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The long-run determination of the real exchange rate.
Evidence from an intertemporal modelling framework
using the dollar-pound exchange rate

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Abstract

This paper develops a model of optimal choice over an array of different assets, including domestic and foreign bonds, domestic and foreign equities and domestic and foreign real money balances in order to examine the determination of the real exchange rate in the long-run. The model is tested empirically using data from the UK and the USA. The results show that all the coefficients of the model are right signed and significant and consequently financial assets may play a significant role in the determination of the real exchange rate.

JEL Classification Codes: F31, G11

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1. Introduction

Trying to estimate the equilibrium real exchange rate remains a major challenge in modern international finance. A fundamental constraint has to do with the fact that the equilibrium real exchange rate (ERER) is not observable. In addition, according to Rogoff (1996) deviations of the real exchange rate from its long-run parity could be linked to the behaviour of macroeconomic fundamentals. In fact, many theoretical models have been constructed based on premise that the ERER is a function of macroeconomic fundamentals. The standard models in the literature on the determination of the ERER emerge from a simple balance of payments equilibrium equation, the so-called statistical equilibrium, see for example McDonald (2000). A simple model that can be extracted from the basic statistical equilibrium real exchange rate equation is the purchasing power parity (PPP) model which implies that the real exchange rate does not change in terms of tradable goods prices but allows for deviations based in price indices made up of both tradable and non-tradable goods. However, the empirical evidence suggests that deviations from PPP can be both substantial and persistent in nature¹ although as shown by Hall *et al* (2013) PPP may well have empirical validity in the long run.

Given that PPP is not able to explain the behaviour of the ERER it has been argued that such a measurement can be derived from an economic model in which macroeconomic fundamentals are explicitly present. Different approaches like the behavioural equilibrium exchange rates (BEER) supported by Clark and MacDonald (1998) and Driver and Westaway (2004) and the fundamental equilibrium exchange rate (FEER) developed by Williamson (1994) have emerged. Once the determination of the ERER has been calculated, the real

¹ This is the well-known 'PPP puzzle' as labelled by Rogoff (1997).

exchange rate misalignment can also be derived. The real exchange rate misalignment reflects the deviation of the real exchange rate from a benchmark (equilibrium) level the calculation of which depends upon the measurement of the ERER. There are several approaches in the literature that have evolved to calculate the real exchange rate misalignment. One approach is based on the PPP doctrine, according to which the real misalignment is reflected by deviations of the real exchange rate from a given PPP level. In the model-oriented approach the real exchange rate misalignment is determined as the deviation of the actual real exchange rate from a theoretically based equilibrium path, which is determined by the behavioural-statistical approach or the simultaneous achievement of internal and external balance.² However, a major drawback of these theoretical approaches has to do with the fact that the real exchange rate misalignments are model dependent.

This paper contributes to the literature by proposing an alternative approach towards the determination of the real exchange rate in the long run. This is of a particular importance for the derivation of the equilibrium real exchange rate and the measurement of real exchange rate misalignments. As opposed to current literature, which is heavily based on various extensions of the balance of payments equilibrium real exchange rate equation, the proposed theoretical framework contributes towards the portfolio balance approach to the determination of the real exchange rate in the long run by constructing a two country model with optimizing agents where wealth is assumed to be allocated optimally in an asset choice set that includes explicitly investment in an array of financial assets. As opposed to other literature³ the model specification introduced in this paper allows the construction of explicit equations for both domestic and foreign real money balances, which can further be utilized in order to generate a

² See for example Sallenave (2010), Edwards(1989) and Alberola and Lopez (2001).

³ See for example Lucas (1982)

relationship that reflects the determination of the real exchange rate in the long-run. In this paper, we show that the theoretical model that we derive is empirically well supported by using the dollar-pound rate indicating that asset prices and returns can play a substantive role in the determination of the real exchange rate in the long run. Although Dellas and Tavlas (2013) have recently shown a theoretical and empirical linkage between exchange rate regimes this differs from our approach which is to show an explicit link between asset prices and the real exchange rate.

The rest of this paper is organised as follows: Section 2 presents the constructed intertemporal optimization model, as a contribution of understanding the determination of the real exchange rate in the long-run. Section 3 discusses the dataset and empirical methodology for examining the predicted relationship. Section 4 discusses the results from the empirical estimations and Section 5 concludes.

2. The Model

An infinitely lived representative agent (individual) is assumed to respond optimally to the economic environment. Utility is assumed to be derived from consumption of goods and from holdings of domestic and foreign real money balances. The consumption basket is assumed to be a composite bundle of goods produced both domestically and in the foreign economy. The presence of real money balances is intended to represent the role of money used in transactions, without addressing explicitly a formal transaction mechanism. This can distinguish money from other assets like interest bearing bonds or stocks.⁴ The representative agent is assumed to maximize the present value of lifetime utility given by:

⁴ A direct way to model the role of money in facilitating transactions would be to develop a time-shopping model after introducing leisure in the utility function. Another approach, commonly found in the literature, allows money balances to finance certain types of purchases through a cash-in-advance (CIA) modelling. For tractability reasons

$$E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t^\alpha)^{1-\sigma}}{1-\sigma} + \frac{X}{1-\varepsilon} \left(\left[\frac{M_t}{P_t} \right]^{\eta_1} \left[\frac{M_t^*}{P_t^*} \right]^{\eta_2} \right)^{1-\varepsilon} \right] \quad (1)$$

where C_t is real consumption of a composite bundle of goods, $\frac{M_t}{P_t}$ and $\frac{M_t^*}{P_t^*}$ are domestic and foreign real money balances respectively, $0 < \beta < 1$ is the individual's subjective time discount factor, σ , ε , X are assumed to be positive parameters, with $0.5 < \sigma < 1$ and $0.5 < \varepsilon < 1$, and $E_t(\cdot)$ the mathematical conditional expectation at time t . For analytical tractability and following Kia's (2006) suggestion, we assume that α , η_1 , and η_2 are all normalized to unity.

