t_{1\text{present}}$? (Figure 1b) Again the parity of reasoning applies to the symmetry of boundary conditions but not to this part of the evolution curve itself, for only the phase $t_1 \rightarrow t_2$ evolves in accordance with the Second law. Consider these scenarios in more detail:

- If we start from the assumption, by the argument from parity, of high entropy states both at the beginning and the end of a system's evolution, then on Boltzmann's assumption the current state of a system, like the universe, is in a lower state of entropy, hence in a more ordered state. Such fluctuations have been dismissed as implausible (Carroll 2010, 221-4; Lebowitz 1994, 142; Price 1996, 35). A Boltzmann fluctuation would require an evolution of a more fibrillated state to a less fibrillated state. From the point of view of typicality, it would mean a change in the ratio of phase space volumes. A Boltzmann fluctuation would mean an atypical switch in the ratio of phase space volumes at some point on the evolution curve. Anti-thermodynamic evolution would be atypical.
- This asymmetry in typicality remains if we postulate an alternative Gold universe, with equal initial and final conditions of low entropy. Now the current state of the system is in a higher state of entropy than the initial condition, which is typical behaviour but the system is supposed to return to a lower state of entropy, hence an atypical, anti-thermodynamic evolution. Such behaviour requires the postulation of constraints in the distant future, attractor states, which by retro-causality would have an effect on the current state. (Schulman 1997) Whether or not evidence for such a future constraint on present conditions exists, another atypical switch in volume ratios occurs.

These typicality considerations do not favour initial over final conditions. They do not require 'perfect aiming' or an appeal to a Loschmidt demon, since they focus on the volume ratio of phase space occupied by respective regions. A Gold universe of either type must assume an atypical switch in volume ratios. This reversal of volume ratios means that a proponent of a Gold universe faces explanatory anomalies.

- If $|I_{pres}|$ is larger than $|I_{final}|$ then the proponent of a Gold universe faces an explanatory anomaly. The ratio $|I_{Mpres}|/|I_{Mfinal}| > 1$ is atypical, whilst the ratio $|I_{Mi}|/|I_{Mpresent}| \ll 1$ remains typical. (Figure 1a)
- If, however, $|I_{Mi}|$ is larger than $|I_{pres}|$, then, in terms of typicality, a proponent of a Gold universe has to find a justification for the fact that the ratio $|I_{Mi}|/|I_{Mpres}| > 1$ is atypical, whilst the ratio $|I_{Mpres}|/|I_{Mfinal}| \ll 1$ remains typical. (Figure 1b)

Under the assumption of the temporal neutrality of the evolution between boundary conditions, Gold universes require a drastic turn-around, at $t_{present}$, of the volume ratio $|I_{Mi}|/|I_{Mfinal}|$ to either $|I_{Mi}|/|I_{Mpres}| > 1$, if a high entropy past is assumed (Figure 1b), or to $|I_{Mpres}|/|I_{Mfinal}| > 1$, if a low entropy future condition is envisaged. (Figure 1a) In both scenarios, the ratios flip at the ‘mid-term’ point – which is the current state – from the typical $\ll 1$ to the atypical > 1 .

But the move from t -symmetric probability to volume-ratio typicality (as an implication of Liouville’s theorem) does not force us to adopt particular boundary conditions. The observed cosmological asymmetry depends solely on the ratio of the initial to the final phase state condition: $|I_{Mi}|/|I_{Mfinal}| \sim 10^{-10^{123}}$, where $|I_{Mfinal}|$ is the total available volume (Lebowitz 1994, 141; cf. D’Abramo 2012, §2) The usual complaint against standard cosmological models is that they can only explain the observed ‘arrow’ of time if they stipulate low-entropy initial conditions. They can be assumed on empirical grounds. But the proponent of typicality can advance plausibility arguments against the Gold universe.

Even on the assumption of the parity of boundary conditions, Gold models face a dynamic switch-over problem in terms of phase space volume ratios. In both scenarios the ratio changes at the ‘mid-term point’ = the current state, from $\ll 1$ to > 1 , on one part of the evolution curve: either in the phase $t_1 \rightarrow t_2$ (Figure 1a) or in the phase $t_0 \rightarrow t_1$ (Figure 1b). On both scenarios the switch-over problem remains unexplained and violates the typicality

requirement. Whilst Gold universes are unlikely on empirical grounds, their explanatory anomalies make them implausible on dynamic grounds.

VI. Conclusion

The t -symmetric statistical argument demands that all microstates ‘consistent with a given macro-state have equal weight.’ (Schulman 1997, §2.6) On this account, it is theoretically possible for 50% of air molecules in a room at normal temperature to travel at 1000mph and 50% to travel at 3000mph to arrive at the average speed of 2000mph, which corresponds to room temperature. But even on the statistical approach this does not mean that the corresponding macro-states are equally probable. The typicality argument can be understood as an argument for the inequality: equally possible \neq equally probable. Hence some macro-states are overwhelmingly more likely than others. In particular (in a departure from the parity-of-reasoning approach) the initial and final macro-conditions are not equally typical. And this typicality can be understood in terms of the volume of the phase space regions, which are occupied by the macro-states. If the ratio $|I_{max}|/|I_{min}|$ is of the order of $10^{10^{123}}$ (see Penrose 1990, Chap. 7), where typicality would lead us to expect that $|I_{max}|$ to be highly fibrillated, then the return to a much smaller volume of phase space is extremely unlikely, not only on physical but also on theoretical grounds. The coherence condition or the ‘perfect aiming’ required for this to happen is minute in a universe, which undergoes cosmological expansion. But typicality arguments can dispense with a *deus ex machina* or ‘perfect aiming’, since they make the evolution towards lower-entropy attractor states atypical because they would require an unexplained switch of volume ratios.

The parity-of-reasoning argument extends the basic assumption of statistical mechanics to the demand that initial and final conditions of a system (like the universe) be treated as equally probable. The favouring of initial over final conditions is unjustified. But on the typicality approach all evolutions are equally possible but not equally probable. Some evolutions are typical whilst other evolutions are atypical. The volume-ratio notion of typicality does not favour initial over final conditions and hence evades the parity-of-reasoning objection. But it turns out that avoidance of temporal chauvinism and double standards (as in a Gold universe) leads to a double standard of its own kind since at $t_{present}$ an

unexpected, unexplained and unjustified reversal of the ratios in the phase space volumes is required. Whilst the Gold universe is already doubtful on empirical grounds these plausibility considerations add further reasons to its unacceptability. If the demand for symmetry is maintained, alternative cosmological models, with pre-big-bang and post-big-bang periods, must be considered. These models retain entropic gradients and temporal asymmetry on part of the evolution curves, even though the multiverse, in which they are embedded, is taken to be symmetric in time.

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