The present value of lifetime utility is assumed to be maximized subject to a sequence of budget constraints given by:

$$y_t + \frac{M_{t-1}}{P_t} + \frac{M_{t-1}^*}{e_t P_t} + \frac{B_{t-1}^D(1+i_{t-1}^D)}{P_t} + \frac{B_{t-1}^F(1+i_{t-1}^F)}{e_t P_t} + \frac{S_{t-1}(P_t^S + d_{t-1})}{P_t} + \frac{S_{t-1}^*(P_t^{S,*} + d_{t-1}^*)}{e_t P_t} = C_t + \frac{M_t}{P_t} + \frac{M_t^*}{e_t P_t} + \frac{B_t^D}{P_t} + \frac{B_t^F}{e_t P_t} + \frac{S_t P_t^S}{P_t} + \frac{S_t^* P_t^{S,*}}{e_t P_t} \quad (2)$$

where y_t is current real income, $\frac{M_{t-1}}{P_t}$ and $\frac{M_{t-1}^*}{e_t P_t}$ are real money balances expressed in current domestic unit terms (with M_{t-1} and M_{t-1}^* domestic and foreign nominal money balances respectively carried forward from last period), e_t the nominal exchange rate defined as the amount of foreign currency per unit of domestic currency and P_t the price index of the composite good consumed domestically. B_{t-1}^D is the amount of domestic currency invested in domestic bonds at $t - 1$ and i_{t-1}^D is the nominal rate of return on the domestic bonds. Similarly, B_{t-1}^F is the amount of foreign currency invested in foreign bonds at $t - 1$ and i_{t-1}^F is the nominal rate of return on the foreign bonds. Both domestic and foreign bonds are assumed to be one period discount bonds paying off one unit of the relevant domestic currency next period. S_{t-1} and S_{t-1}^* denote the number of domestic and foreign shares respectively purchased at $t - 1$,

the specification expressed by Equation (1) is adopted in this paper. See Walsh (2003) for the various approaches in modelling the role of money in the utility function.

P_t^S and $P_t^{S,*}$ denote the domestic and the foreign share prices respectively and d_{t-1} and d_{t-1}^* the value of the domestic and foreign dividends earned.⁵

The agent is assumed to observe the total real wealth and then proceed with an optimal consumption and portfolio allocation plan. The right hand side of equation (2) indicates that total real wealth is allocated at time t among real consumption of the composite good (C_t), real domestic and foreign money balances $(\frac{M_t}{P_t}, \frac{M_t^*}{e_t P_t})$, real domestic and foreign bond holdings $(\frac{B_t^D}{P_t}, \frac{B_t^F}{e_t P_t})$, and real domestic and foreign equity holdings $(\frac{S_t P_t^S}{P_t}, \frac{S_t^* P_t^{S,*}}{e_t P_t})$.⁶

The representative agent is assumed to maximize equation (1) subject to equation (2). In order to get an analytical solution for the intertemporal maximization problem, the Hamiltonian equation is constructed and the following necessary first order conditions are derived:

$$\beta^t U_{c,t} - \lambda_t = 0 \quad (3)$$

$$\beta^t U_{\frac{M}{P},t} \frac{1}{P_t} - \lambda_t \frac{1}{P_t} + E_t \left[\lambda_{t+1} \frac{1}{P_{t+1}} \right] = 0 \quad (4)$$

$$\beta^t U_{\frac{M^*}{P^*},t} \frac{1}{P_t^*} - \lambda_t \frac{1}{e_t P_t} + E_t \left[\lambda_{t+1} \frac{1}{e_{t+1} P_{t+1}} \right] = 0 \quad (5)$$

$$-\lambda_t \frac{1}{P_t} + E_t \left[\lambda_{t+1} \frac{1}{P_{t+1}} (1 + i_t^D) \right] = 0 \quad (6)$$

$$-\lambda_t \frac{1}{e_t P_t} + E_t \left[\lambda_{t+1} \frac{1}{e_{t+1} P_{t+1}} (1 + i_t^F) \right] = 0 \quad (7)$$

$$-\lambda_t \frac{P_t^S}{P_t} + E_t \left[\lambda_{t+1} \frac{1}{P_{t+1}} (P_{t+1}^S + d_t) \right] = 0 \quad (8)$$

$$-\lambda_t \frac{P_t^{S,*}}{e_t P_t} + E_t \left[\lambda_{t+1} \frac{1}{e_{t+1} P_{t+1}} (P_{t+1}^{S,*} + d_t^*) \right] = 0 \quad (9)$$

where λ_t the costate variable, $U_{c,t}$, the marginal utility from consumption and $U_{\frac{M}{P},t}$, $U_{\frac{M^*}{P^*},t}$ the

marginal utilities from domestic and foreign real money balances respectively.

⁵ It is assumed that the individual collects his dividend first and then goes out in the financial market to trade. In other words, the stock market opens after the realization of dividends.

⁶ All variables are expressed in real domestic terms.

It is further assumed that the representative agent consumes according to the following constant elasticity of substitution (CES) composite:

$$C_t = [\alpha \frac{1}{\Theta} (C_t^h)^{\frac{\Theta-1}{\Theta}} + (1-\alpha) \frac{1}{\Theta} (C_t^f)^{\frac{\Theta-1}{\Theta}}]^{\frac{\Theta}{\Theta-1}} \quad (10)$$

Where C_t^h, C_t^f represent consumption of domestically produced goods and foreign imported goods respectively. The degree of home bias in preferences is given by parameter $\alpha \in [0,1]$ and can be perceived as a natural index of the degree of openness of the economy. Parameter $\Theta > 1$ measures the substitutability between domestic and foreign goods.

Defining P_t^h and P_t^f as the price indexes of domestically produced goods and goods produced in the foreign economy (all expressed in units of domestic currency), the utility based consumer price index (CPI) of the composite good consumed domestically is given by:

$$P_t = [\alpha (P_t^h)^{1-\Theta} + (1-\alpha) (P_t^f)^{1-\Theta}]^{\frac{1}{1-\Theta}} \quad (11)$$

Given that the nominal exchange rate e_t is the amount of foreign currency per unit of domestic currency we can write the domestic price equivalent (P_t^f) of the price index of the goods produced in the foreign economy (P_t^{f*}) as $P_t^f = \frac{P_t^{f*}}{e_t}$ and the foreign currency equivalent of the price index of domestically produced goods (P_t^h) as $P_t^{h*} = P_t^h e_t$.

Following Galí and Monacelli (2004) a simplifying assumption is introduced namely that there is no distinction between foreign CPI (P_t^*) and the price index of the goods produced in the foreign economy (P_t^{f*}) i.e. $P_t^{f*} = P_t^{*7}$. The intuition of this is that PPP does hold for foreign

⁷ This assumption is also employed in deriving equations 14 and 15 on page 9.

(tradable) goods. This is not the case however for the domestic aggregate CPI. Assuming that the price index of domestically (non-traded) produced goods increases (given P_t^*, P_t^{f*}) domestic consumers move towards foreign goods and a nominal depreciation is induced. Given the nominal depreciation, P_t^f will increase but given its composition P_t will increase more than the nominal depreciation i.e. PPP fails to hold.

Consequently, the terms of trade T_t and the real exchange rate q_t are defined respectively as:

$$T_t = \frac{P_t^f}{P_t^h} = \frac{\frac{P_t^{f*}}{e_t}}{P_t^h} = \frac{P_t^{f*}}{e_t P_t^h} = \frac{P_t^{f*}}{P_t^{h*}} \quad (12)$$

$$q_t = \frac{\frac{P_t^*}{e_t}}{P_t} = \frac{P_t^*}{e_t P_t} \quad (13)$$

q_t denotes the real exchange rate defined as $q_t = \frac{P_t^*}{e_t P_t}$ where P_t and P_t^* the price indexes of the composite bundles of goods consumed domestically and in the foreign economy. A rise in q_t represents a real depreciation while a fall represents a real appreciation.

The static optimal allocation of total (composite) consumption leads to the following symmetric isoelastic demand functions for both domestic and foreign goods respectively⁸:

$$C_t^h = \alpha \left(\frac{T_t}{q_t} \right)^\Theta C_t \quad (14)$$

$$C_t^f = (1 - \alpha) (q_t)^{-\Theta} C_t \quad (15)$$

⁸ Details of the formal derivation are available from the authors by request.

Rewriting equation (14) and equation (15) in terms of real total consumption of the composite bundle consumed in the domestic economy leads to equations (16) and (17):

$$C_t = \frac{c_t^h}{\alpha \left(\frac{T_t}{q_t}\right)^\theta} \quad (16)$$

$$C_t = \frac{C_t^f}{(1 - \alpha)q_t^{-\theta}} \quad (17)$$

Dividing equation (5) by equation (7) and using equation (3) yields equation (18):

$$\frac{U_{M^*,t}}{P^{*,t}} + U_{C,t}(1 + i_t^F)^{-1}q_t = U_{C,t} q_t \quad (18)$$

Equation (18) implies that the marginal benefit of holding additional foreign real money balances at t must equal the marginal utility from consuming units of the domestic composite bundle of goods at time t . Note that the total marginal benefit of holding money at time t is equal to the marginal utility from holding real money balances at t , as reflected by $\frac{U_{M^*,t}}{P^{*,t}}$, and the marginal utility from the consumption of the composite bundle of goods, given by $U_{C,t}$.

Equation (18) can be rearranged in order to express the intratemporal marginal rate of substitution of composite domestic consumption for foreign real money balances as a function of the foreign bond return and the real exchange rate i.e. $\frac{U_{M^*,t}}{U_{C,t}} = \{1 - [(1 + i_t^F)^{-1}]\}q_t$.

Dividing equation (5) by equation (9) and using equation (3) yields equation (19):⁹

$$U_{M^*,t} + U_{C,t} \left[\frac{P_{t+1}^{S,*} + d_t^*}{P_t^{S,*}} \right]^{-1} q_t = U_{C,t} q_t \quad (19)$$

In a similar vein, equation (19) can be rearranged to express the intratemporal marginal rate of substitution of composite domestic consumption for foreign real money balances as a function of the expected foreign stock return and the real exchange rate i.e.

⁹ For notational simplicity we drop the mathematical conditional expectation $E_t(\cdot)$.

$$\frac{U_{M^*}^*}{U_{C,t}} = \left\{ 1 - \left[\frac{P_{t+1}^{S,*} + d_t^*}{P_t^{S,*}} \right]^{-1} \right\} q_t.$$

Dividing equation (4) by equation (6) and using equation (3) yields equation (20):

$$U_{\frac{M}{P},t} + U_{c,t}(1 + i_t^D)^{-1} = U_{c,t} \quad (20)$$

Equation (20) implies that the marginal benefit of holding additional domestic real money balances at time t must equal the marginal utility from consuming units of the domestic composite bundle of goods at time t . This can be rearranged to express the intratemporal marginal rate of substitution of composite domestic consumption for domestic real money

balances as a function of the domestic bond return i.e. $\frac{U_M}{U_{C,t}} = \{1 - [(1 + i_t^D)^{-1}]\}$.

Finally, by dividing equation (4) by equation (8) and using equation (3) yields equation (21):

$$U_{\frac{M}{P},t} + U_{c,t} \left(\frac{P_{t+1}^S + d_t}{P_t^S} \right)^{-1} = U_{c,t} \quad (21)$$

Equation (21) can be rearranged to express the intratemporal marginal rate of substitution of composite domestic consumption for domestic real money balances as a function of the

expected domestic stock return i.e. $\frac{U_M}{U_{C,t}} = \left\{ 1 - \left[\frac{P_{t+1}^S + d_t}{P_t^S} \right]^{-1} \right\}$

Using equation (1) the marginal utility of consumption of the composite bundle of goods can be derived as follows:

$$U_{c,t} = \beta^t (C_t)^{-\sigma} \quad (22)$$

The marginal utilities for foreign and domestic real money balances are given respectively as:

$$U_{\frac{M^*}{P^*},t} = \beta^t X \left(\frac{M_t}{P_t} \right)^{1-\varepsilon} \left(\frac{M_t^*}{P_t^*} \right)^{-\varepsilon} \quad (23)$$

$$U_{\frac{M}{P},t} = \beta^t X \left(\frac{M_t^*}{P_t^*} \right)^{1-\varepsilon} \left(\frac{M_t}{P_t} \right)^{-\varepsilon} \quad (24)$$

Equations (18), (22), (23) and (17) imply that:

$$m_t^* = (1 - \alpha)^{-\frac{\sigma}{\varepsilon}} (C_t^f)^{\frac{\sigma}{\varepsilon}} (q_t)^{\left[\frac{\sigma\theta-1}{\varepsilon}\right]} X^{\frac{1}{\varepsilon}} (m_t)^{\frac{(1-\varepsilon)}{\varepsilon}} \left[\frac{i_t^F}{1 + i_t^F} \right]^{-\frac{1}{\varepsilon}} \quad (25)$$

Equations (19), (22), (23) and (17) imply that:

$$m_t^* = (1 - \alpha)^{-\frac{\sigma}{\varepsilon}} (C_t^f)^{\frac{\sigma}{\varepsilon}} (q_t)^{\left[\frac{\sigma\theta-1}{\varepsilon}\right]} X^{\frac{1}{\varepsilon}} (m_t)^{\frac{(1-\varepsilon)}{\varepsilon}} \left[1 - \left(\frac{P_{t+1}^{S,*} + d_t^*}{P_t^{S,*}} \right)^{-1} \right]^{-\frac{1}{\varepsilon}} \quad (26)$$

Equations (20), (22), (24) and (16) imply that:

$$m_t = \alpha^{-\frac{\sigma}{\varepsilon}} (C_t^h)^{\frac{\sigma}{\varepsilon}} (q_t)^{\frac{\sigma\theta}{\varepsilon}} (T_t)^{\frac{-\sigma\theta}{\varepsilon}} X^{\frac{1}{\varepsilon}} (m_t^*)^{\frac{1-\varepsilon}{\varepsilon}} \left[\frac{i_t^D}{1 + i_t^D} \right]^{-\frac{1}{\varepsilon}} \quad (27)$$

Finally, equations (21), (22), (24) and (16) imply that:

$$m_t = \alpha^{-\frac{\sigma}{\varepsilon}} (C_t^h)^{\frac{\sigma}{\varepsilon}} (q_t)^{\frac{\sigma\theta}{\varepsilon}} (T_t)^{\frac{-\sigma\theta}{\varepsilon}} X^{\frac{1}{\varepsilon}} (m_t^*)^{\frac{1-\varepsilon}{\varepsilon}} \left[1 - \left(\frac{P_{t+1}^S + d_t}{P_t^S} \right)^{-1} \right]^{-\frac{1}{\varepsilon}} \quad (28)$$

Equations (25) to (28) reflect the demand equations for domestic and foreign real money balances that is, m_t and m_t^* respectively as implied by the economic model. This system of equations can be used in order to solve explicitly for the determinants of the real exchange rate. Substituting equation (26) into equation (27) and equation (28) into equation (25) yields equation (29):¹⁰

$$lq_t = \delta_1(lM_t) + \delta_2(lM_t^*) + \delta_3(lr_t) + \delta_4(lr_t^*) + \delta_5(lP_t^{FS,*}) + \delta_6(lP_t^S) \quad (29)$$

$$\text{Where: } \delta_1 = -\left[\frac{2\varepsilon-1}{\varepsilon}\right]; \delta_2 = \left[\frac{2\varepsilon-1}{\varepsilon}\right]; \delta_3 = -\left[\frac{2\varepsilon-1}{\varepsilon}\right]; \delta_4 = \left[\frac{2\varepsilon-1}{\varepsilon}\right]; \delta_5 = \left[\frac{1-\varepsilon}{\varepsilon}\right]; \delta_6 = -\left[\frac{1-\varepsilon}{\varepsilon}\right];$$

Where lq_t is the log of the real exchange rate; lM_t is the log of the domestic nominal money supply; lM_t^* is the log of the foreign nominal money supply; $lr_t = li_t^h - lP_t$ and $lr_t^* =$

¹⁰ A l before a variable denotes log. See Appendix I for the full derivation of Equation (29) along with the various assumptions employed. Appendix II presents a table with all variables employed in the construction of the theoretical model.

$li^* - lP_t^*$ are proxies for the real interest rate in the domestic and foreign economy

respectively (with $i_t^h = \left[\frac{i_t^D}{1+i_t^D} \right]$, $i_t^* = \left[\frac{i_t^F}{1+i_t^F} \right]$) and $lP_t^{FS,*} = lP_t^{S,*} - le_t$.

The predictions of the model are that:

$$\delta_1 < 0 ; \delta_2 > 0 ; \delta_3 < 0 ; \delta_4 > 0 ; \delta_5 > 0 ; \delta_6 < 0$$

In addition, the following restrictions (as implied by the economic model) are assumed to hold. These restrictions are imposed on the long-run co-integrating vector for the real exchange rate as derived in Section 3.

$$\delta_2 = -\delta_1 ; \delta_3 = \delta_1 ; \delta_4 = -\delta_3 ; \delta_6 = -\delta_5$$

3. Long-Run Empirical Methodology and Results

In order to test empirically the validity of the economic predictions implied by equation (29) in the long-run, a Vector Error Correction Model (VECM) of the following form is employed¹¹.

$$\Delta\chi_t = \Gamma_1^m \Delta\chi_{t-1} + \Gamma_2^m \Delta\chi_{t-2} + \dots + \Gamma_{k-1}^m \Delta\chi_{t-k+1} + \Pi\chi_{t-m} + \varepsilon_t \quad (30)$$

where $\chi_t = (lq_t, lM_t, lM_t^*, lr_t, lr_t^*, lP_t^{FS,*}, lP_t^S)$ a (7x1) vector of variables, m denotes the lag placement of the ECM term, Δ denotes the difference, and $\Pi = a\beta'$ with a and β (pxr) matrices with $r < p$, where p the number of variables and r the number of stationary co-integrated relationships.

To test for co-integration among a set of integrated variables the Full Information Maximum Likelihood (FIML) approach is employed as proposed by Johansen (1988, 1991).¹² Having uniquely identified potential co-integrating vectors, stationarity among the variables can be

¹¹ Some of the advantages of the VECM are that it reduces the multicollinearity effect in time series, that the estimated coefficients can be classified into short-run and long-run effects, and that the long-run relationships of the selected macroeconomic series are reflected in the level matrix Π and so can be used for further co-integration analysis. See Juselius (2006).

¹² The main advantage of such an approach is that it is asymptotically efficient since the estimates of the parameters of the short-run and long-run relationships are carried out in a single estimation process. In addition, through the FIML procedure potential co-integrating relationships can be derived in an empirical model with more than two variables.

tested, while imposing specific restrictions. The above methodology is applied to test for a potential long-run relationship among the macroeconomic variables depicted by equation (29).

To test the model quarterly time series data for the United Kingdom and the USA are employed for the period 1982 to 2011 for the variables depicted by equation (29)¹³. The UK and the USA were selected in the analysis as both economies have financial systems based on financial markets rather than on the banking sector as in most European economies. The beginning of the sample period was employed because in the early 1980's the UK fundamentally changed the definitions of its monetary aggregates (*M2* definition of money supply in the UK now corresponds to *M1* in the USA) and both the UK and the USA deregulated their financial markets.¹⁴

In the empirical equation (29) lq_t is the log of the UK bilateral real exchange rate defined as dollars per pound, lM_t is the log of the UK nominal money supply (*M2*), lM_t^* is the log of the USA nominal money supply (*M1*), lP_t^S and $lP_t^{S,*}$ are the main stock market indices in the UK and the USA (FTSE 100 and DJIA respectively), le_t is the bilateral nominal exchange rate defined as dollars per pound, li_t^h is the log of $\frac{i_t^D}{1+i_t^D}$ where i_t^D is the three month rate on the UK Treasury securities and li_t^* is the log of $\frac{i_t^F}{1+i_t^F}$ where i_t^F is the three month USA Treasury bill rate, lP_t the log of the CPI in the UK and lP_t^* the log of the CPI in the USA.

In order to proceed with the VECM analysis the time series employed were tested first for stationarity. **Table 1** presents the results from the Augmented Dickey-Fuller (ADF) test under

¹³ Data are collected from Datastream.

¹⁴ Data from the United States are used as a proxy for foreign variables and data from the UK as proxies for domestic variables.

the null of a unit root. Evidence suggests (given the various levels of significance) that the first differences of the variables appear to be stationary as opposed to their levels. Consequently, the variables can be considered to be integrated of order one, i.e. $I(1)$, and co-integration among the variables is possible.¹⁵

Table 1. Augmented Dickey-Fuller test for a unit root.

Variable	Test in levels		Test in differences	
	No Trend	Trend	No Trend	Trend
lq_t	-0.86(0)	-3.61(3)	-9.24(1)*	-9.21(1)*
LM_t	4.46(2)	-2.79(0)	-8.42(0)*	-9.69(0)*
LM_t^*	0.29(1)	1.76(1)	-4.54(0)*	-4.48(0)*
lr_t	-0.03(1)	-2.15(1)	-6.20(0)*	-6.31(0)*
lr_t^*	1.70(5)	0.32(5)	-4.15(4)*	-4.55(4)*
$lP_t^{FS,*}$	-2.38(0)	-2.24(0)	-10.13(0)*	-10.22(0)*
lP_t^S	2.44(1)	1.19(1)	-8.25(0)*	-8.50(0)*

Note: Entries in parenthesis indicate the lag length based on SIC maxlag=12.

(*) indicates that the test is significant at all critical values.

Before testing for the co-integration rank, the appropriate lag length for the underlying empirical VECM model must be specified. Given the Lagrangian multiplier (LM) test for serial correlation of the residuals, 3 lags were employed for the model.¹⁶ The Johansen (1995) procedures were then applied to test for the co-integration rank. From the trace test, two co-integrating vectors were employed. **Table 2** presents the results of the trace test.

Table 2. Results of co-integration test

No of co-integrated relationships	Trace		
	Statistic	5% Critical Value	1% Critical Value
None **	144.7384	109.99	119.80
At most 1 *	83.78000	82.49	90.45
At most 2	57.02115	59.46	66.52
At most 3	37.00038	39.89	45.58
At most 4	19.06144	24.31	29.75
At most 5	9.458046	12.53	16.31
At most 6	0.768347	3.84	6.51

*(**) denotes rejection of the hypothesis at the 5% (1%) level

¹⁵ For robustness purposes we have also performed the Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) test with stationarity under the null. The KPSS also suggests that the variables are integrated of order one i.e. $I(1)$.

¹⁶ The AIC, SBA, HQ tests are employed for the lag order selection. Beginning with the lowest lag suggested by the tests (based on the SBC criterion) the serial correlation of the residuals is tested using the Lagrangian multiplier (LM) test.

The rank of the Π -matrix was found to be $r = 2$ implying that statistically a discrimination among two conditionally independent stationary relations is possible. The two unrestricted co-integration relations are uniquely determined but the question remains on whether they are meaningful for economic interpretation. Consequently, Johansen and Juselius (1994) identifying restrictions were imposed to distinguish among the vectors and ensure the uniqueness of the coefficients. By taking a linear combination of the unrestricted β vectors, it is always possible to impose $r - 1$ just identifying restrictions and one normalization on each vector without changing the likelihood function. Although the normalization process can be done arbitrarily it is generally accepted practice to normalize on a variable that is representative of a particular economic relationship. Since the purpose of the paper is to identify a possible long-run determination of the real exchange rate, the first co-integrating vector is normalized with respect to the real exchange rate. Additional restrictions (as implied by the economic model) are also imposed, namely that $\delta_2 = -\delta_1, \delta_3 = \delta_1, \delta_4 = -\delta_3$ and $\delta_6 = -\delta_5$.

In addition, all foreign variables, i.e. LM_t^*, lr_t^* and $LP_t^{FS,*}$ are treated as weakly exogenous variables, thus long run forcing in the co-integrating space. This can be justified under the assumption that the UK is a small open economy, as such domestic policy decisions or more generally domestic economic activity do not have a significant impact on the evolution of foreign variables. Consequently, treating all variables as jointly endogenously determined would lead to inappropriate inference.

The Chi-squared value (with 9 degrees of freedom) turns out to be 9.50 with P value of 0.39. Consequently, all restrictions are jointly accepted, the system is identified and according to Theorem 1 of Johansen and Juselius (1994) and the rank condition is satisfied.

Table 3. Long-Run Co-integrating Relationship (constrained coefficients)

$$lq_t = 1.880 - 0.583(IM_t) + 0.583(IM_t^*) - 0.583(lr_t) + 0.583(lr_t^*) + 0.831(IP_t^{FS,*}) - 0.831(IP_t^S)$$

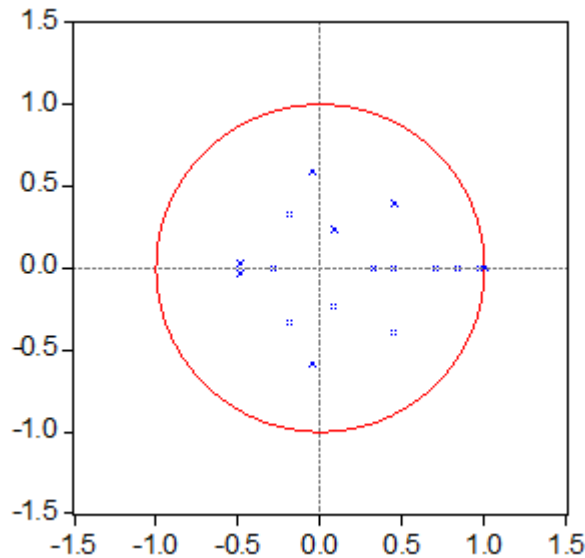
(-3.810) (3.810) (-3.810) (3.810) (2.348) (-2.348)

Note: *t* statistics in brackets

All constraint coefficients are statistically significant at 5% level and correctly signed in accordance with the predictions of the model

Table 3 reports the constraint coefficients from the long-run co-integrating relationship normalized with respect to lq_t ¹⁷. All variables are statistically significant and correctly signed in accordance with the predictions of the theoretical model. To test the stability of the VECM model the inverse roots of the characteristic AR polynomial are reported in **Figure 1**. The analysis confirms that the VECM is stable since the inverted roots of the model lie inside the unit circle. Having established that the VECM is stable the identified long-run co-integrating relationship, normalized on the real exchange rate, can be interpreted.

Figure1. Inverse roots of AR characteristic polynomial



¹⁷ The second co-integrating vector with unconstrained coefficients is available upon request

4. Economic Interpretation of Results

The economic model predicts that an expansionary monetary policy in the UK in a form of an increase in the nominal money supply will result in a real appreciation of the long run real exchange rate i.e. $\delta_1 < 0$. The estimated coefficient for the domestic (UK) nominal money supply LM_t , as depicted in Table 3, is also negative supporting the prediction of the model. The prediction of the model regarding the increase in the money supply is because in the long run the price level will accommodate the increase in the nominal money supply (given that money neutrality holds) and assuming that the Purchasing Power Parity need not hold in the long run the real exchange rate appreciates¹⁸. In a similar manner, the model predicts real exchange rate depreciation after an increase in the foreign (USA) nominal money supply LM_t^* ($\delta_2 > 0$). The coefficient for the foreign money supply comes with a positive sign, thus providing evidence in favour of the theoretical model.

The model predicts that an increase in the real interest rate lr_t results in a long run real exchange rate appreciation i.e. $\delta_3 < 0$. The estimated coefficient in **Table 3** for lr_t is also negative supporting the prediction of the model. An explanation is that an increase in the real interest rate will increase the demand of domestic currency, which induces both a nominal and real appreciation of the domestic currency in the long run. Likewise, the model predicts a real depreciation after an increase in the real foreign interest rate lr_t^* i.e. $\delta_4 > 0$. This prediction is also borne out in our empirical test of the model.

¹⁸ Following Galí and Monacelli (2004) a simplifying assumption is introduced namely that there is no distinction between foreign CPI (P_t^*) and the domestic price level for the foreign economy (P_t^{f*}) i.e. $P_t^{f*} = P_t^*$ (an assumption also employed in deriving equations 14 and 15). When the price index of domestically produced goods increases (given P_t^*, P_t^{f*}) domestic consumers move towards foreign goods and a nominal depreciation is induced. Given the nominal depreciation, P_t^f will increase but given its composition P_t^f will increase more than the nominal depreciation i.e. PPP fail to hold.

Finally, the model predicts that an increase in the domestic (UK) share price index will lead into a real appreciation of the long run real exchange rate i.e. $\delta_6 < 0$, which is confirmed in our results. An explanation is that as the price of equities increases the implied increase in portfolio risk may induce investors to adjust towards safer assets, including money. Consequently the demand for real money balances increases and the interest rate adjusts in order to satisfy equilibrium in the money market. The increase in the interest rate induces capital inflows and results in both a nominal and real appreciation. Similarly, an increase in the foreign (USA) stock market index leads to a real depreciation of the exchange rate i.e. $\delta_5 > 0$ which is also confirmed by our results.

5. Concluding Remarks

This paper contributes towards the theoretical determination of the real exchange rate by constructing an intertemporal optimization model, which incorporates investment in an array of assets such as domestic and foreign bonds, domestic and foreign stocks, and domestic and foreign real money balances. Such an approach to the determination of the real exchange rate in the long run has been neglected in the current literature, which is heavily based on the BEER and FEER models as well as on other extensions of the basic balance of payment equilibrium approach.

The basic predictions of the model are borne out empirically suggesting that asset prices and returns play an important role in the determination of the long run real exchange rate and its evolution. The model suggests that an increase in the domestic money supply, an increase in the domestic real interest rate and an increase in the domestic economy's stock market will lead into a real exchange rate appreciation in the long run. Given the importance of the role of the real exchange rate for policy makers and the functioning of open economies our contribution provides an alternative framework to much of the existing literature.

Our results suggest that future research would benefit from incorporating a range of asset prices when considering the equilibrium real exchange rate. There is also scope for future research to consider how mispricing of financial assets may also have feedback effects on the real exchange rate and hence on the real economy. It would also be interesting to compare the results of our model with the alternative methods of modelling the real exchange rate to see the extent of any quantitative and qualitative differences.

APPENDIX I

The derivation of the real exchange rate equation

Substituting equation (26) into equation (27) and equation (28) into equation (25) in the text the following equation is derived:

$$\frac{m_t}{m_t^*} = \frac{\alpha^{-\frac{\sigma}{\varepsilon}} (C_t^h)^{\frac{\sigma}{\varepsilon}} (q_t)^{\frac{\sigma\theta}{\varepsilon}} (T_t)^{-\frac{\sigma\theta}{\varepsilon}} X^{\frac{1}{\varepsilon}} \left\{ (1-\alpha)^{-\frac{\sigma}{\varepsilon}} (C_t^f)^{\frac{\sigma}{\varepsilon}} (q_t)^{\frac{\sigma\theta-1}{\varepsilon}} X^{\frac{1}{\varepsilon}} (m_t)^{\frac{(1-\varepsilon)}{\varepsilon}} \left[1 - \left(\frac{P_{t+1}^{S,*} + d_t^*}{P_t^{S,*}} \right)^{-1} \right]^{-\frac{1}{\varepsilon}} \right\}^{\frac{1-\varepsilon}{\varepsilon}} \left[\frac{i_t^D}{1+i_t^D} \right]^{-\frac{1}{\varepsilon}}}{(1-\alpha)^{-\frac{\sigma}{\varepsilon}} (C_t^h)^{\frac{\sigma}{\varepsilon}} (q_t)^{\frac{\sigma\theta-1}{\varepsilon}} X^{\frac{1}{\varepsilon}} \left\{ \alpha^{-\frac{\sigma}{\varepsilon}} (C_t^h)^{\frac{\sigma}{\varepsilon}} (q_t)^{\frac{\sigma\theta}{\varepsilon}} (T_t)^{-\frac{\sigma\theta}{\varepsilon}} X^{\frac{1}{\varepsilon}} (m_t^*)^{\frac{1-\varepsilon}{\varepsilon}} \left[1 - \left(\frac{P_{t+1}^S + d_t}{P_t^S} \right)^{-1} \right]^{-\frac{1}{\varepsilon}} \right\}^{\frac{1-\varepsilon}{\varepsilon}} \left[\frac{i_t^F}{1+i_t^F} \right]^{-\frac{1}{\varepsilon}}}$$

which simplifies to:

$$\frac{m_t}{m_t^*} = \left(\frac{\alpha}{1-\alpha} \right)^{-\frac{\sigma}{\varepsilon}} \left(\frac{C_t^h}{C_t^f} \right)^{\frac{\sigma}{\varepsilon}} (q_t)^{\frac{\sigma\theta}{\varepsilon}} (q_t)^{-\frac{\sigma\theta-1}{\varepsilon}} (S_t)^{-\frac{\sigma\theta}{\varepsilon}} \left(\frac{1-\alpha}{\alpha} \right)^{\frac{\sigma}{\varepsilon} \frac{(1-\varepsilon)}{\varepsilon}} \left(\frac{C_t^f}{C_t^h} \right)^{\frac{\sigma}{\varepsilon} \frac{(1-\varepsilon)}{\varepsilon}} (q_t)^{-\frac{1-\sigma\theta}{\varepsilon} \frac{1-\varepsilon}{\varepsilon}} (q_t)^{-\frac{\sigma\theta}{\varepsilon} \frac{1-\varepsilon}{\varepsilon}} (S_t)^{\frac{\sigma\theta}{\varepsilon} \frac{1-\varepsilon}{\varepsilon}} \Omega$$

$$\text{where } \Omega = \frac{\left((m_t)^{\frac{1-\varepsilon}{\varepsilon}} \right)^{\frac{1-\varepsilon}{\varepsilon}} \left\{ \left[1 - \left(\frac{P_{t+1}^{S,*} + d_t^*}{P_t^{S,*}} \right)^{-1} \right]^{-\frac{1}{\varepsilon}} \right\}^{\frac{1-\varepsilon}{\varepsilon}} \left[\frac{i_t^D}{1+i_t^D} \right]^{-\frac{1}{\varepsilon}}}{\left((m_t^*)^{\frac{1-\varepsilon}{\varepsilon}} \right)^{\frac{1-\varepsilon}{\varepsilon}} \left\{ \left[1 - \left(\frac{P_{t+1}^S + d_t}{P_t^S} \right)^{-1} \right]^{-\frac{1}{\varepsilon}} \right\}^{\frac{1-\varepsilon}{\varepsilon}} \left[\frac{i_t^F}{1+i_t^F} \right]^{-\frac{1}{\varepsilon}}}$$

$$\frac{m_t}{m_t^*} = \left(\frac{C_t^h}{C_t^f} \right)^{-\frac{\sigma}{\varepsilon}} (T_t)^{\frac{\sigma\theta}{\varepsilon}} \left(\frac{C_t^h}{C_t^f} \right)^{\frac{\sigma}{\varepsilon}} (q_t)^{\frac{\sigma\theta}{\varepsilon} - \frac{\sigma\theta-1}{\varepsilon}} (T_t)^{-\frac{\sigma\theta}{\varepsilon}} (q_t)^{-\frac{1-\sigma\theta}{\varepsilon} \frac{1-\varepsilon}{\varepsilon}} (q_t)^{-\frac{\sigma\theta}{\varepsilon} \frac{1-\varepsilon}{\varepsilon}} \Omega$$

$$\frac{m_t}{m_t^*} = (q_t)^{\frac{2\varepsilon-1}{\varepsilon^2}} \Omega \quad (A.1)$$

Dividing equation (6) with equation (8) yields that: $\frac{1}{P_t^S} = \frac{1+i_t^D}{P_{t+1}^S + d_t}$, which implies that:

$$P_t^S - [P_{t+1}^S + d_t] = -[P_{t+1}^S + d_t] \frac{i_t^D}{1+i_t^D} \quad (A.2)$$

In a similar manner dividing equation (7) with equation (9) implies that:

$$P_t^{S,*} - [P_{t+1}^{S,*} + d_t^*] = -[P_{t+1}^{S,*} + d_t^*] \frac{i_t^F}{1+i_t^F} \quad (A.3)$$

Using Equations (A.2) and (A.3) and dividing equation (8) with equation (9) implies that $\frac{P_t^S}{P_t^{S,*}} =$

$\frac{e_{t+1} P_{t+1}^S + d_t}{e_t P_{t+1}^{S,*} + d_t^*}$, Equation (A.1) becomes

$$\frac{m_t}{m_t^*} = (q_t)^{\left[\frac{2\varepsilon-1}{\varepsilon^2}\right]} (m_t)^{\left[\frac{(1-\varepsilon)^2}{\varepsilon^2}\right]} (m_t^*)^{\left[-\frac{(1-\varepsilon)^2}{\varepsilon^2}\right]} [P_{t+1}^{S,*} + d_t^*]^{-\left[\frac{1-\varepsilon}{\varepsilon^2}\right]} (i_t^*)^{-\left[\frac{1-\varepsilon}{\varepsilon^2}\right]} e_t^{-\left[\frac{1-\varepsilon}{\varepsilon^2}\right]} P_t^S^{-\left[\frac{1-\varepsilon}{\varepsilon^2}\right]} e_{t+1}^{\left[\frac{1-\varepsilon}{\varepsilon^2}\right]} P_t^{S,*\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}$$

$$[P_{t+1}^S + d_t]^{\left[\frac{1-\varepsilon}{\varepsilon^2}\right]} (i_t^h)^{\left[\frac{1-\varepsilon}{\varepsilon^2}\right]} (i_t^h)^{-\left[\frac{1}{\varepsilon}\right]} (i_t^*)^{\left[\frac{1}{\varepsilon}\right]}$$

Where $(i_t^h) = \left[\frac{i_t^D}{1+i_t^D}\right]$ and $(i_t^*) = \left[\frac{i_t^F}{1+i_t^F}\right]$

Taking logs of all variables we obtain equation A.4:¹⁹

$$\left[\frac{2\varepsilon-1}{\varepsilon^2}\right] (lm_t) - \left[\frac{2\varepsilon-1}{\varepsilon^2}\right] (lm_t^*) + \left[\frac{2\varepsilon-1}{\varepsilon^2}\right] (lr_t - lr_t^*) + \left[\frac{\varepsilon}{\varepsilon^2}\right] (lq_t) \left[\frac{1-\varepsilon}{\varepsilon^2}\right] (le_t + lP_t^S - lP_t^{S,*}) = 0$$

$$lq_t = -\left[\frac{2\varepsilon-1}{\varepsilon}\right] (lm_t) + \left[\frac{2\varepsilon-1}{\varepsilon}\right] (lm_t^*) - \left[\frac{2\varepsilon-1}{\varepsilon}\right] (lr_t) + \left[\frac{2\varepsilon-1}{\varepsilon}\right] (lr_t^*) + \left[\frac{1-\varepsilon}{\varepsilon}\right] (lP_t^{S,*} - le_t) - \left[\frac{1-\varepsilon}{\varepsilon}\right] (lP_t^S) \quad (\text{A.4})$$

Equation (A.4) corresponds to equation (29) in the text.

¹⁹ Following the fact that $\frac{P_t^S}{P_t^{S,*}} = \frac{e_{t+1} P_{t+1}^S + d_t}{e_t P_{t+1}^{S,*} + d_t^*}$ and assuming that capital and consumption are homogeneous goods. A l before a variable denotes log.

APPENDIX II

Variable	Explanation
C_t	Real consumption of a composite bundle of goods
$m_t = \frac{M_t}{P_t}$	Domestic real money balances, with M_t domestic nominal money balances and P_t the consumer price index of the composite good consumed domestically.
$m_t^* = \frac{M_t^*}{P_t^*}$	Foreign real money balances, with M_t^* foreign nominal money balances and P_t^* the consumer price index of the composite good consumed in the foreign economy.
y_t	Real income
e_t	Nominal exchange rate (amount of foreign currency per unit of domestic currency)
B_t^D	Amount of domestic currency invested in domestic bonds
B_t^F	Amount of foreign currency invested in foreign bonds
i_t^D	Nominal rate of return on domestic bonds
i_t^F	Nominal rate of return on foreign bonds
S_t	Number of domestic shares purchased
S_t^*	Number of foreign shares purchased
P_t^S	Domestic share price
$P_t^{S,*}$	Foreign share price
d_t	Value of domestic dividend earned
d_t^*	Value of foreign dividend earned
$U_{c,t}$	Marginal utility from consumption
$U_{\frac{M}{P},t}$	Marginal utility from domestic real money balances
$U_{\frac{M^*}{P^*},t}$	Marginal utility from foreign real money balances
C_t^h	Consumption of domestically produced goods
C_t^f	Domestic consumption of foreign imported goods
P_t^h	The price index of domestically produced goods
P_t^f	Price index of goods produced in the foreign economy (expressed in units of domestic currency)
P^{f*}	Price index of goods produced in the foreign economy
P^{h*}	Foreign currency equivalent of the price index of domestically produced goods
T_t	Terms of trade

q_t	Real exchange rate – a rise represents a real depreciation a fall represents a real appreciation
i_t^h	$\left[\frac{i_t^D}{1 + i_t^D} \right]$
i_t^*	$\left[\frac{i_t^F}{1 + i_t^F} \right]$
$lP_t^{FS,*}$	$lP_t^{S,*} - l e_t$ (l denotes log)
$l r_t$	$l i_t^h - l P_t$ (l denotes log)
$l r_t^*$	$l i_t^* - l P_t^*$ (l denotes log)

References

- Alberola, E. and Lopez, H. (2001). Internal and External Exchange Rate Equilibrium in a co-integration Framework: An Application to the Spanish Peseta. *Spanish Economic review*, 3, 23-40.
- Baxter, M. and King R. (1999) Measuring business cycles: approximate band-pass filters for economic time series, *Review of Economics and Statistics*, 81, 575-593.
- Clark, P. and MacDonald, R. (1998) Exchange Rates and Economic Fundamentals: A Methodological Comparison of BEERs and FEER's, *IMF Working Paper WP98/67*.
- Dellas H. and Tavals G (2013) Exchange Rate Regimes and Asset Prices. *Journal of International Money and Finance*, 38, pp. 85-94.
- Driver, R.L. and Westaway, P.F. (2004) Concepts of Equilibrium Exchange Rates, *Bank of England, Working Paper No. 248*.
- Edwards, S. (1989) *Real Exchange Rates, Devaluation and Adjustment: Exchange Rate Policy in Developing Countries*, Cambridge, Massachusetts: MIT Press.
- Galí J, and Monacelli T. 2004. Monetary Policy and Exchange Rate Volatility in a Small Open Economy. *Review of Economic Studies*, 72, 704-734.
- Hall S.G.; Hondroyiannis G.; Kenjegaliev A.; Swamy P.A.V.B. and Tavlas G.S. (2013) "Is the Relationship between Exchange Rates and Prices Homogenous." *Journal of International Money and Finance*, 31, 411-38.
- Johansen, S. (1988) Statistical Analysis in Co-integrated Vectors, *Journal of Economic Dynamics and Control*, 12, 231-54.
- Johansen, S. (1991) Estimation and Hypothesis Testing of Co-integration Vectors in Gaussian Vector Autoregressive Models, *Econometrica*, 52, 389-402.
- Johansen, S. and Juselius, K. (1994) Identification of the Long-Run and Short-Run Structure: an Application to the ISLM Model, *Journal of Econometrics*, 63, 7-36.
- Johansen, S. (1995) *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models*, *Advanced Texts in Econometrics*, Oxford University Press, Oxford.

Juselious, K. (2006) *The Cointegrated VAR Model: Methodology and Applications*, Oxford University Press, Oxford

Kia, A. (2006) Deficits, Debt Financing, Monetary Policy and Inflation in Developing Countries: Internal or External Factors? Evidence from Iran, *Journal of Asian Economics*, 17, 879-903.

Lucas, R.E. (1982) Interest Rates and Currency Prices in a Two-Country World, *Journal of Monetary Economics*, 10, 335-359.

MacDonald, R. (2000) Concepts to Calculate Equilibrium Exchange Rates: An Overview, *Deutsche Bundesbank* No. 2000,03.

Rogoff, K. (1997) The Purchasing Power Parity Puzzle, *Journal Of Economic Literature*, 34, 647-668.

Sallenave, A. (2010) Real Exchange Rate Misalignment and Economic Performance for the G20 Countries, *International Economics*, 121, 59-80.

Walsh, C.E. (2003) *Monetary Theory and Policy*, Massachusetts Institute of Technology, Massachusetts.

Williamson, J. (1994) Estimates of FEERs, in Williamson, ed., *Estimating Equilibrium Exchange Rates*, Washington D.C.: Institute for International Economics.